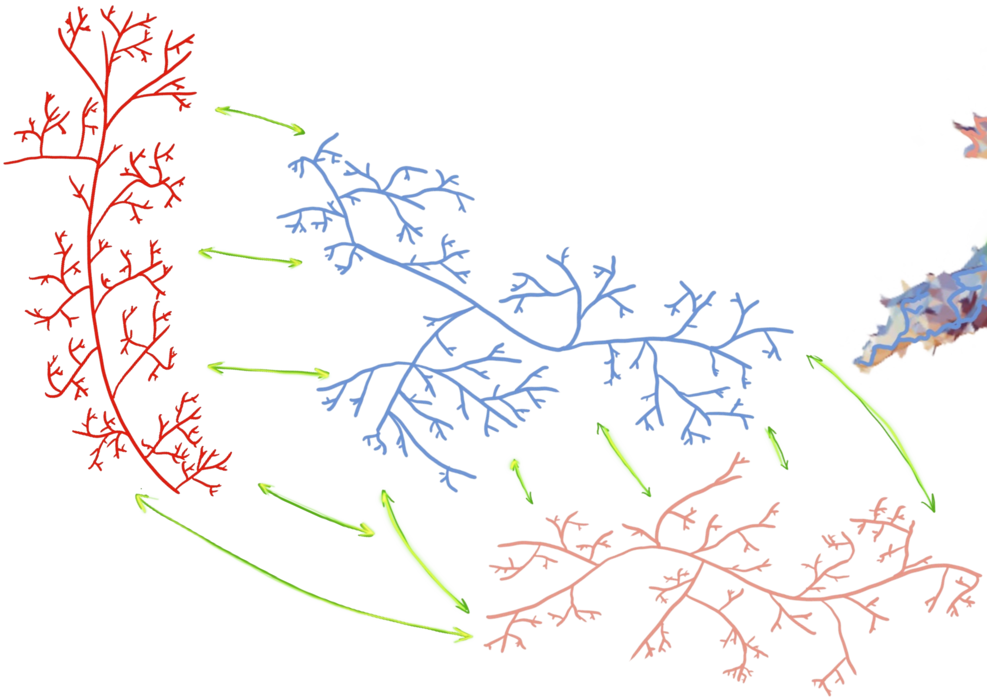
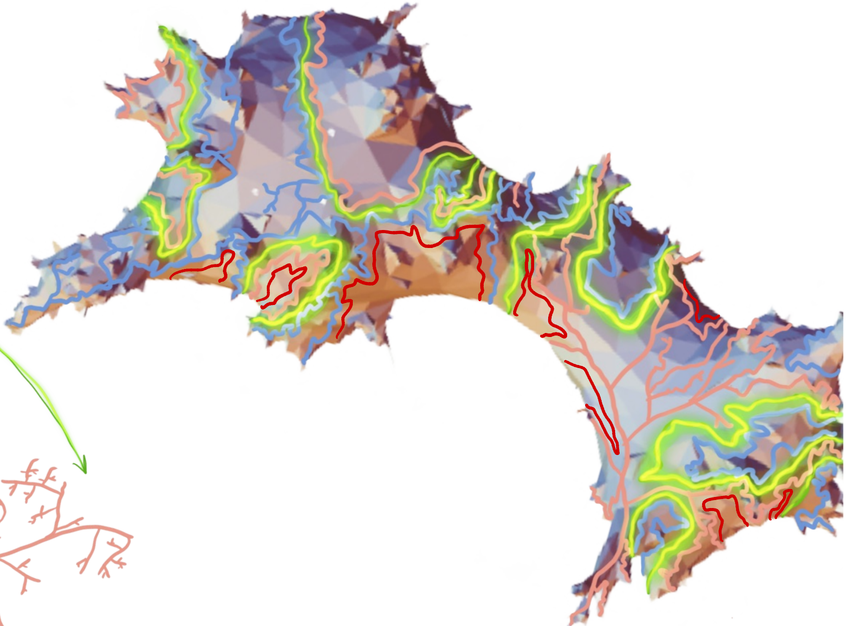
Scale invariant random geometries from mating of trees

by Alicia Castro
in collab w/ T. Budd





Institute for Mathematics, Astrophysics and Particle Physics
Radboud University

Basics of Quantum Gravity

16 May 2023 to 16 November 2023
Europe/Zurich timezone

Enter your search term

- General Information
- Lecturers
- Schedule
- Registration
- Participant List
- Organizing Committee

Support

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lupetruzziello@unisa.it

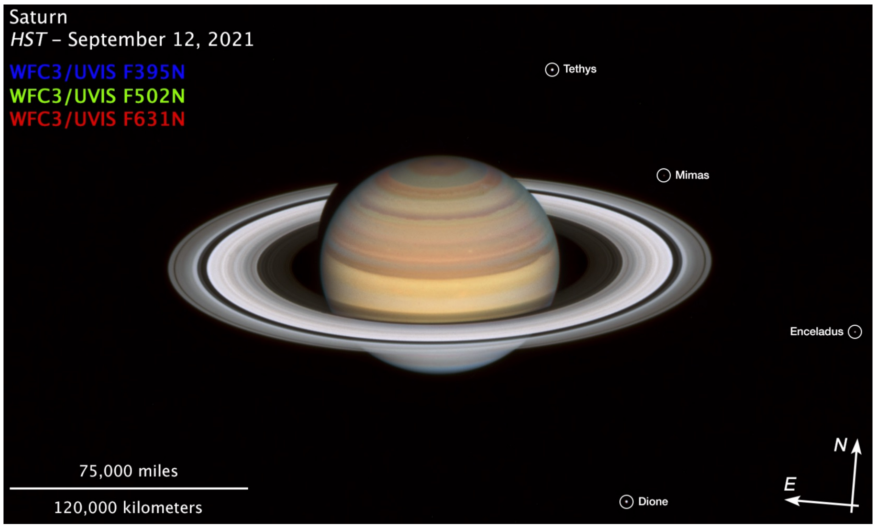
isqg.org

The **Basics of Quantum Gravity** online school is a series of lectures on the key concepts and techniques involved in the research of Quantum Gravity. It is dedicated to young researchers who are tackling this great challenge in theoretical physics. The goal is to bridge several paths and approaches to quantum gravity and provide young researchers with the tools to work on all of these frameworks and utilize various formalisms.

The launching lecture for this event will take place on May 16th. The event will run throughout Europe's spring and summer of 2023, featuring lectures on String Theory, Loop Gravity, the perturbative and non-perturbative renormalization flow of General Relativity, and quantum information in Gravity. Additional lectures on other topics will be planned for the fall and winter.

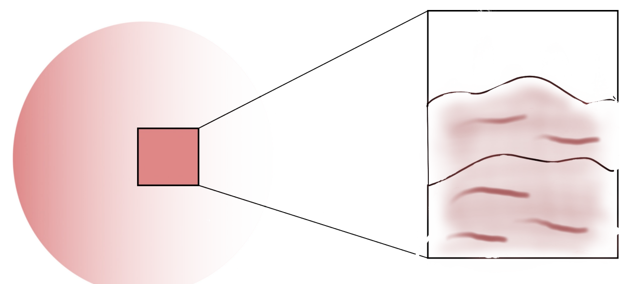
This is an initiative by the International Society for Quantum Gravity. It will be followed by a series of workshops on more advanced topics.

Quantum Gravity Approaches



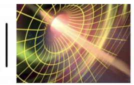
Classical Gravity works here

+ quantum perturbations + very high energy

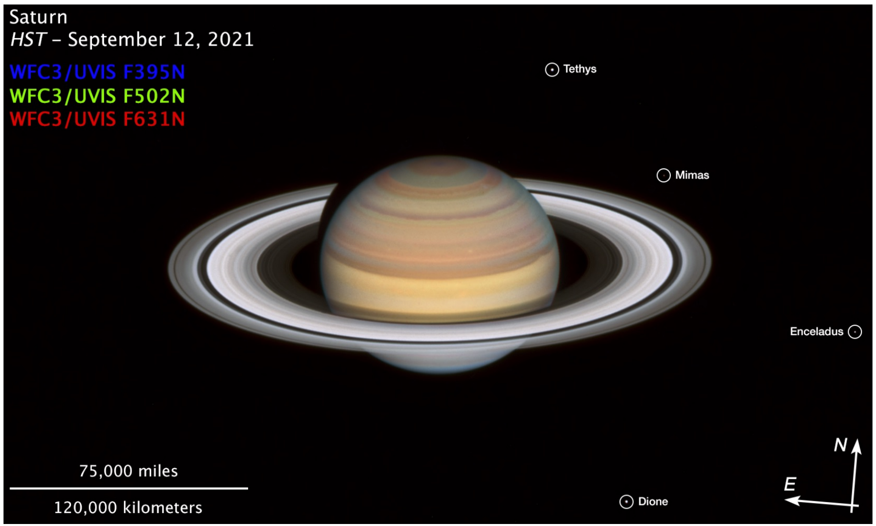


Quantum Gravity APPROACHES

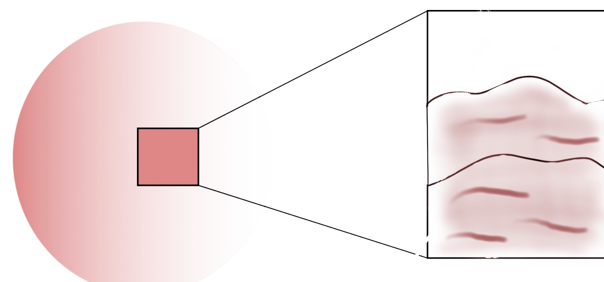
non-renormalizable

$Prob($  $|$  $) = ?$

No predictivity!

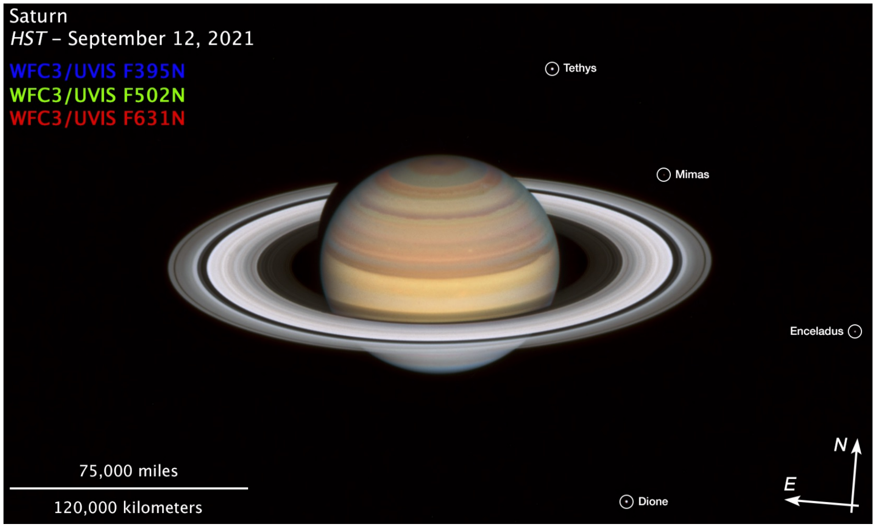


\+ quantum perturbations + very high energy



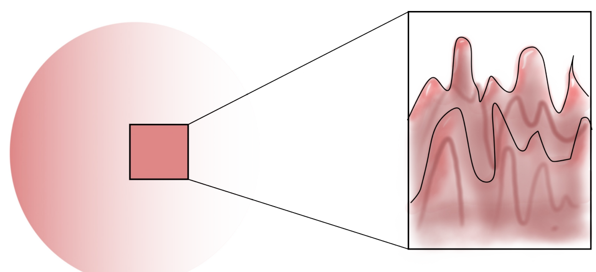
Classical Gravity works here

Quantum Gravity APPROACHES



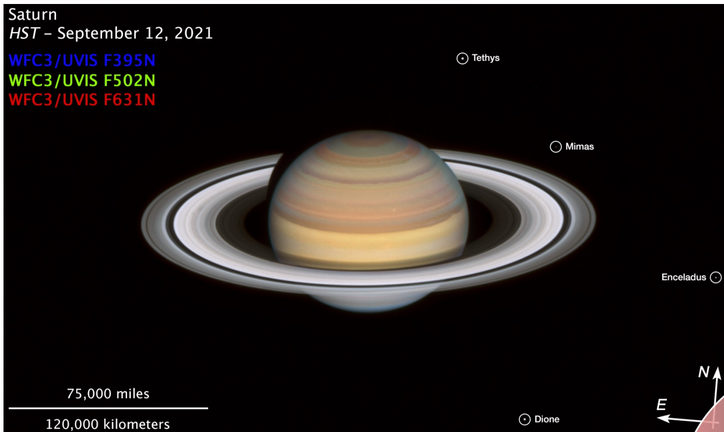
ASYMPTOTIC SAFETY

+ non-perturbative! + very high energy

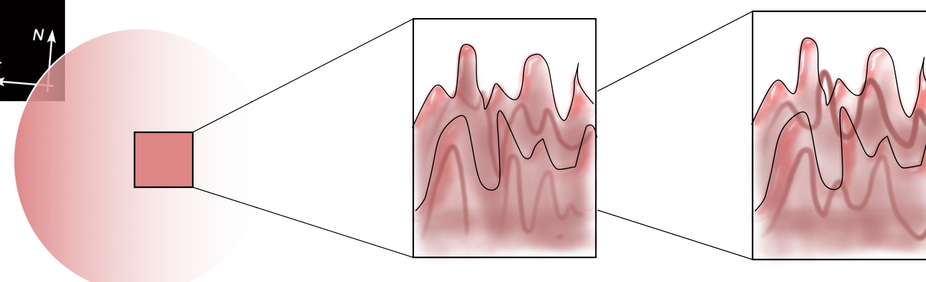


Quantum Gravity Approaches

Asymptotic Safety



+ non-perturbative! + very high energy

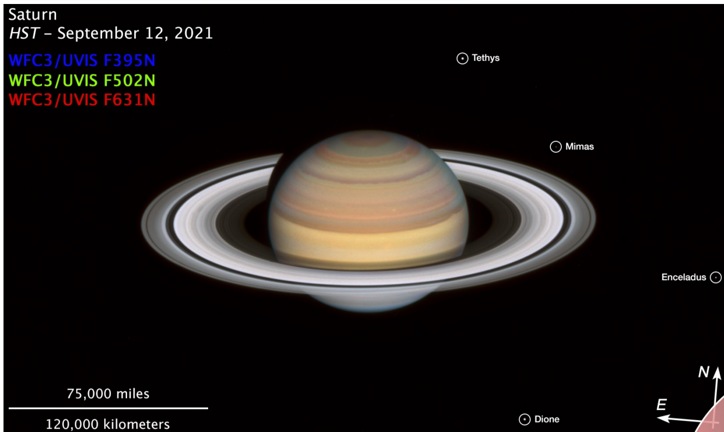


fixed point of RG flow

scale invariance = Predictive!

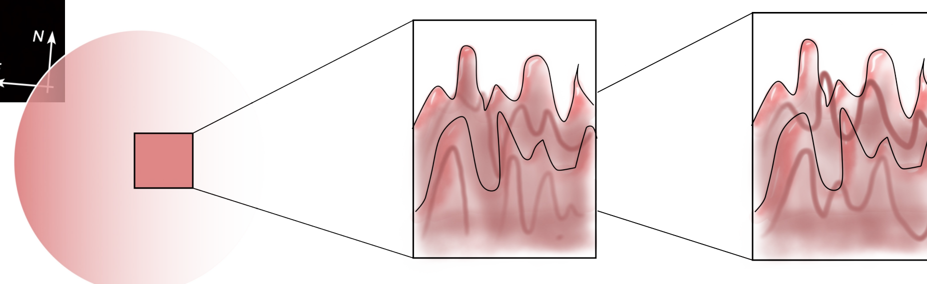
Quantum Gravity APPROACHES

ASYMPTOTIC SAFETY



DYNAMICAL TRIANGULATIONS

+ non-perturbative! + very high energy

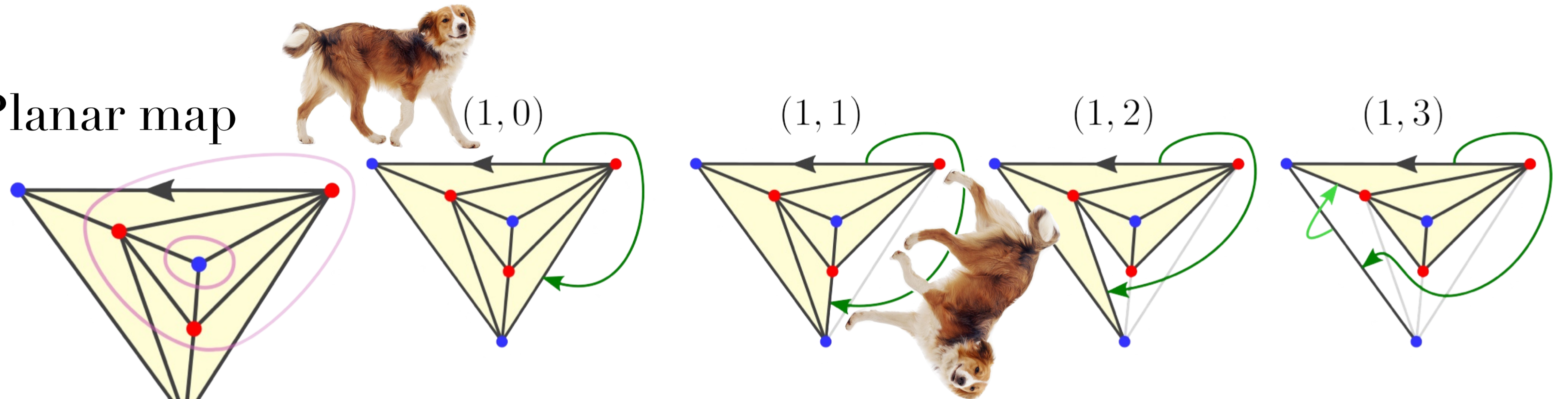


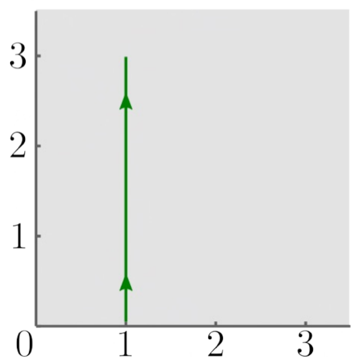
Spacetime @ fixed point

AS review [Eichhorn, 1810.07615]
CDT review[Loll 1905.08669]

How to study discrete surfaces ?

Planar map

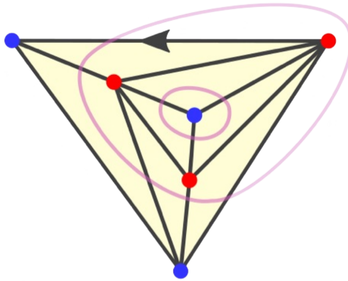


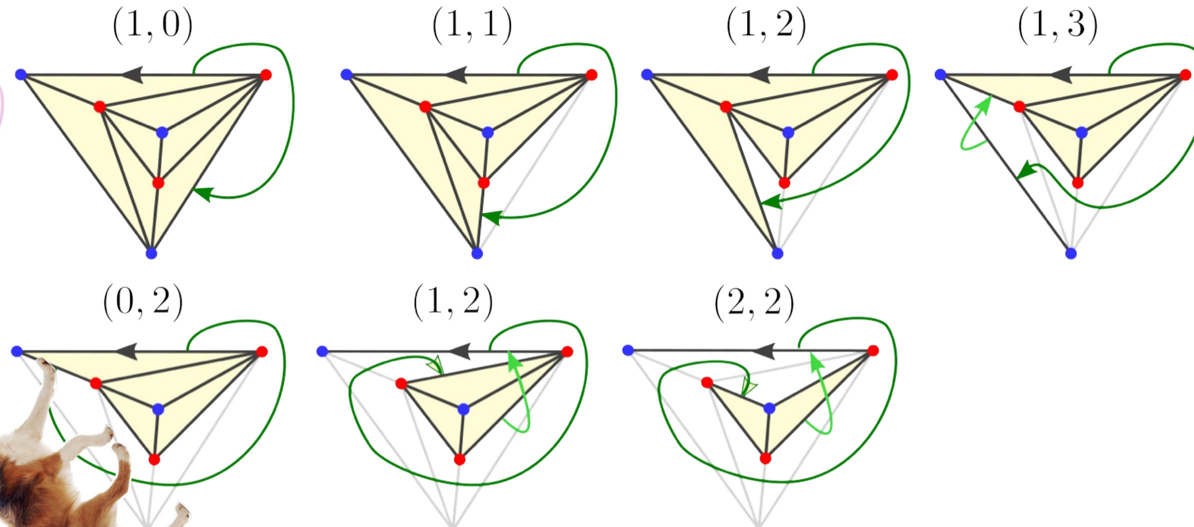


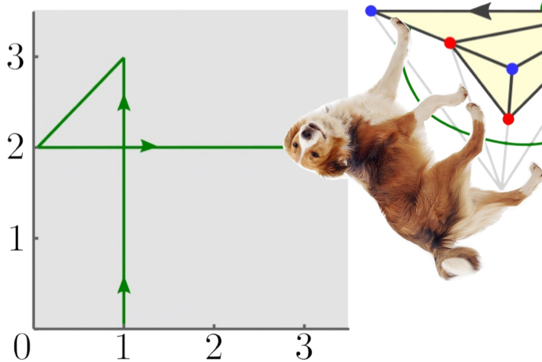
Walk

How to study discrete surfaces ?

Planar map

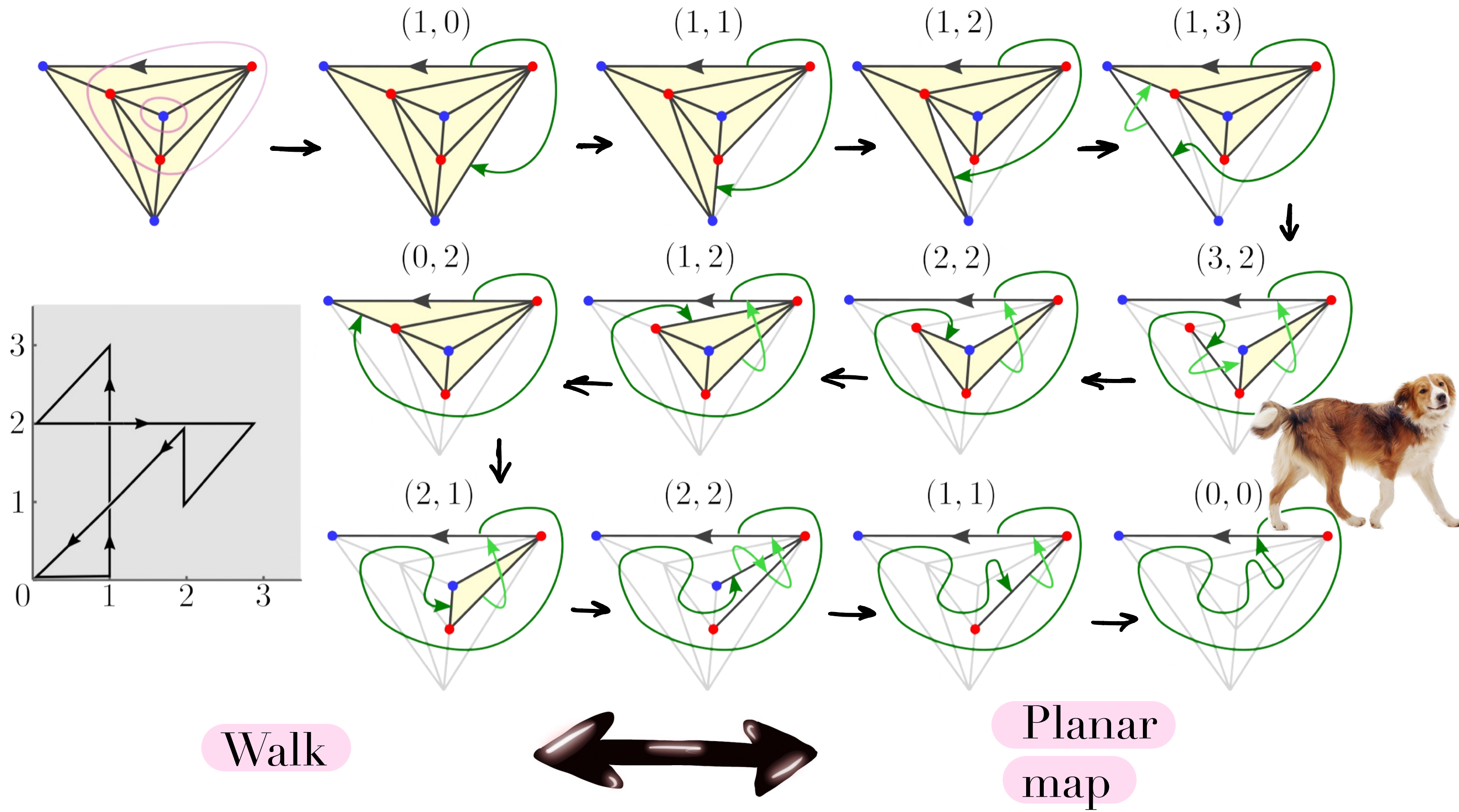




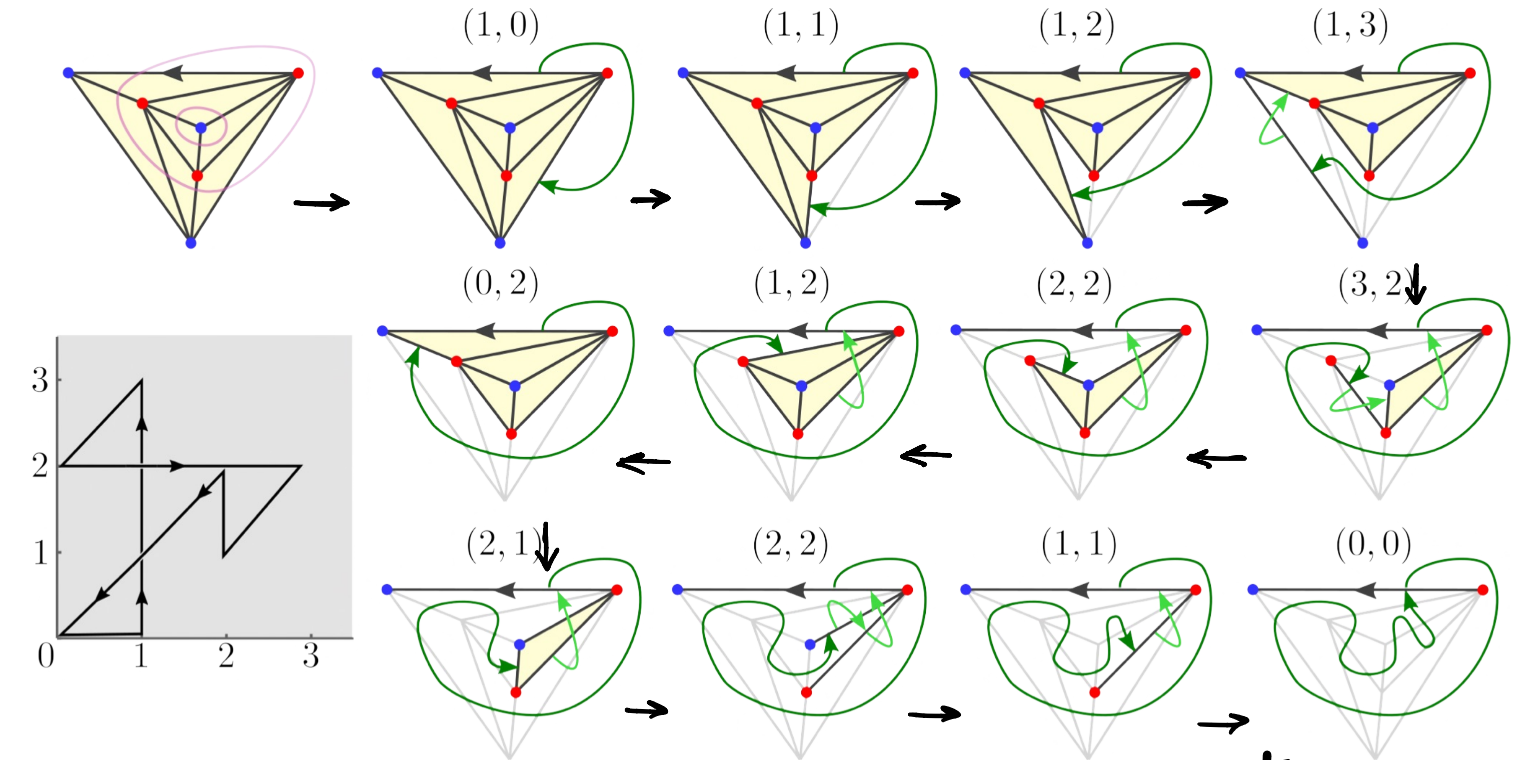


Walk

How to study discrete surfaces ?



How to study discrete surfaces ?



count Walks n steps

$$\mathcal{Z}_n = \frac{4^n}{(n+1)(2n+1)}\binom{3n}{n} \overset{n\to\infty}{\sim} \sqrt{\frac{3}{16\pi}}\, 3^{3n} n^{-5/2}.$$

count Planar maps n vertices.

Kreweras '65
Bernardi 0605320
Gwyne, Holden, Sun 1905.06757

Random walk

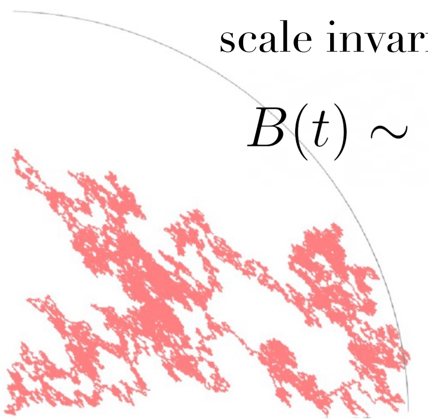
Random planar map

scaling limit
$n \to \infty$

Brownian
Motion

scale invariant!

$$B(t) \sim \frac{1}{a} B(a^2 t)$$



Random walk

Random planar map

scaling limit
$n \to \infty$

?

Random walk ⟷ Random planar map

Liouville Quantum Gravity

$$S_L = \frac{1}{4\pi}\int d^2x\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a\phi\partial_b\phi + Q\hat{R}\phi + 4\pi\hat{\mu}e^{\gamma\phi}\right)$$

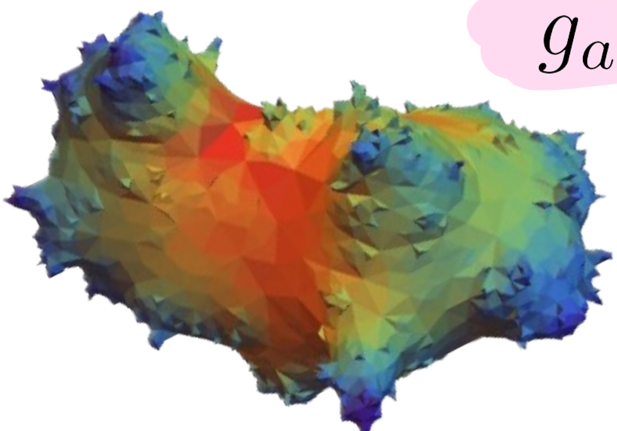
gaussian free field

Path integral
e^{-S_L}

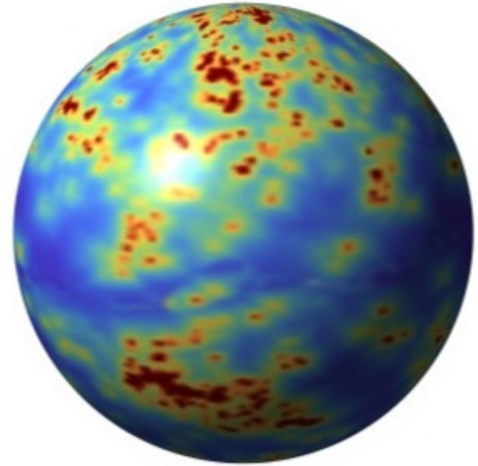
$\hat{\mu} > 0$

$Q = \frac{\gamma}{2} + \frac{2}{\gamma}$ $\qquad \gamma \in (0,2)$

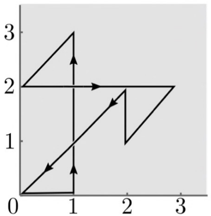
$$g_{ab} = e^{\gamma\phi}\hat{g}_{ab}$$



scaling limit

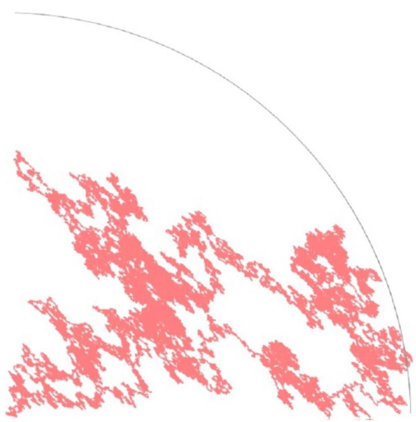


Random walk

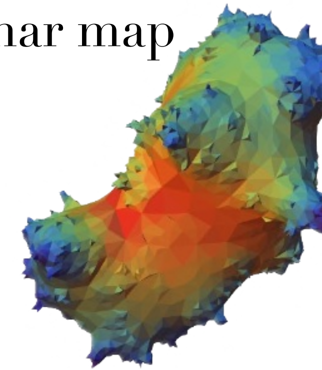


scaling limit
$n \to \infty$

Brownian Motion



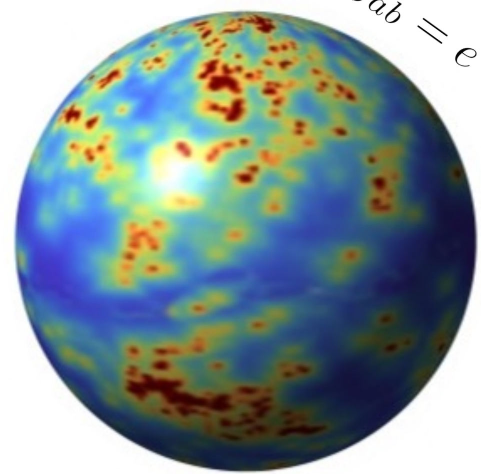
Random planar map



scaling limit
$n \to \infty$

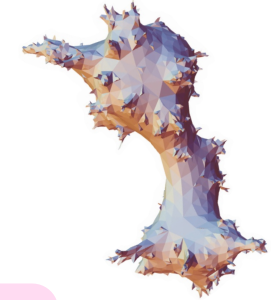
Random Surface

$g_{ab} = e^{\gamma\phi}\hat{g}_{ab}$



Random walk

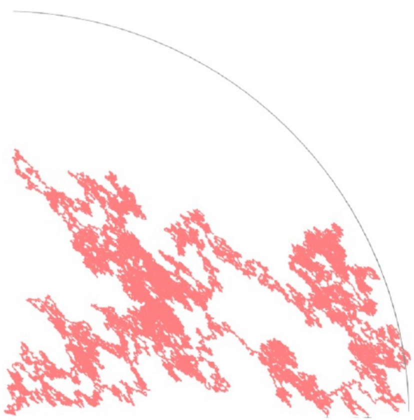
Random graph

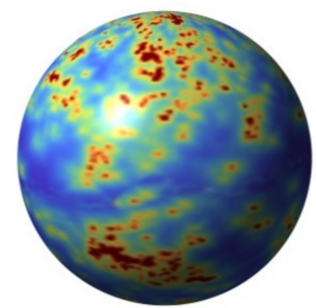


Brownian Motion

Random Surface

?



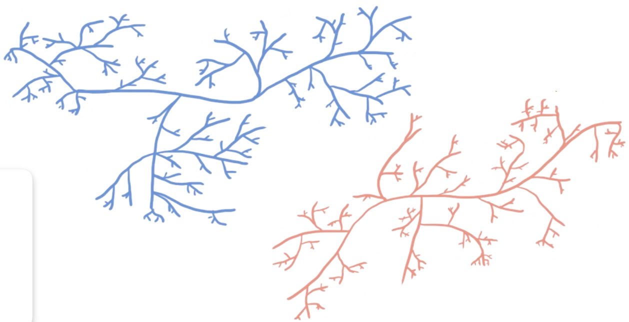


scale invariant?

$$Z_n^* \overset{n\to\infty}{\sim} C\kappa^n n^{\gamma_s - 2}$$

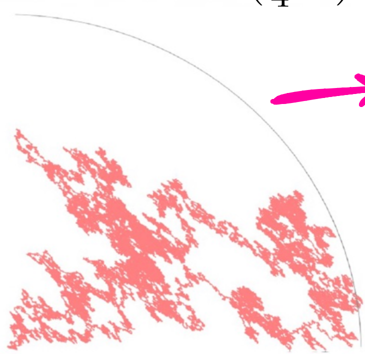
The answer: Mating of Trees

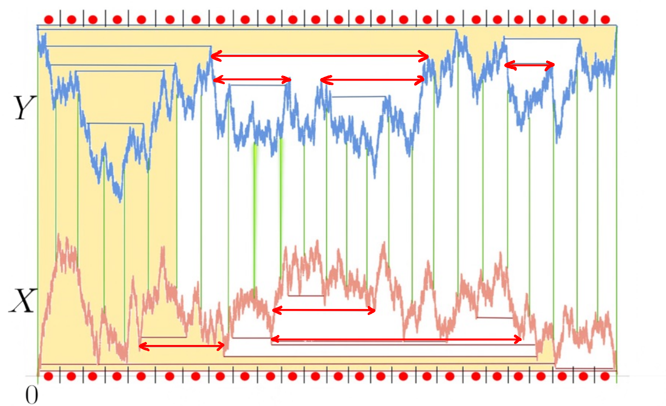
tree bijection!



2D Brownian Motion

$$Cov(X,Y) = -\cos\left(\frac{\pi}{4}\gamma^2\right)$$





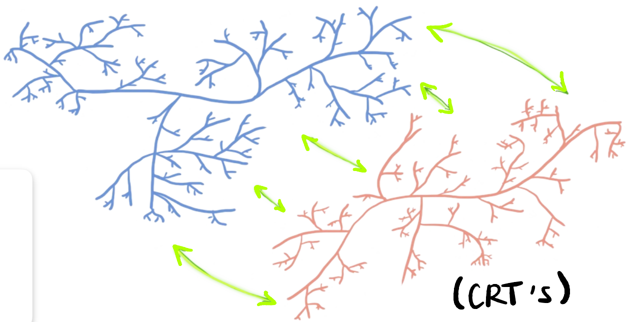
Duplantier, Miller & Sheffield [1409.7055]

The answer: Mating of Trees

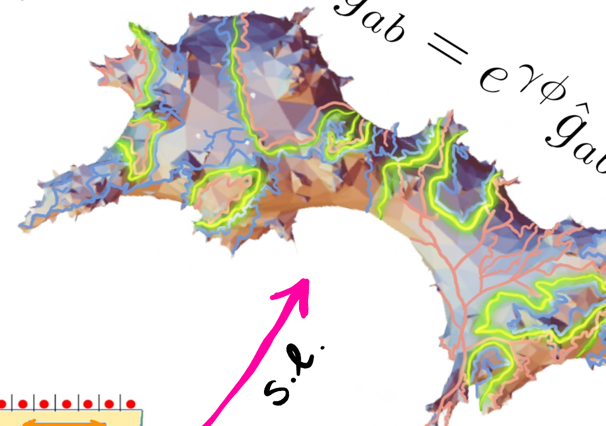
Scale Invariant Random Surface

$$\gamma_s = 1 - 4/\gamma^2$$

string susceptibility



(CRT's)



$$g_{ab} = e^{\gamma\phi}\hat{g}_{ab}$$

2D Brownian Motion

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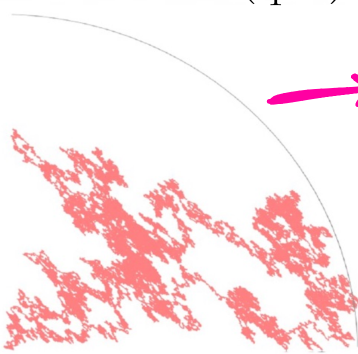
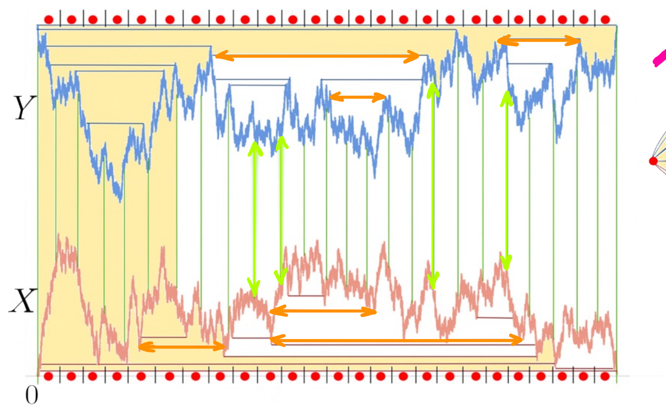
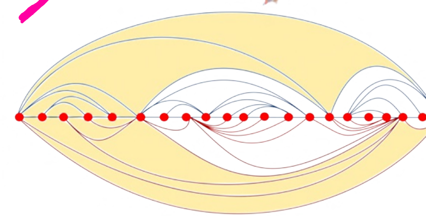


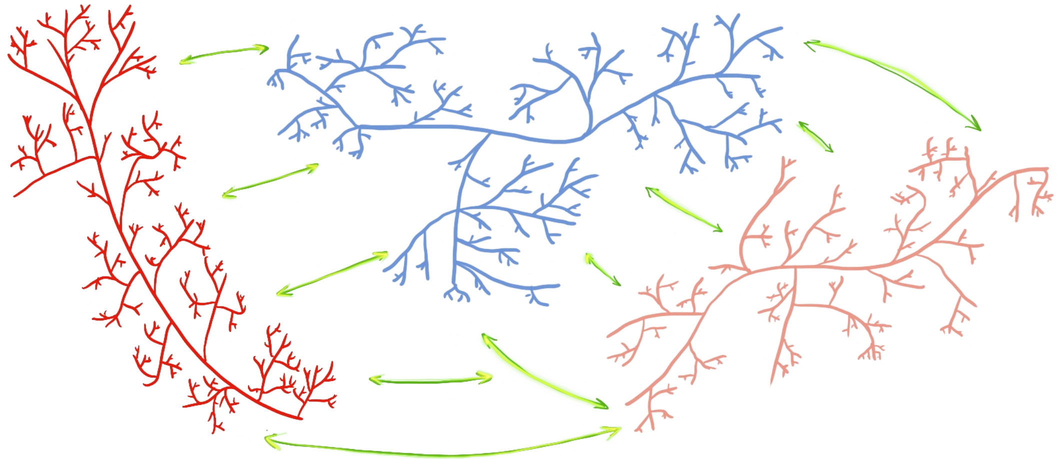


s.l.

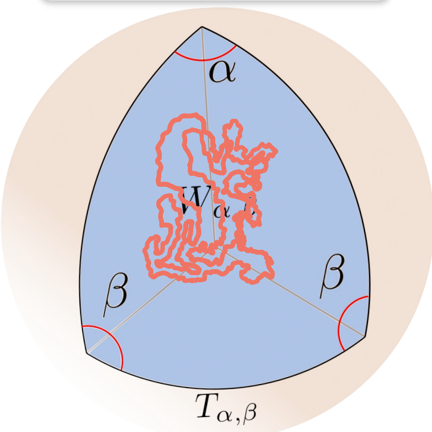


mated-CRT map

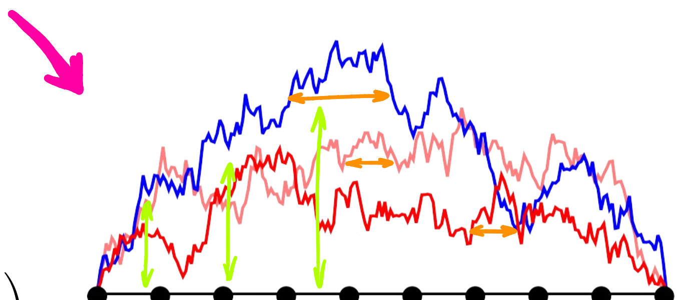
GENERALIZATION



3D Brownian Motion

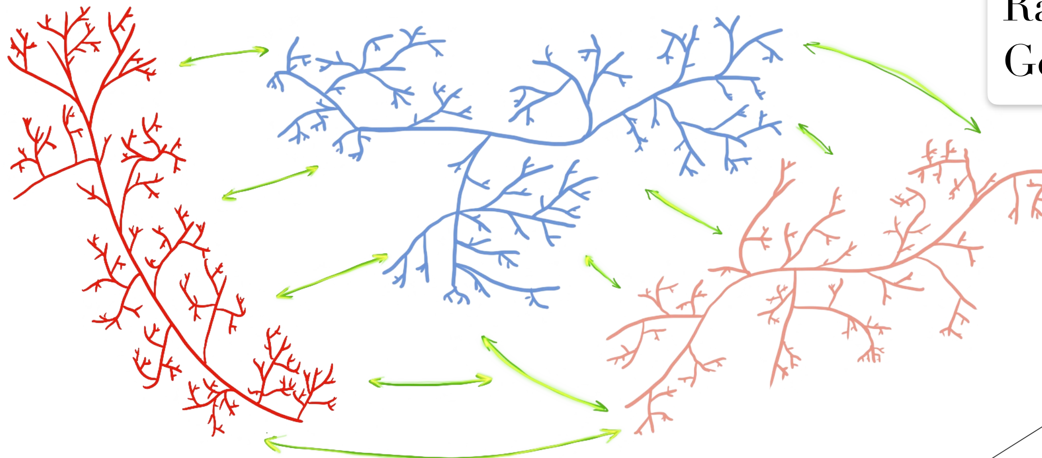


$$\begin{pmatrix} 1 & -\cos(\alpha) & -\cos(\gamma) \\ -\cos(\alpha) & 1 & -\cos(\beta) \\ -\cos(\gamma) & -\cos(\beta) & 1 \end{pmatrix}$$



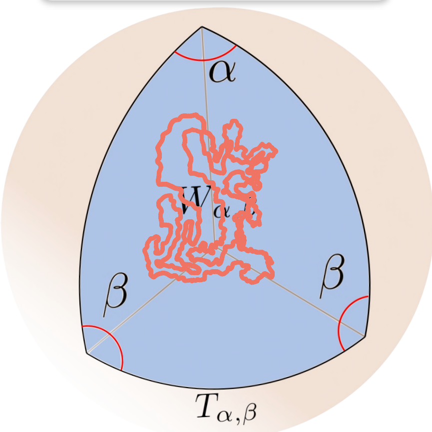
Budd & Castro 2207.05355

GENERALIZATION

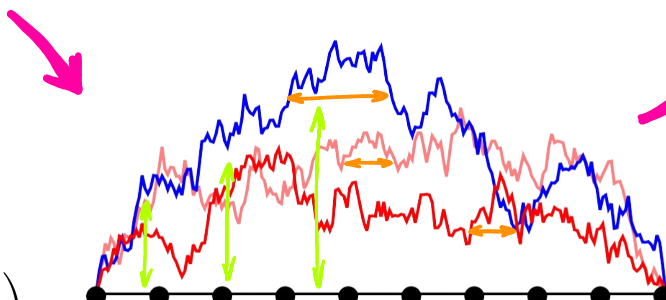


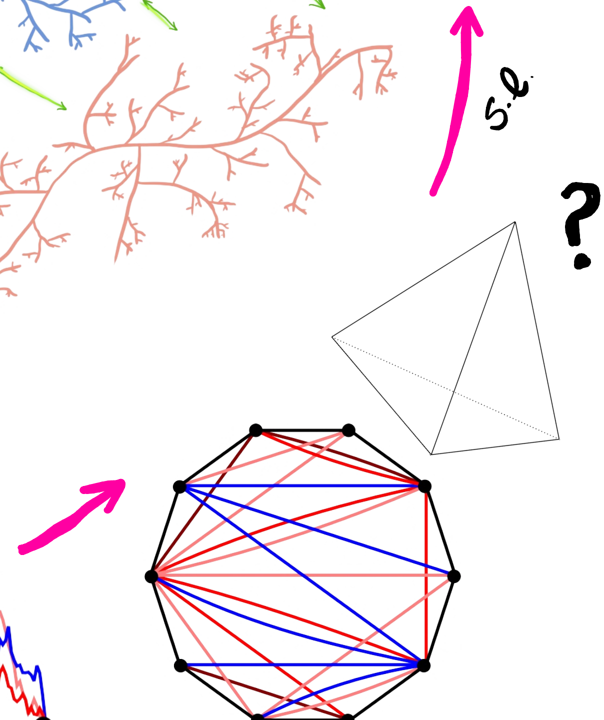
Scale Invariant Random Geometry ?

3D Brownian Motion



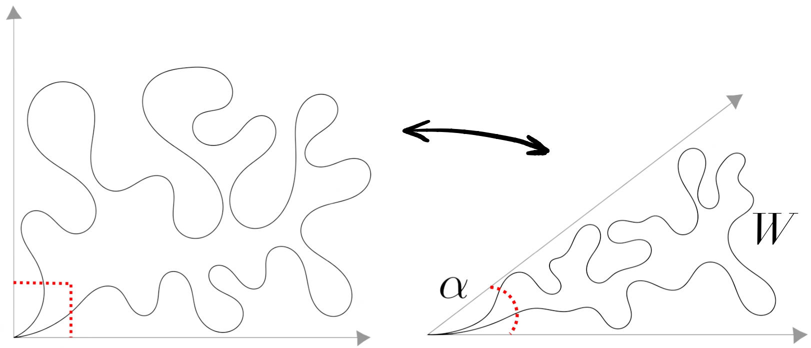
$$\begin{pmatrix} 1 & -\cos(\alpha) & -\cos(\gamma) \\ -\cos(\alpha) & 1 & -\cos(\beta) \\ -\cos(\gamma) & -\cos(\beta) & 1 \end{pmatrix}$$





Budd & Castro 2207.05355

How to simulate Brownian excursions



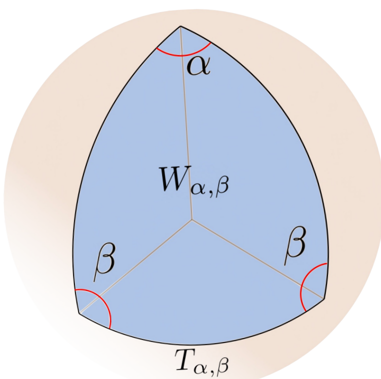
Correlated   Uncorrelated

Probability density : standard BM started at x remains in wedge after time t

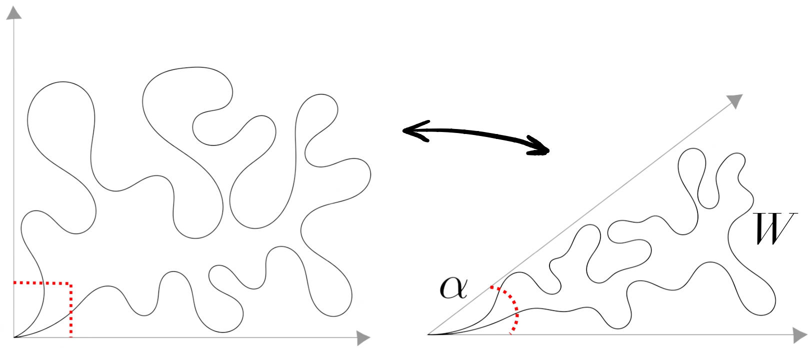
$P_t^{\mathbf{C}}(x,y)$

angular

$$\begin{cases} L_{\mathbb{S}^{d-1}} m_i(\tilde{x}) = -\lambda_i m_i(\tilde{x}) & \text{for } \tilde{x} \in W \cap \mathbb{S}^{d-1}, \\ m_i(\tilde{x}) = 0 & \text{for } \tilde{x} \in \partial W \cap \mathbb{S}^{d-1}. \end{cases}$$



How to simulate Brownian excursions



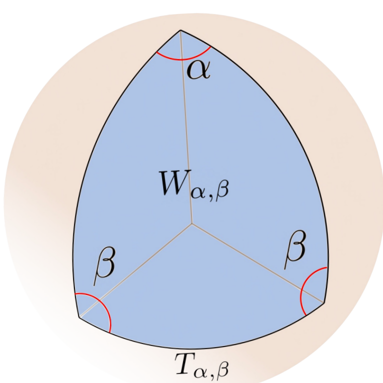
Probability density : standard BM started at x remains in wedge after time t

$$P_t^{\mathbf{C}}(x,y) = \boxed{c\,t^{\gamma_s - 2}} + O(t^{\gamma_s - 2})$$

$$\begin{cases} L_{\mathbb{S}^{d-1}} m_i(\tilde{x}) = -\lambda_i m_i(\tilde{x}) & \text{for } \tilde{x} \in W \cap \mathbb{S}^{d-1}, \\ m_i(\tilde{x}) = 0 & \text{for } \tilde{x} \in \partial W \cap \mathbb{S}^{d-1}. \end{cases}$$

$$\gamma_s = 1 - \sqrt{\lambda_1 + \left(\frac{d}{2} - 1\right)^2}$$

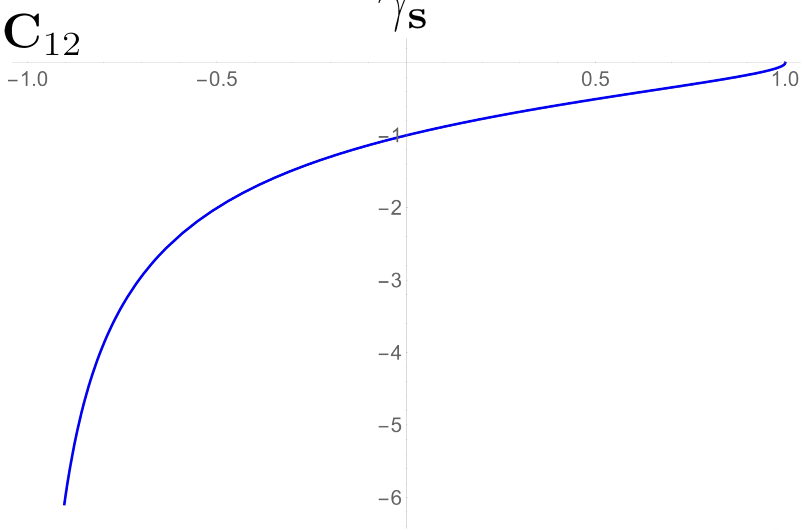
"string susceptibility"



Bañuelos & Smith '97

CRITICAL EXPONENTS: STRING SUSCEPTIBILITY

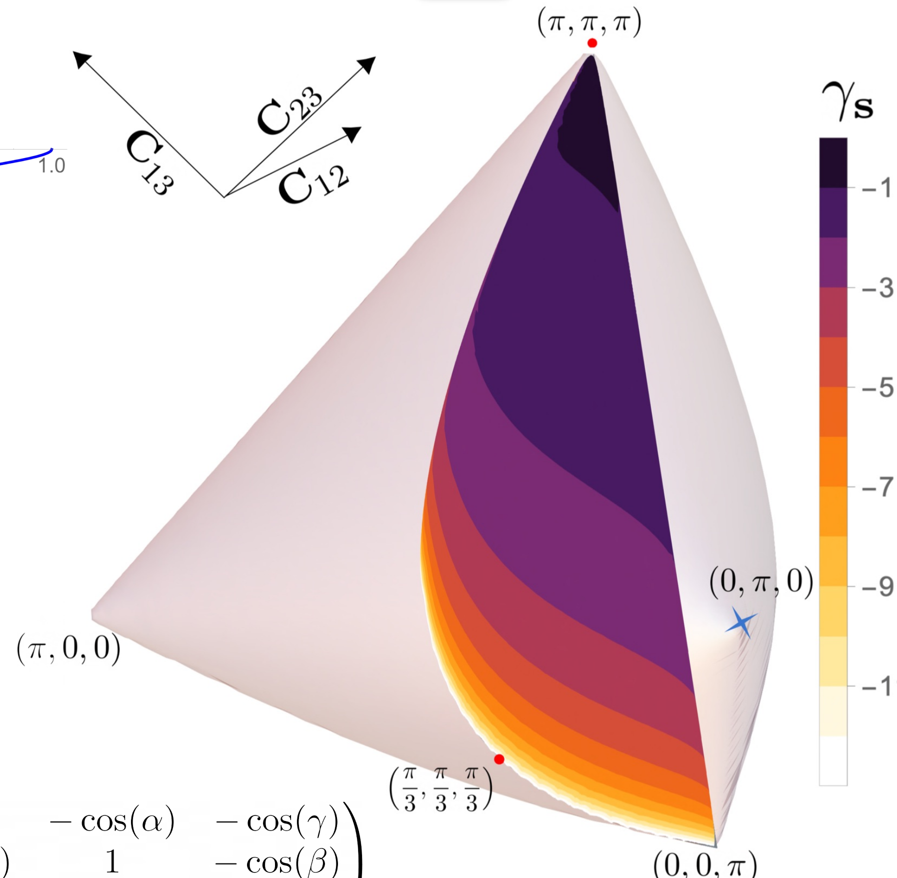
2D



$$\mathcal{C} = \begin{pmatrix} 1 & -\cos\alpha \\ -\cos\alpha & 1 \end{pmatrix}$$

$$\alpha = \frac{\pi}{4}\gamma^2$$

3D



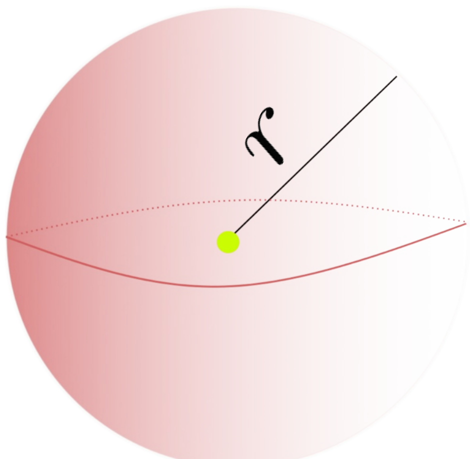
$$\mathbf{C} = \begin{pmatrix} 1 & -\cos(\alpha) & -\cos(\gamma) \\ -\cos(\alpha) & 1 & -\cos(\beta) \\ -\cos(\gamma) & -\cos(\beta) & 1 \end{pmatrix}$$

HAUSDORFF DIMENSION

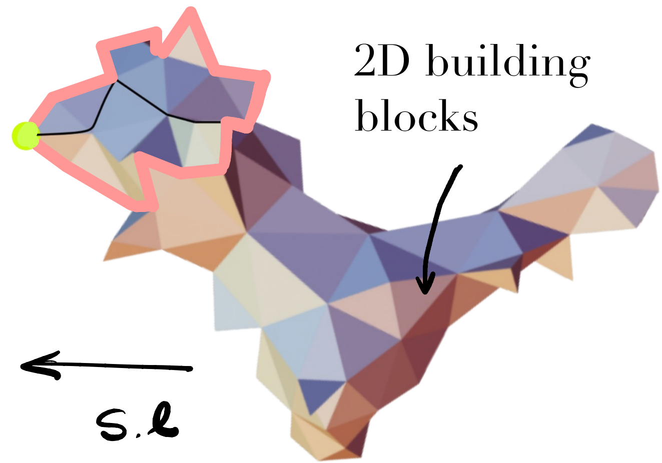
volume / length $\rightarrow$ $Vol \sim \ell^{d_H}$

e.g. $Area_{S^2} \sim r^2$

$\#$ of vertices $\sim \left(\begin{matrix}\text{graph}\\ \text{distance}\end{matrix}\right)^{d_H}$



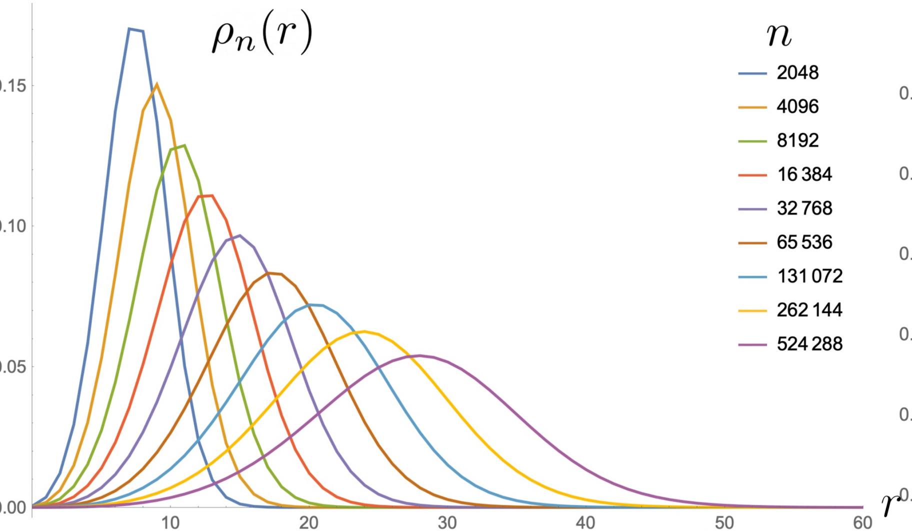
$\leftarrow$ s.ℓ

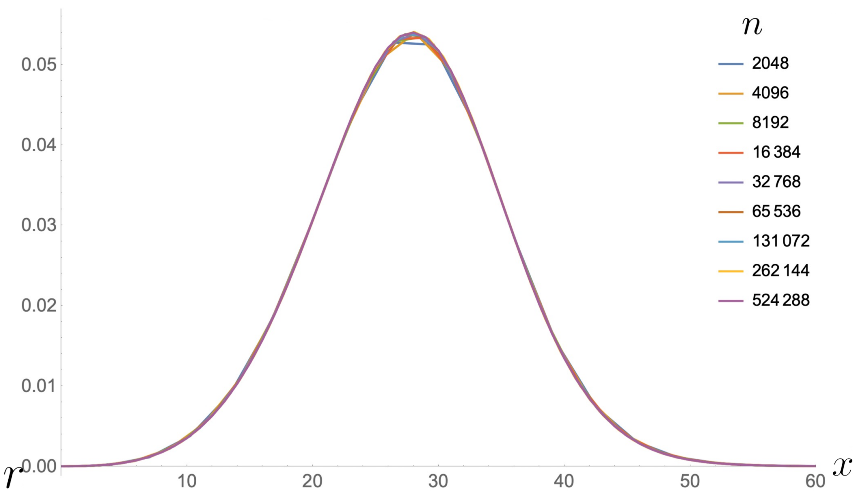


2D building blocks

HAUSDORFF DIMENSION

$$\lim_{n\to\infty} n^{1/d_H}\rho_n(n^{1/d_H}x) = \rho(x)$$

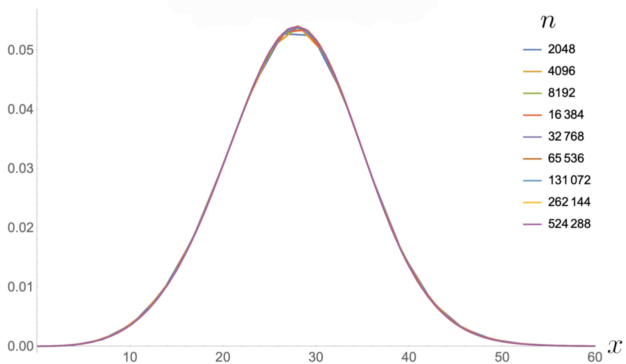


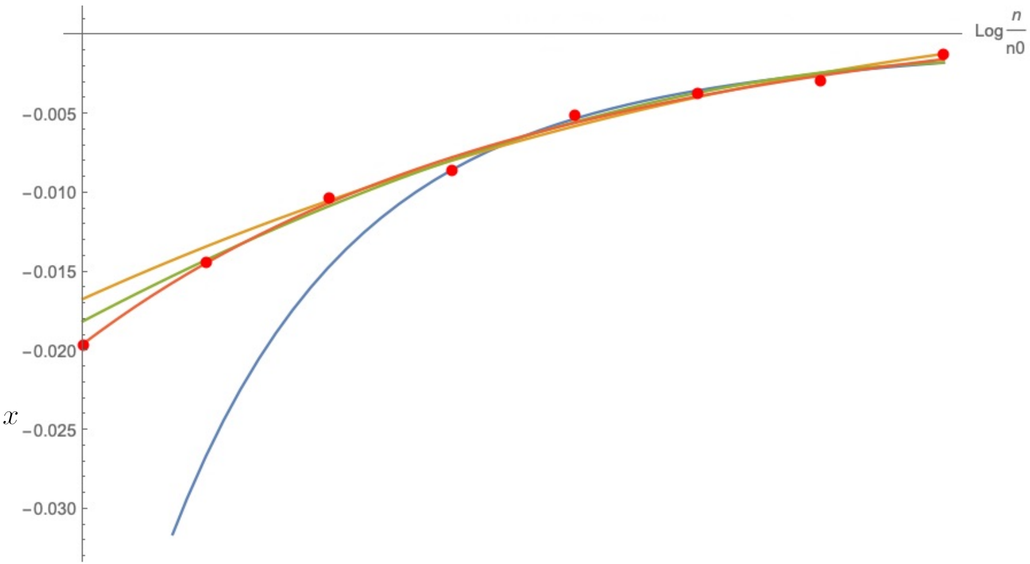


distance
histograms

HAUSDORFF DIMENSION

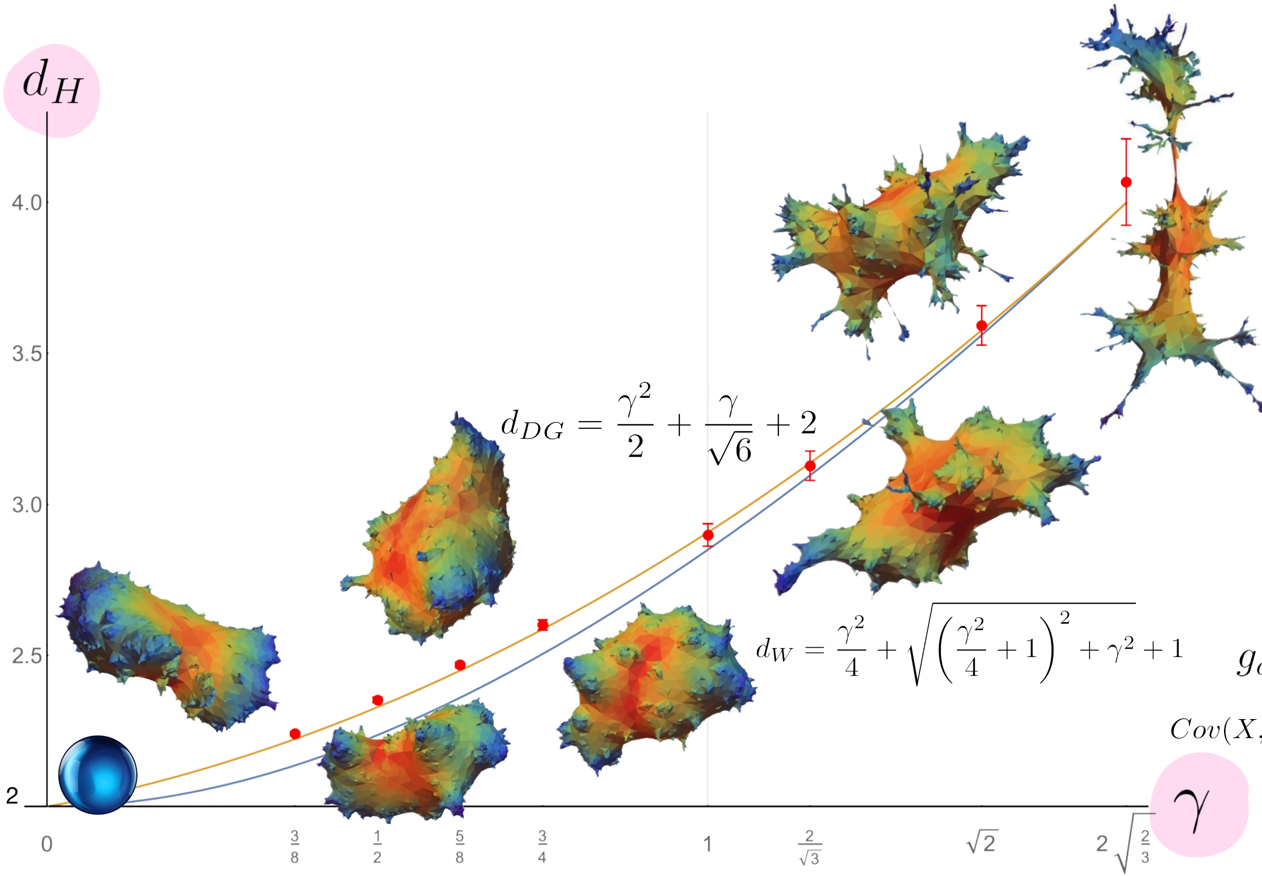
$$\lim_{n\to\infty} n^{1/d_H} \rho_n(n^{1/d_H} x) = \rho(x)$$





$$\left(\frac{n}{n_0}\right)^{-1/d_H} \left(a + b\left(\frac{n}{n_0}\right)^{-\delta}\right)$$

HAUSDORFF DIMENSION MEASUREMENTS



$$d_{DG} = \frac{\gamma^2}{2} + \frac{\gamma}{\sqrt{6}} + 2$$

$$d_W = \frac{\gamma^2}{4} + \sqrt{\left(\frac{\gamma^2}{4} + 1\right)^2 + \gamma^2} + 1$$

+ formal LQG bounds

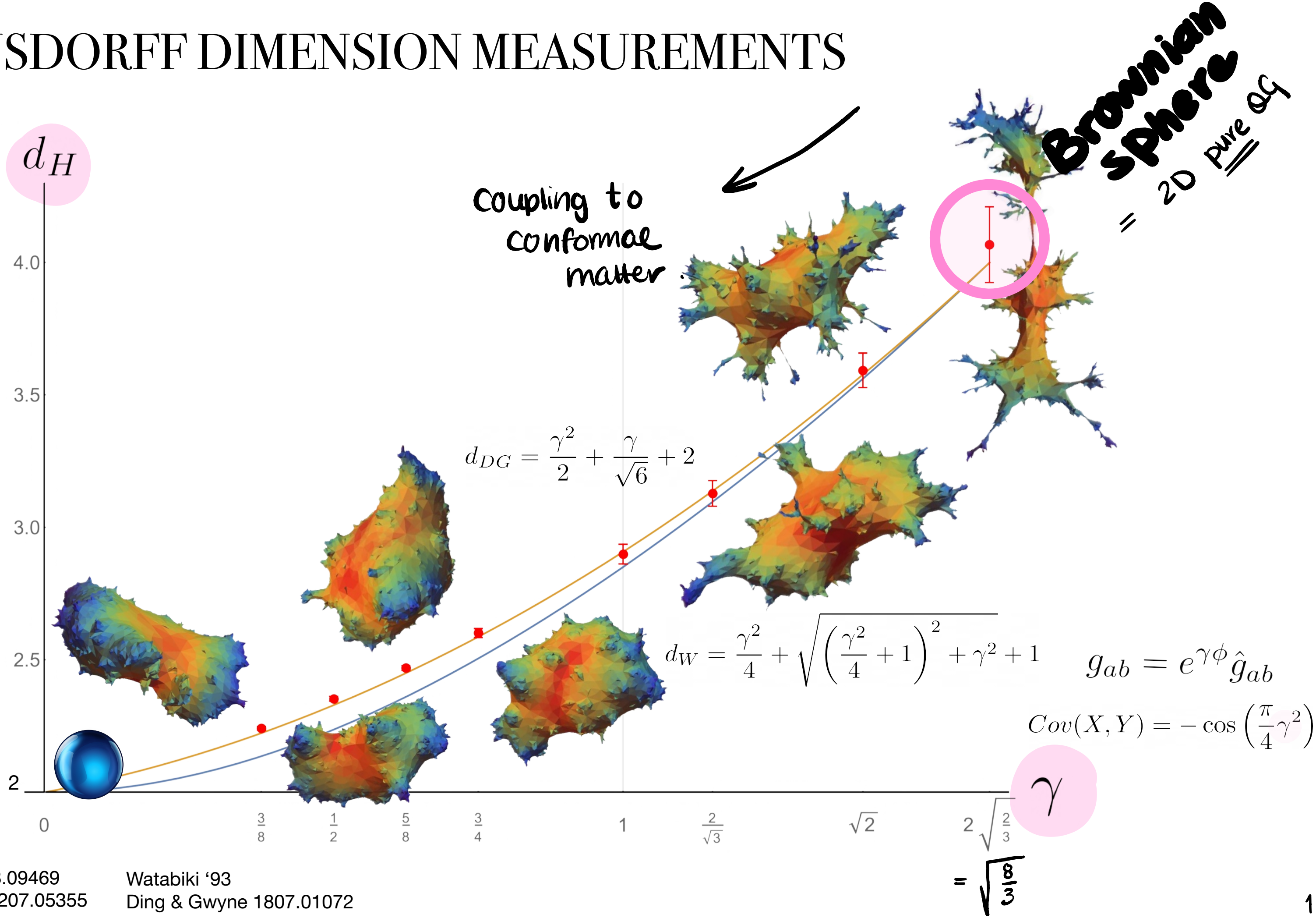
$$g_{ab} = e^{\gamma\phi}\hat{g}_{ab}$$

$$Cov(X,Y) = -\cos\left(\frac{\pi}{4}\gamma^2\right)$$

Budd et. al. 1908.09469
Budd & Castro 2207.05355

Watabiki '93
Ding & Gwyne 1807.01072

HAUSDORFF DIMENSION MEASUREMENTS



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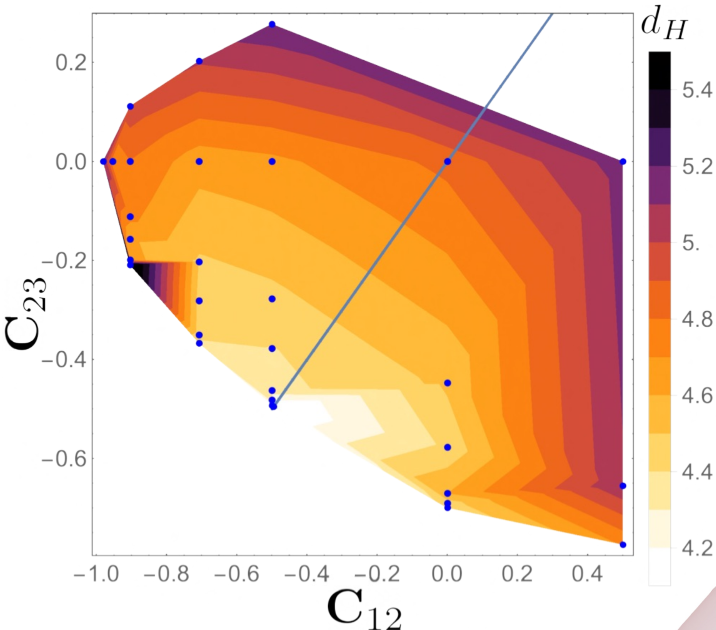

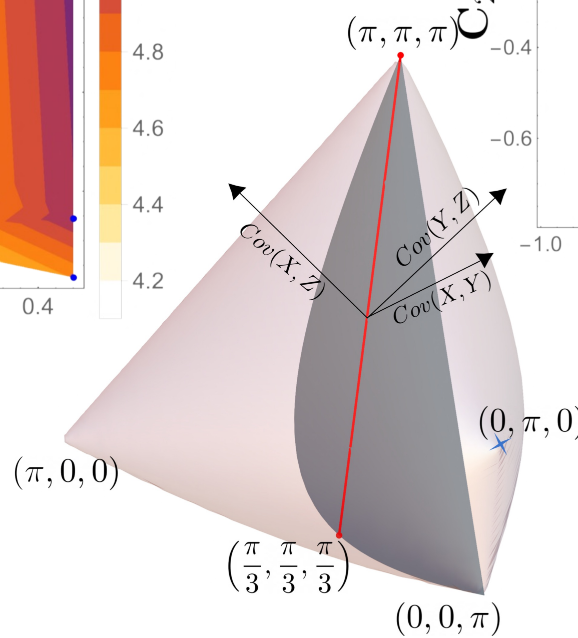

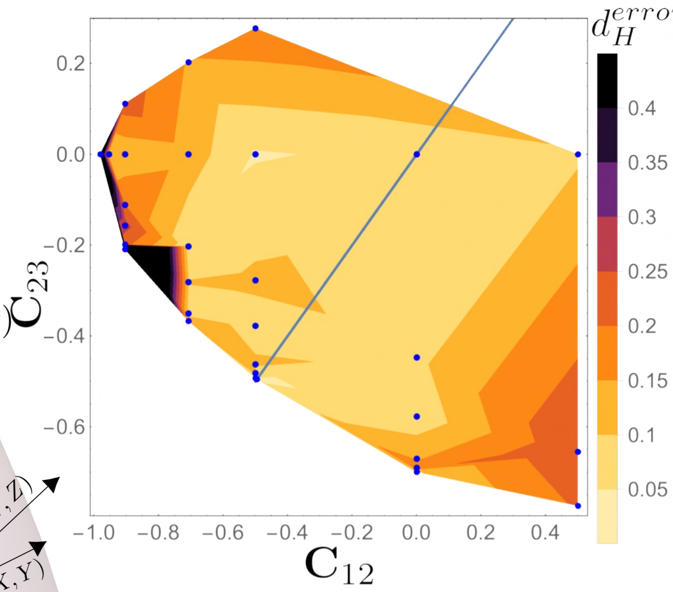

Budd & Castro 2207.05355

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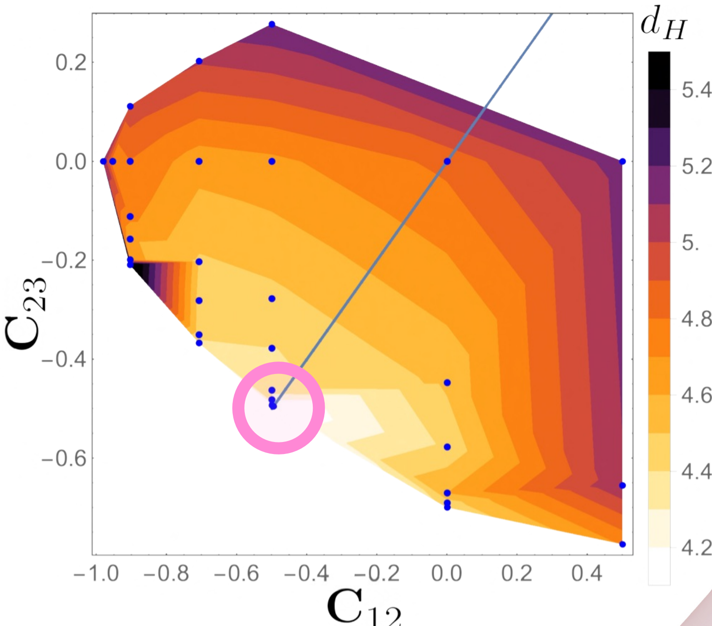

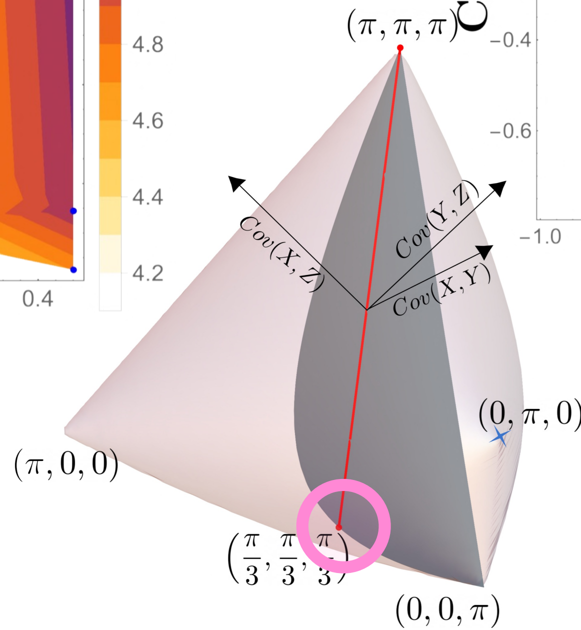

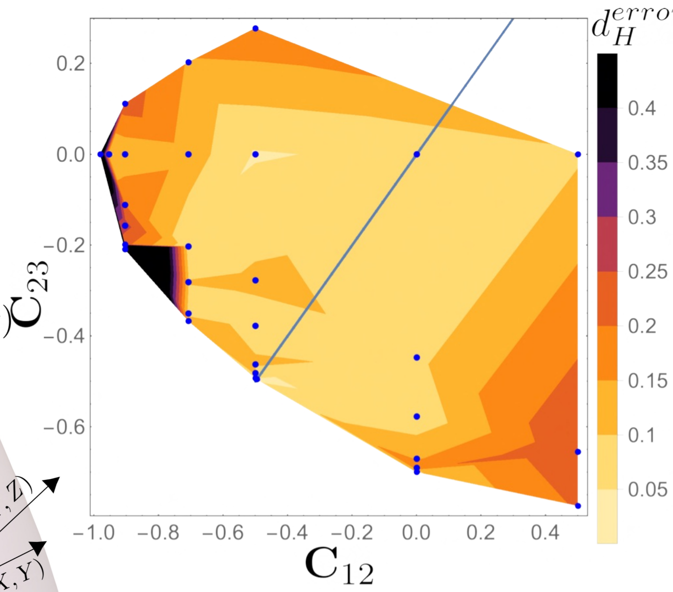

Budd & Castro 2207.05355

Outlook

Math

- New universality classes of higher dimensional scale-invariant random geometries
- more evidence for Hausdorff dimension predictions
- Topology?
- Random walk for special points

Physics

- Asymptotic Safety fixed point?
- matching critical exponents

  spectral dimension measurements
  Hausdorff dimension in AS
- Field theory of scaling limit?
  analogous to LQG