

Decorated maps and quadrant tandem walks

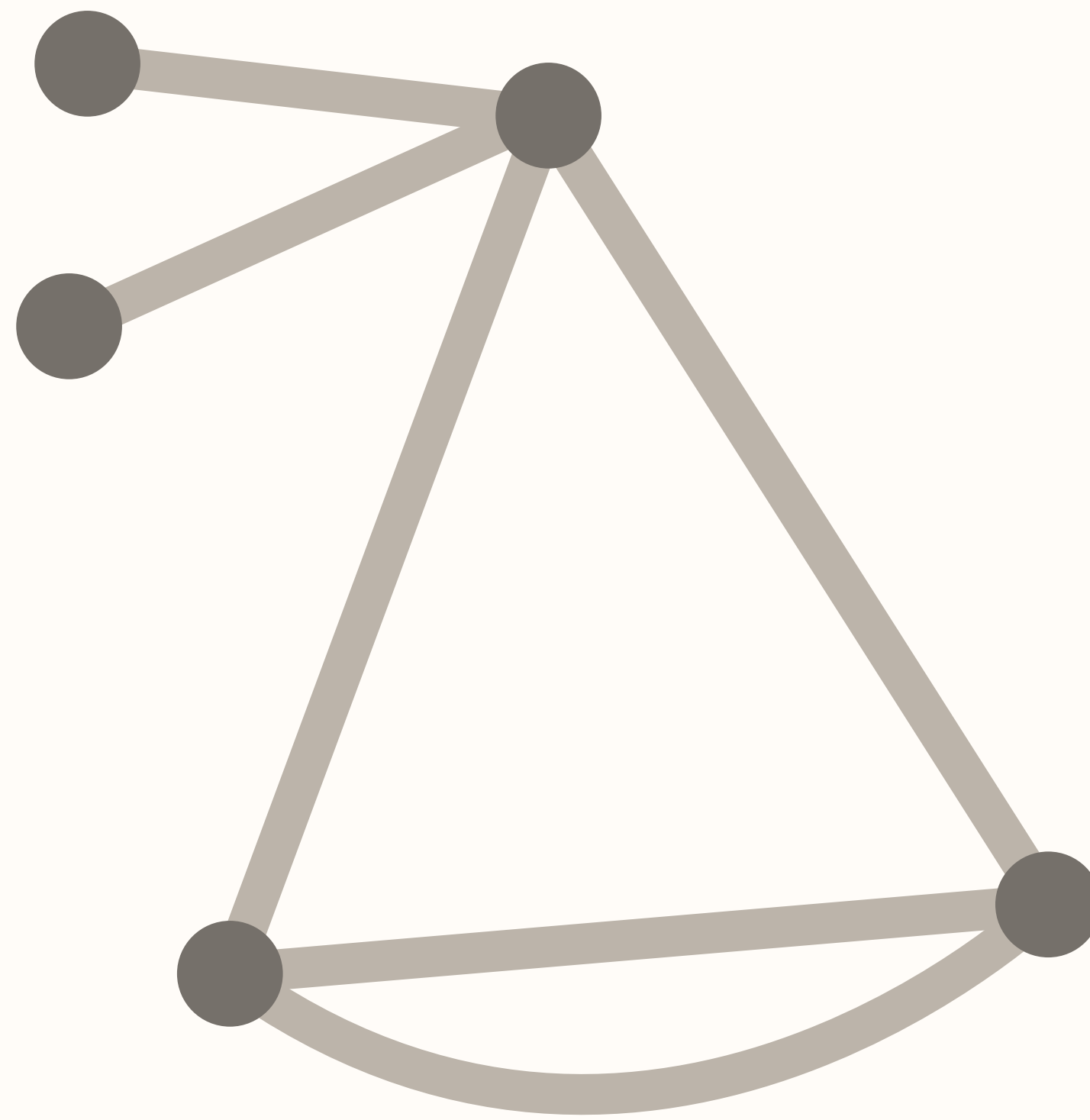
Éric Fusy, Erkan Narmanli, and Gilles Schaeffer



ÉCOLE
POLYTECHNIQUE

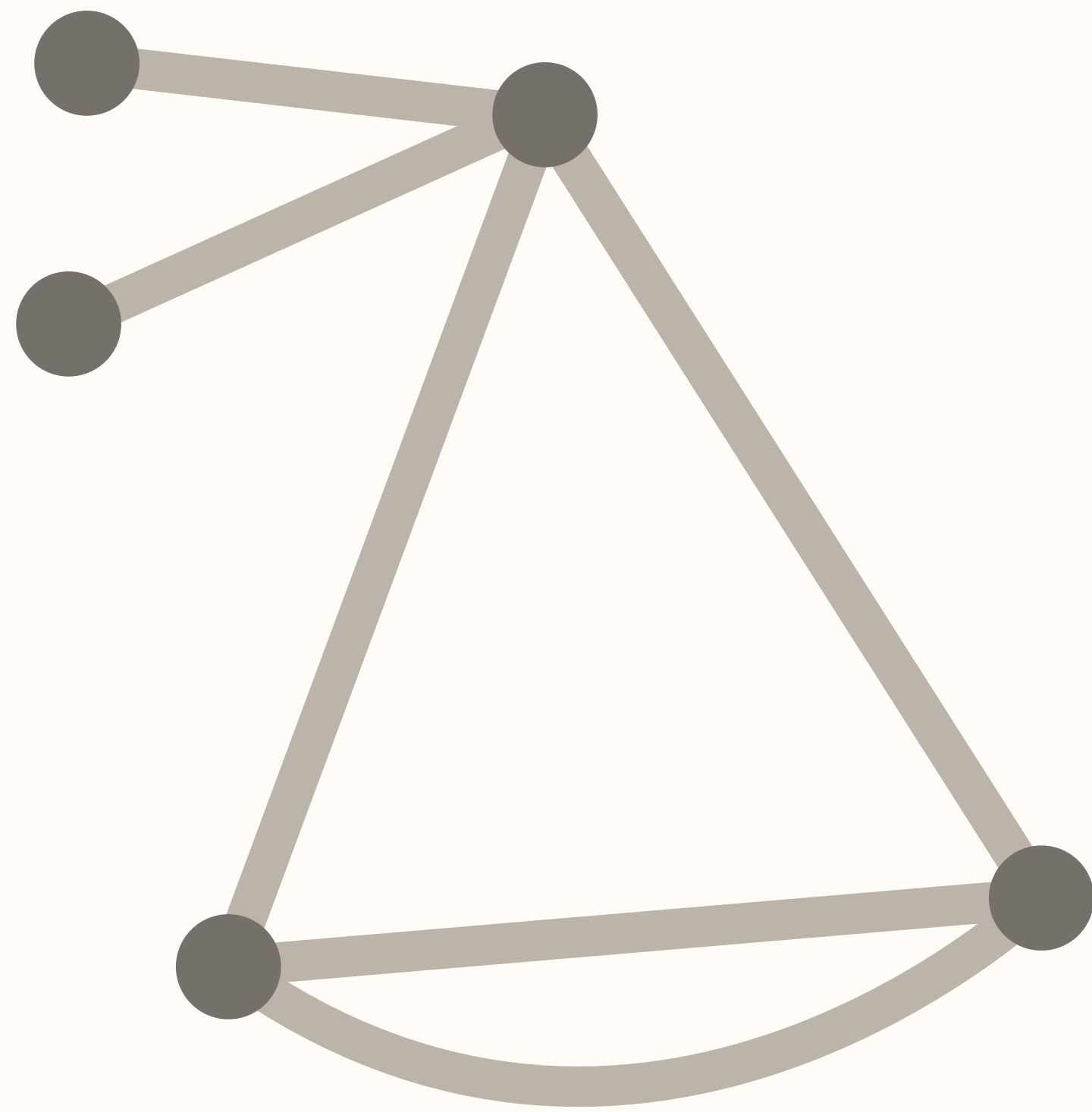
Decorated maps

Graphs



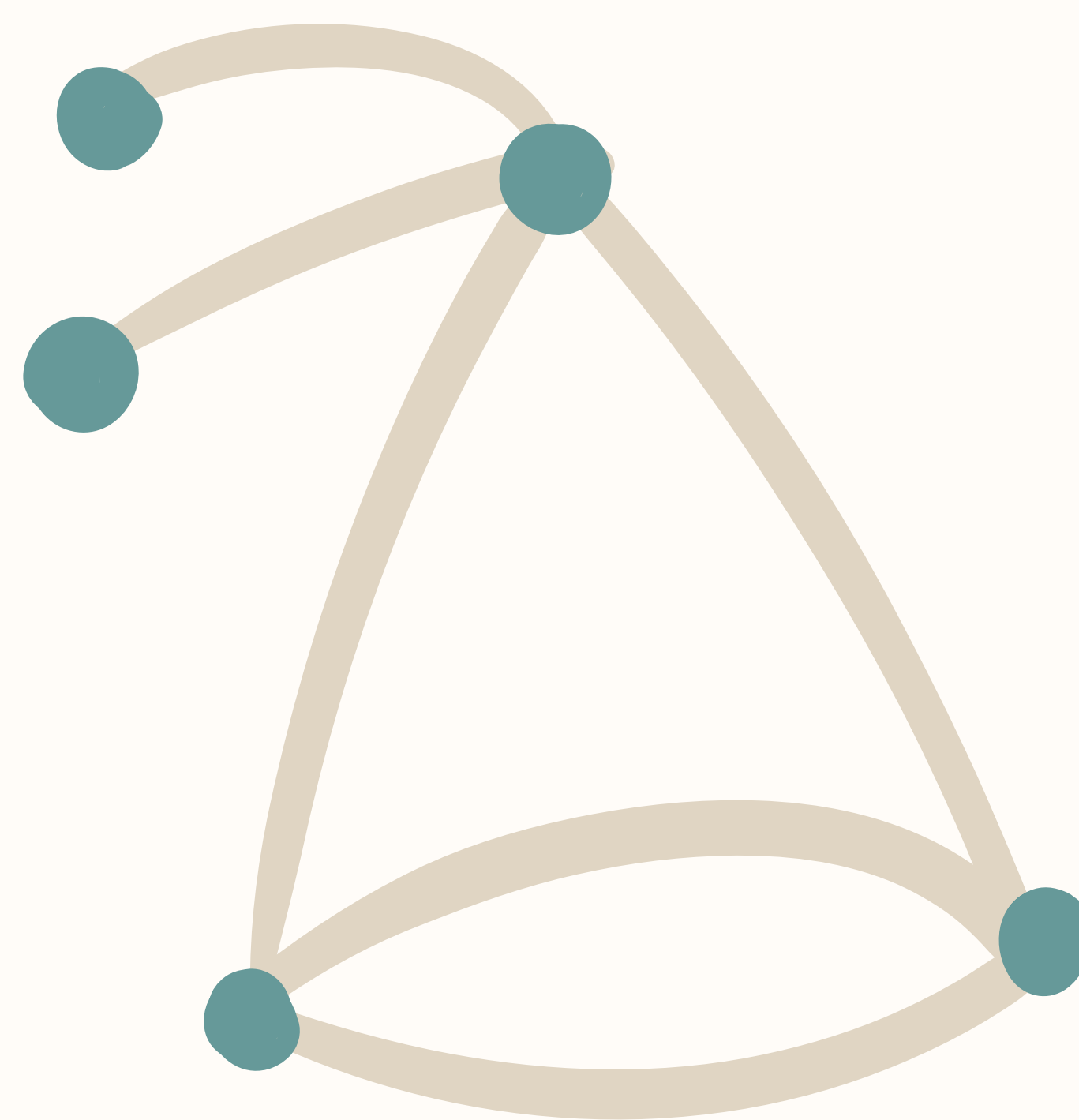
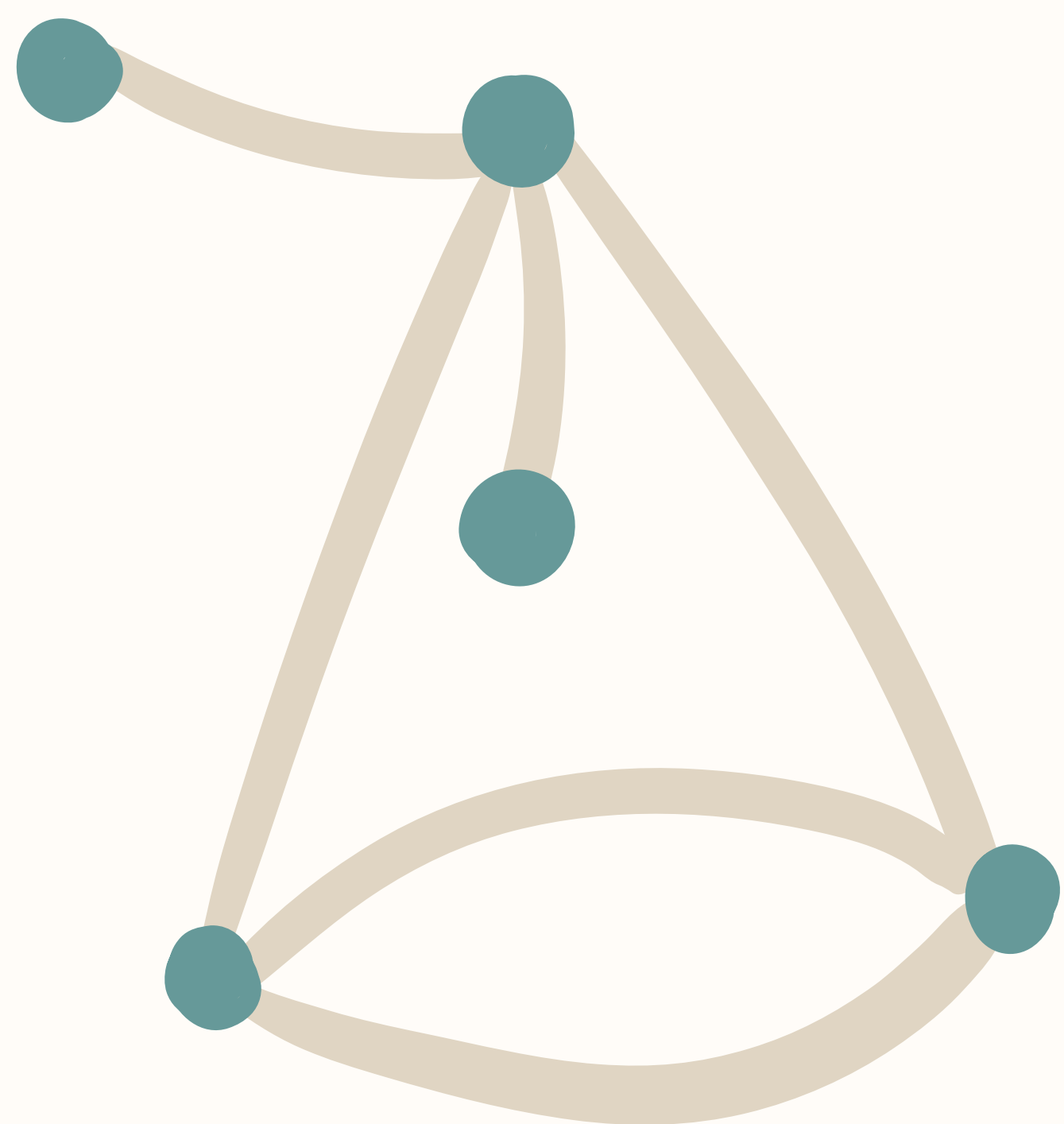
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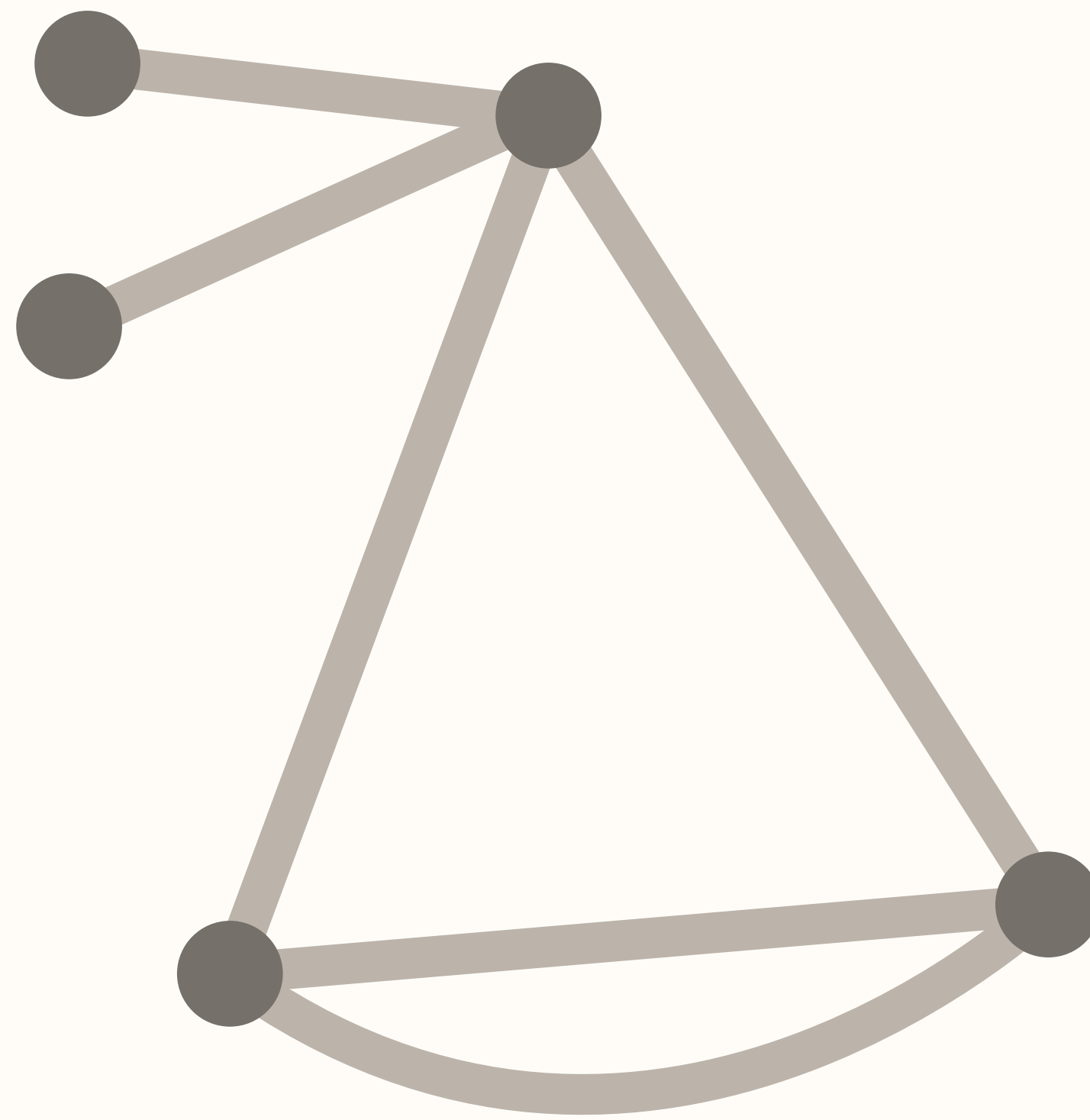
Maps

embedding on the plane



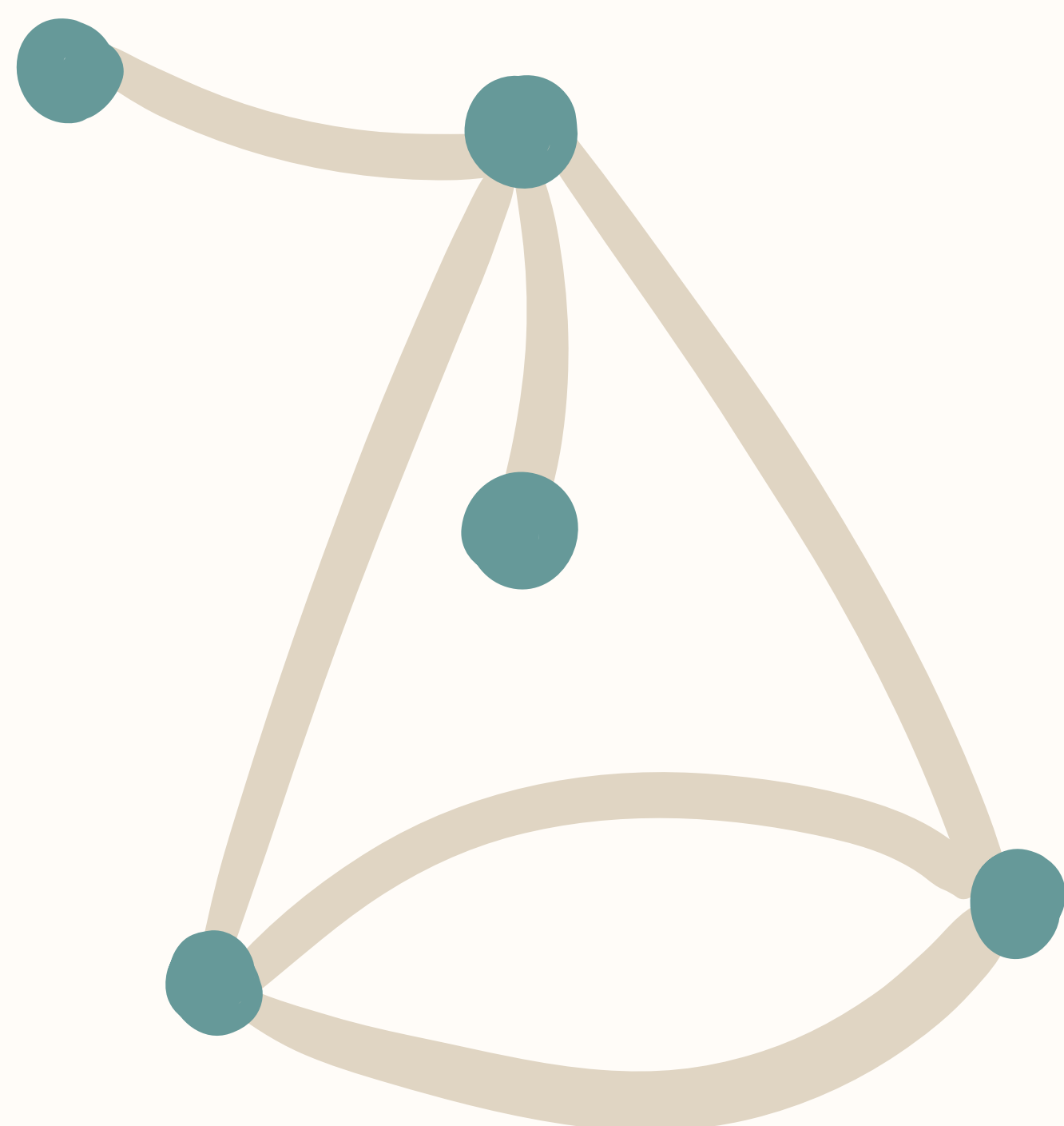
Decorated maps

Graphs

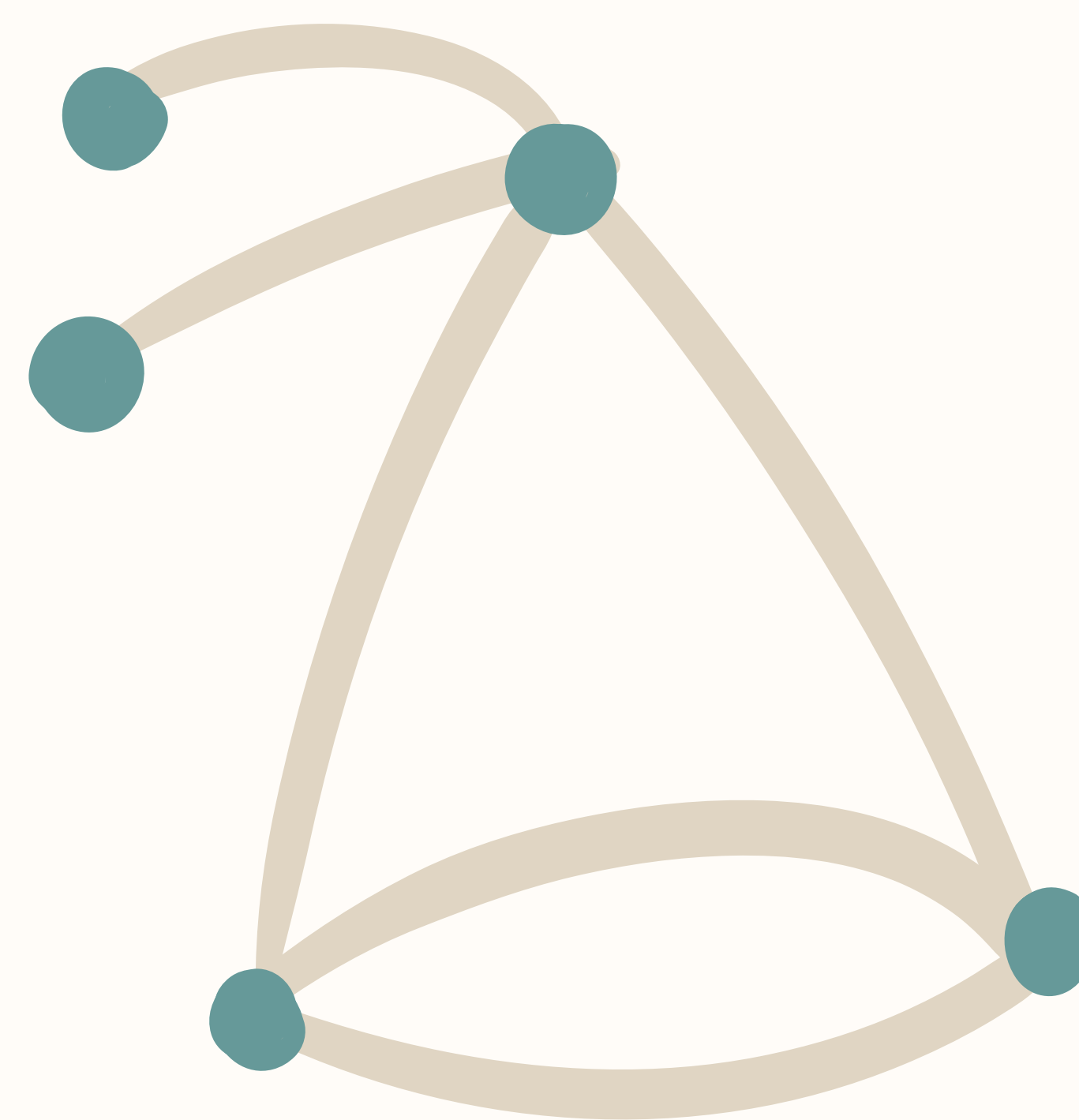


Maps

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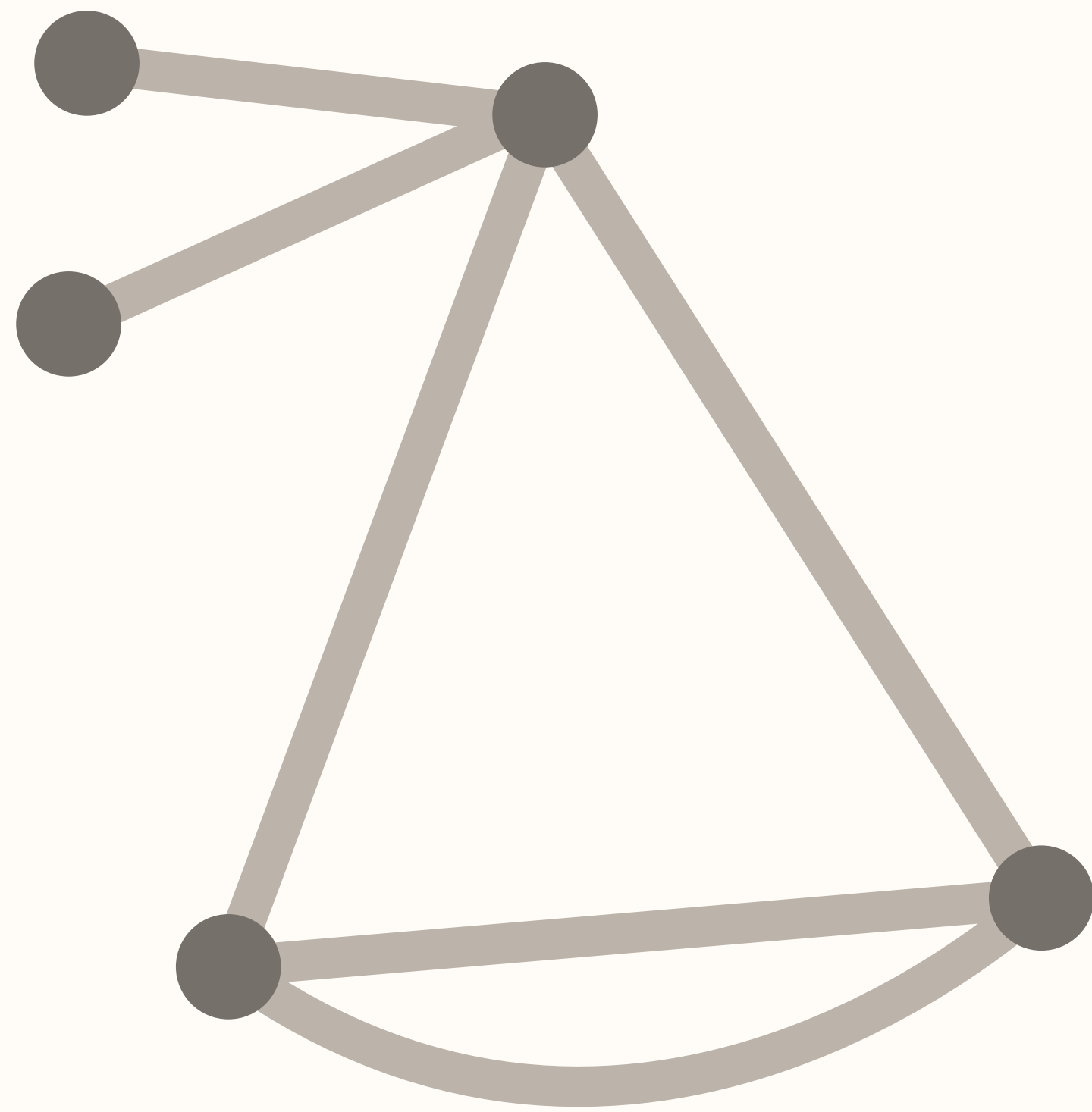


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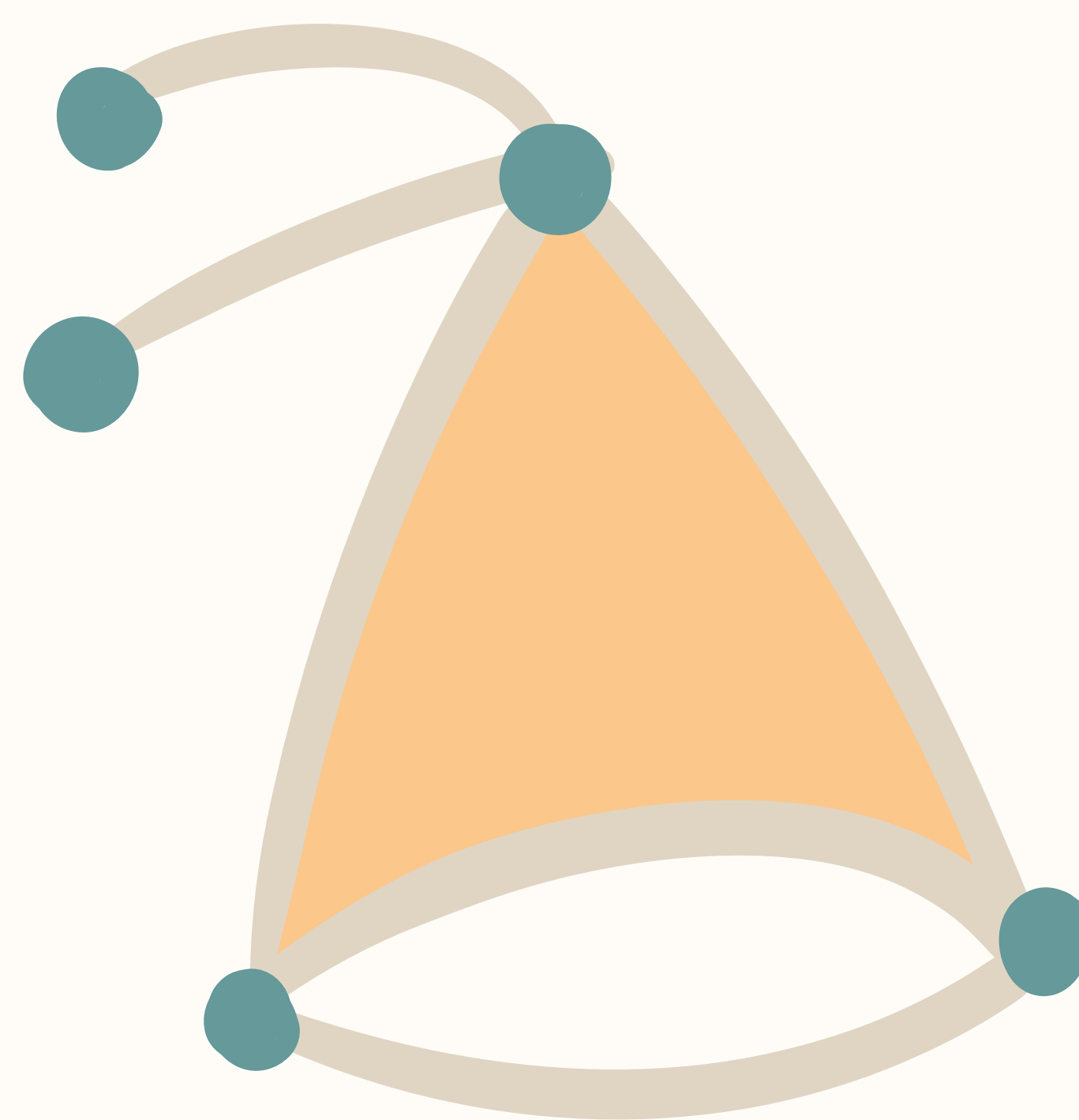
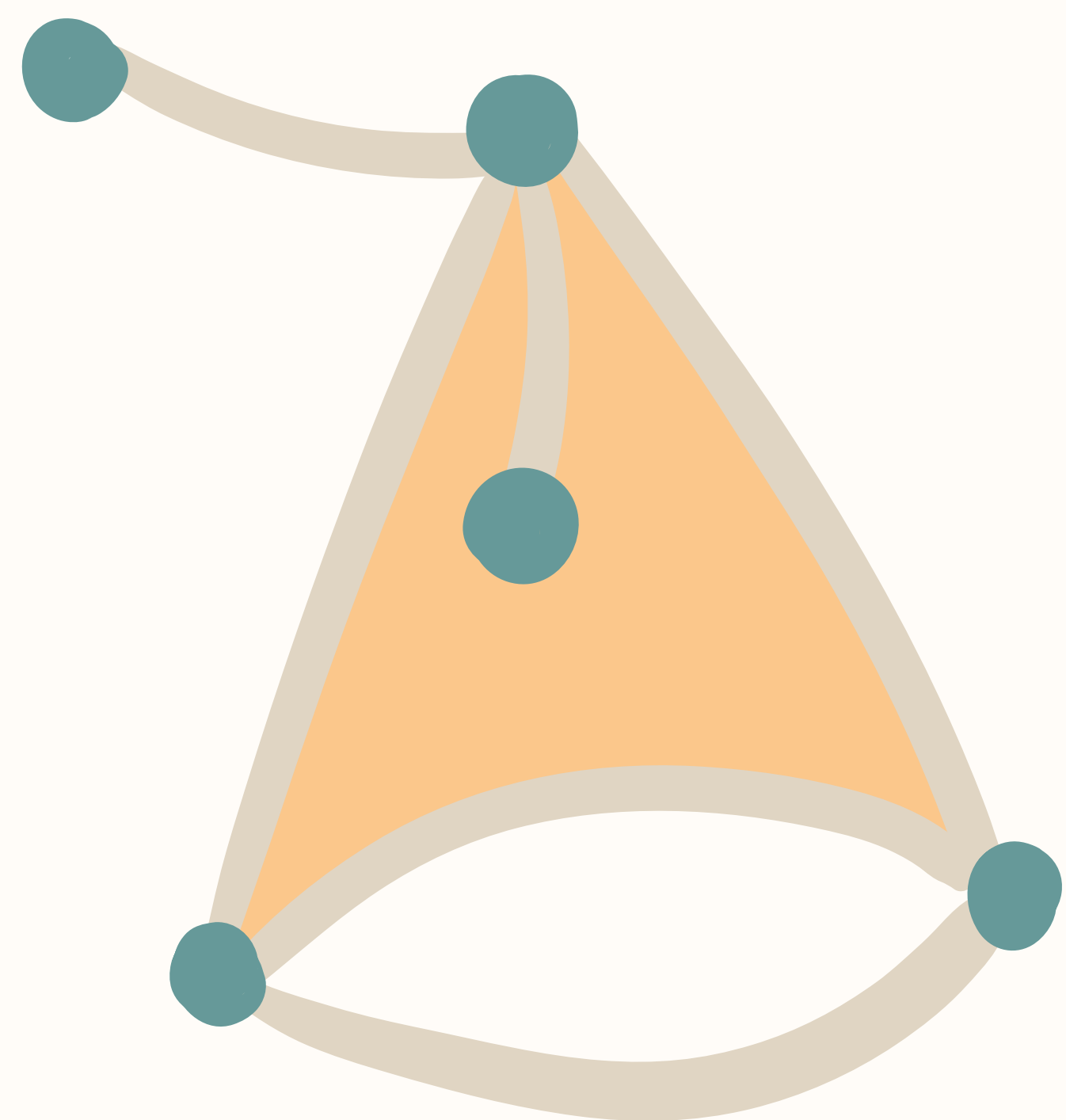
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Graphs



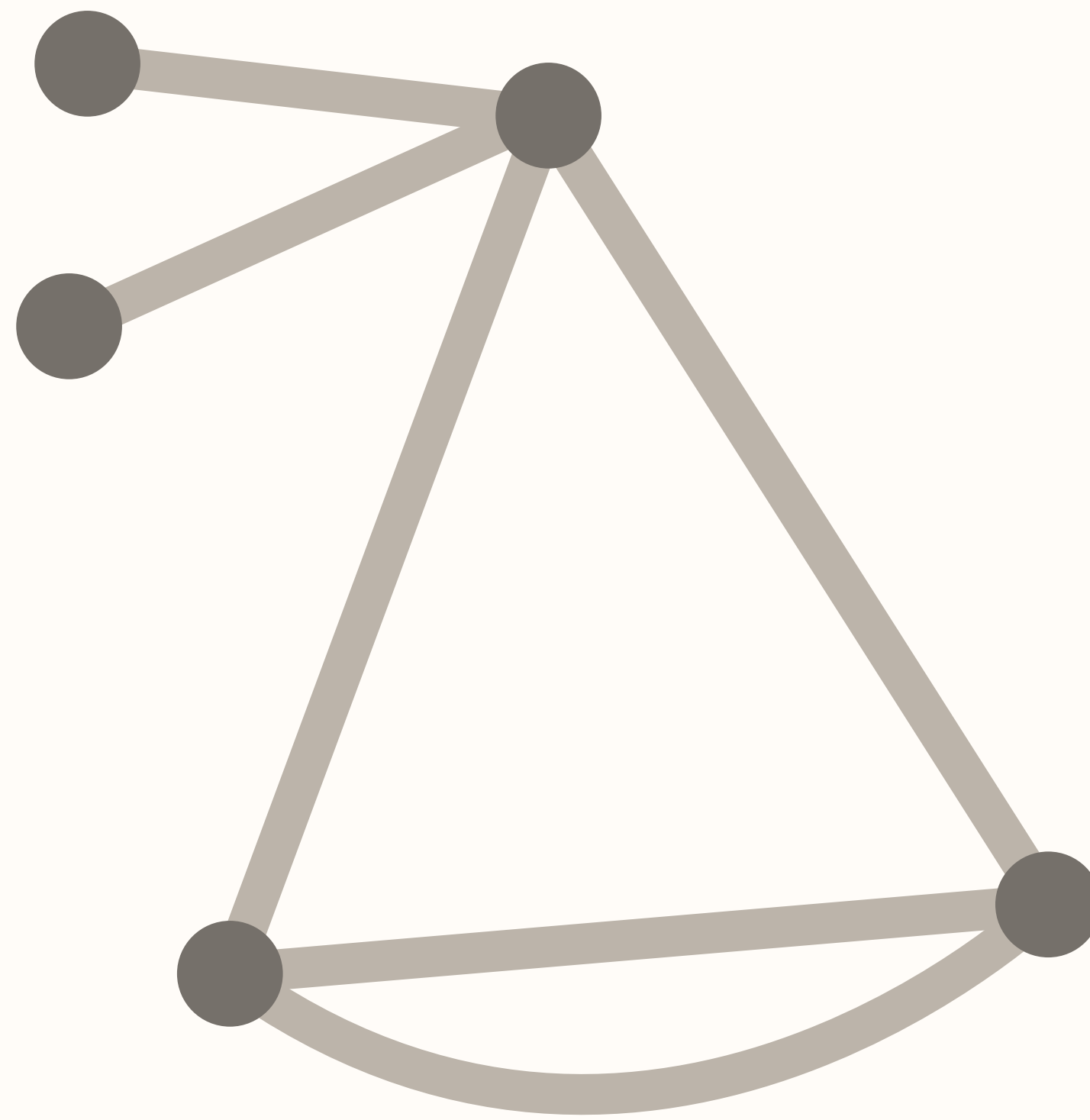
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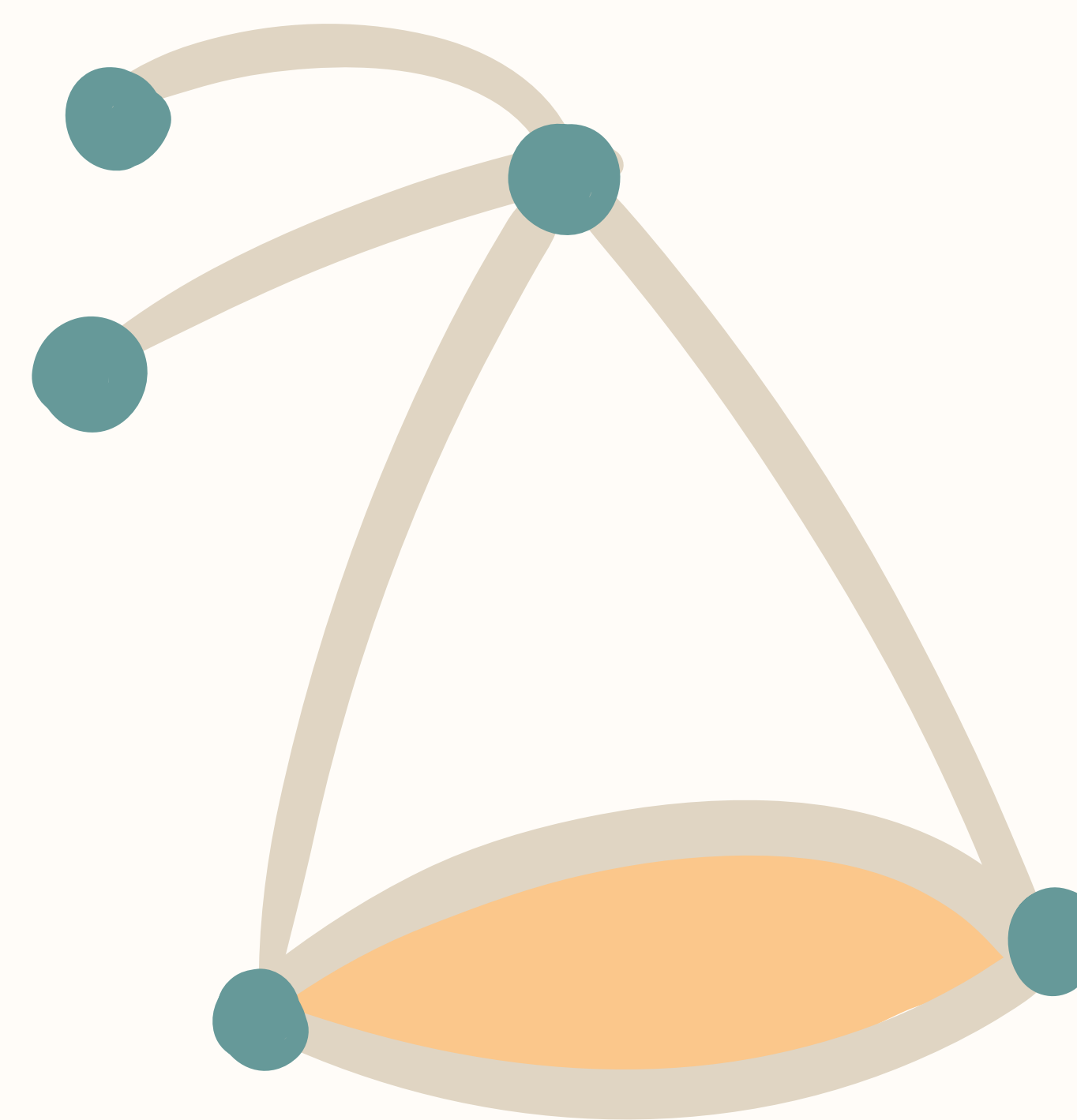
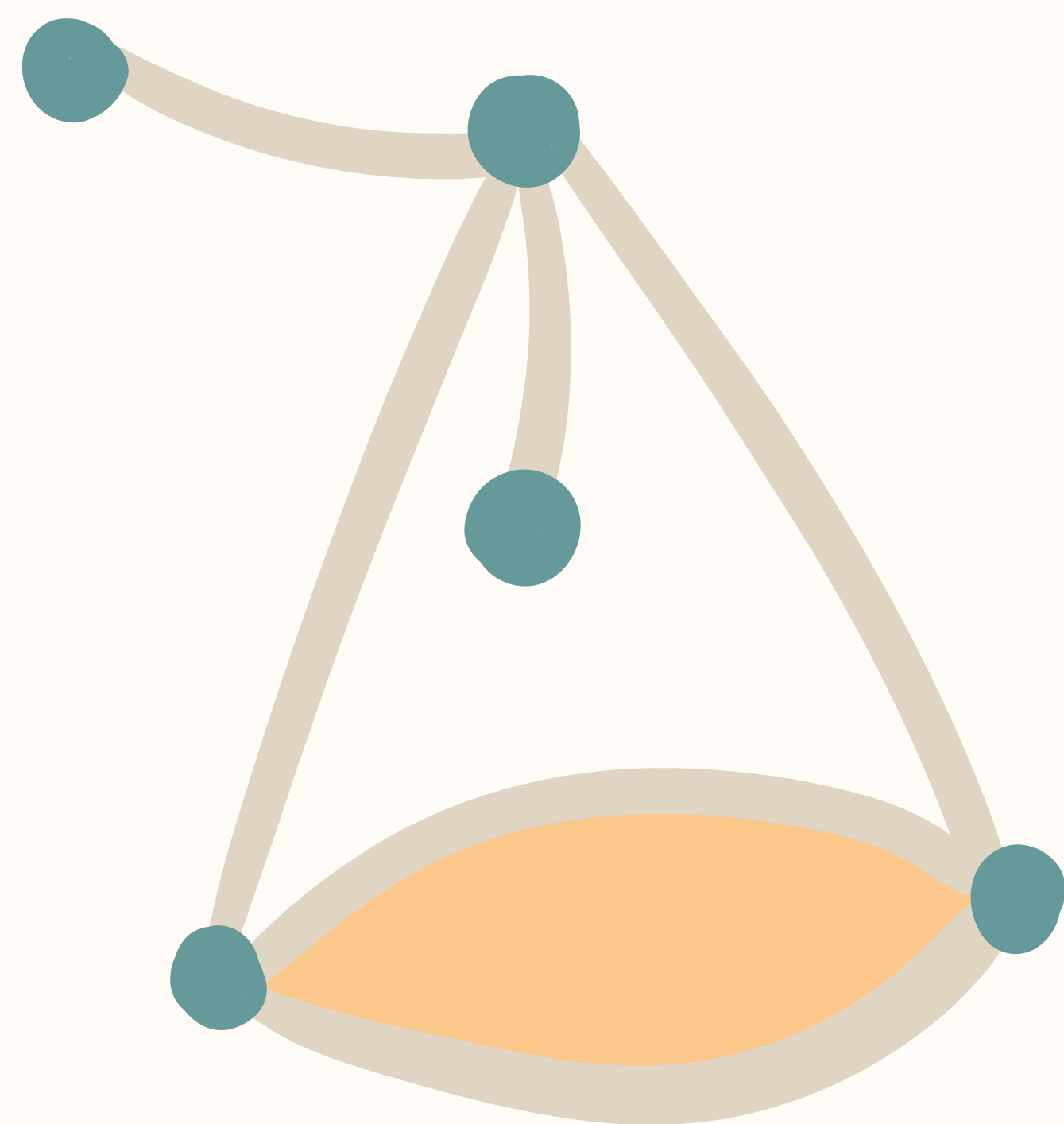
Decorated maps

Graphs



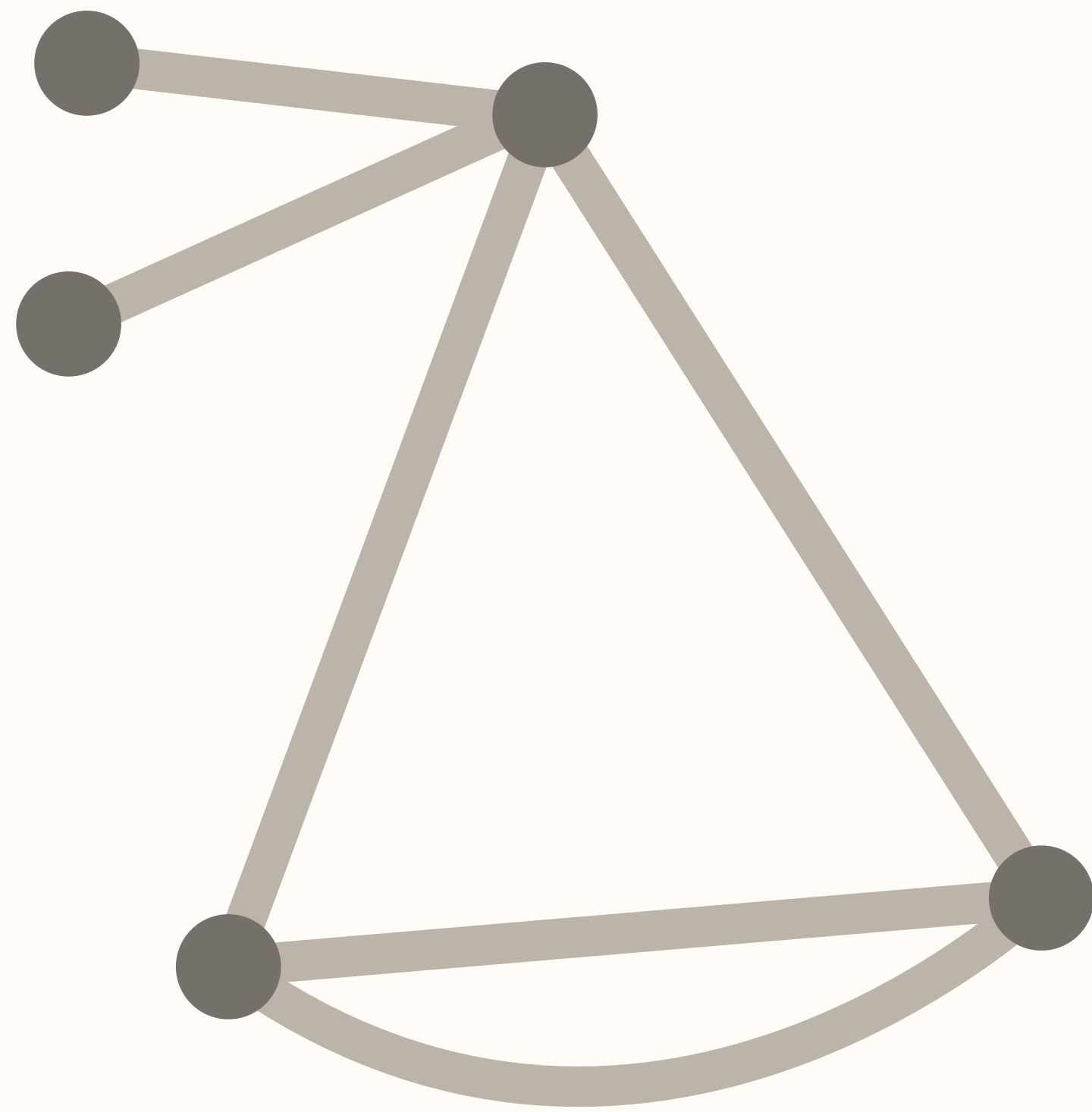
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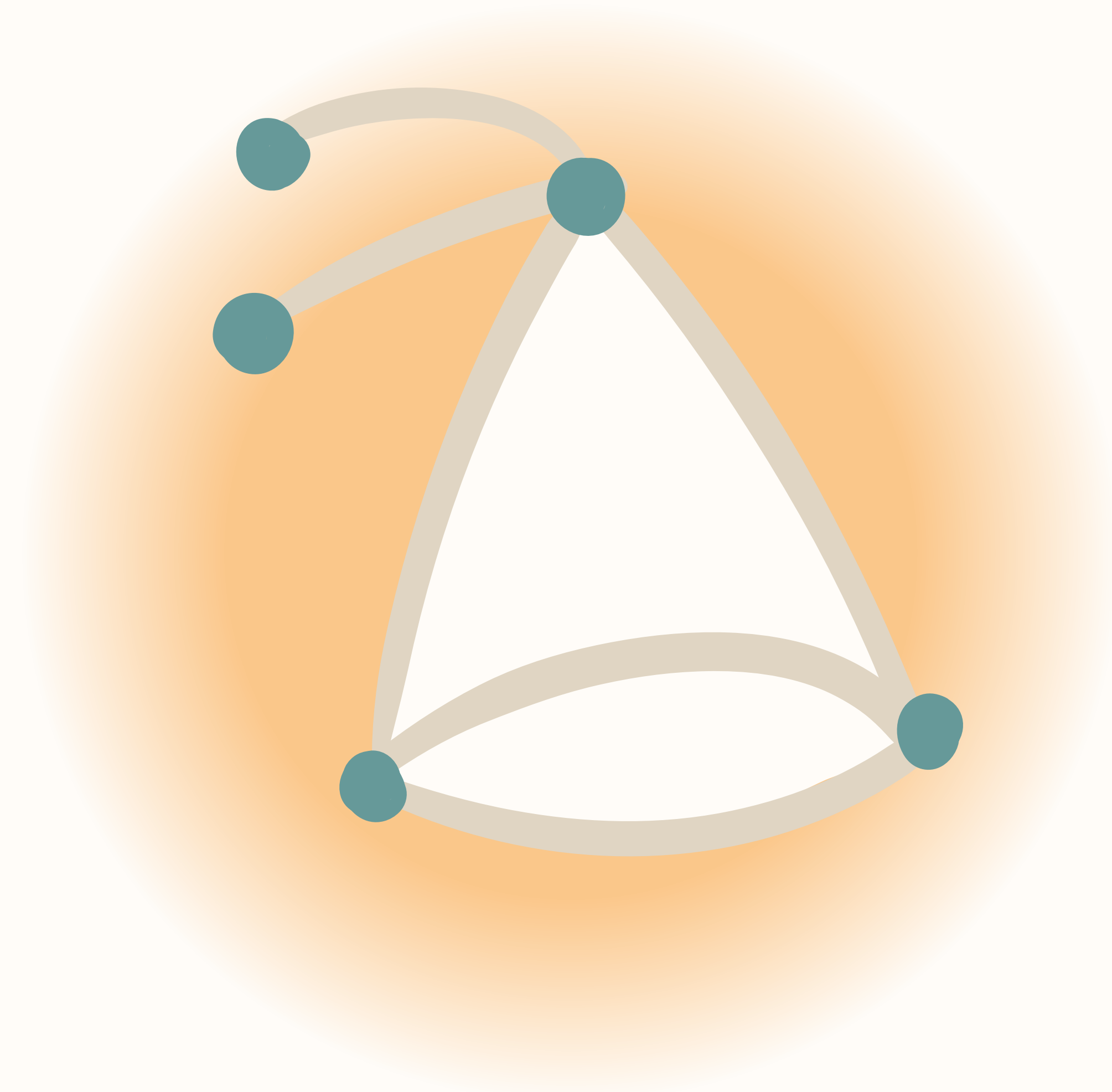
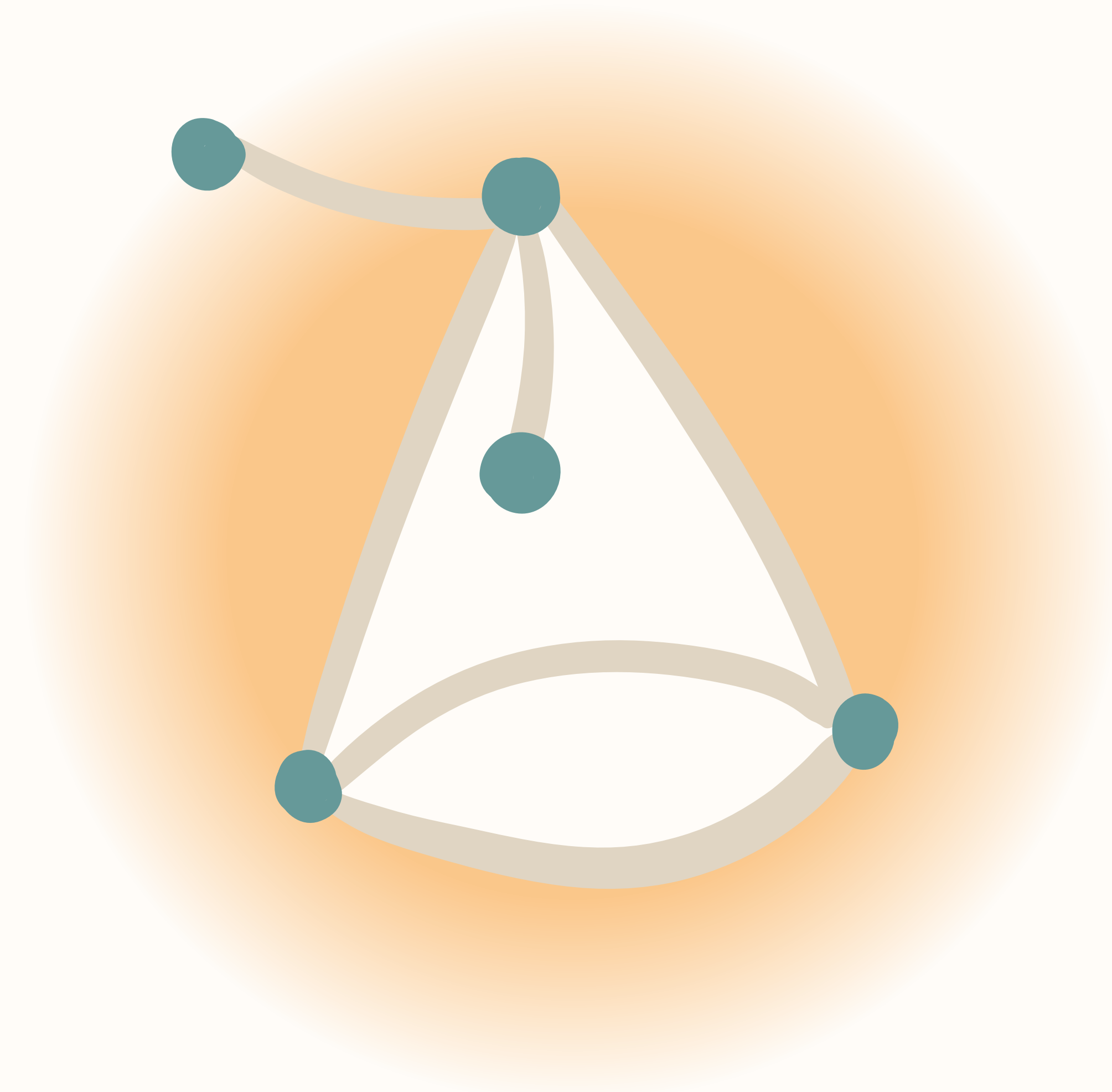
Decorated maps

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Maps

embedding on the plane



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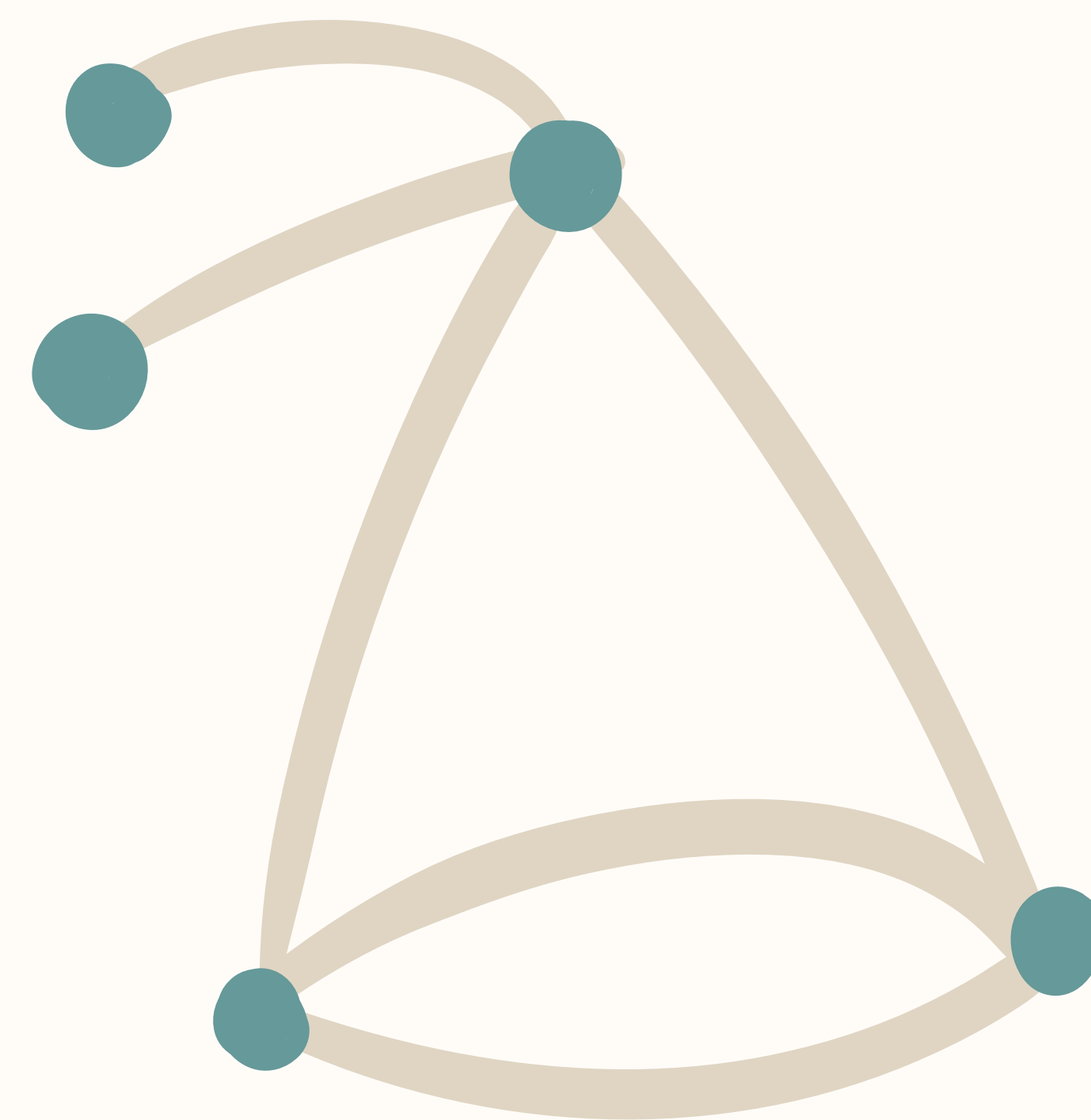
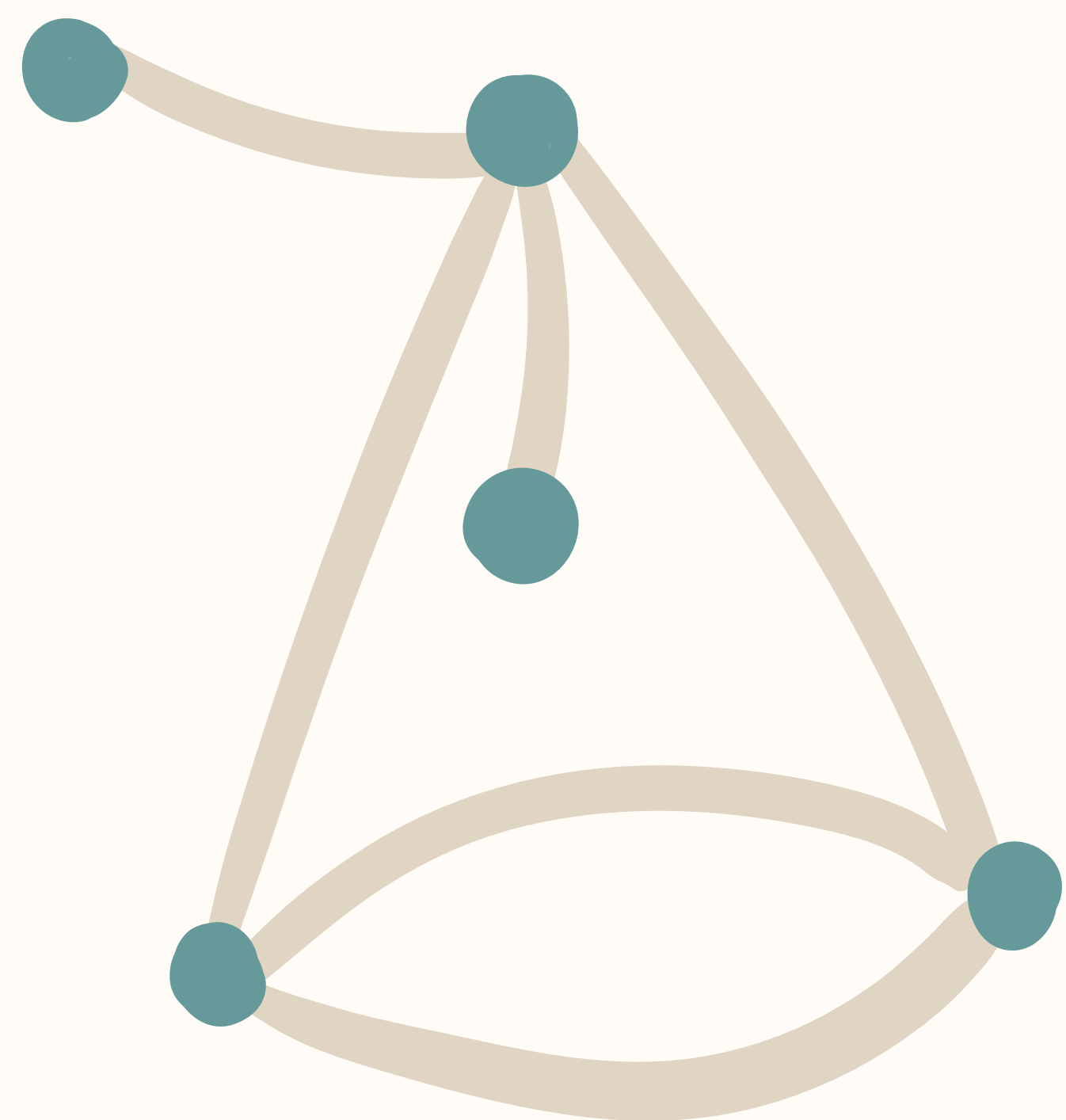
→ *exact enumeration formulas*

→ *universal critical exponent*

maps with n edges : $\kappa \cdot \gamma^n n^{-5/2}$

Maps

embedding on the plane



Decorated maps

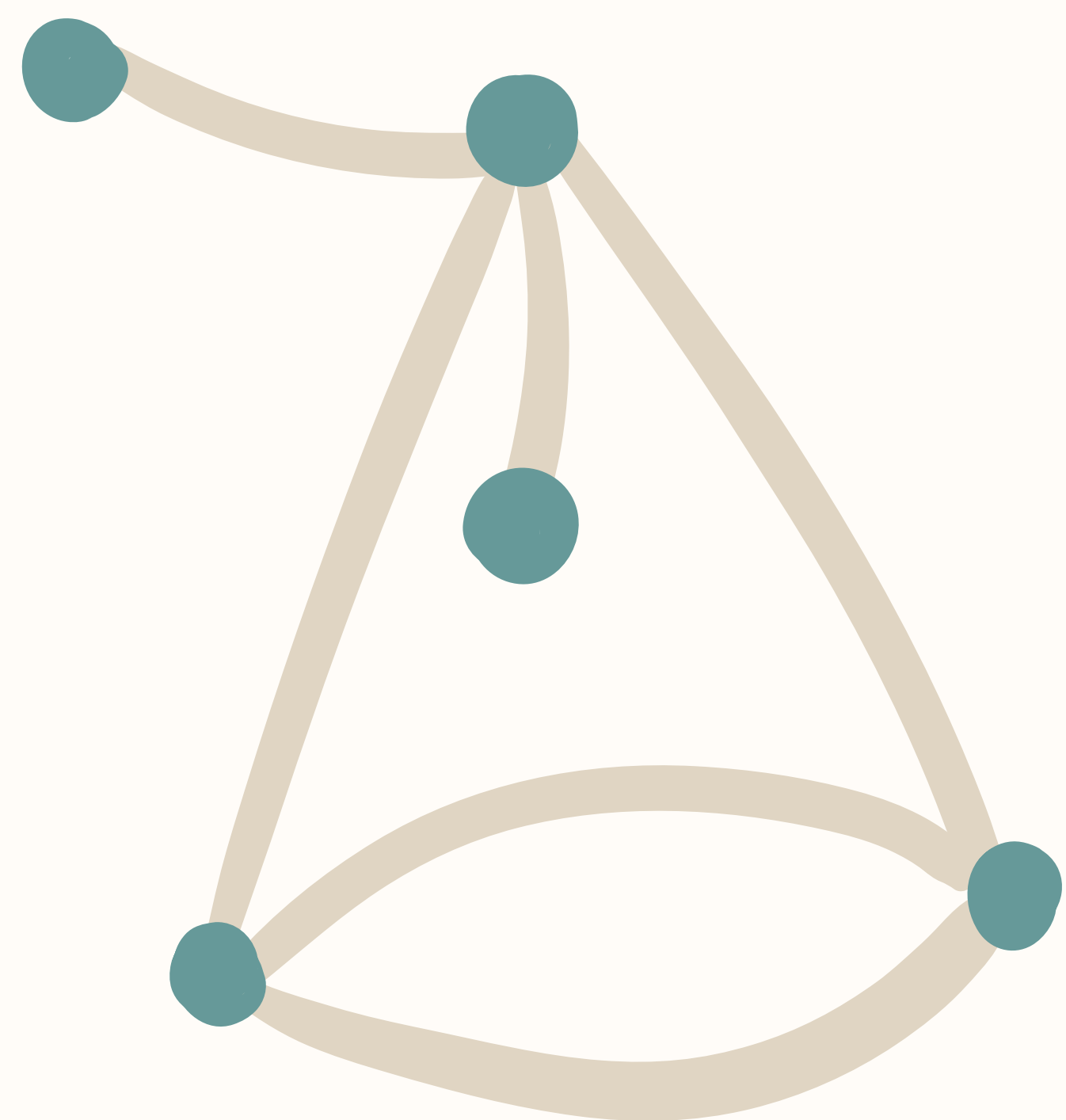
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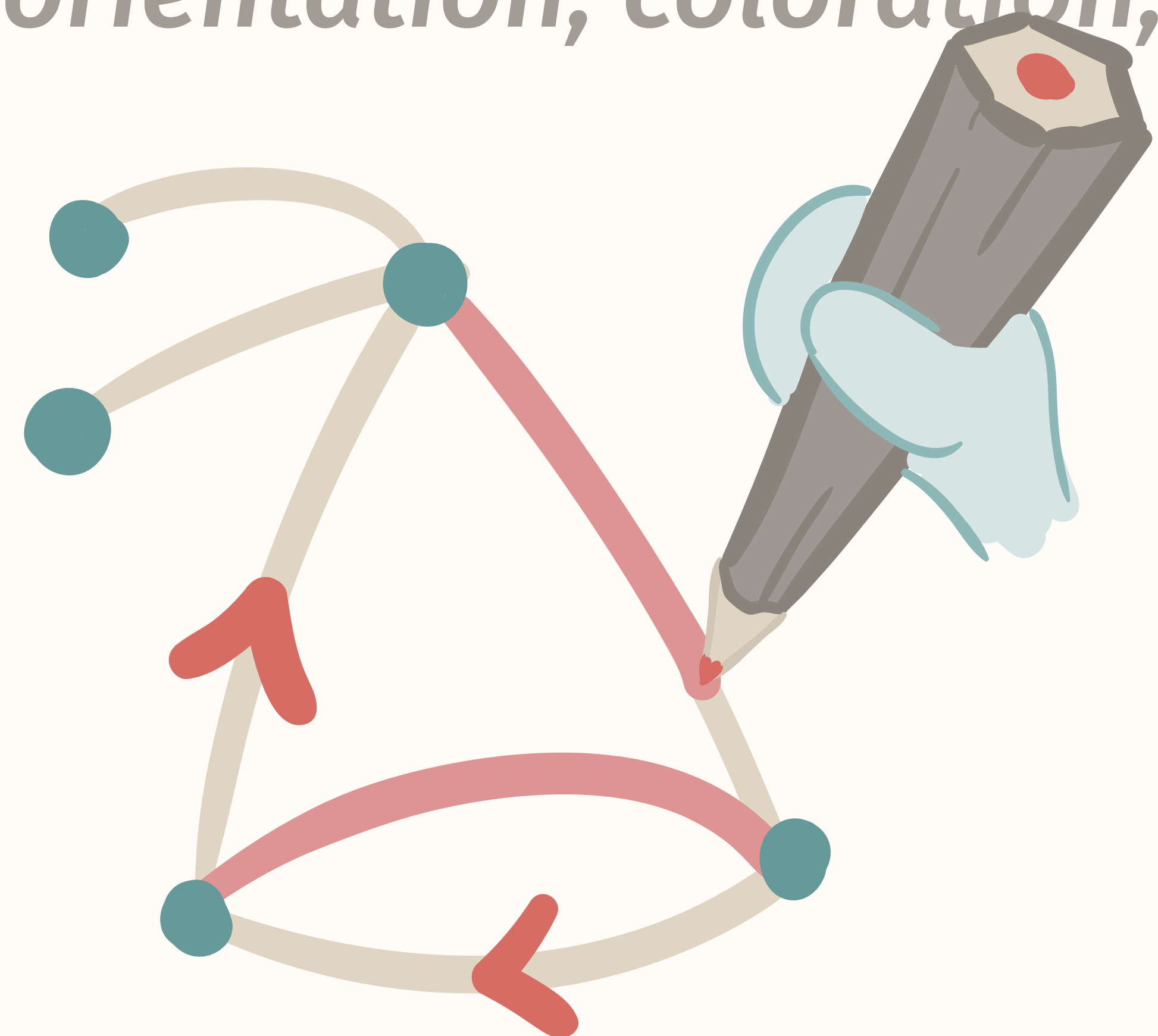
Maps

embedding on the plane



Decorated maps

orientation, coloration, etc.



Decorated maps

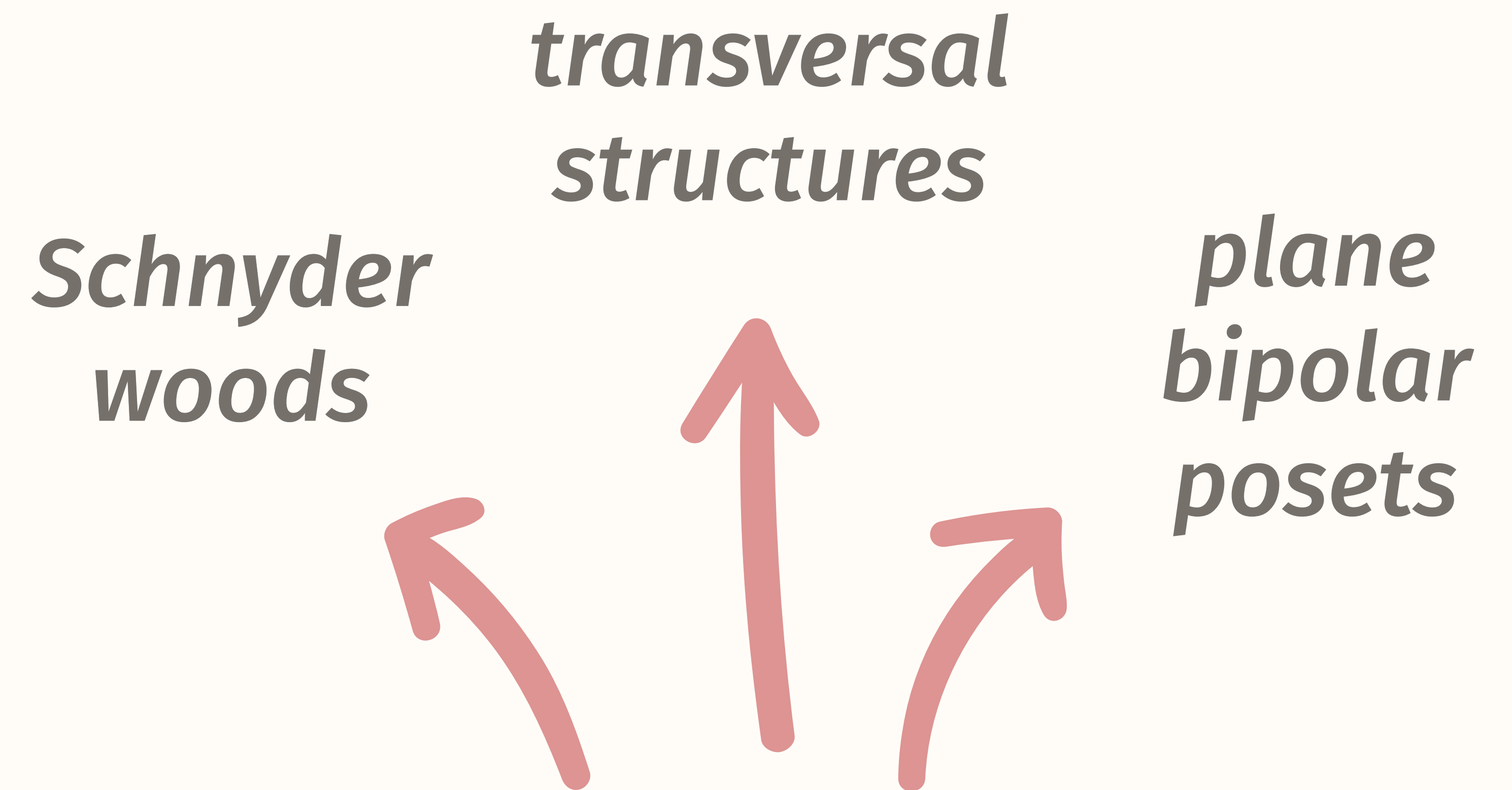
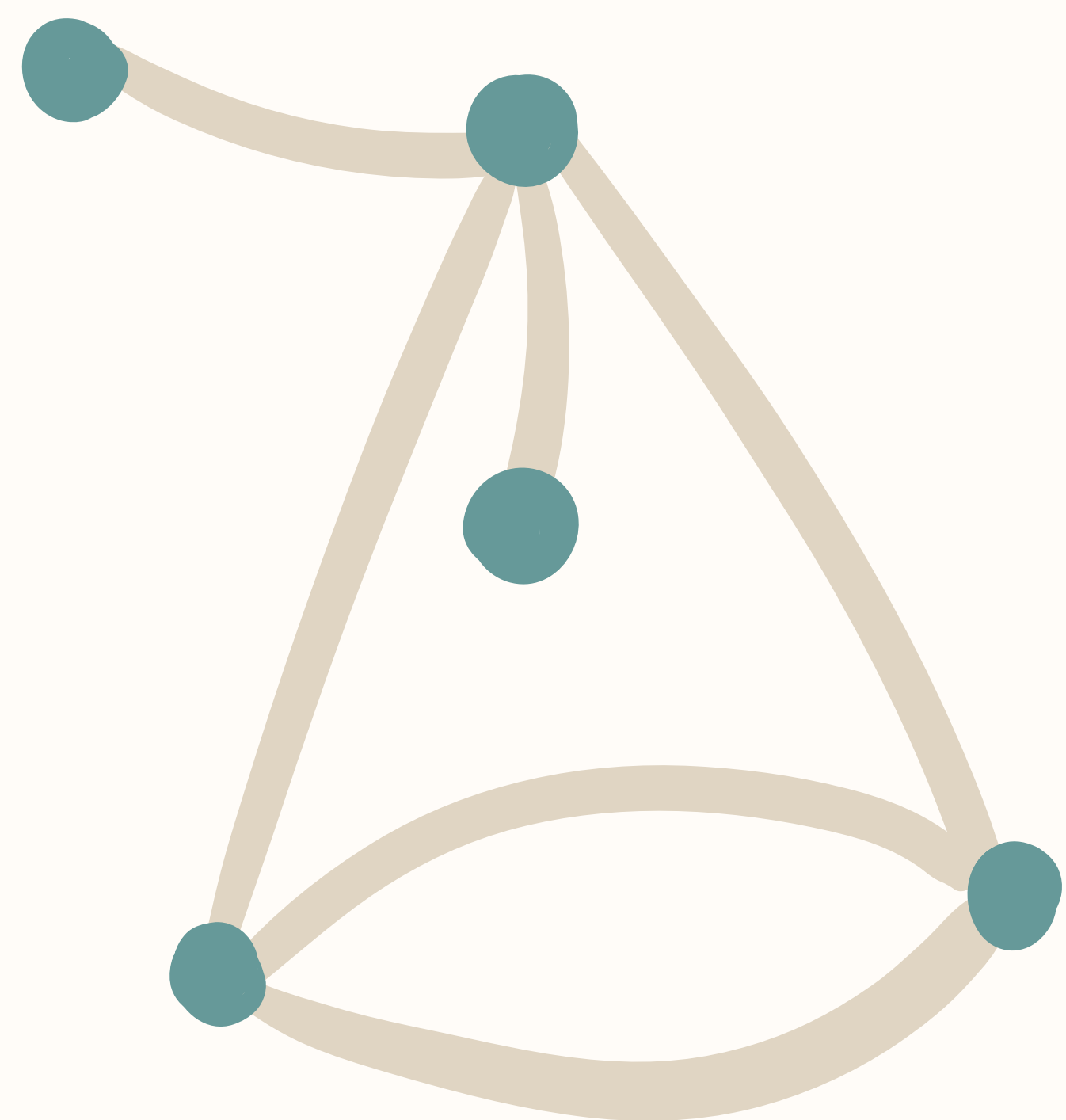
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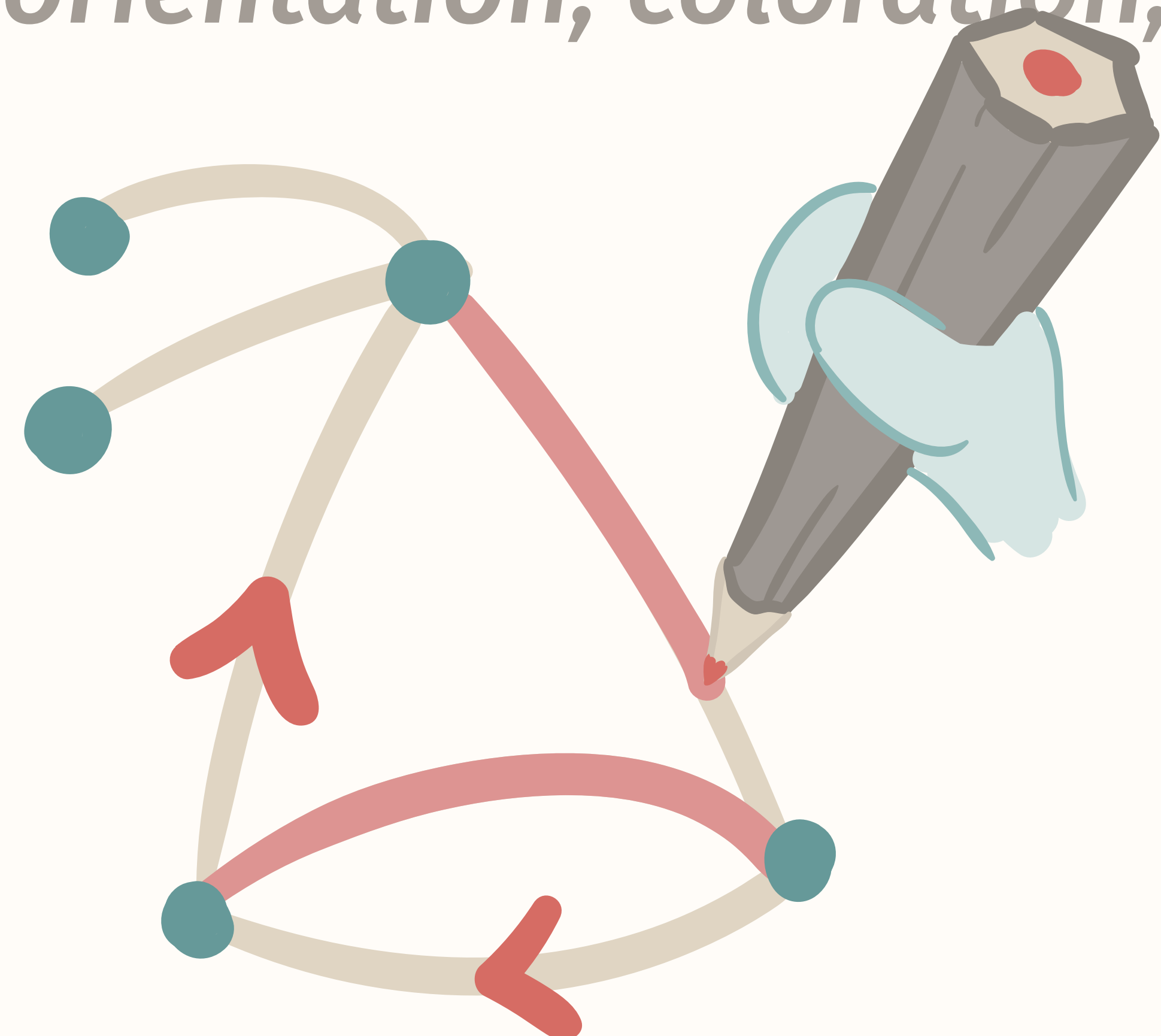
Maps

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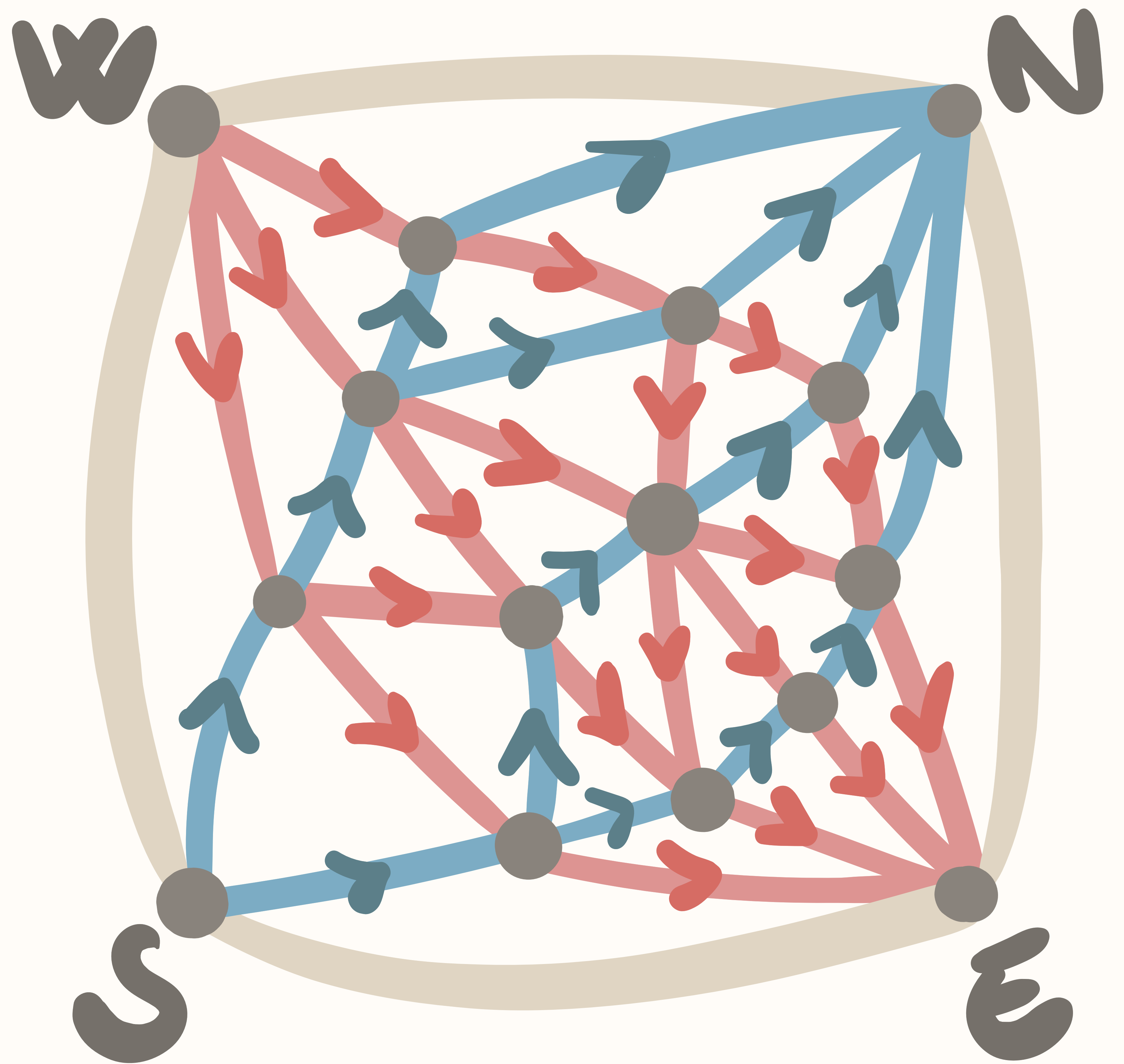


Decorated maps

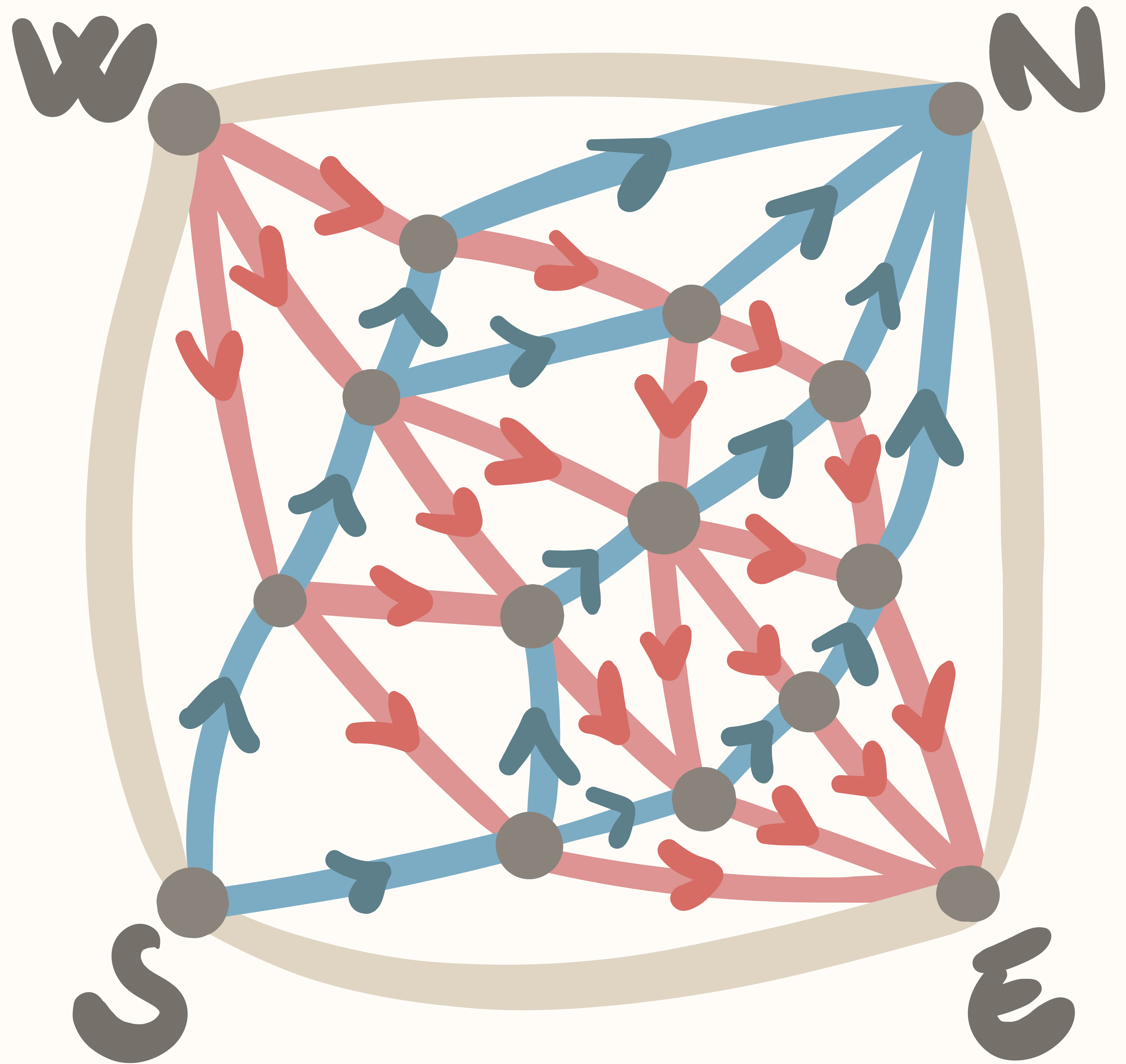
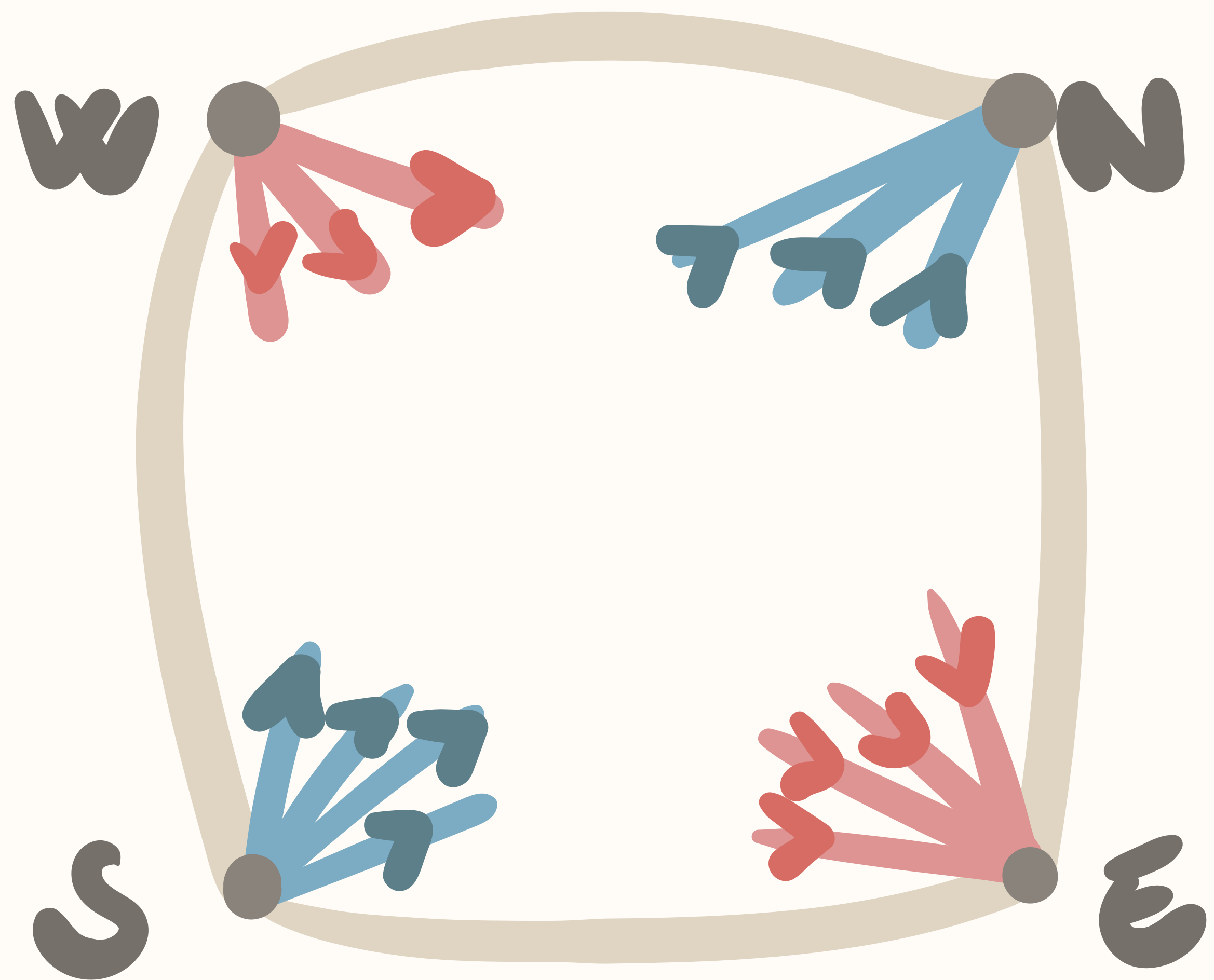
orientation, coloration, etc.



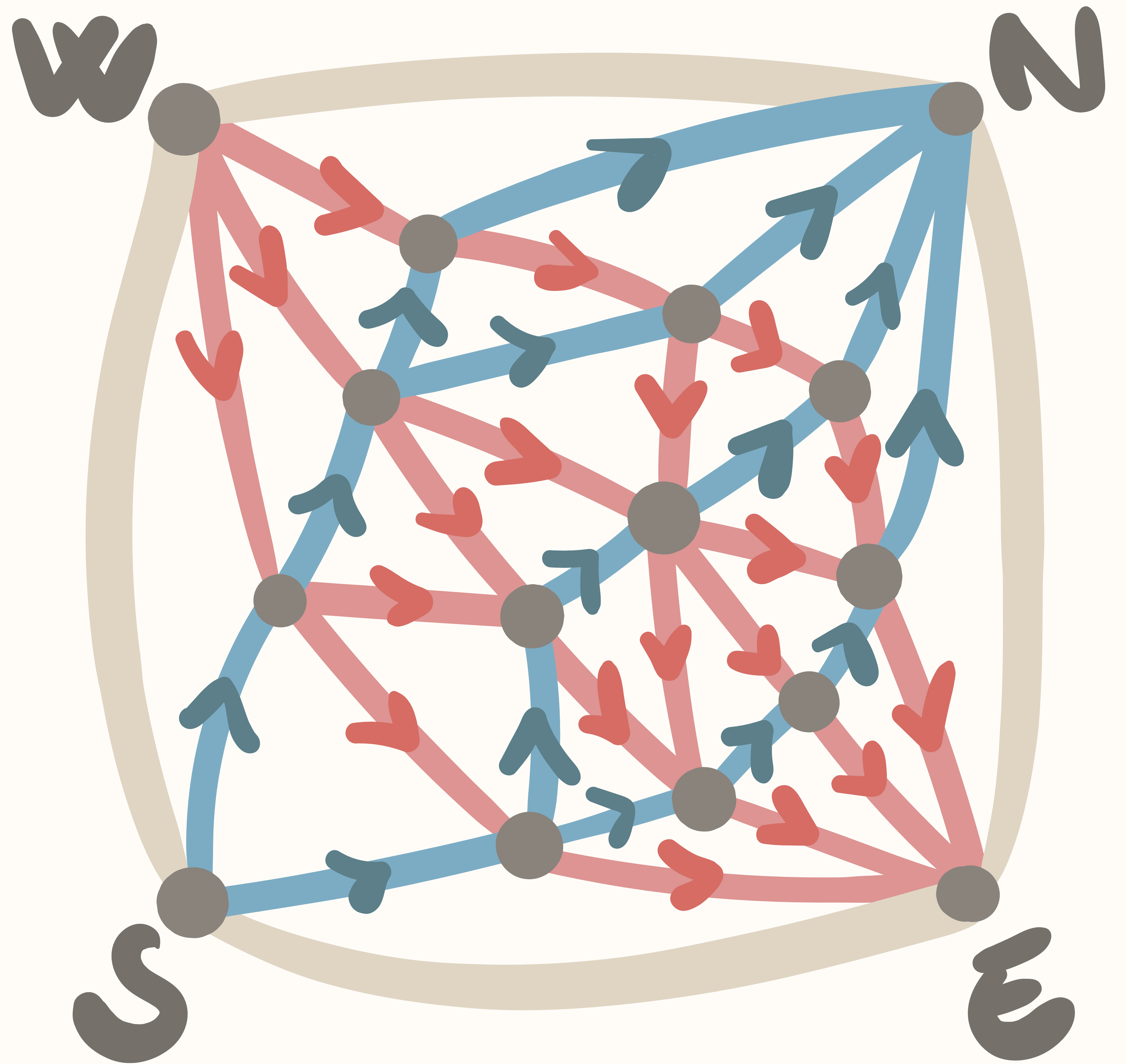
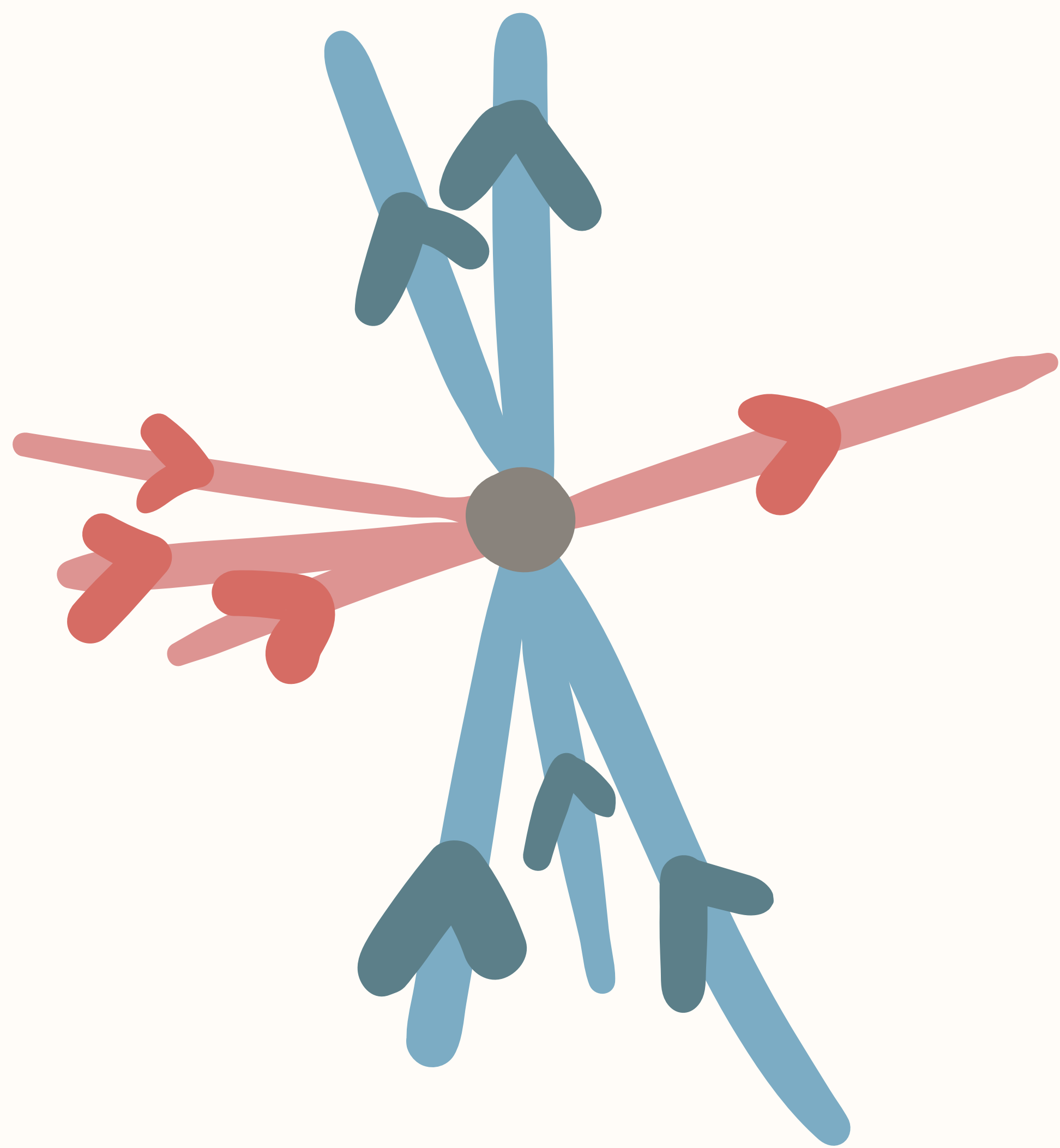
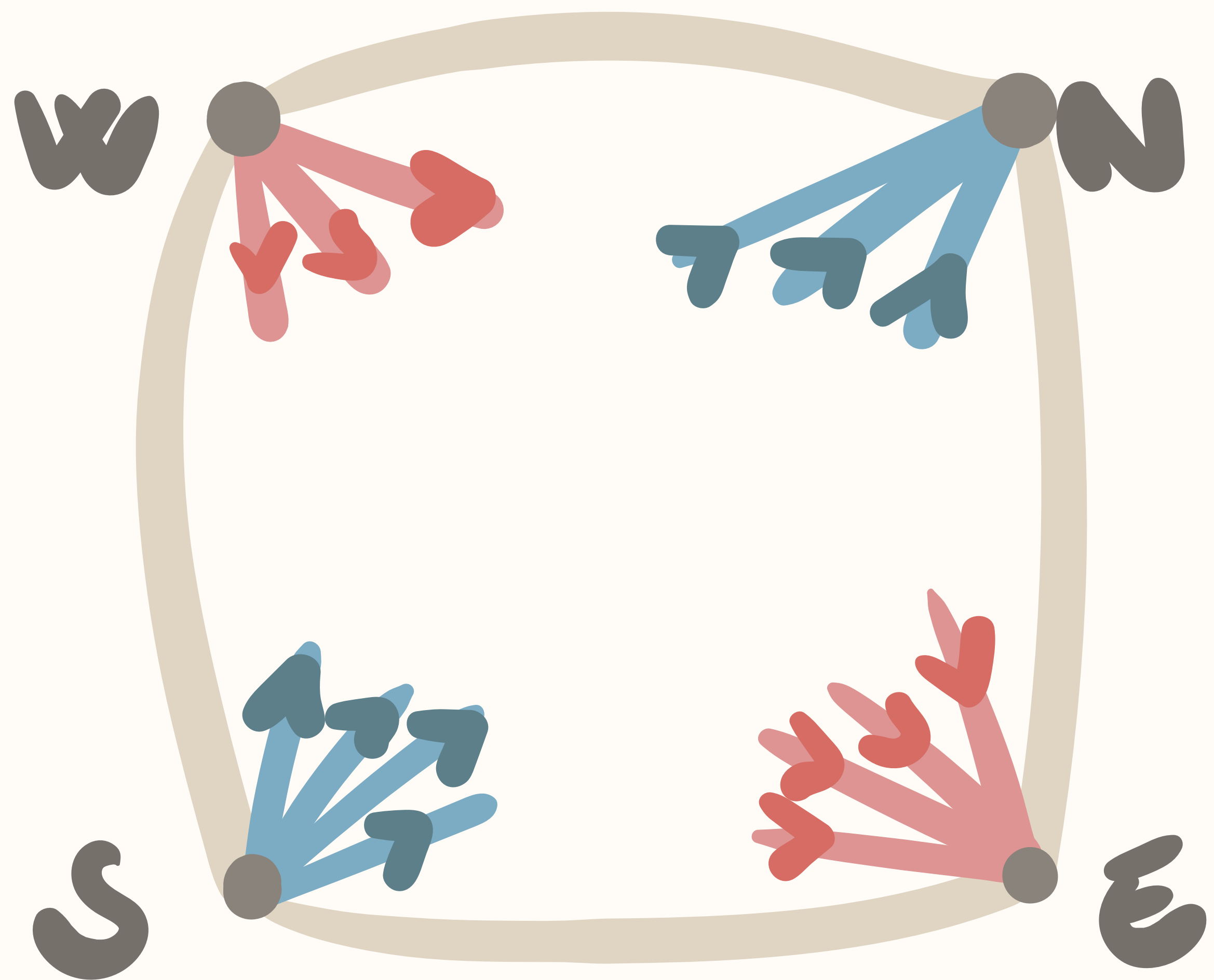
Transversal structures



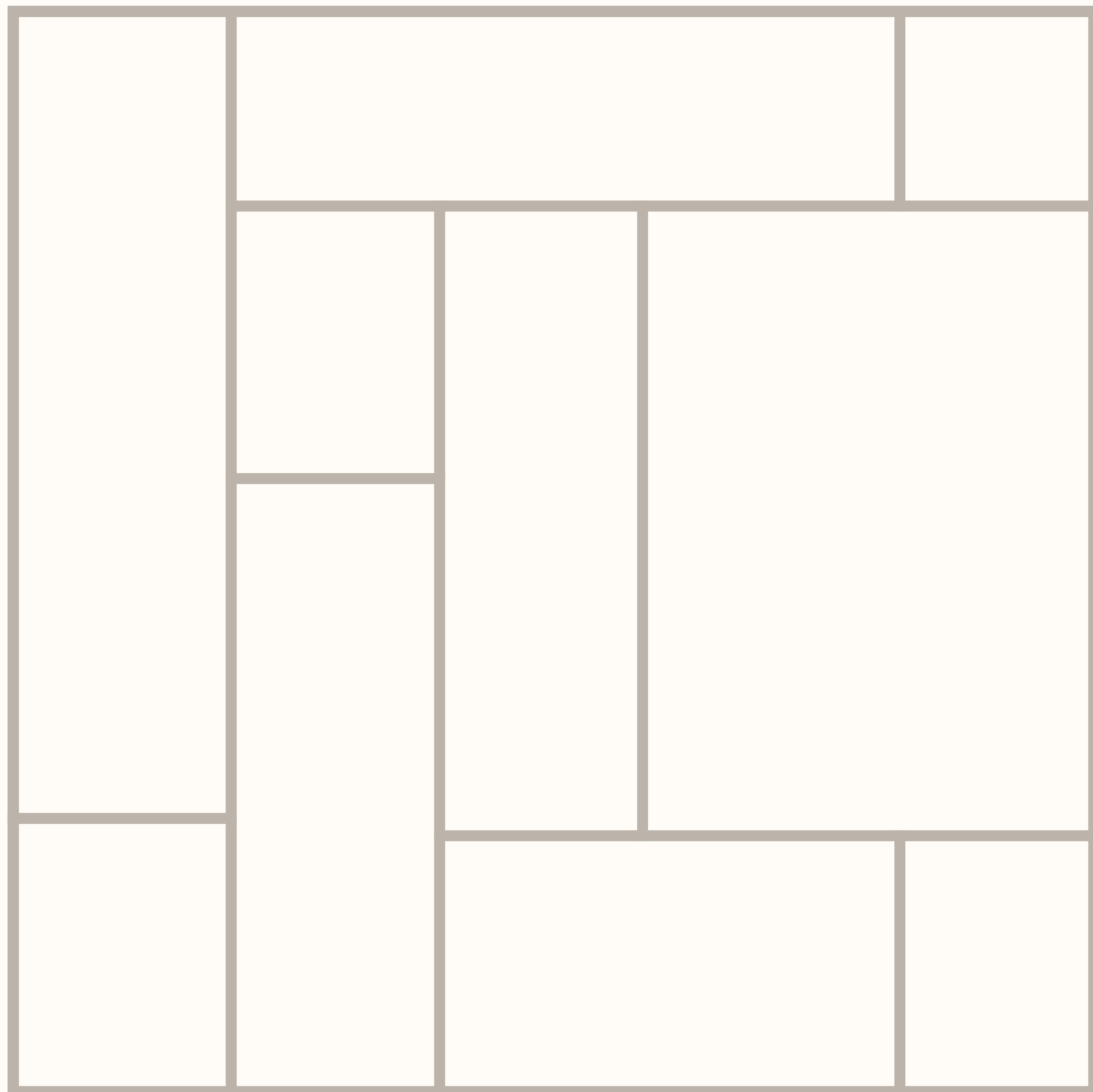
Transversal structures



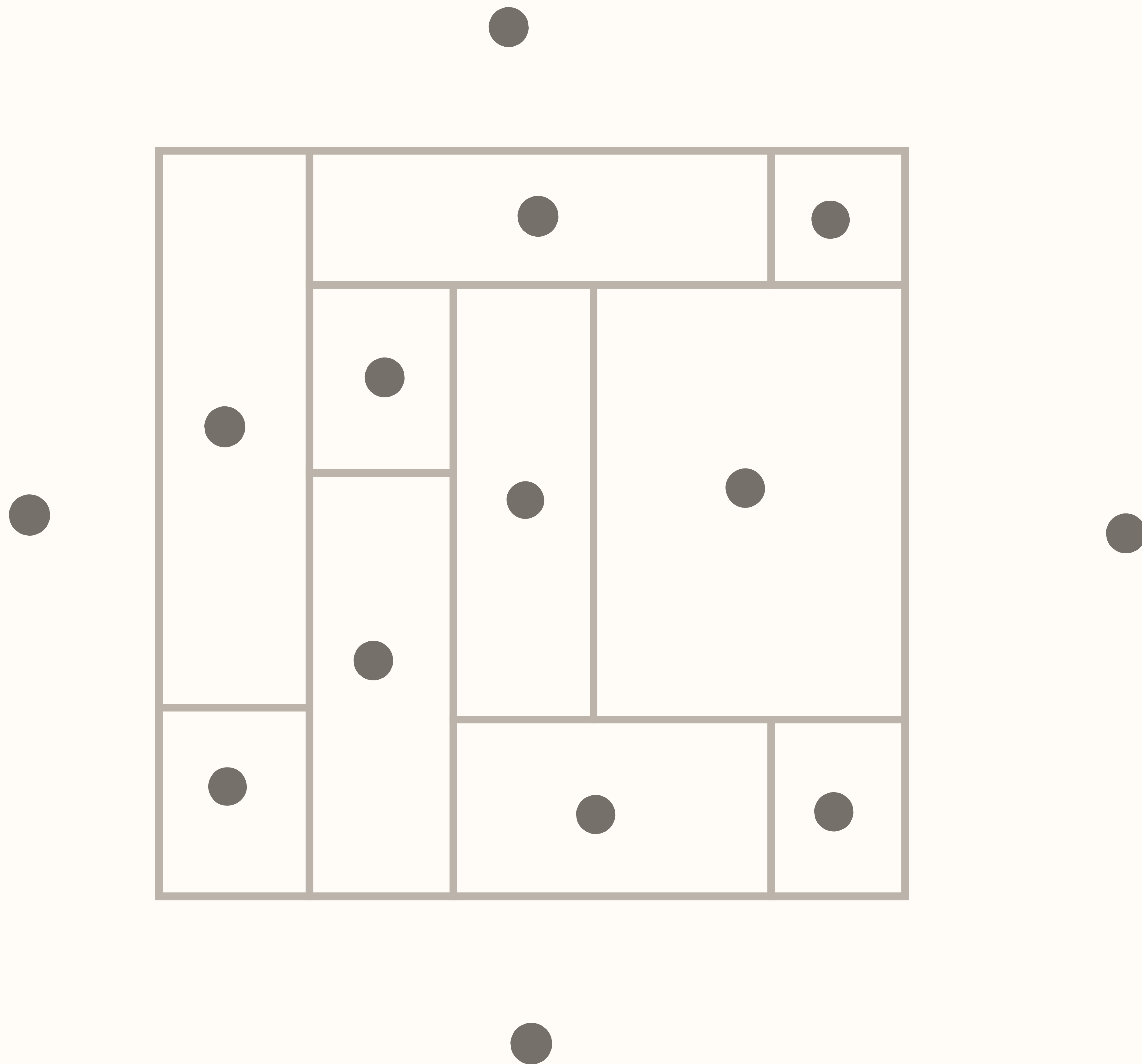
Transversal structures



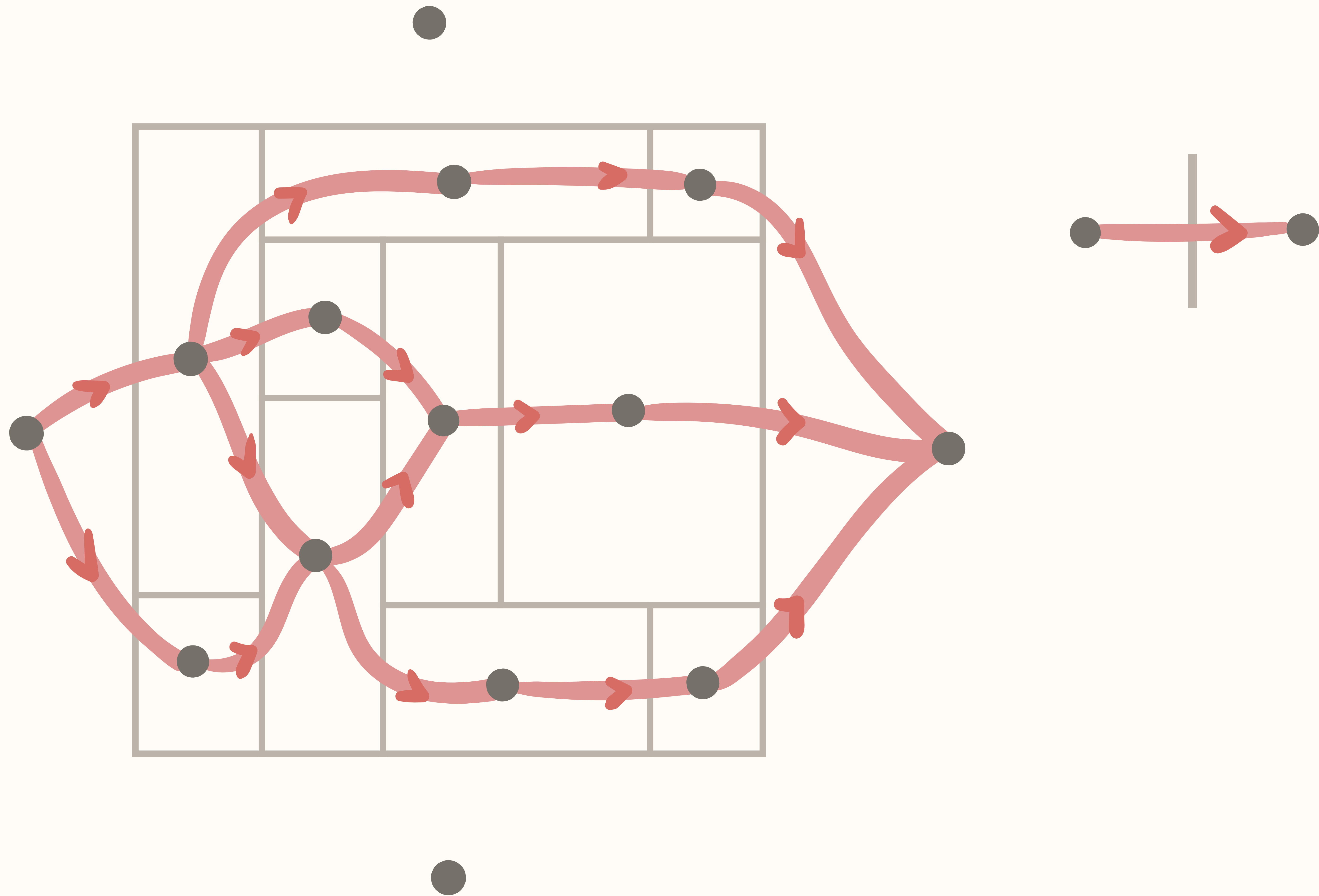
Link with rectangular tilings



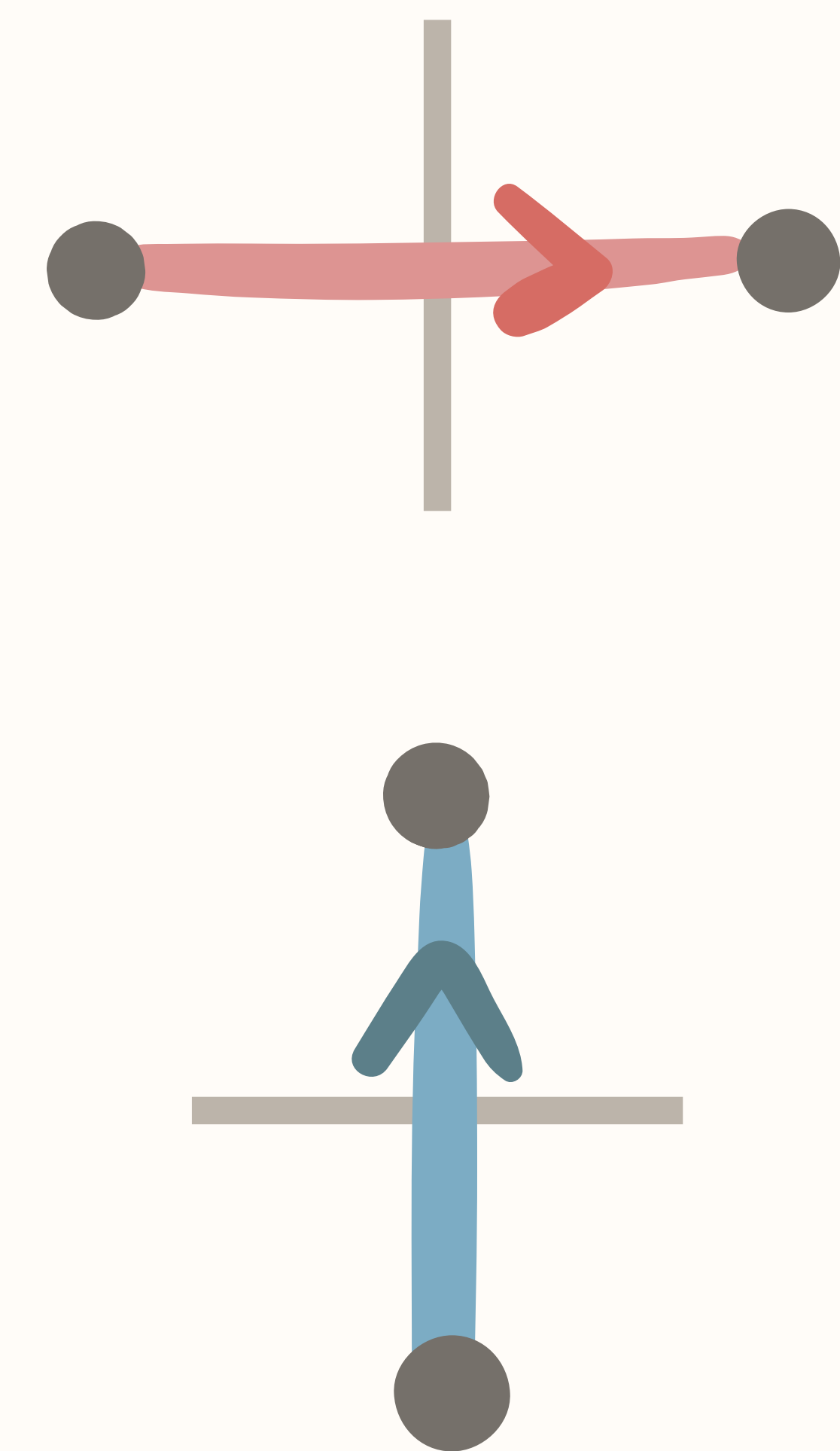
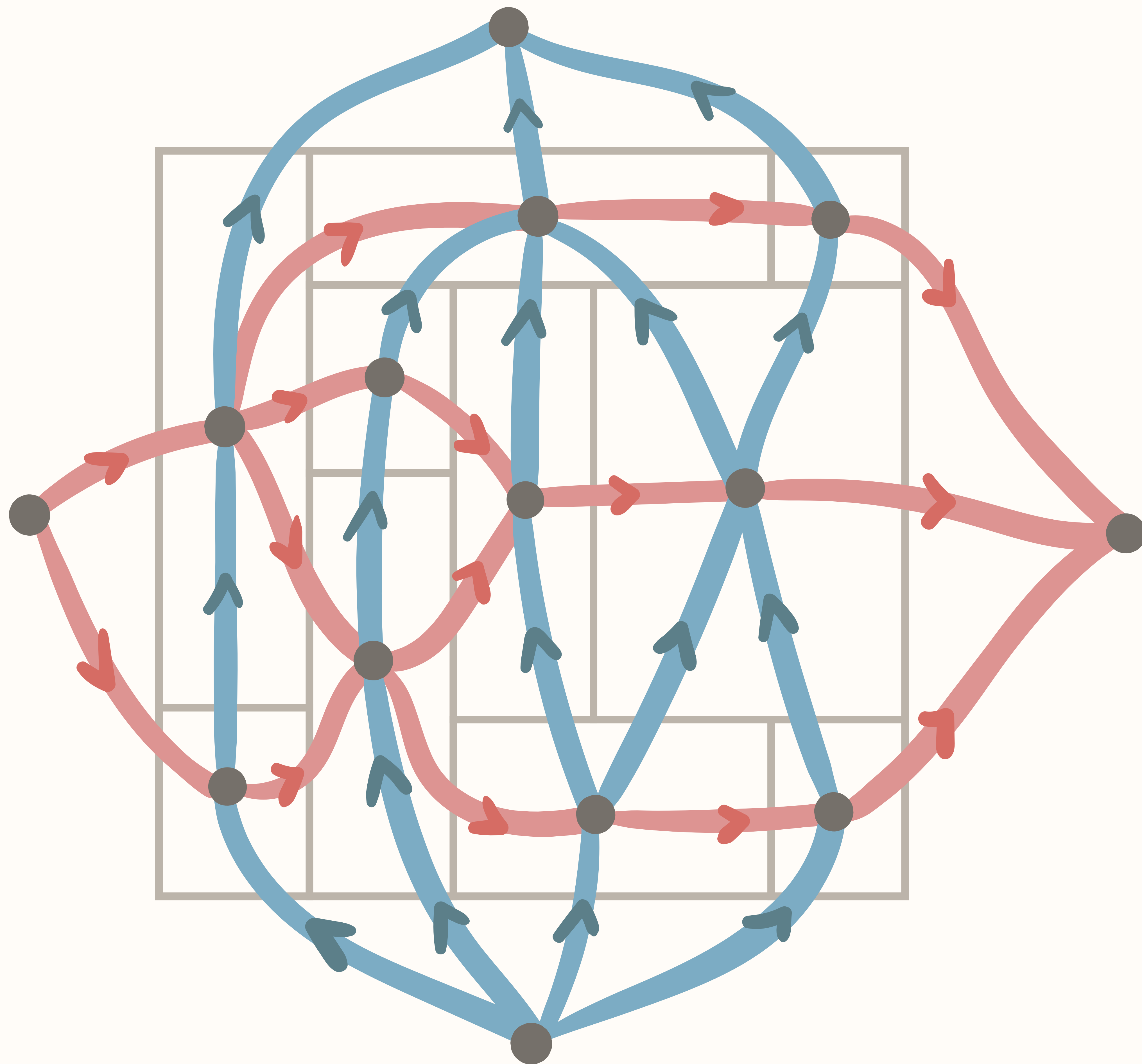
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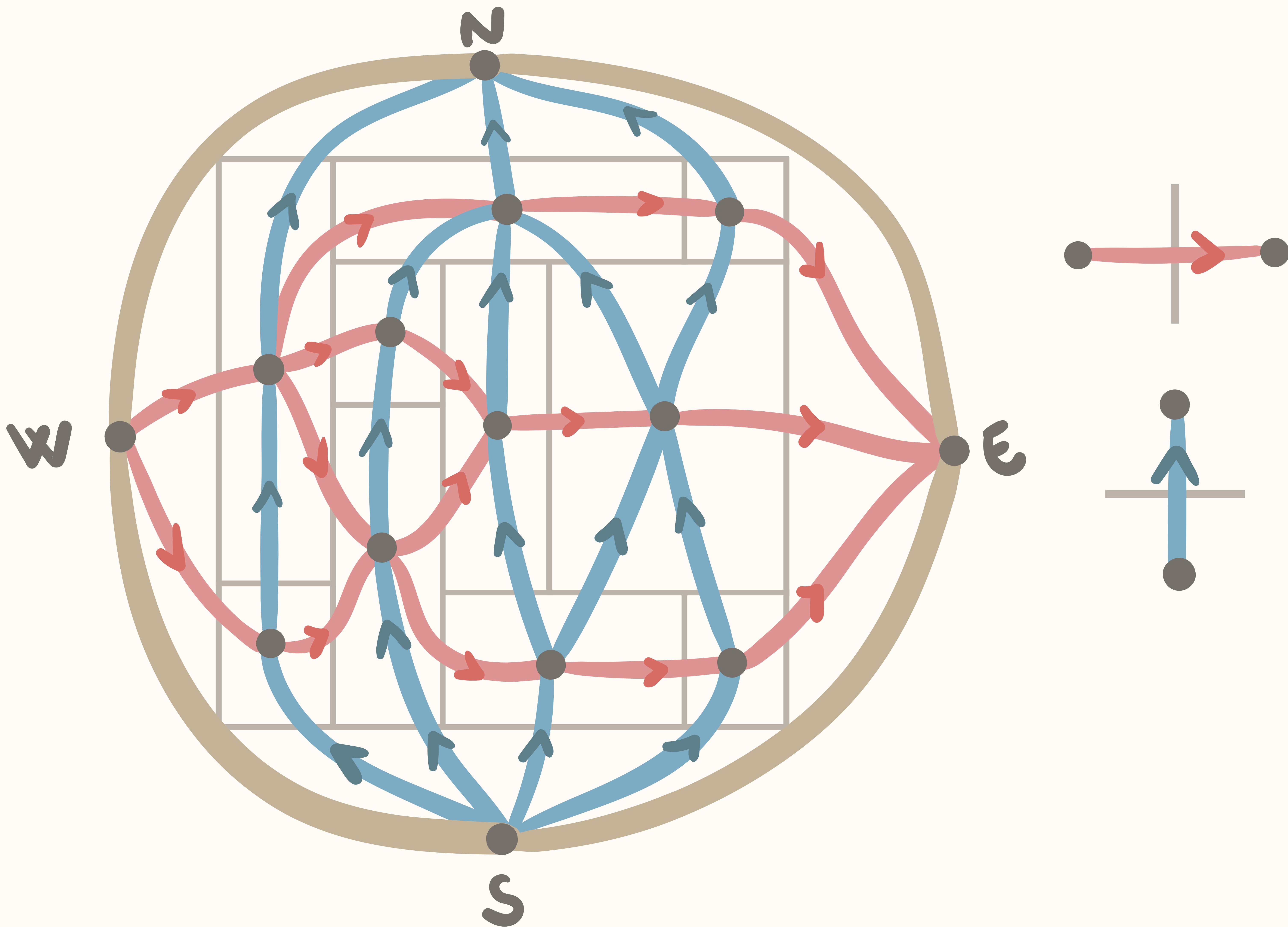
Link with rectangular tilings



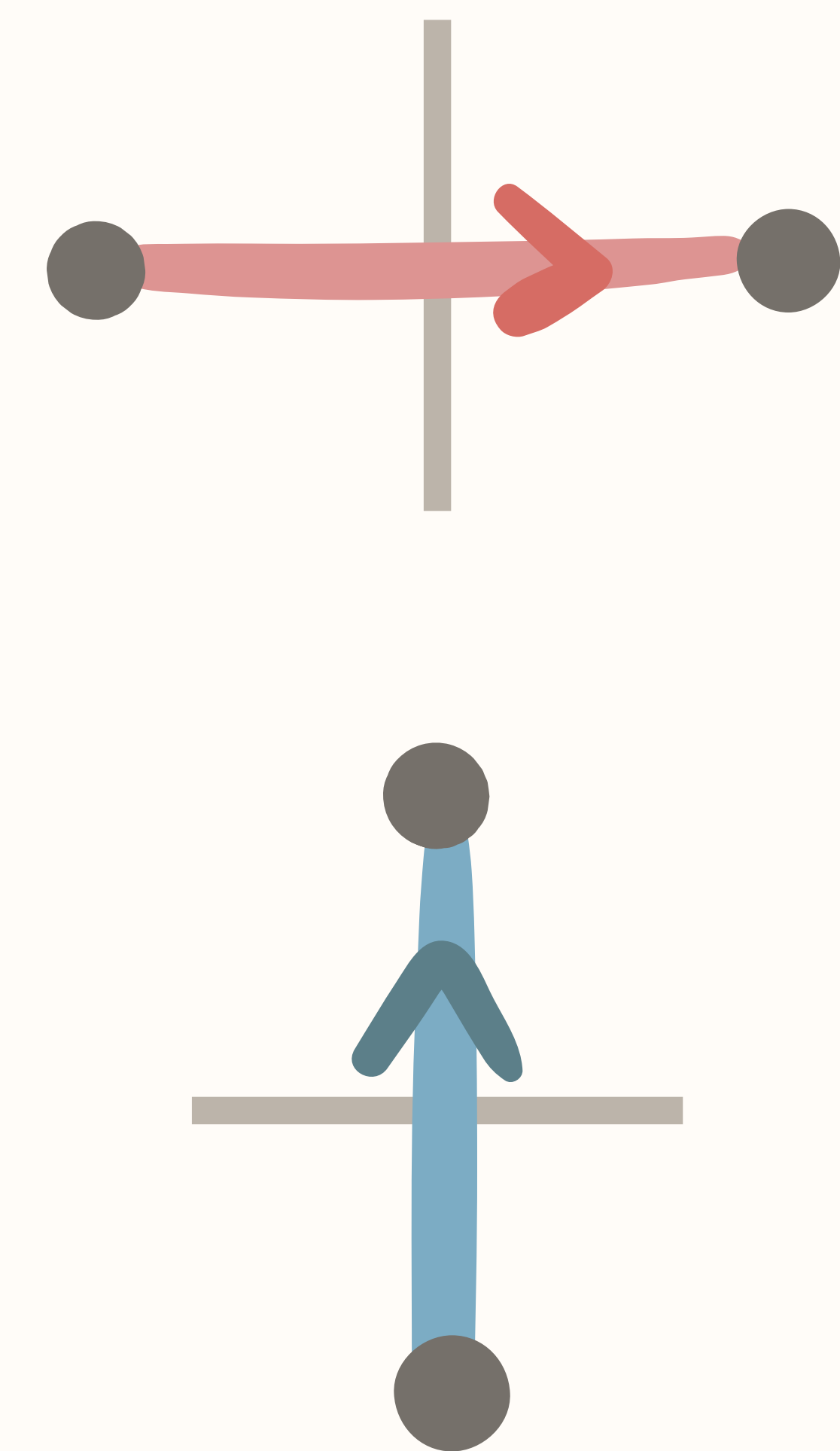
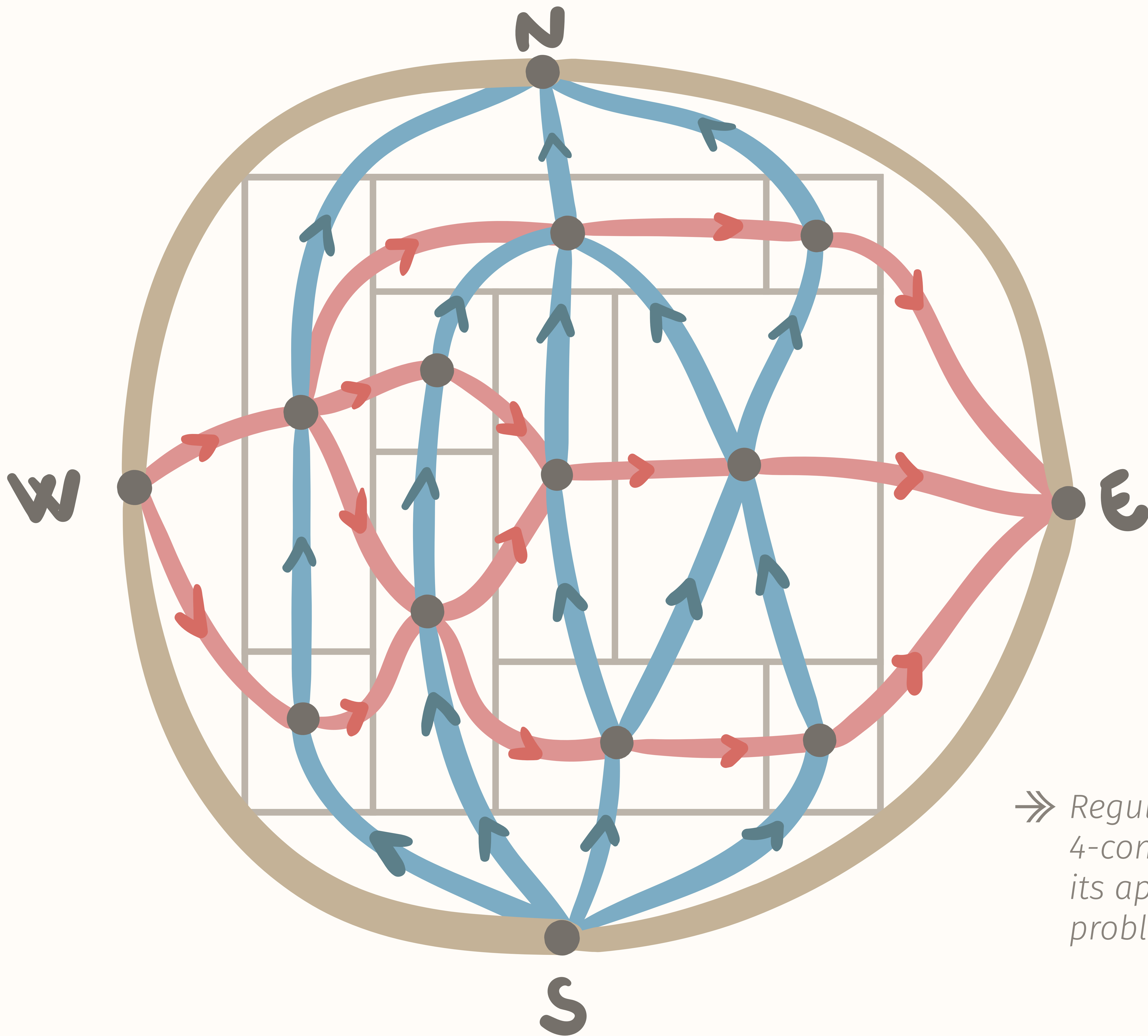
Link with rectangular tilings



Link with rectangular tilings



Link with rectangular tilings



⇒ Regular edge labellings of 4-connected plane graphs and its applications in graph drawing problems, G. Kant & X. He (1997)

Sommaire

The KMSW Bijection

1. Application to three maps model

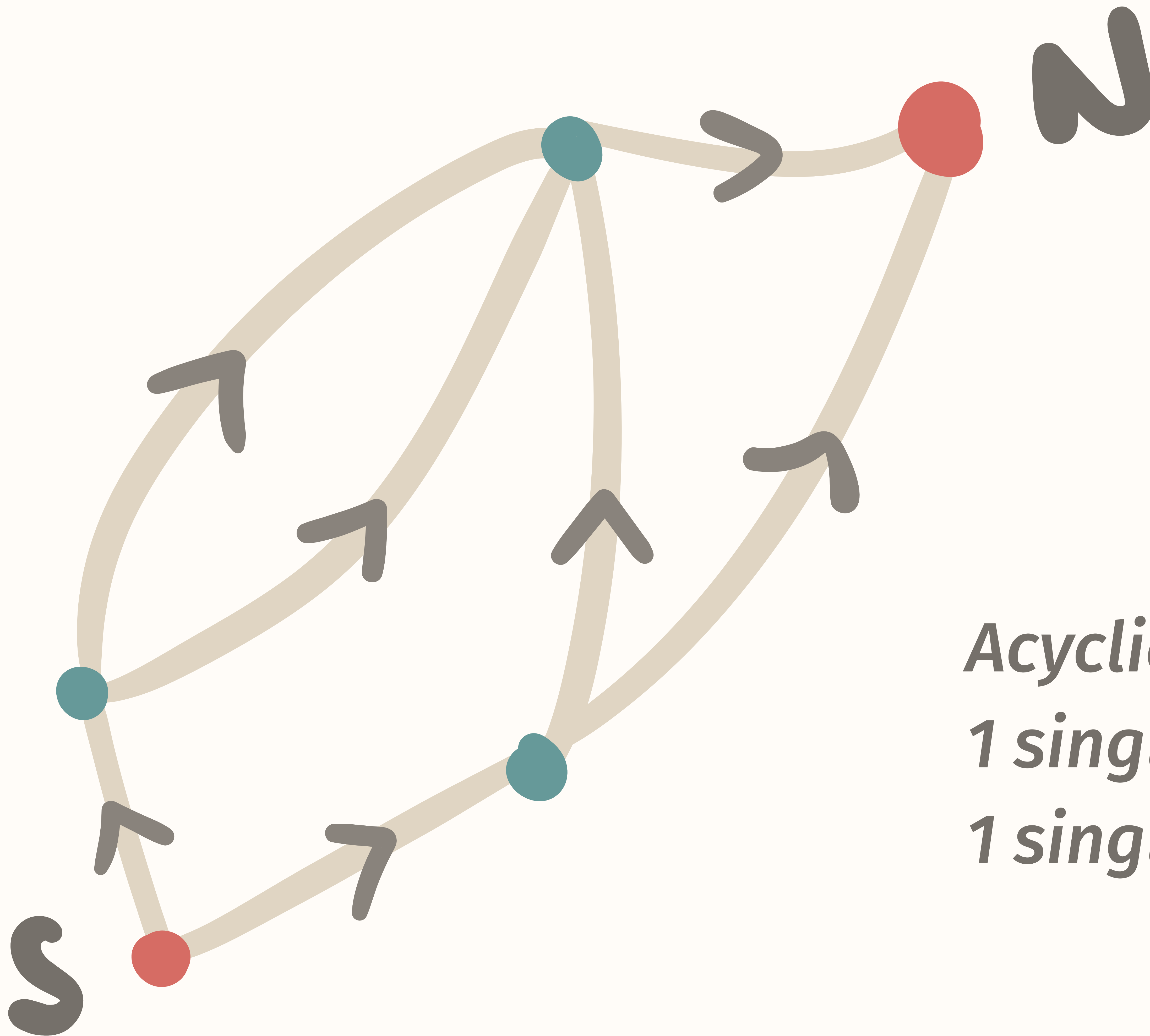
- a. Plane bipolar posets*
- b. Plane bipolar posets by vertices*
- c. Transversal structures*
- d. Asymtotoxic*

2. Interlude : plane permutations

3. Application to corner polyhedra

- a. Via polyhedral orientations*
- b. Via Schnyder colorings*
- c. Asymtotoxic*

Plane bipolar orientation

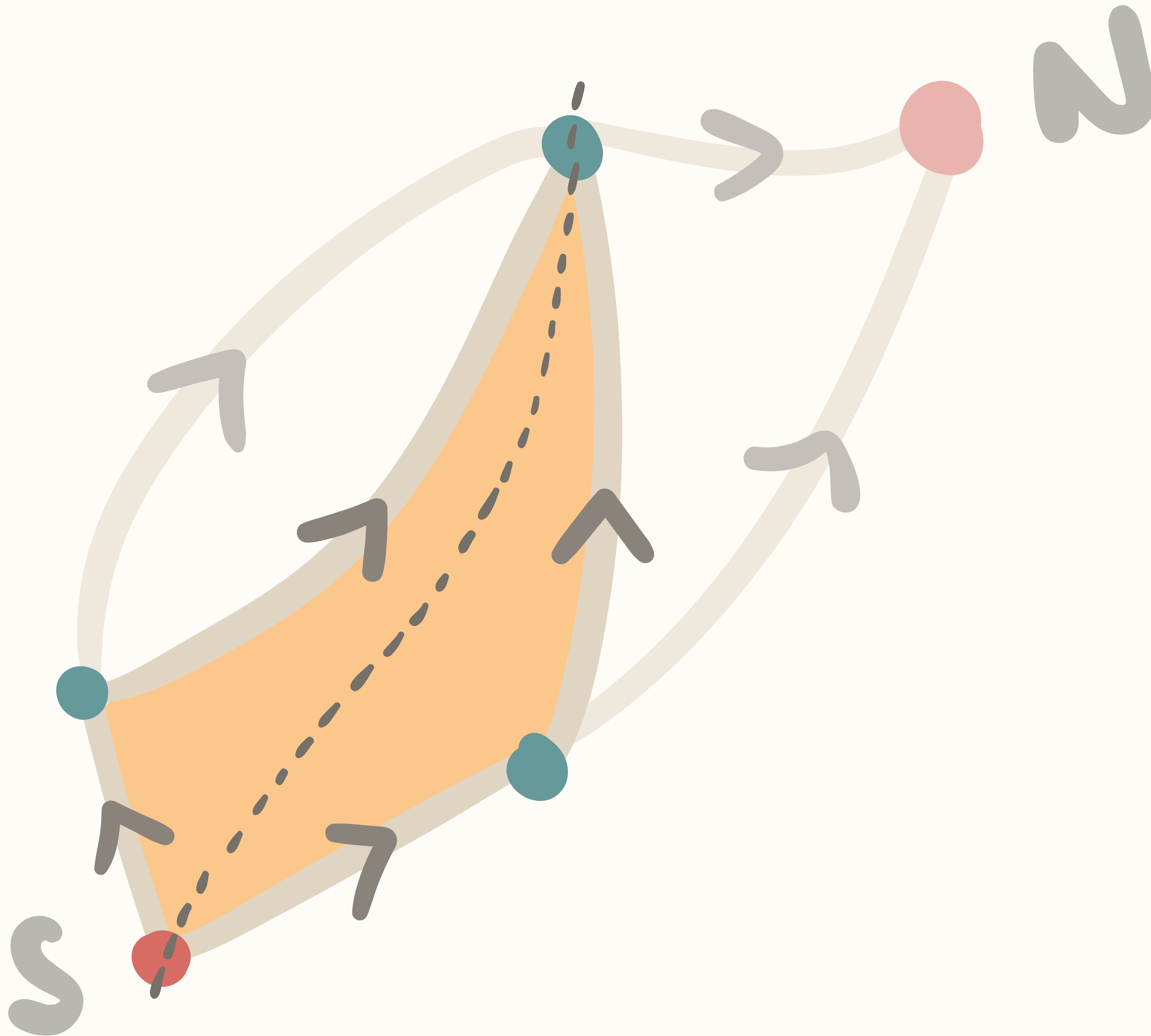


Acyclic

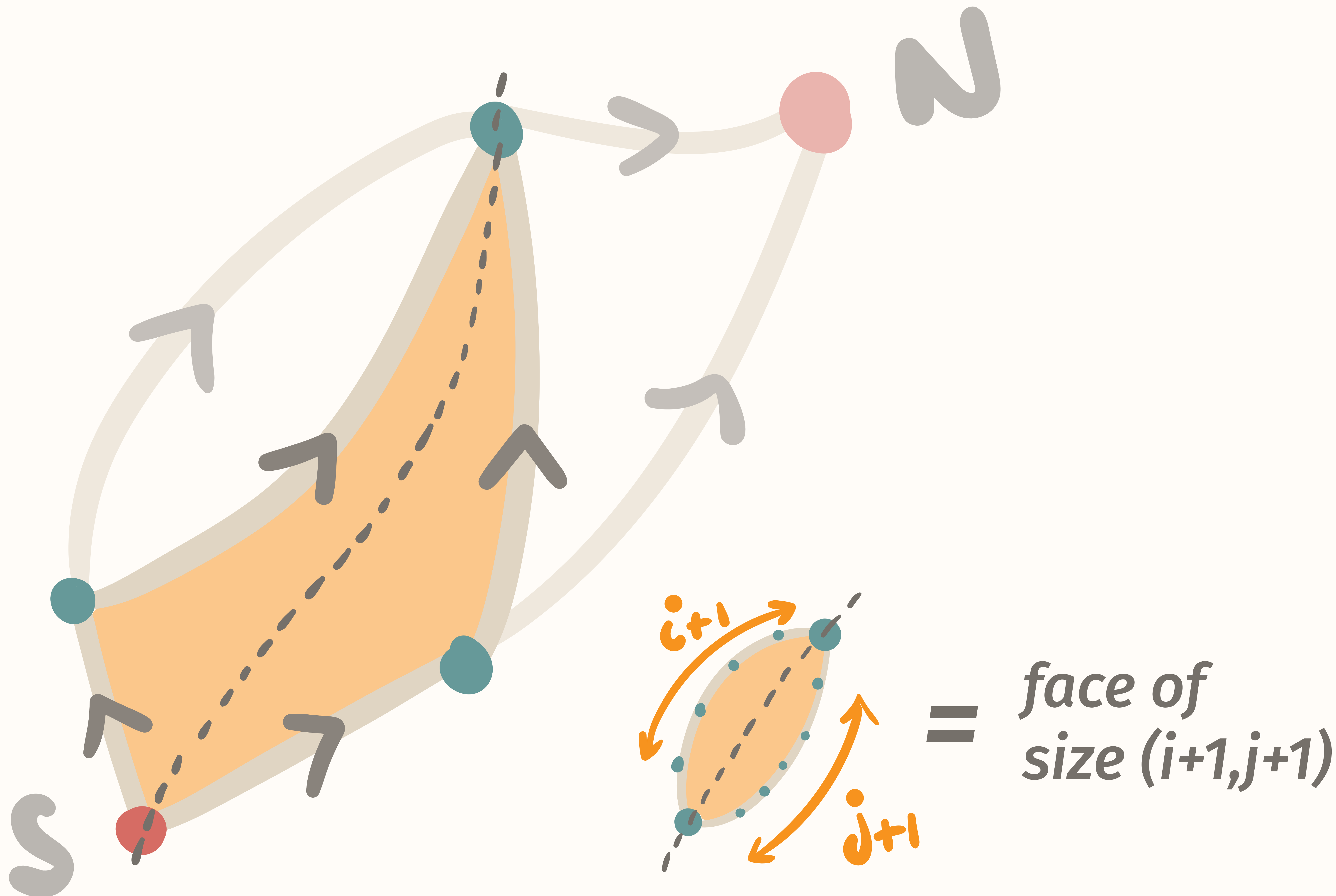
1 single source S

1 single sink N

Plane bipolar orientation



Plane bipolar orientation



The KMSW bijection

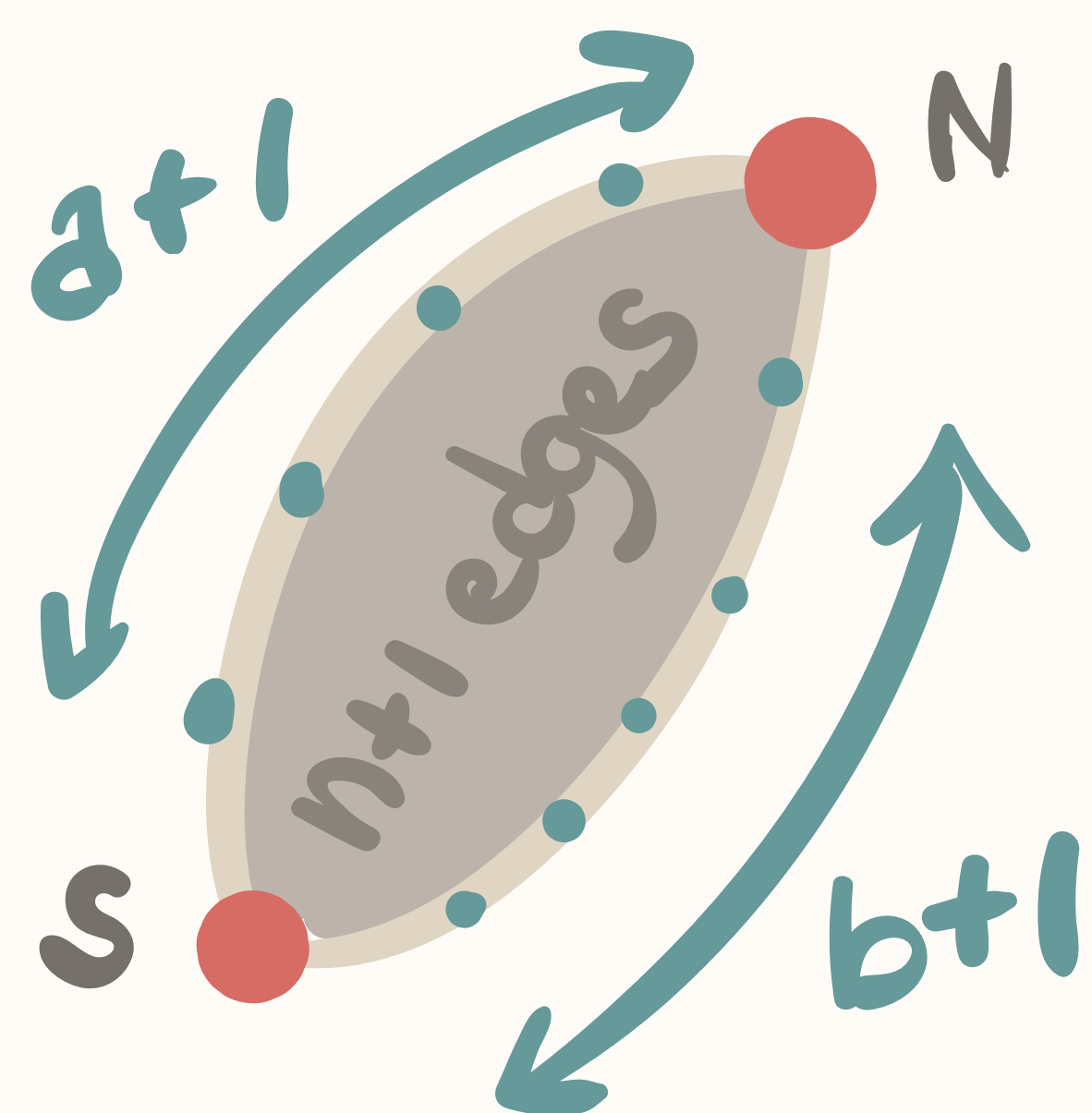
*Plane bipolar
orientations*



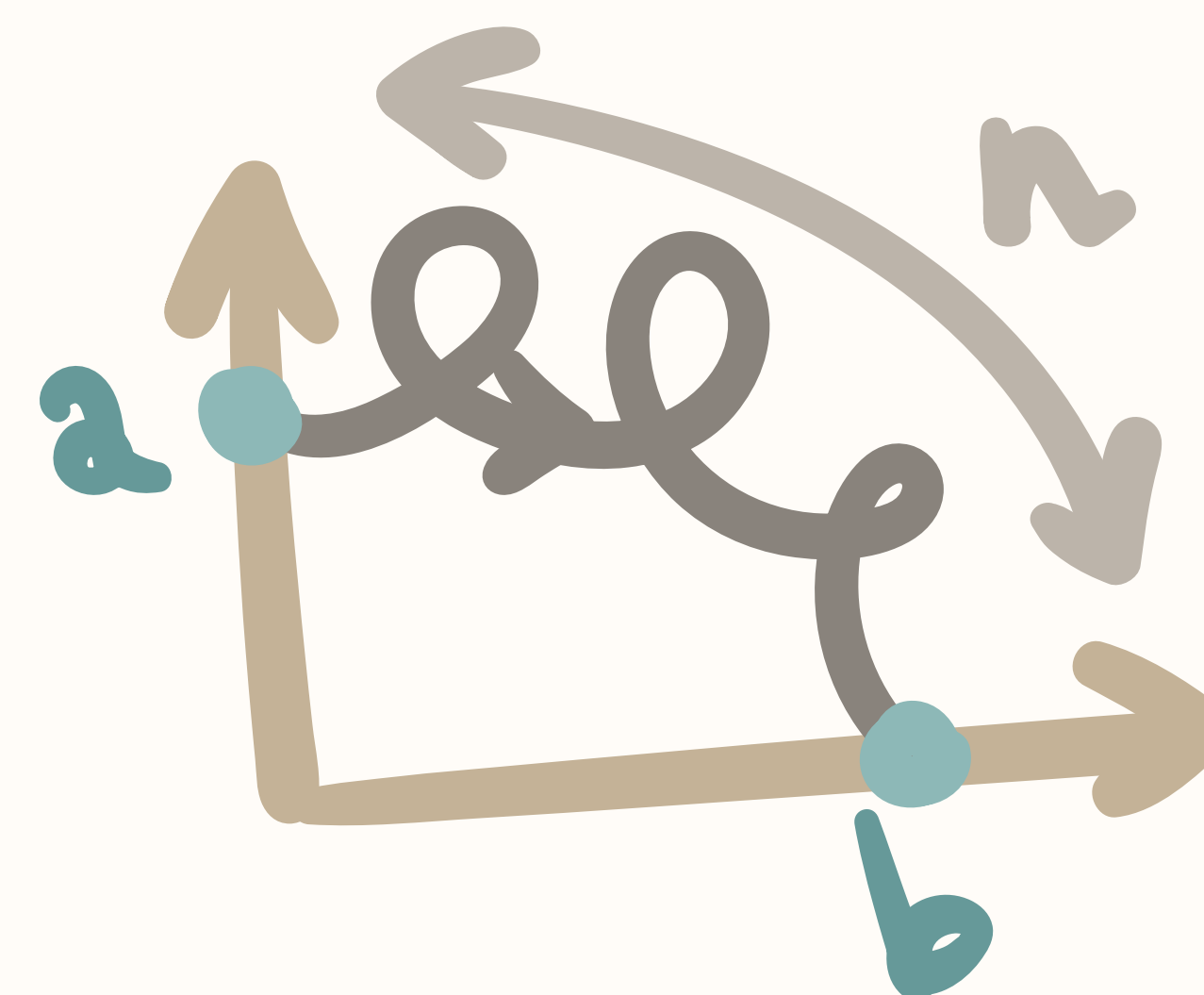
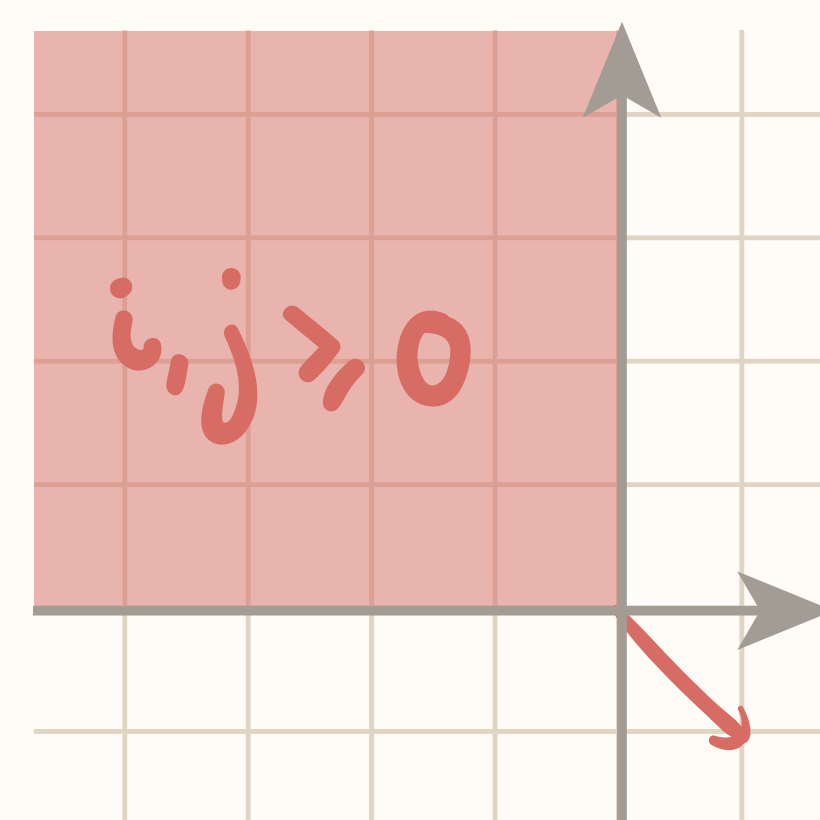
*tandems walks
in the quarter plane*

The KMSW bijection

Plane bipolar orientations



tandems walks in the quarter plane

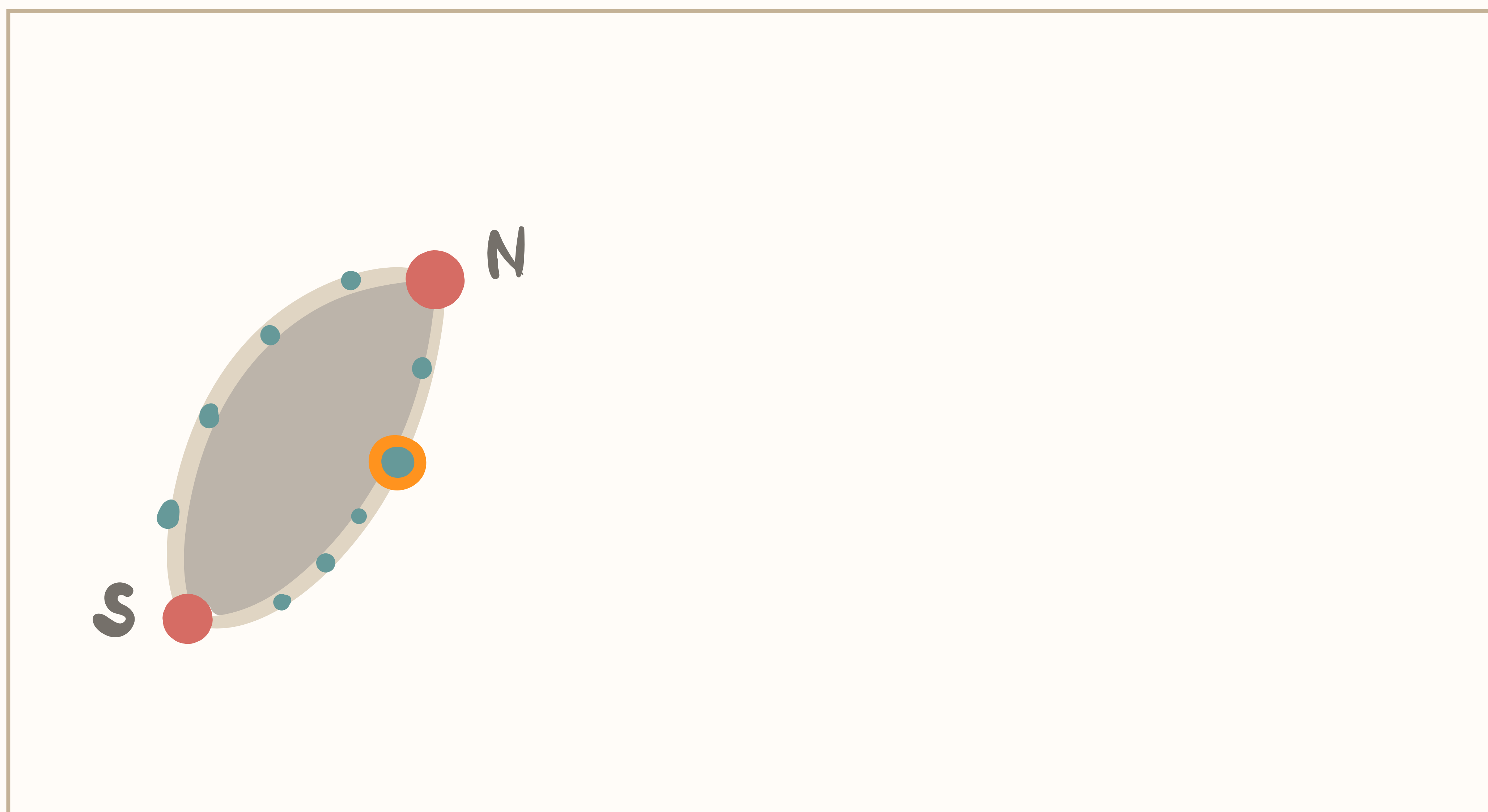
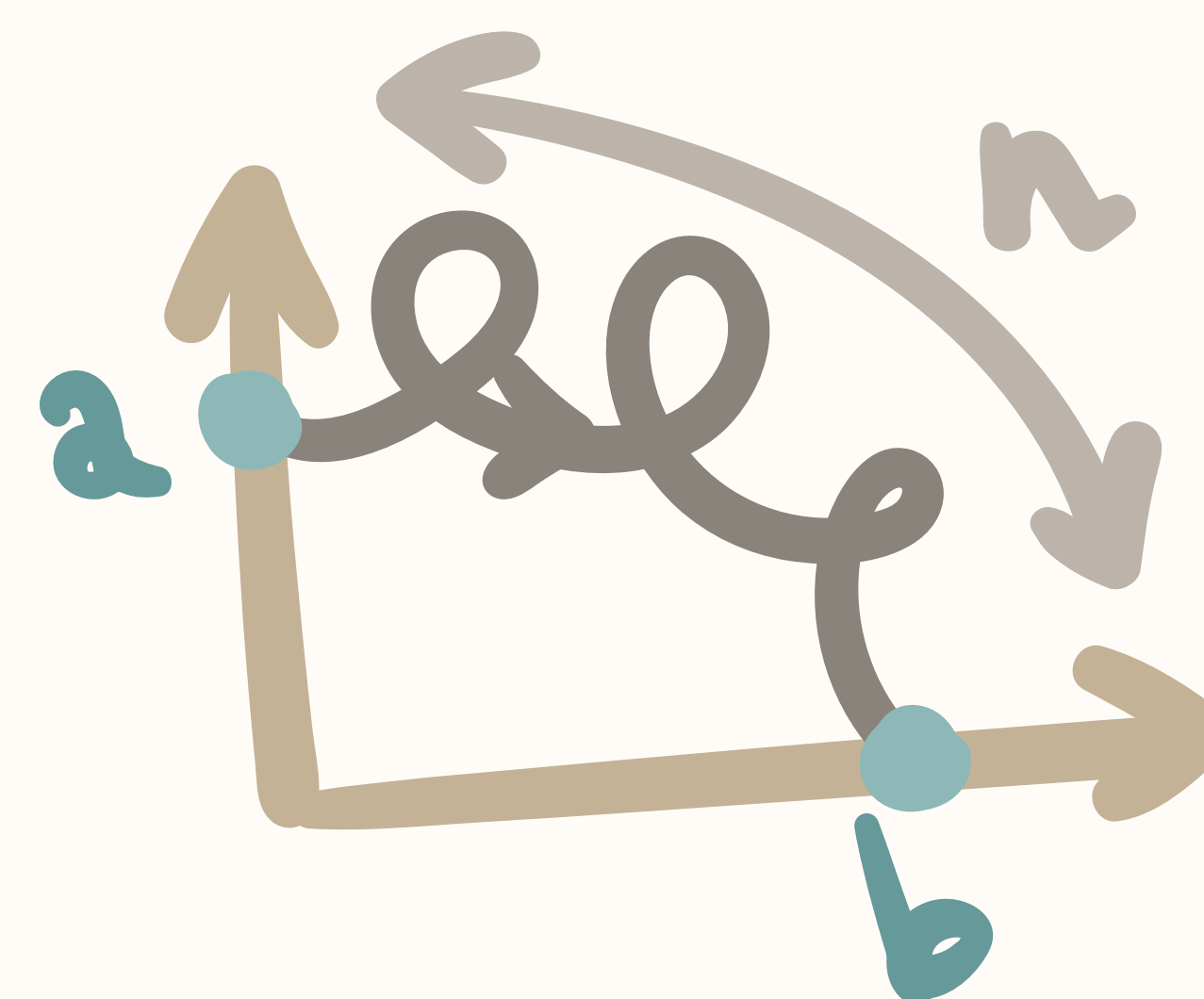
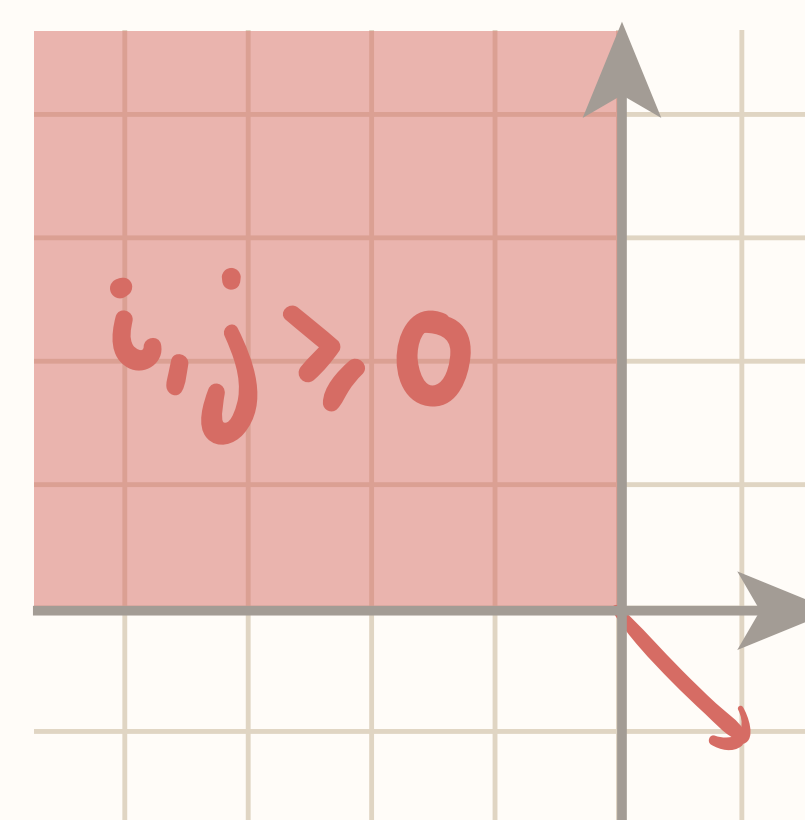
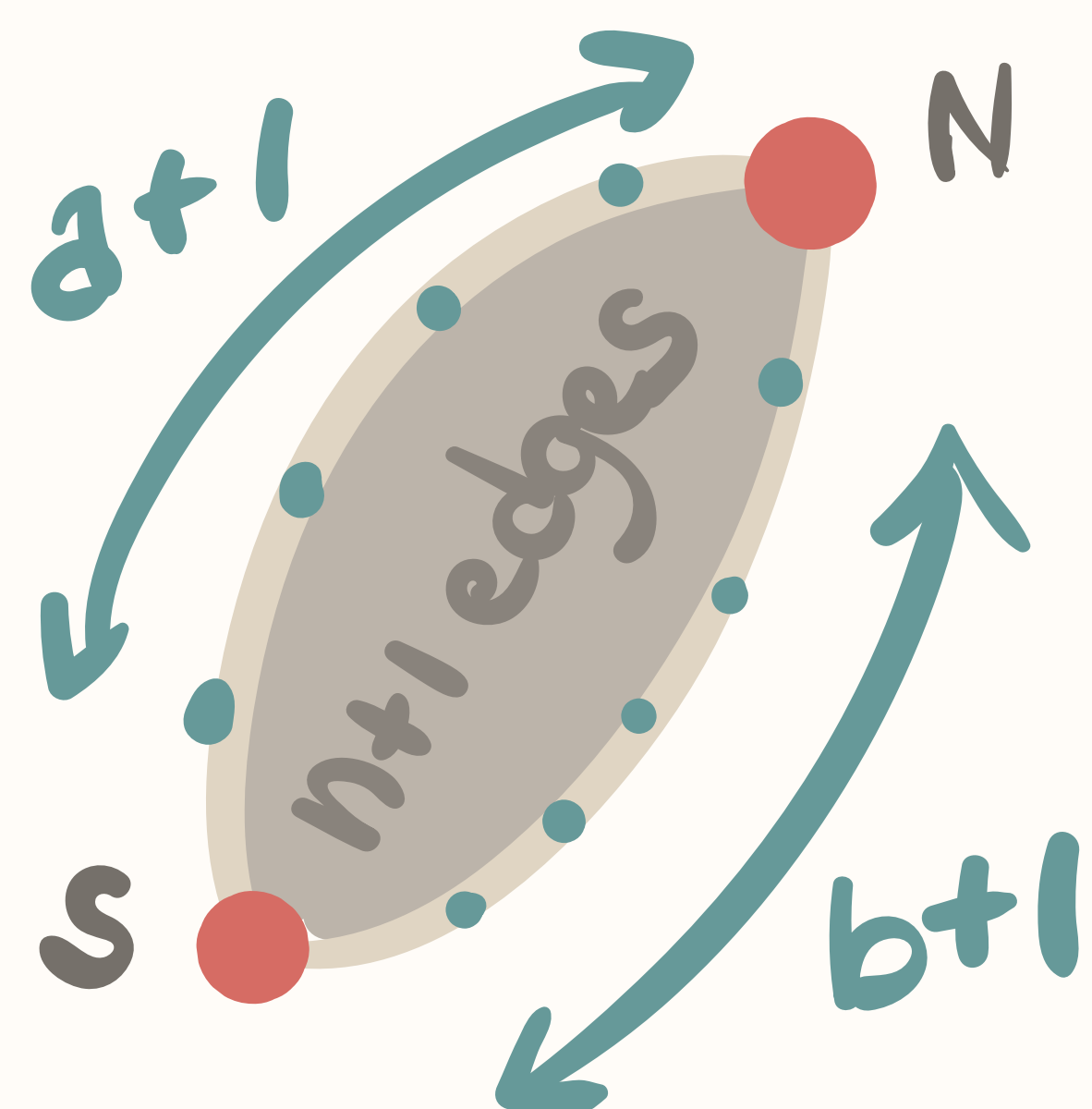


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

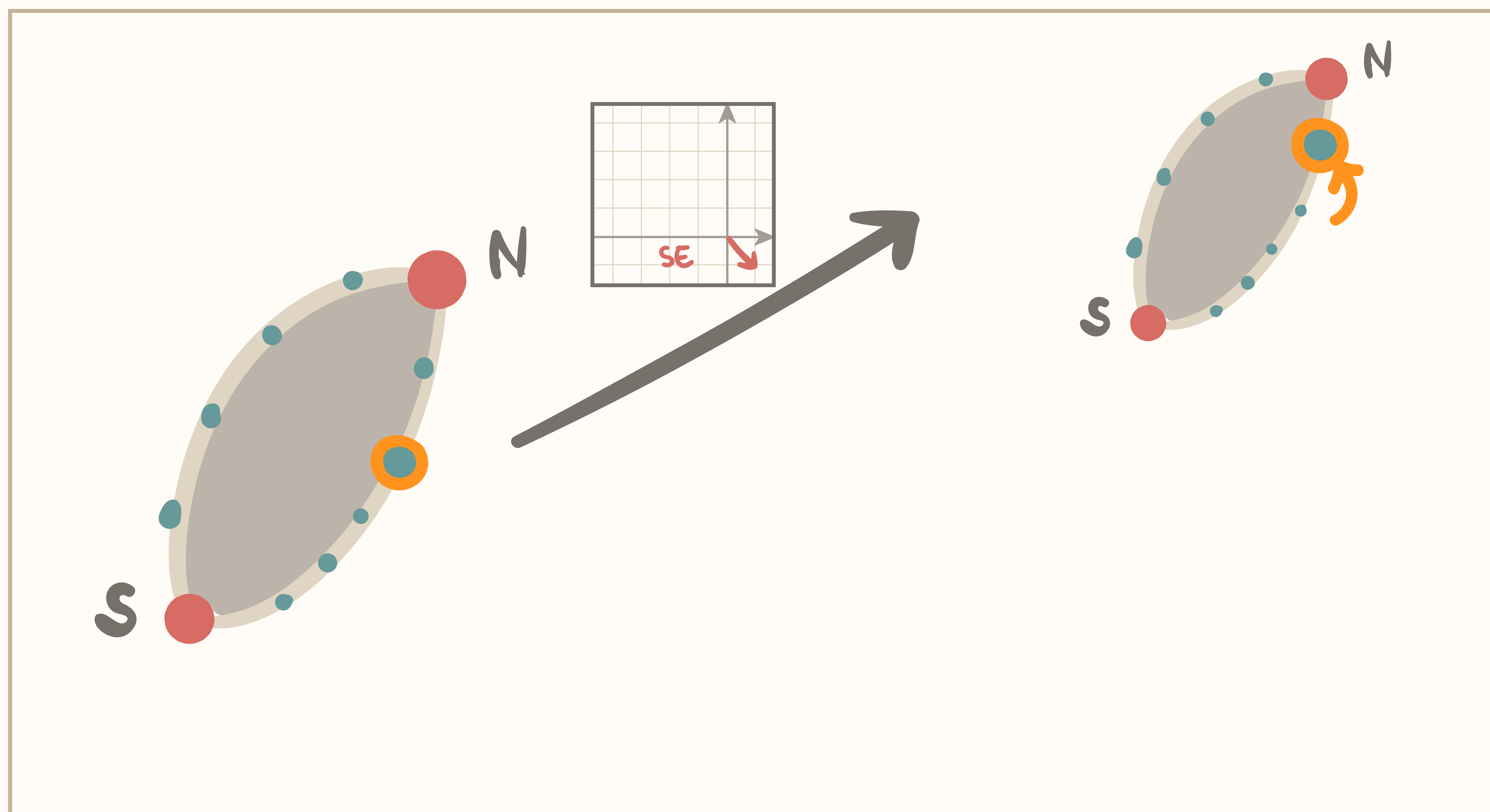
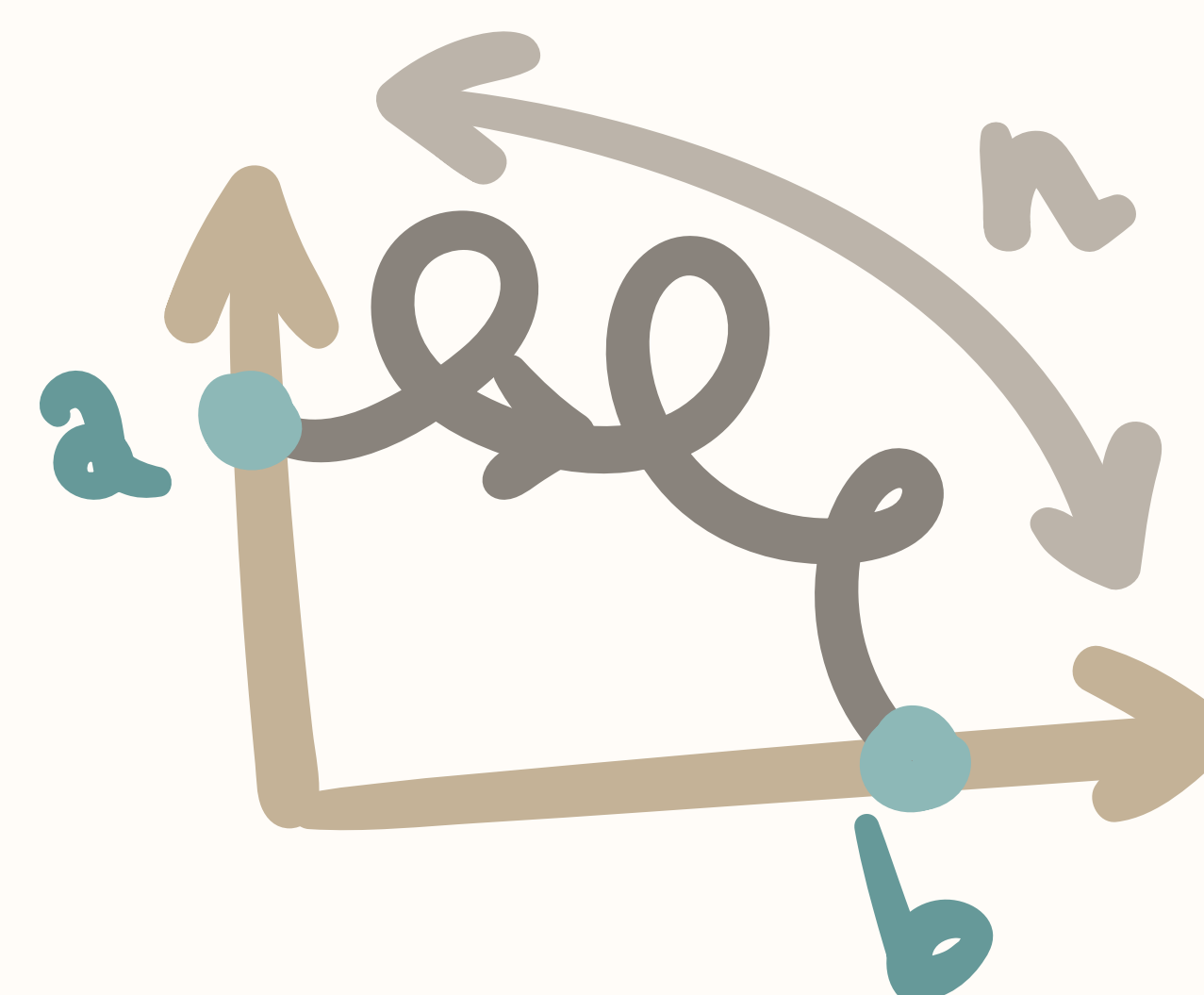
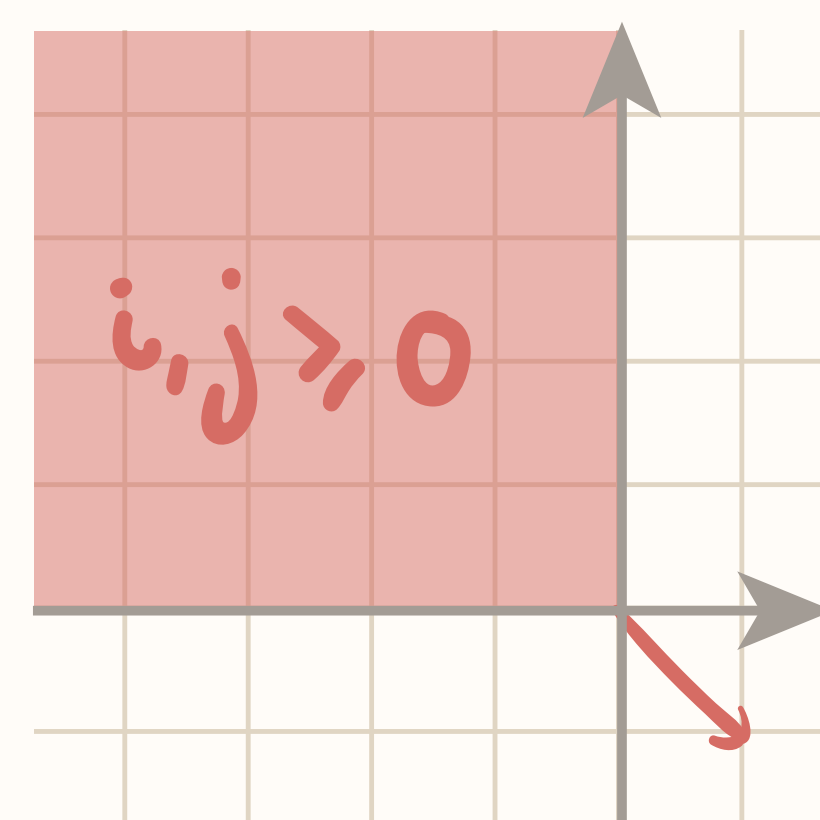
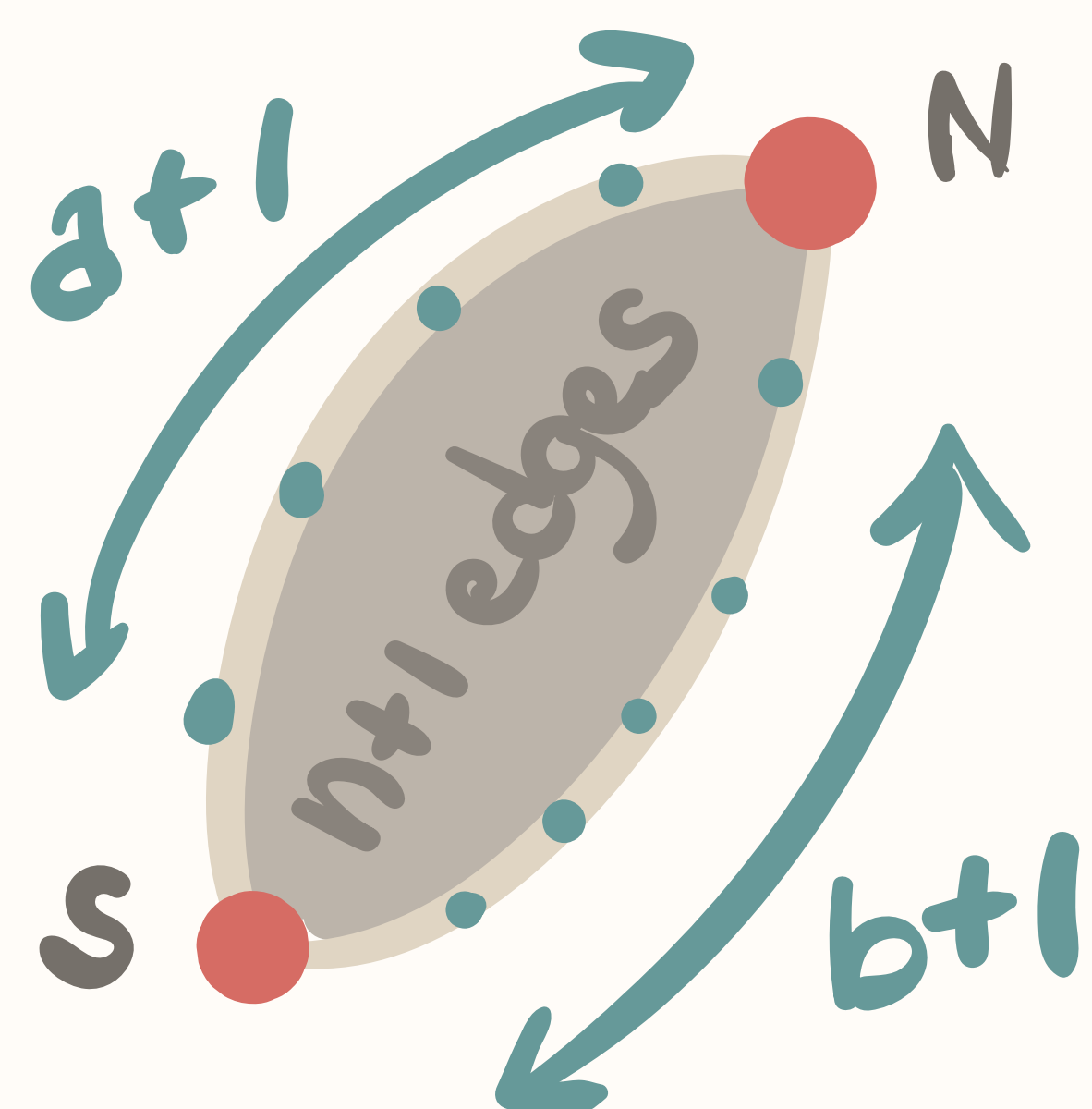


The KMSW bijection

Plane bipolar orientations



tandem walks in the quarter plane

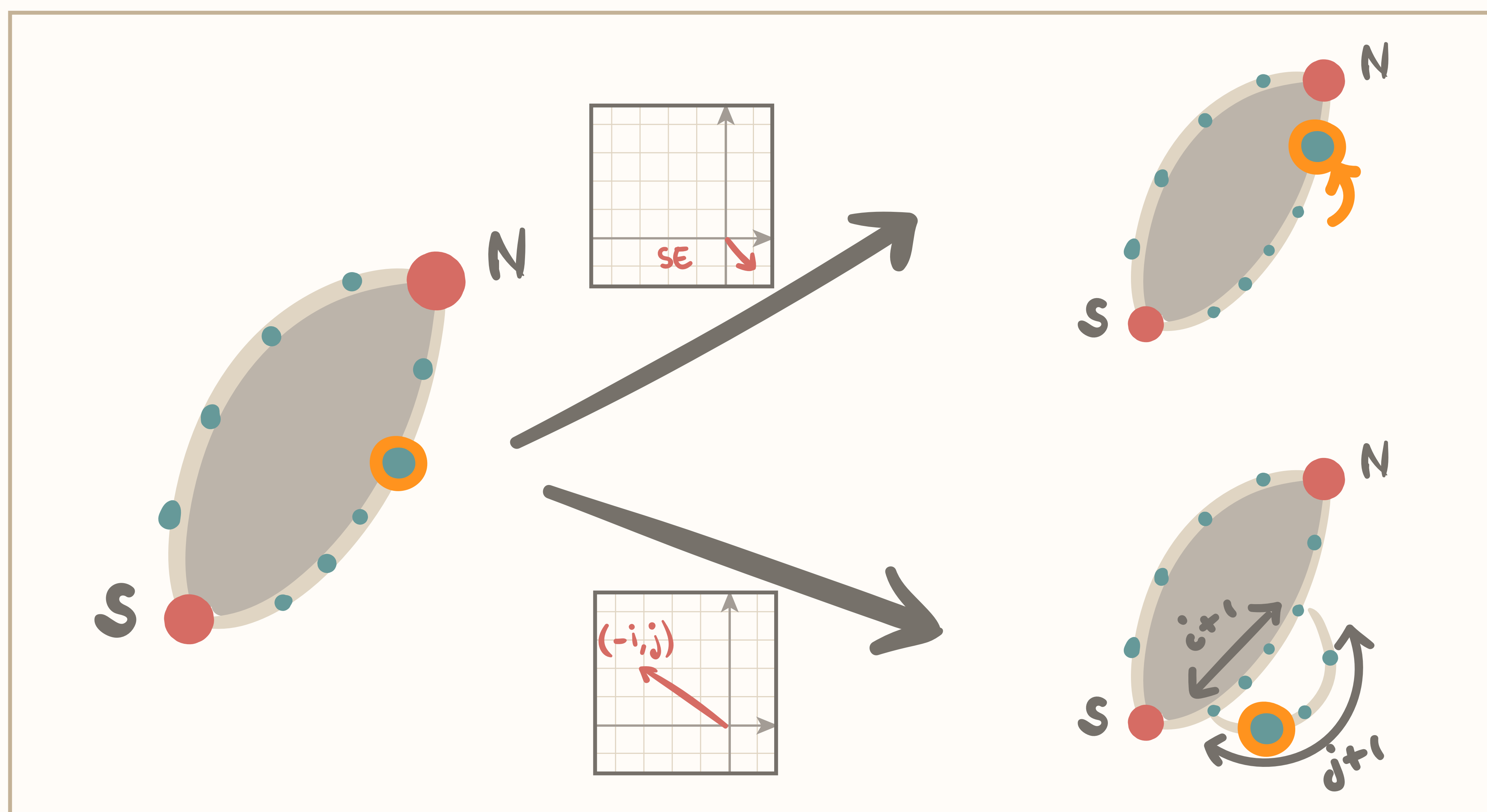
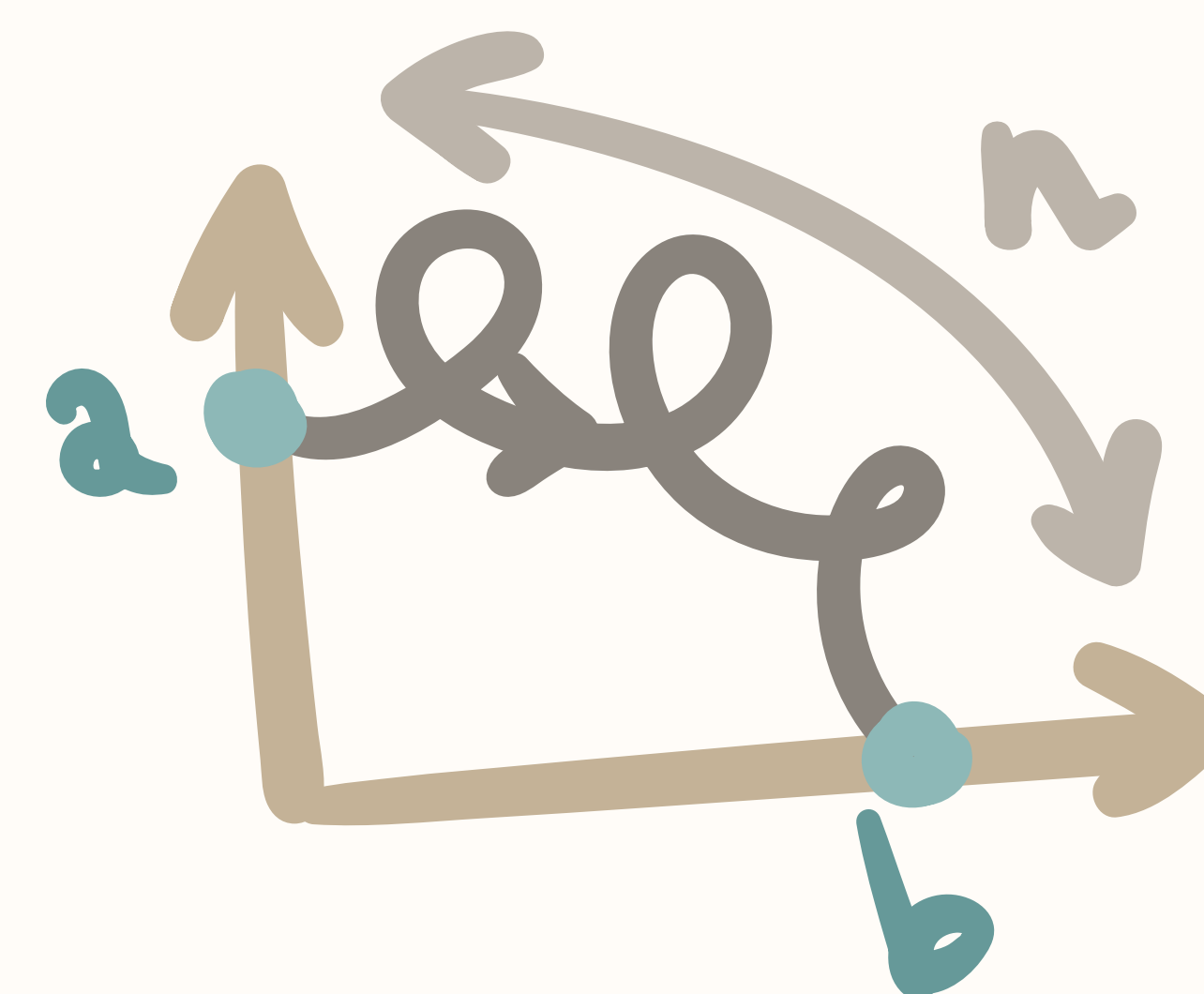
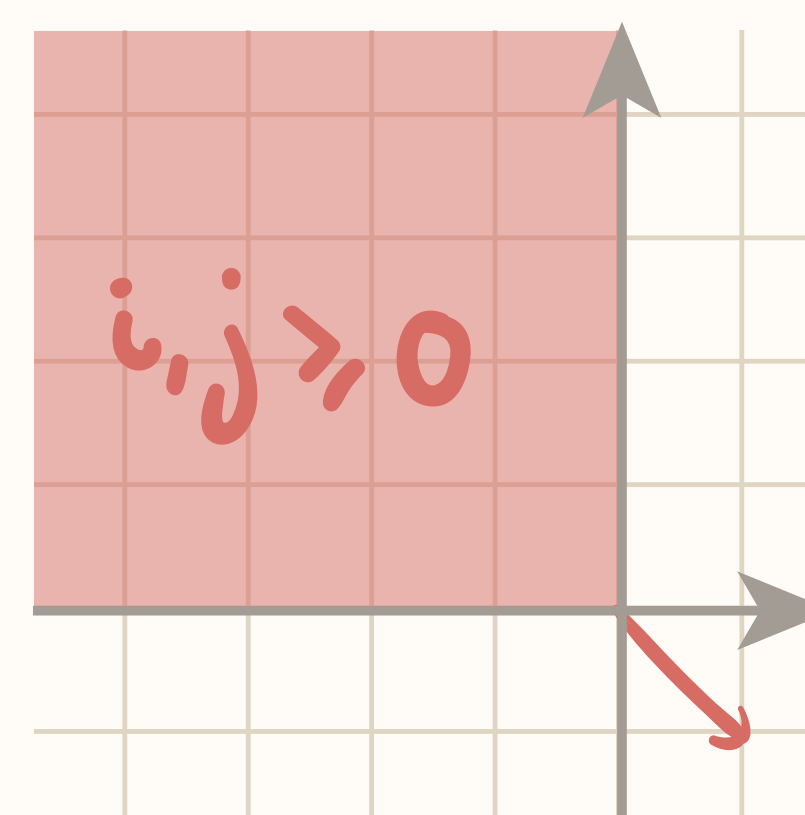
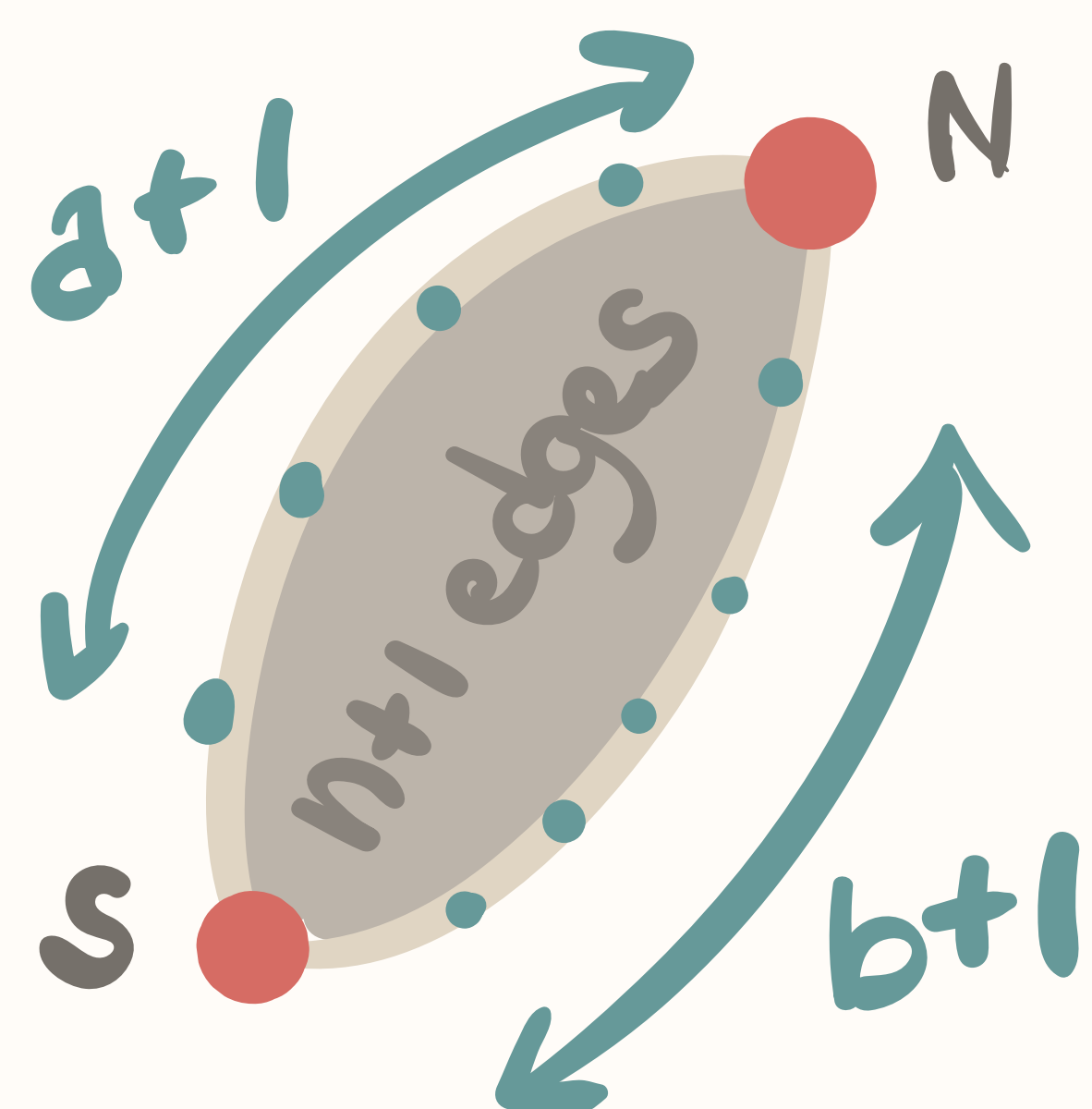


The KMSW bijection

Plane bipolar orientations



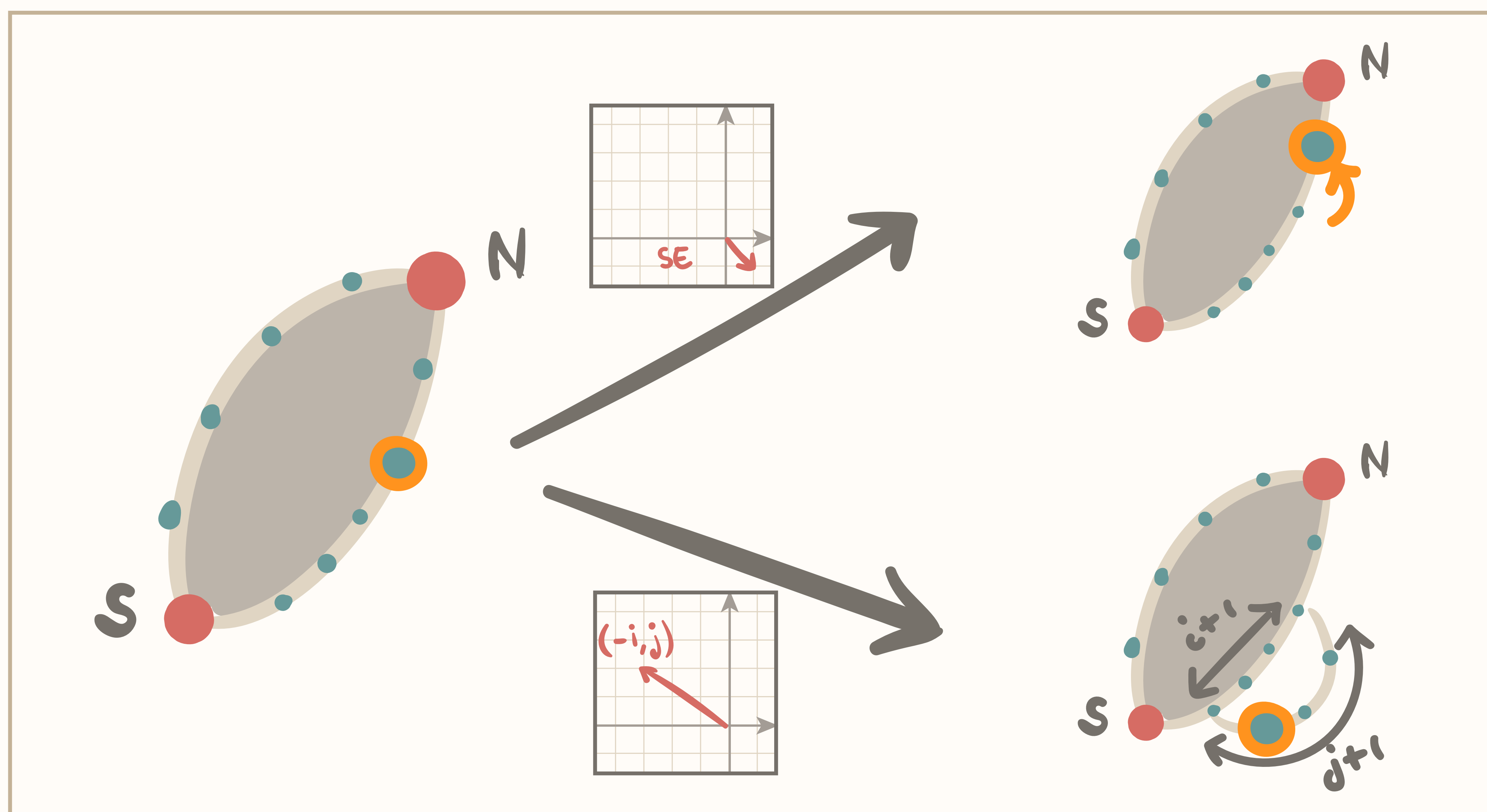
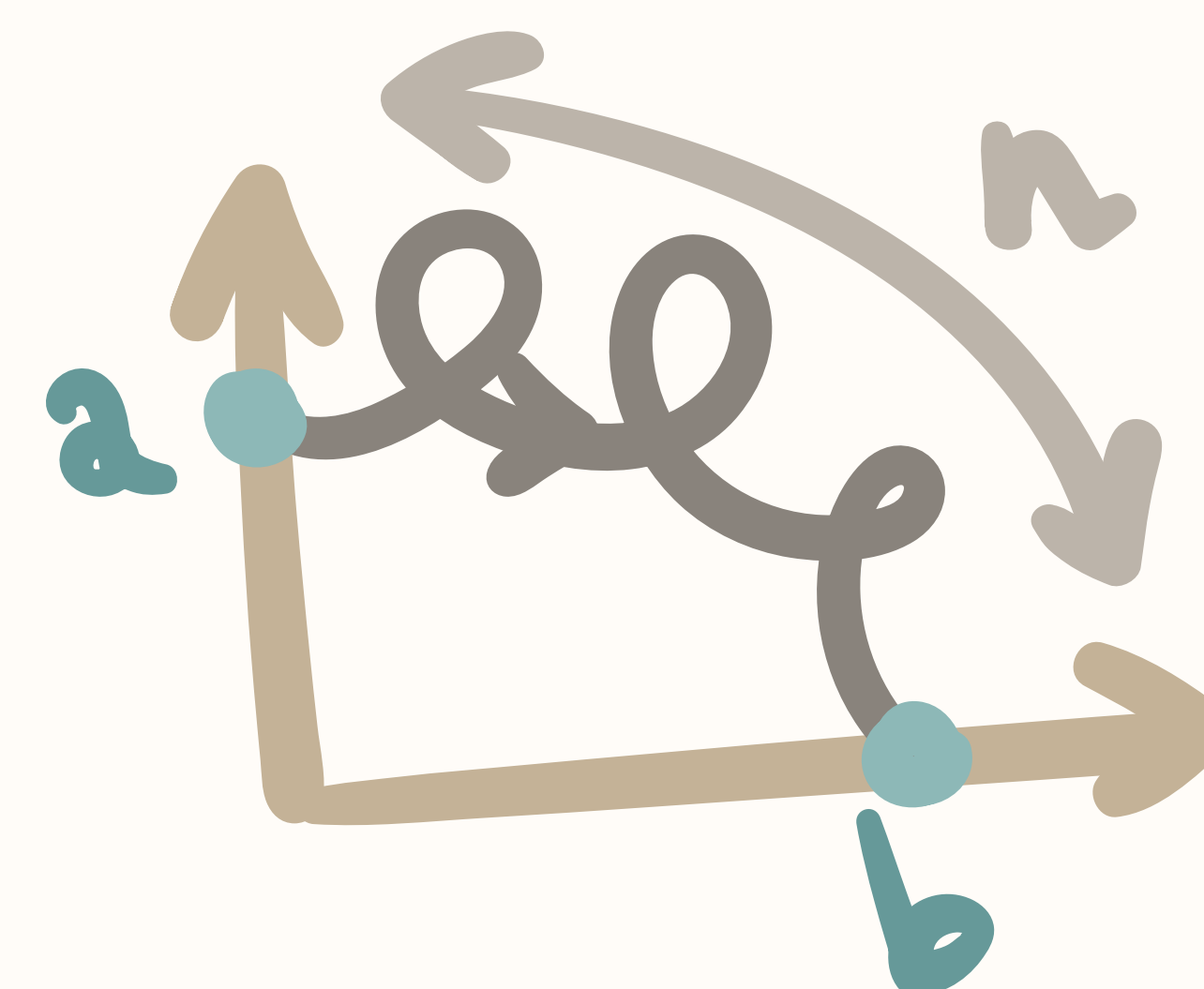
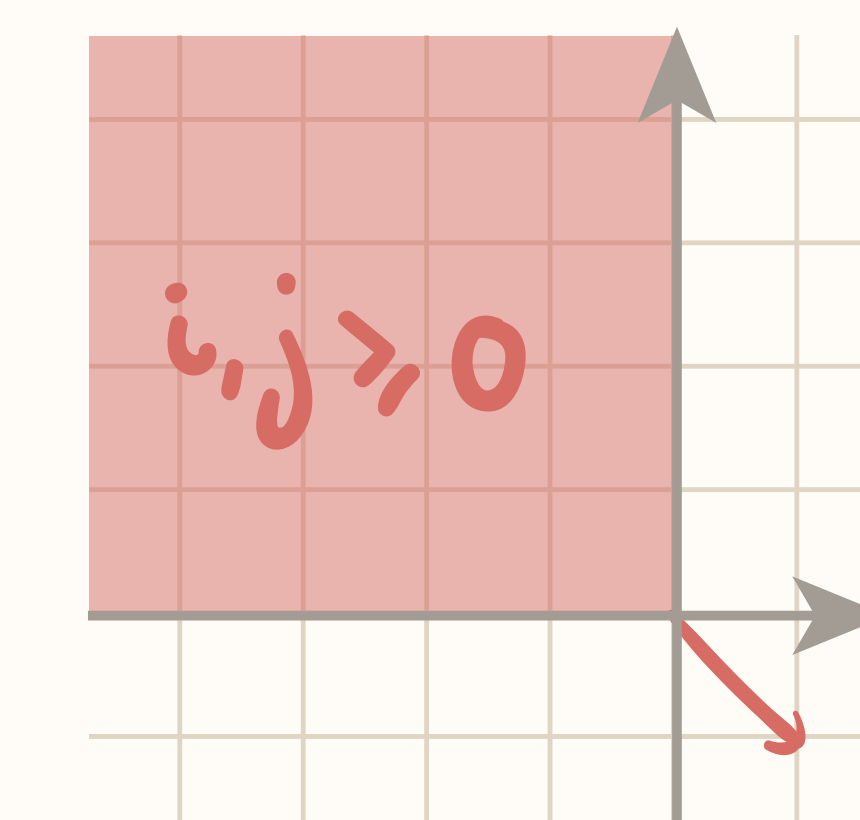
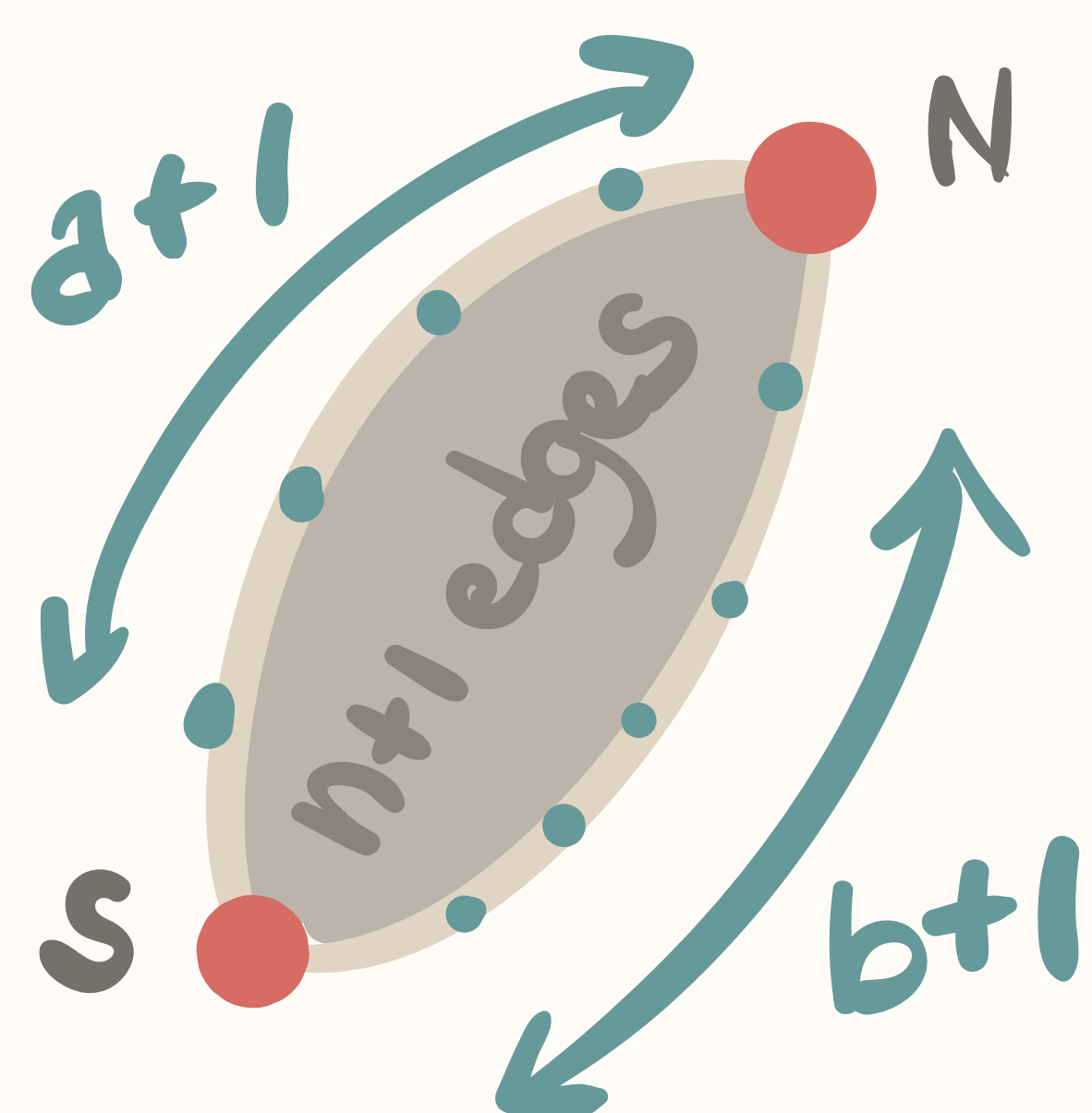
tandem walks in the quarter plane



The KMSW bijection

Plane bipolar orientations

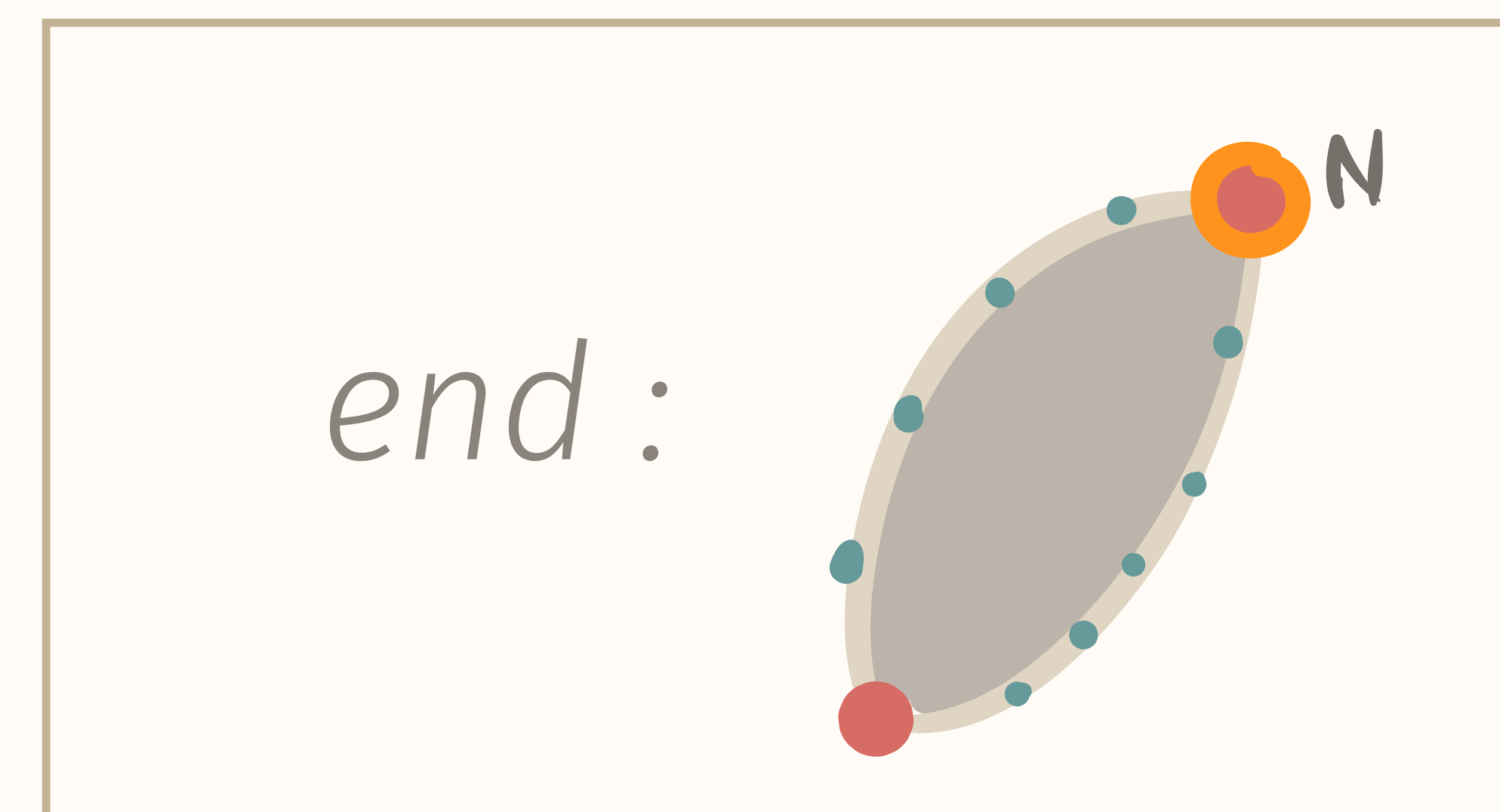
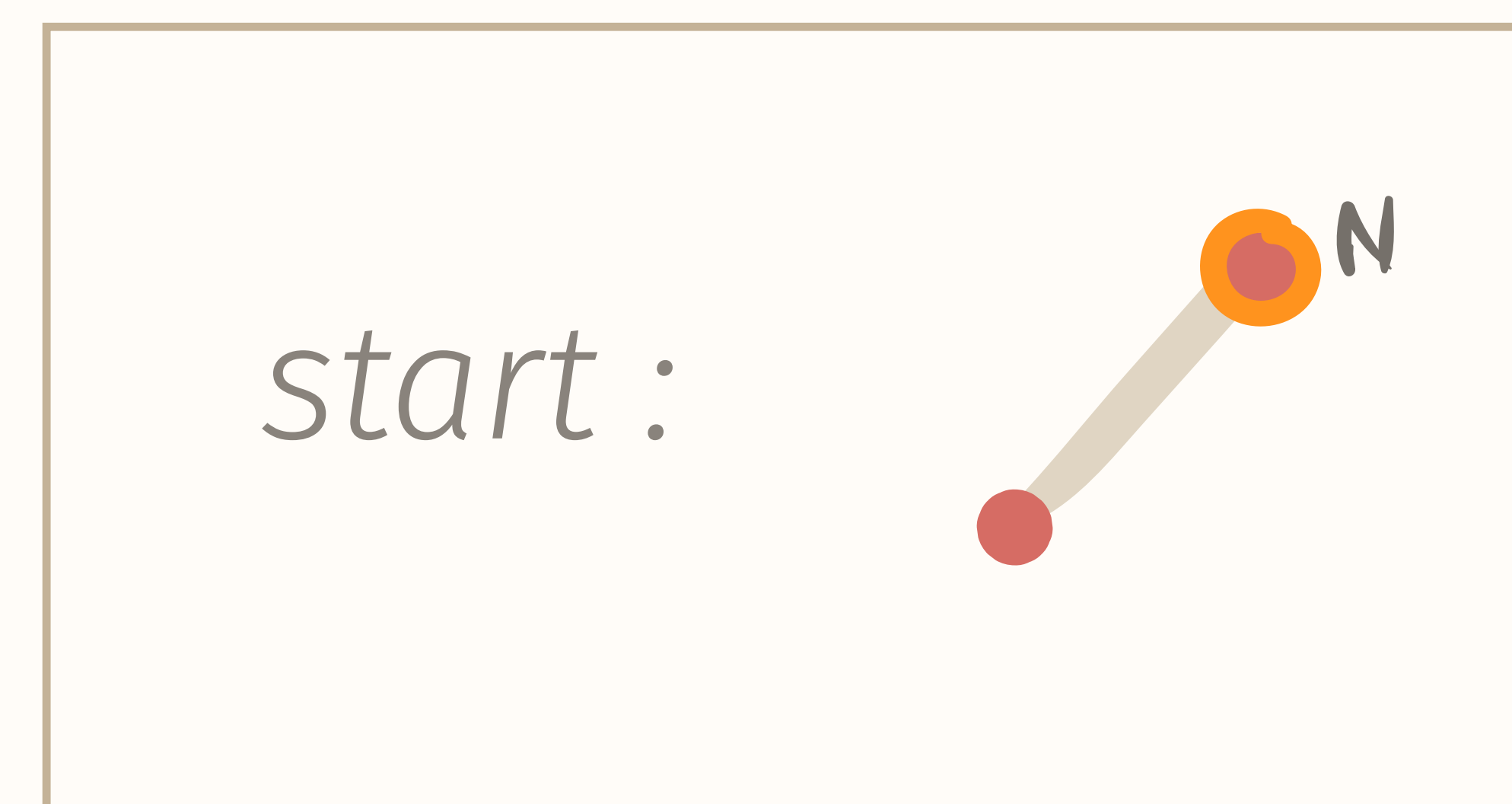
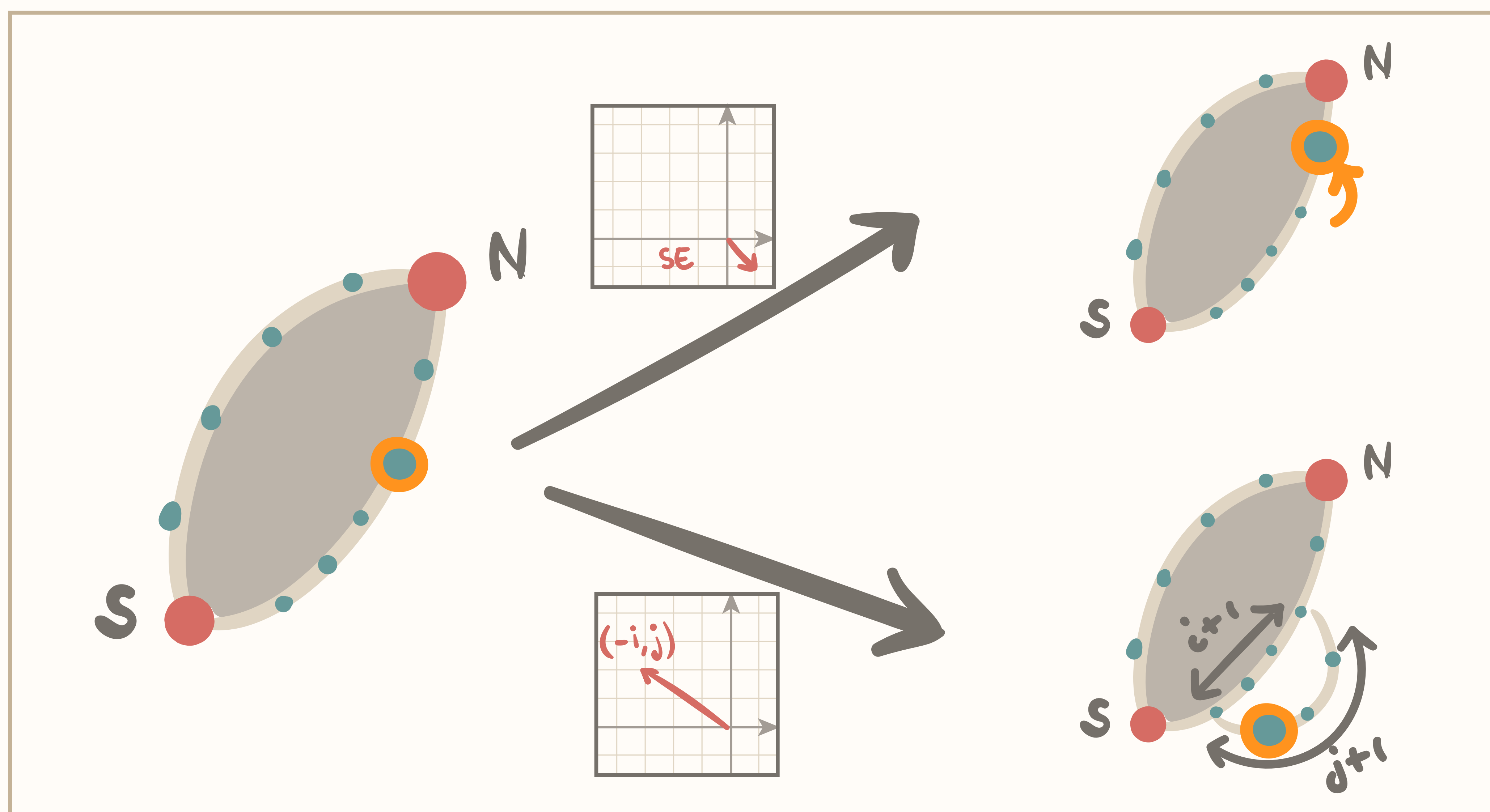
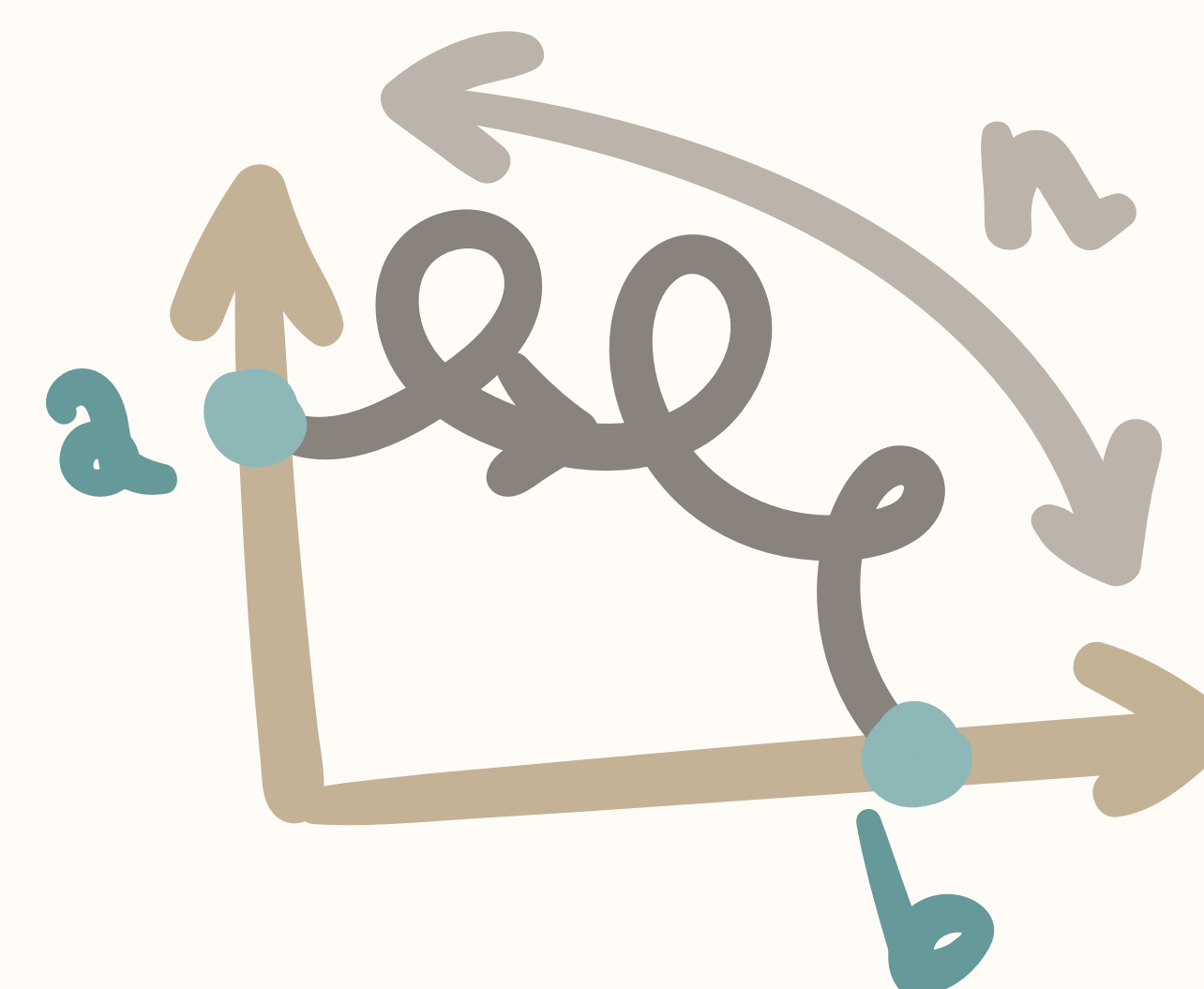
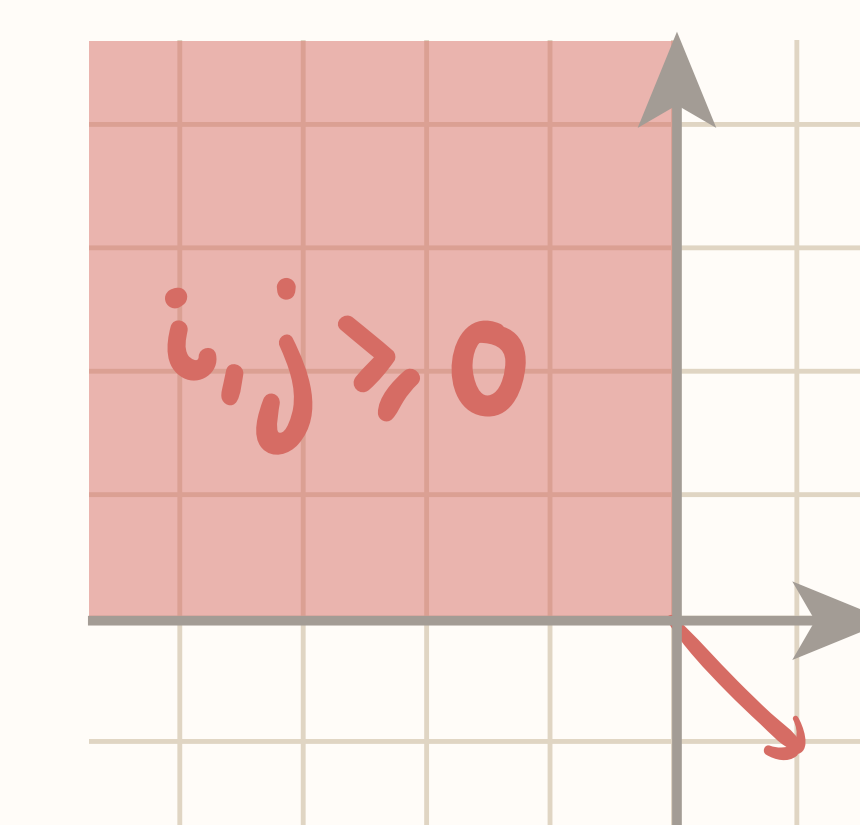
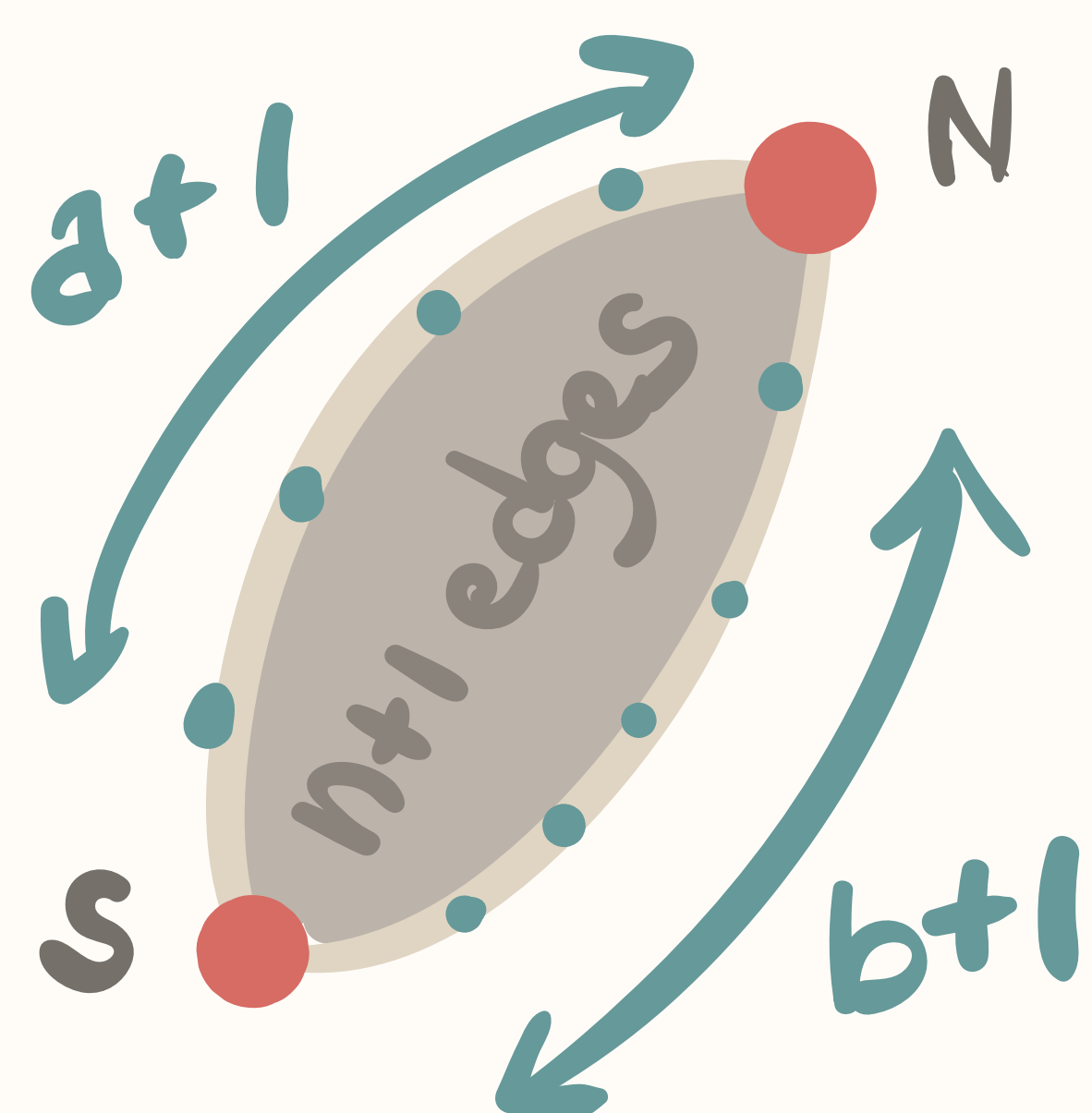
tandems walks in the quarter plane



The KMSW bijection

Plane bipolar orientations

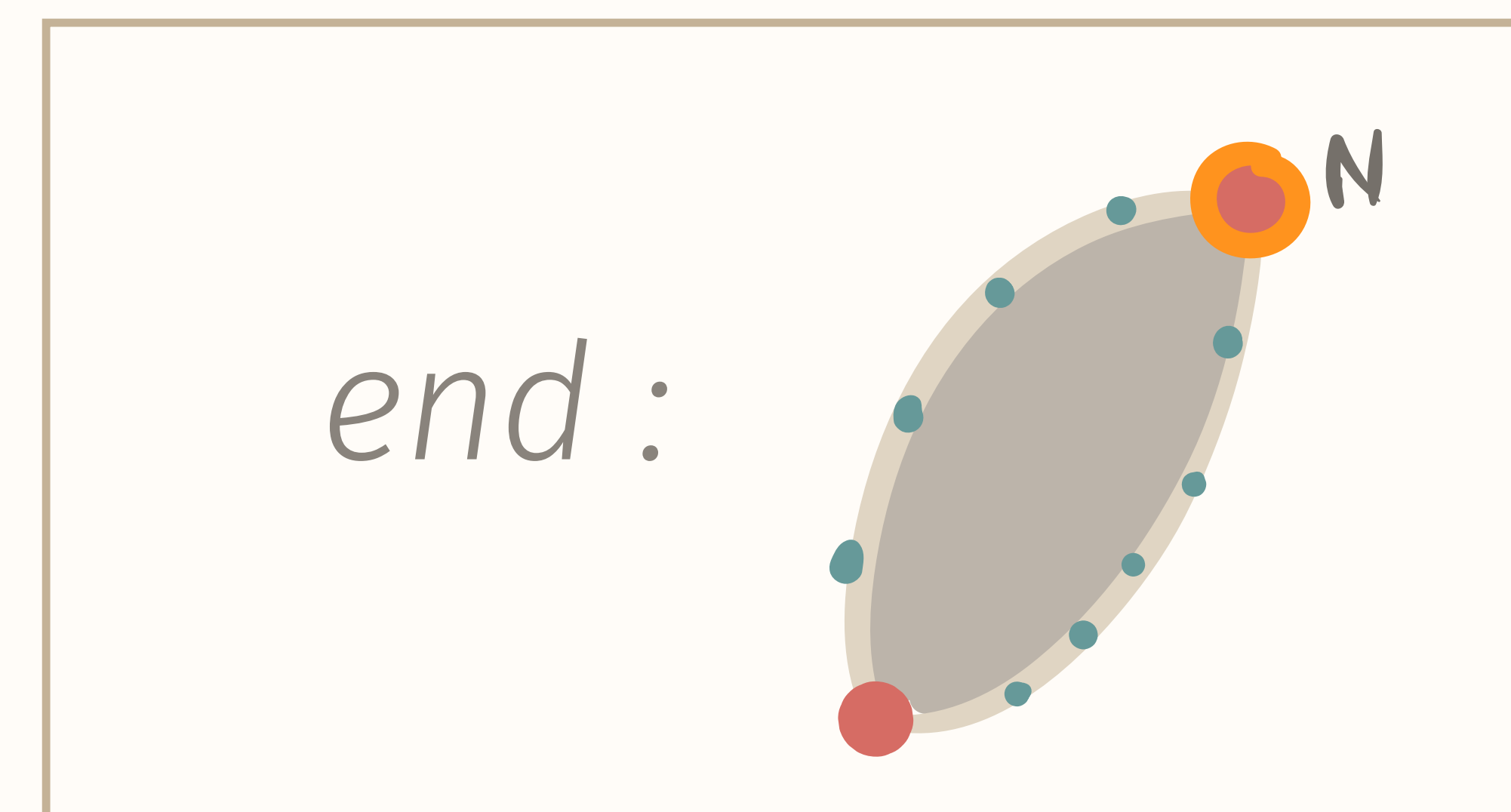
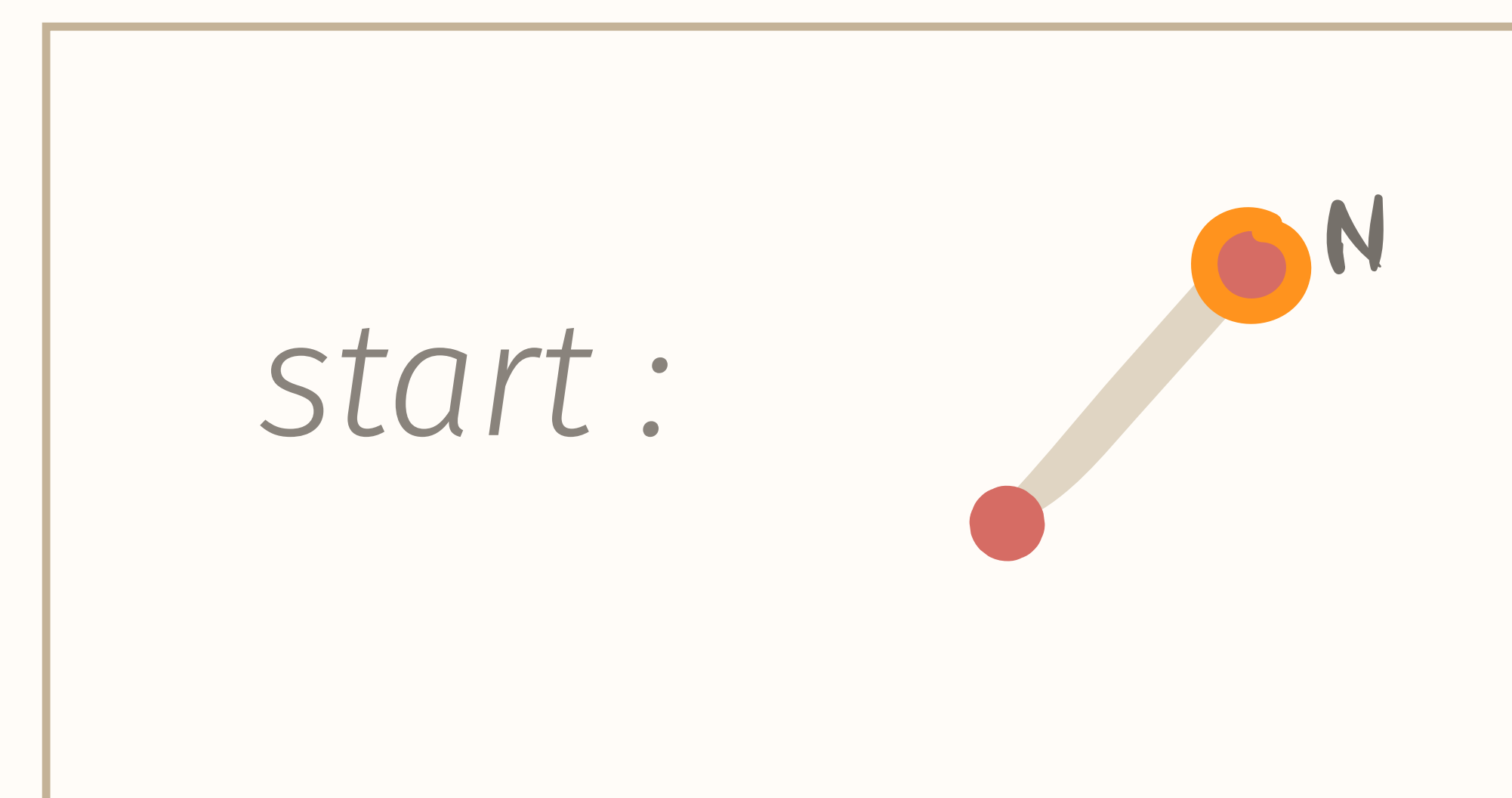
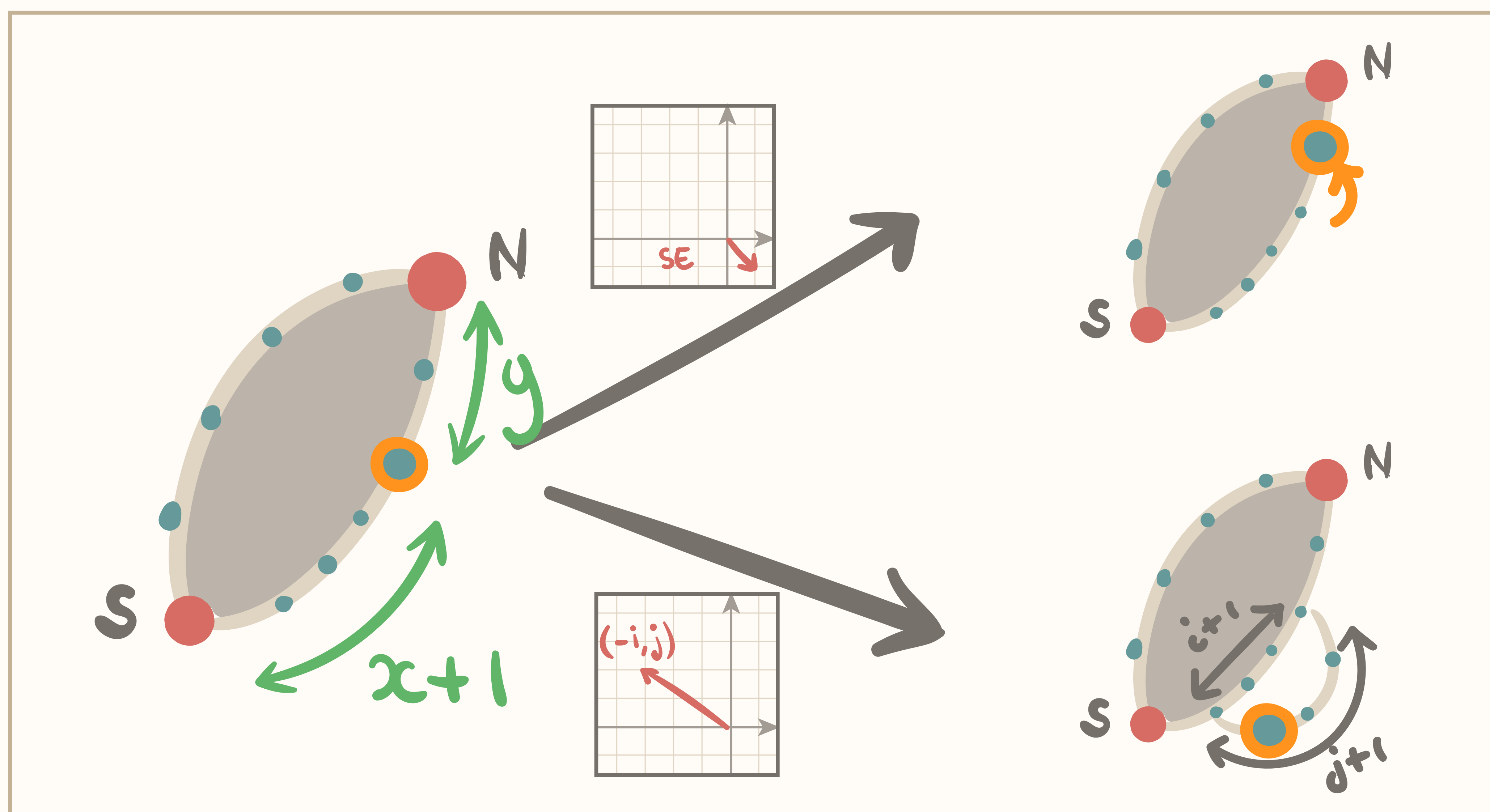
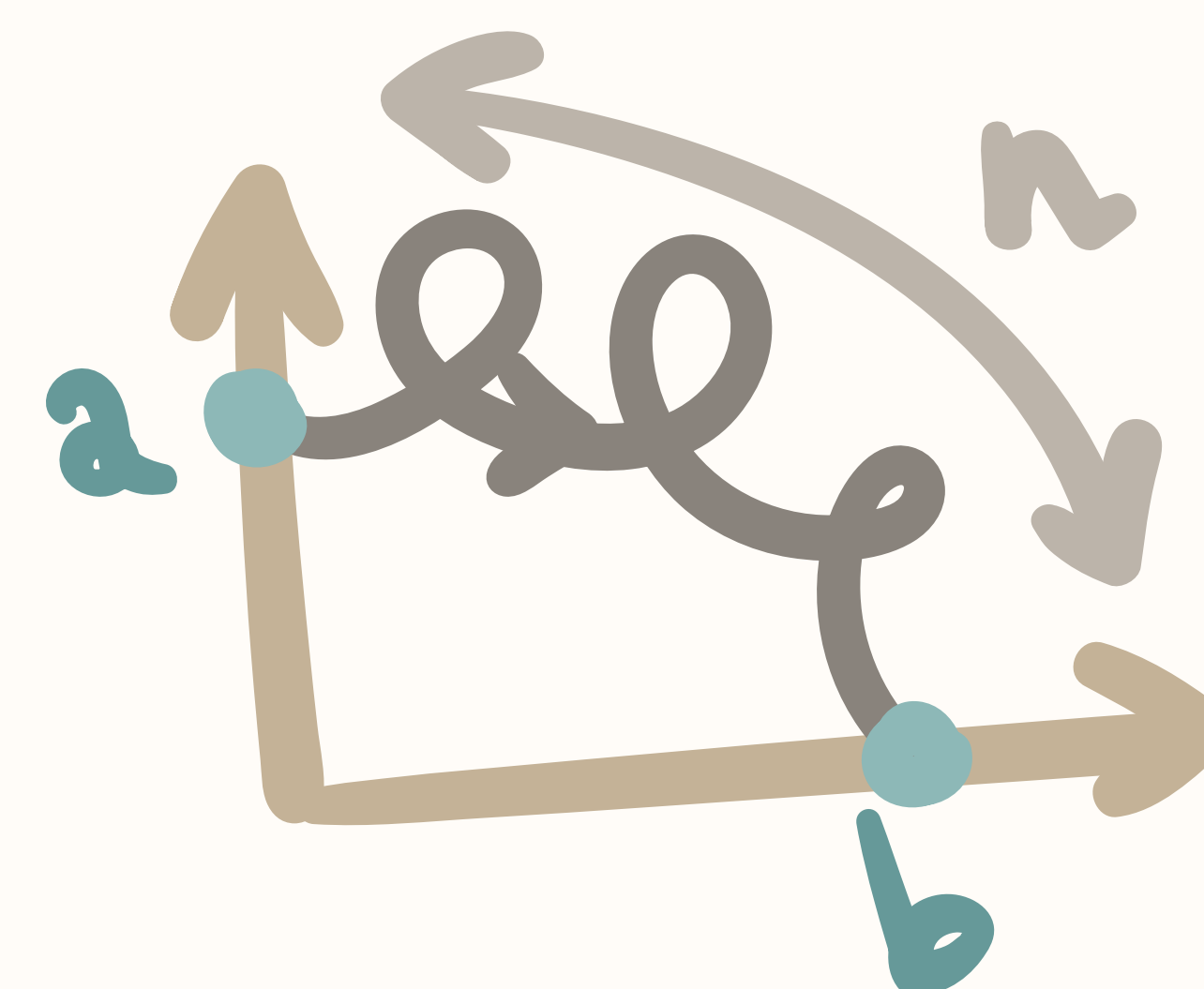
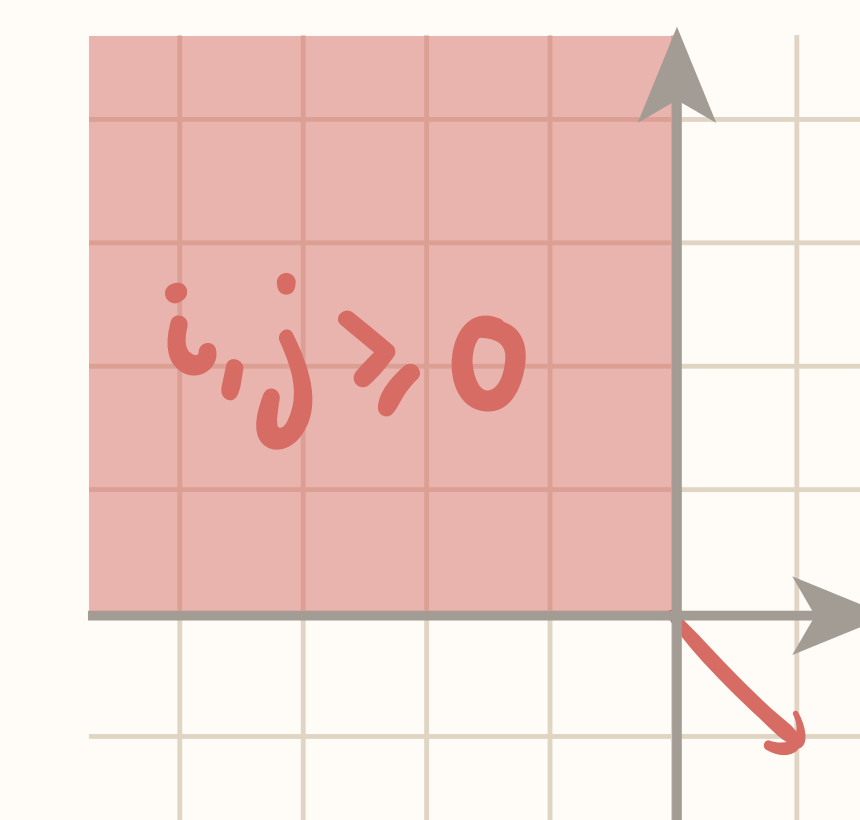
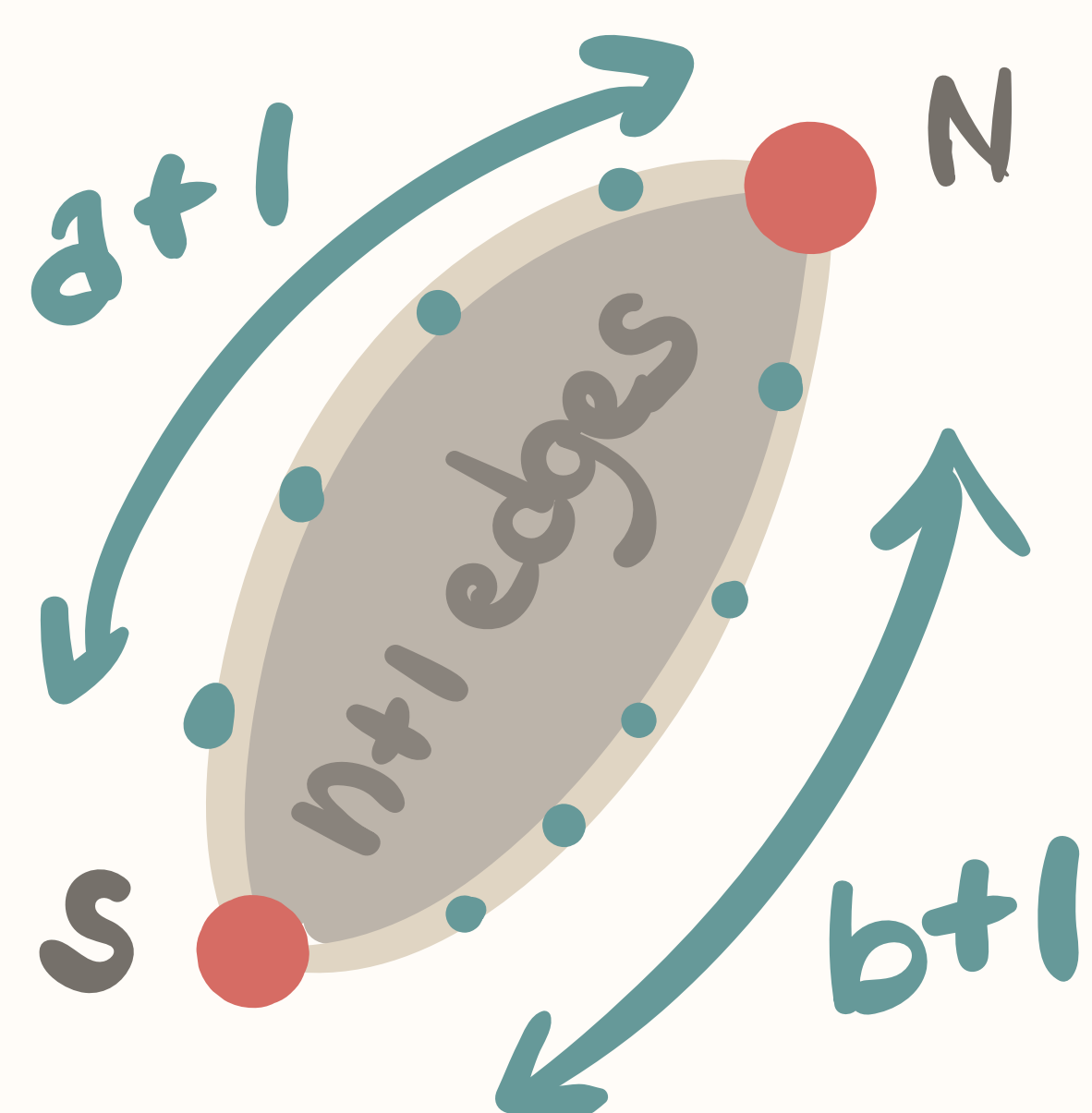
tandems walks in the quarter plane



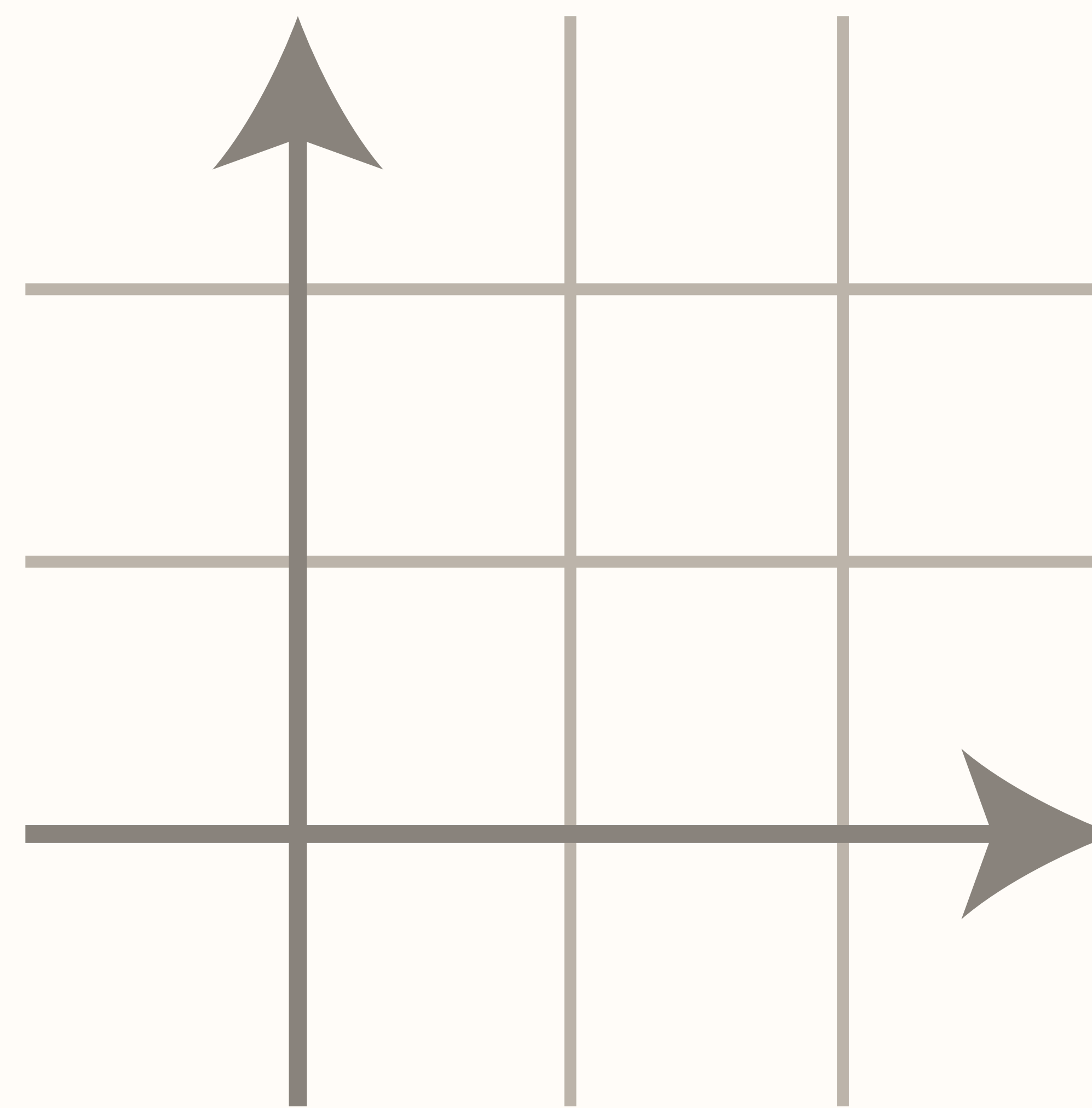
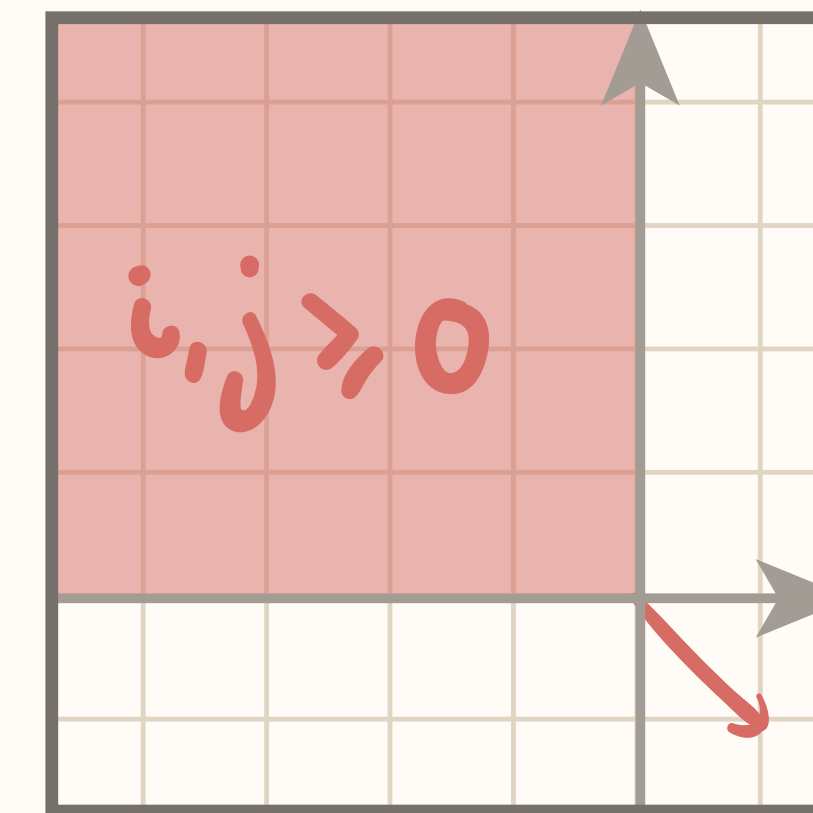
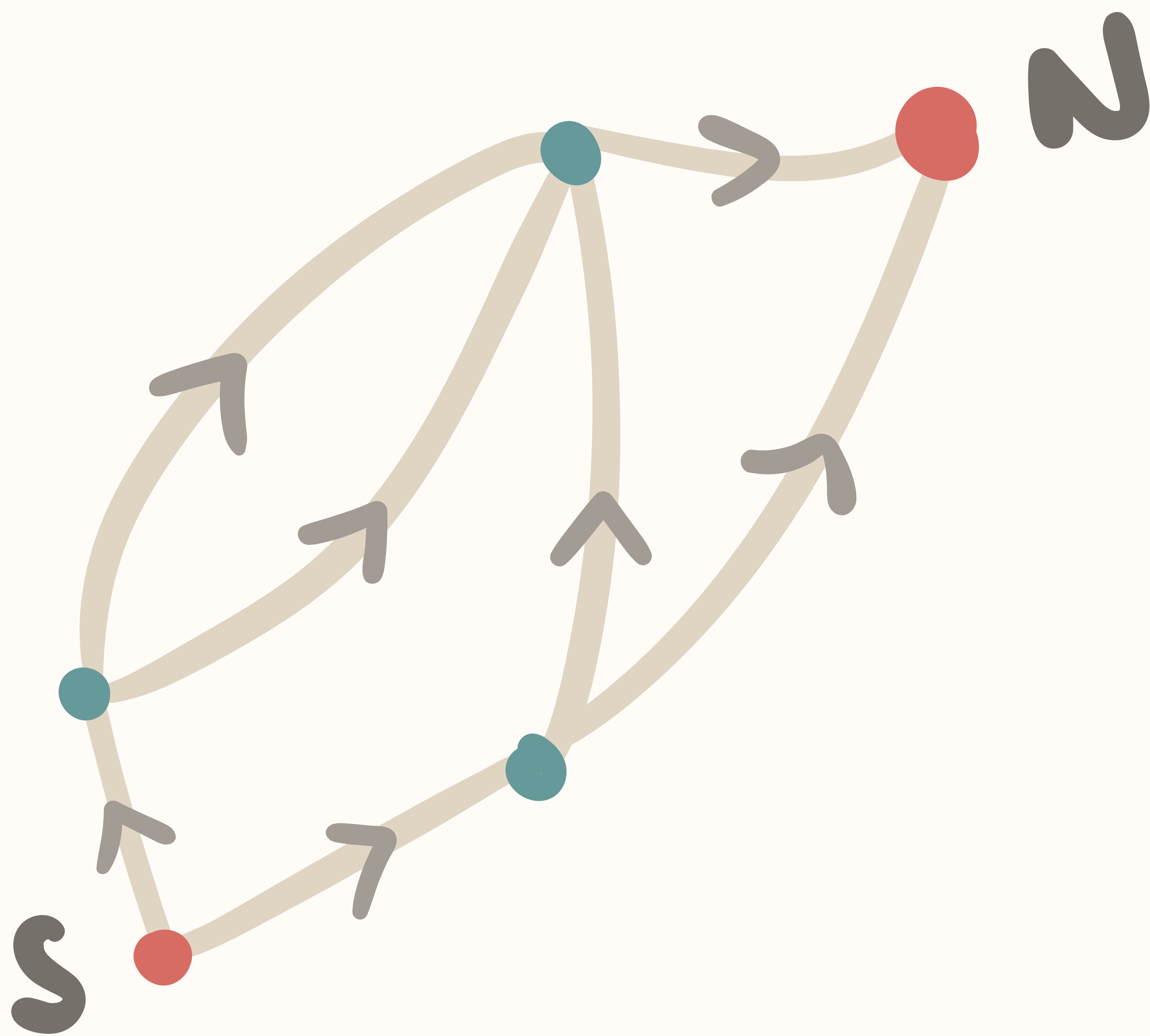
The KMSW bijection

Plane bipolar orientations

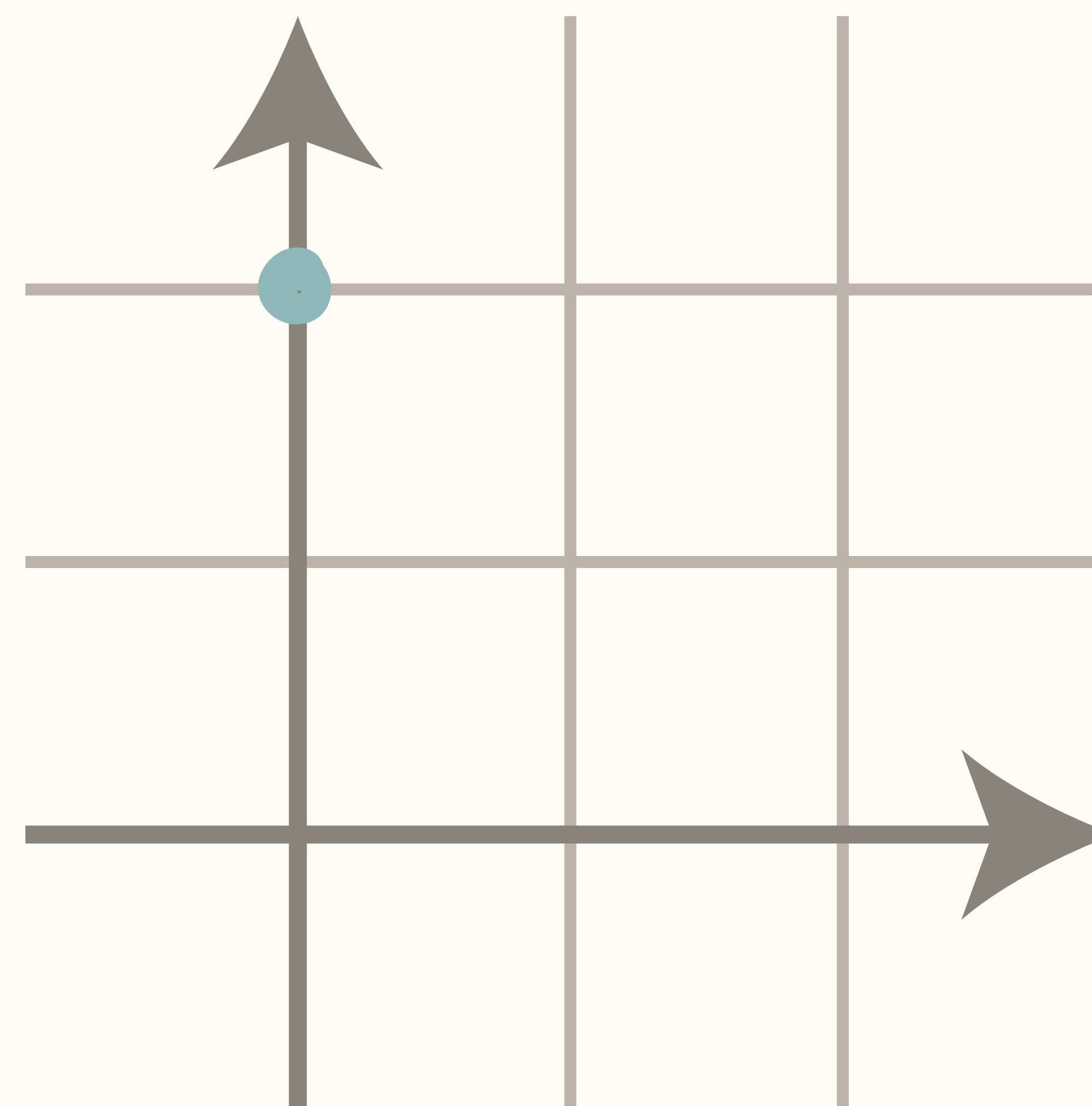
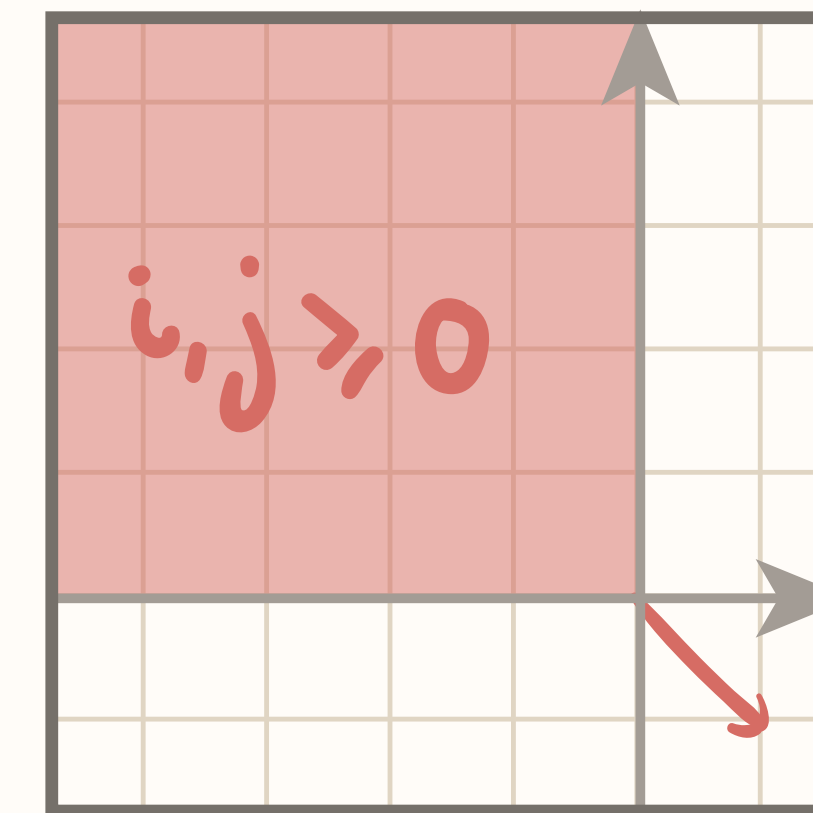
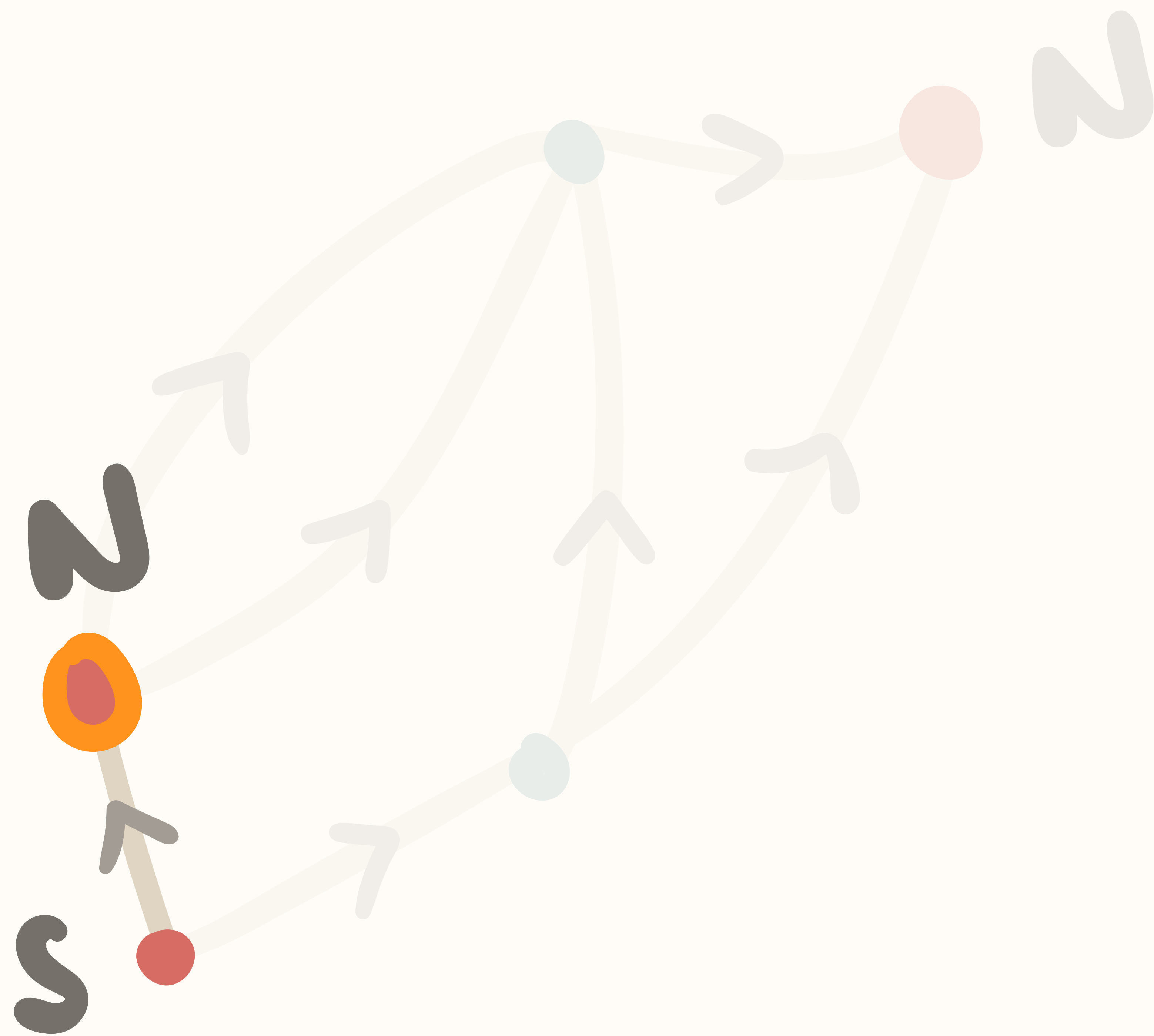
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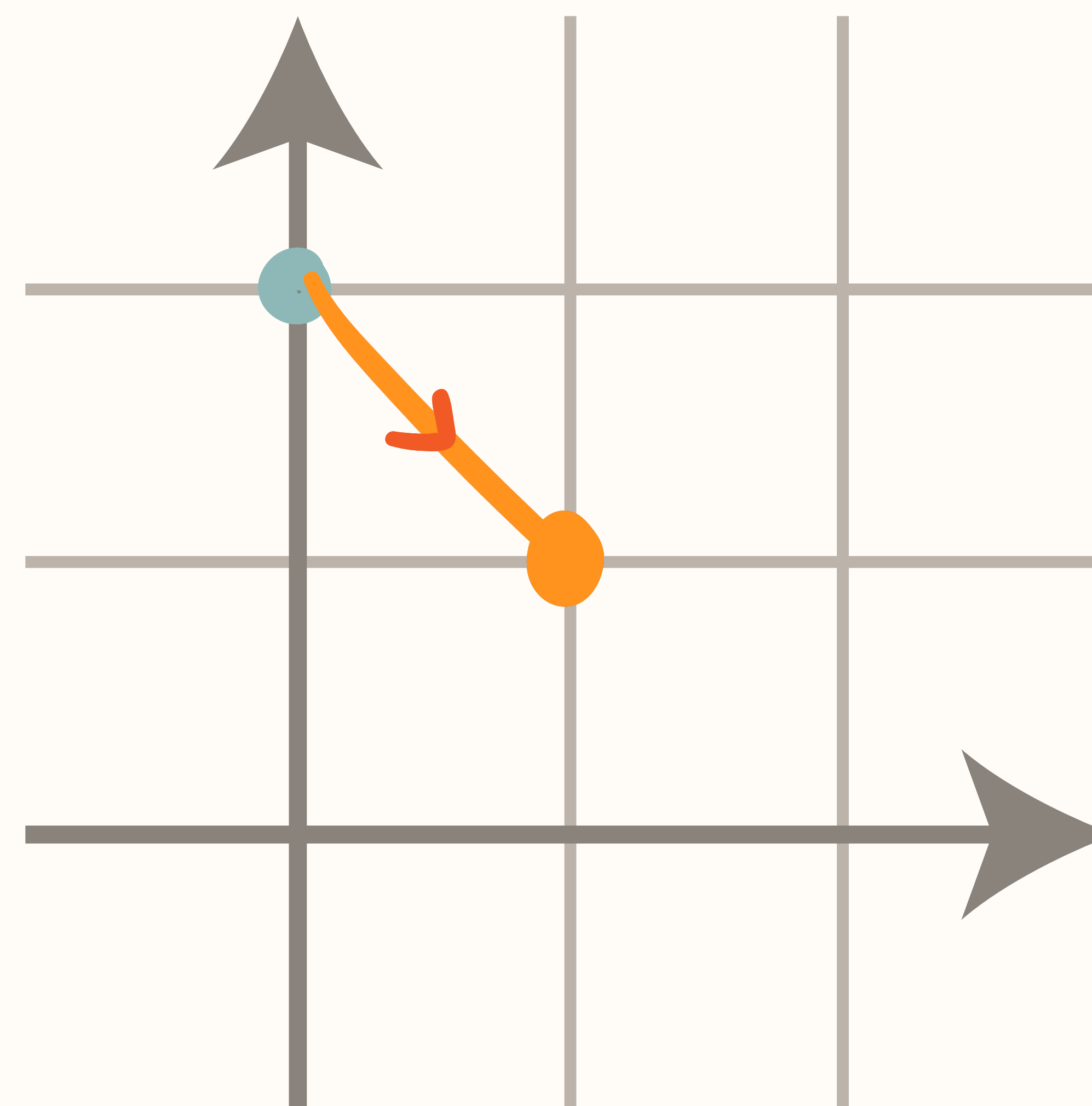
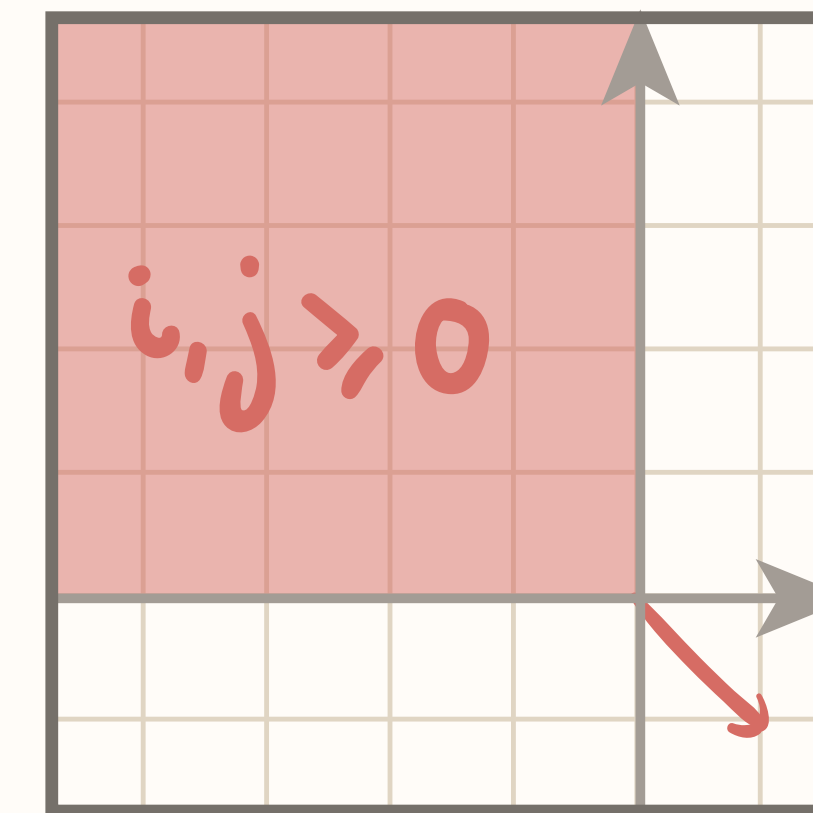
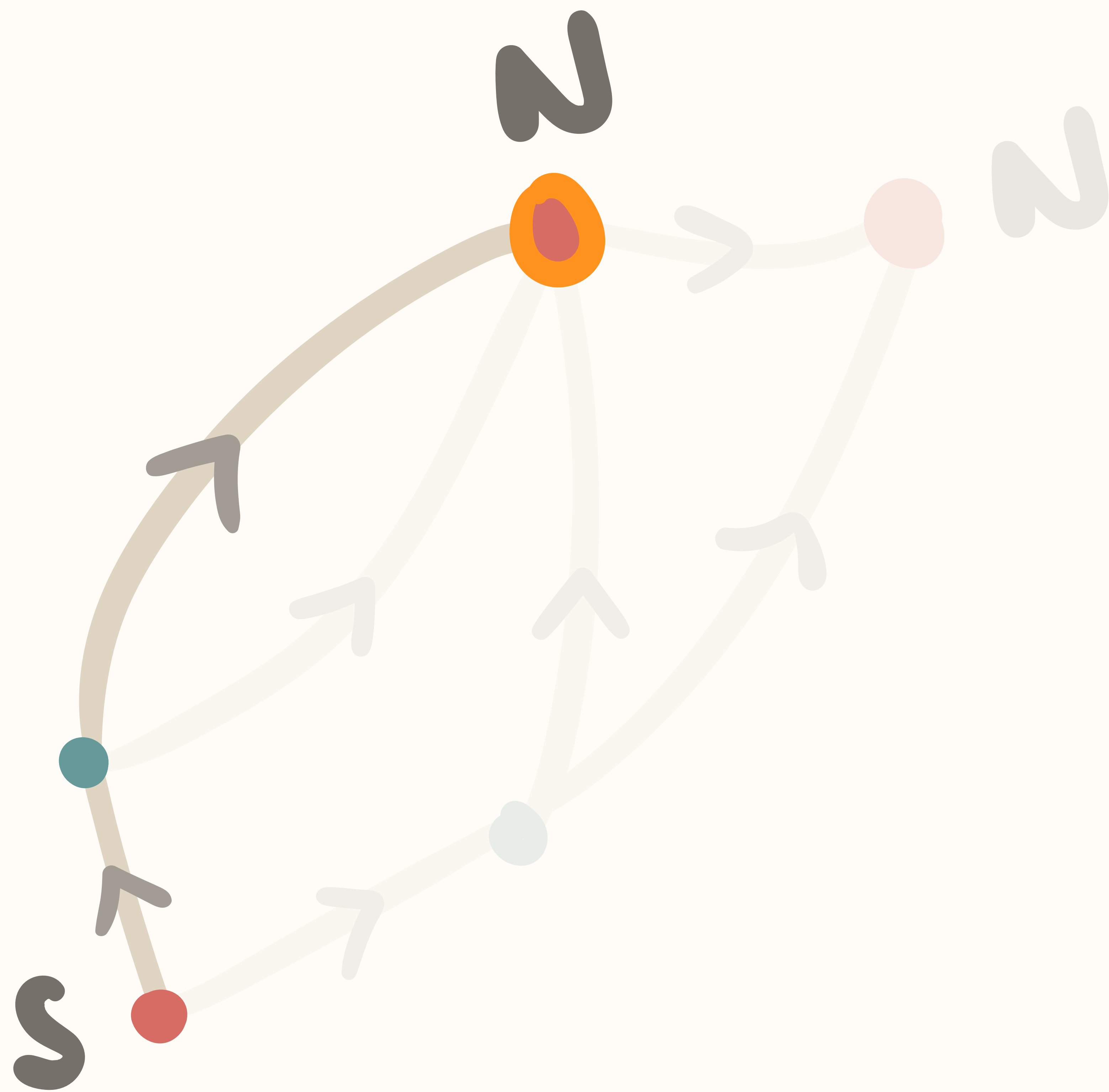
KMSW bijection example



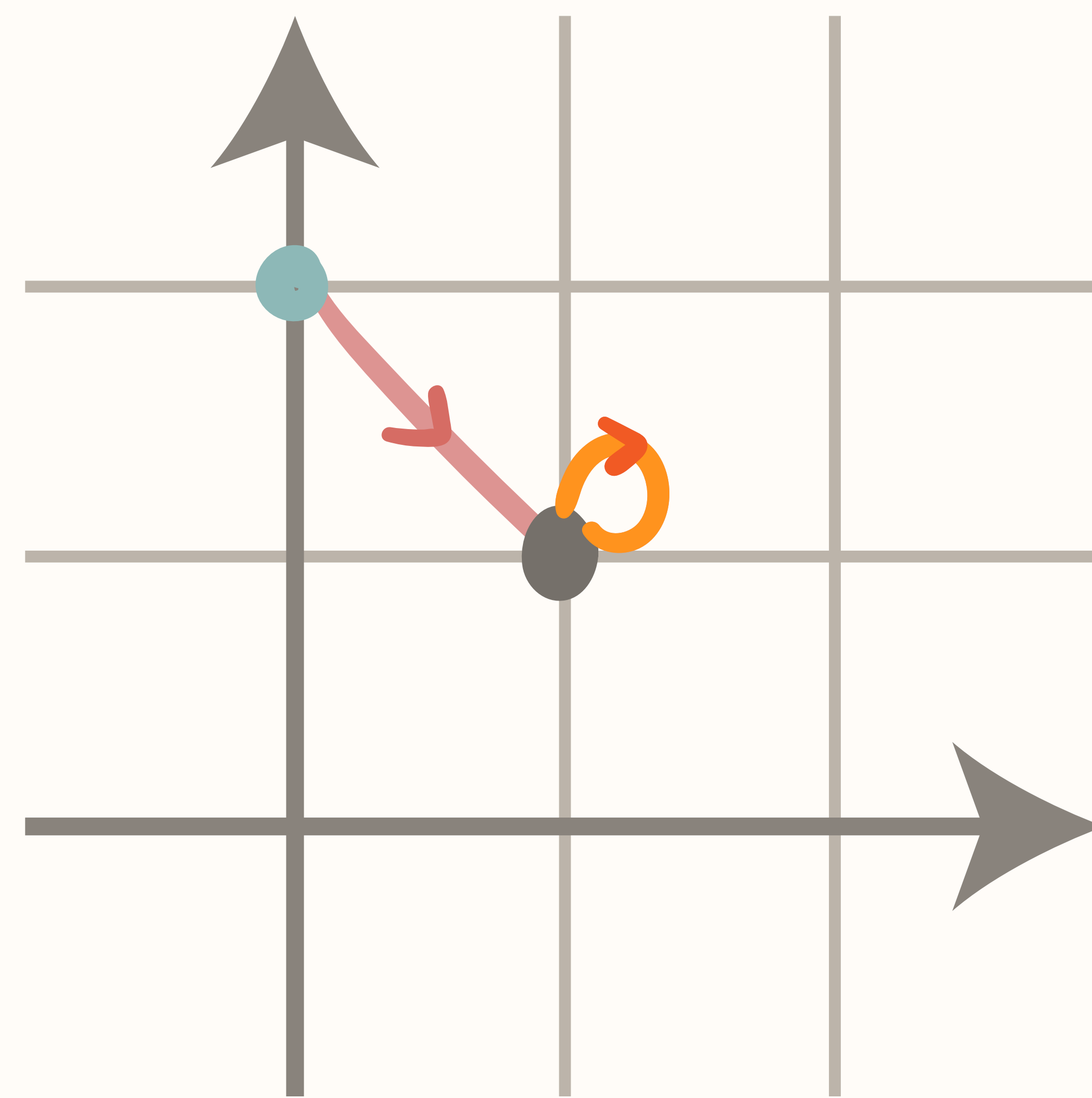
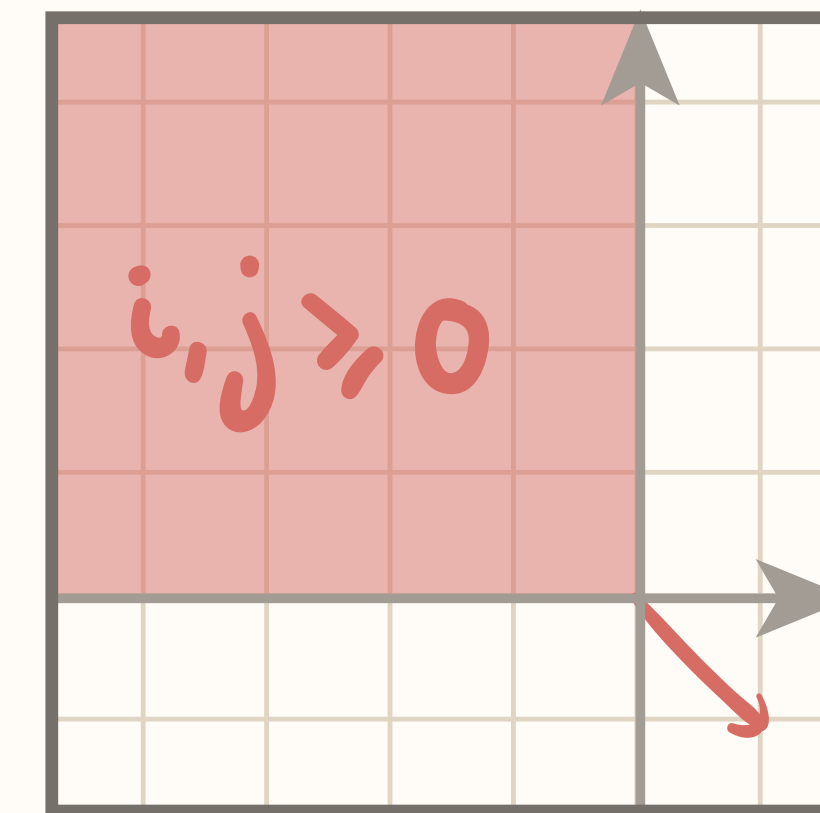
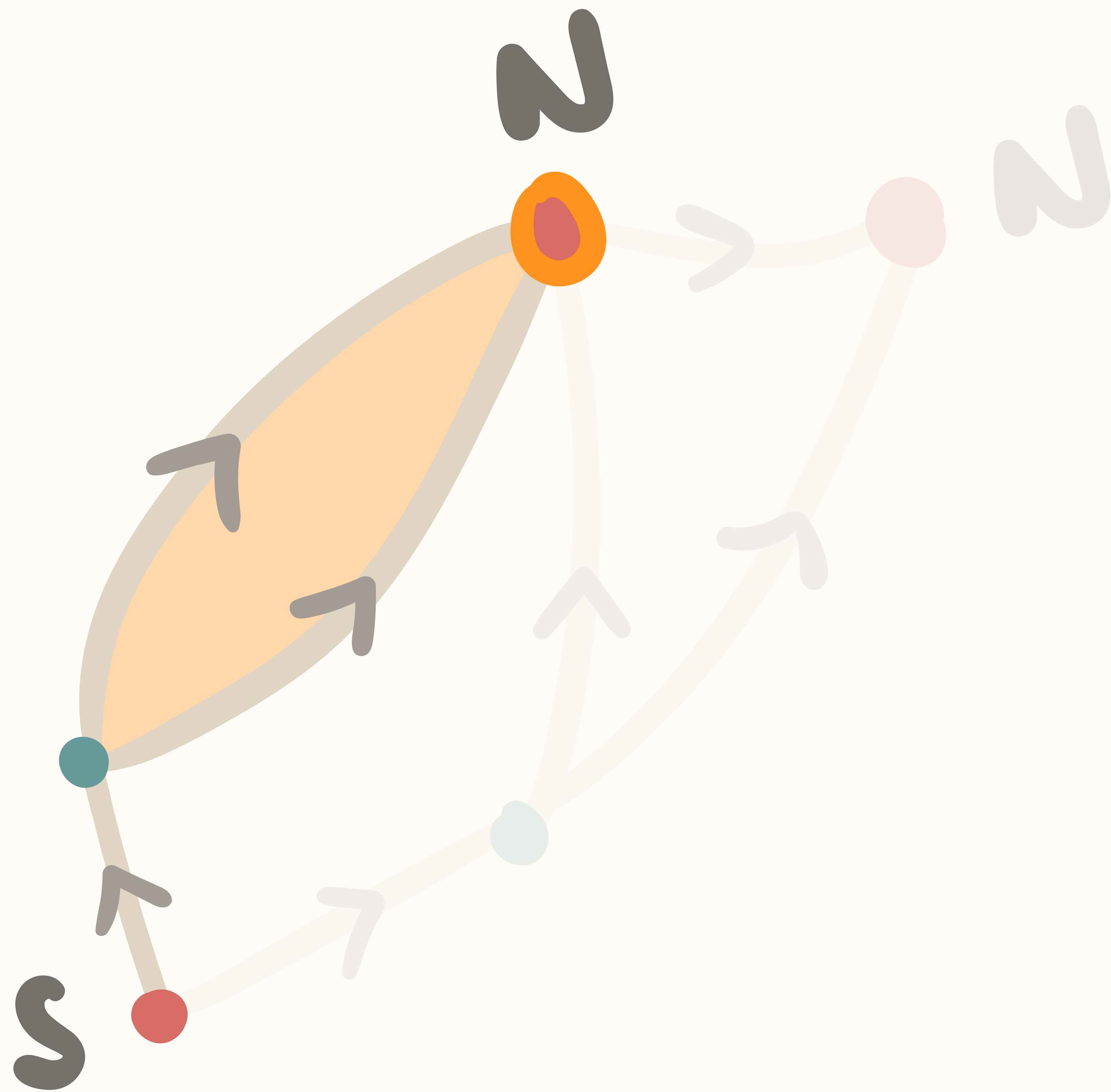
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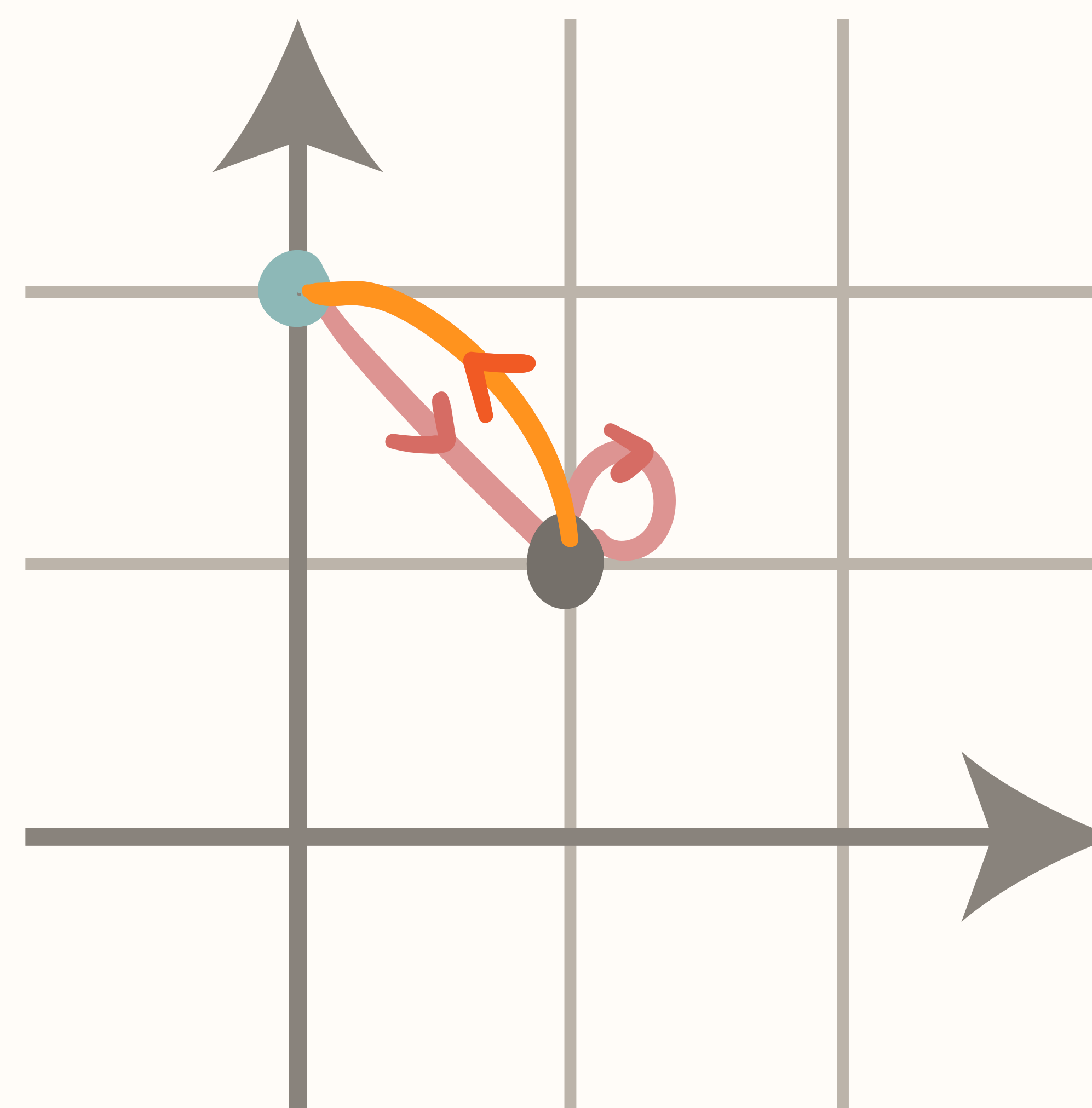
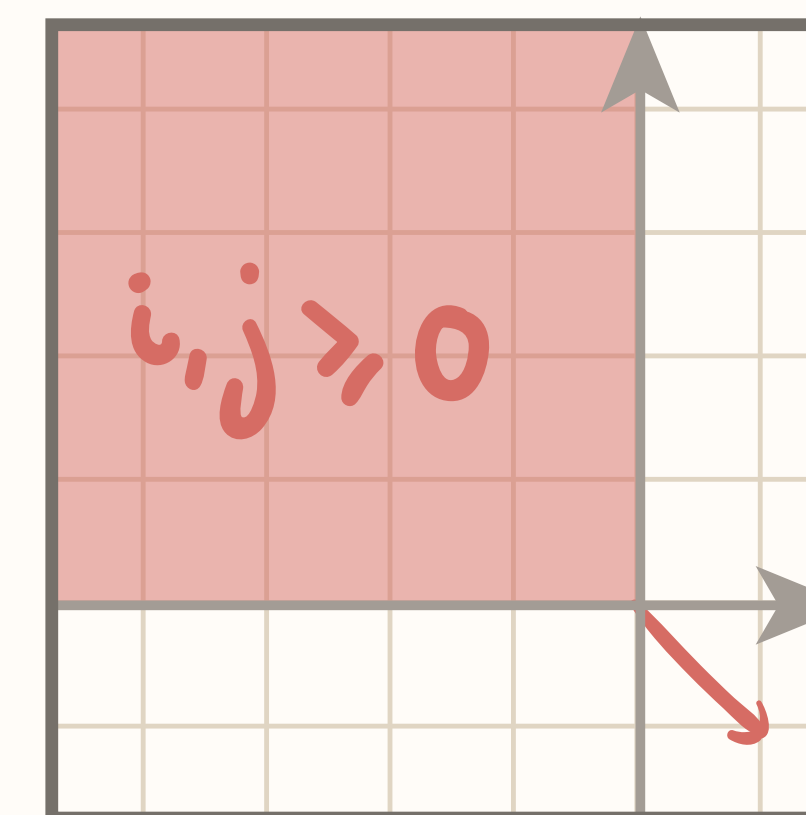
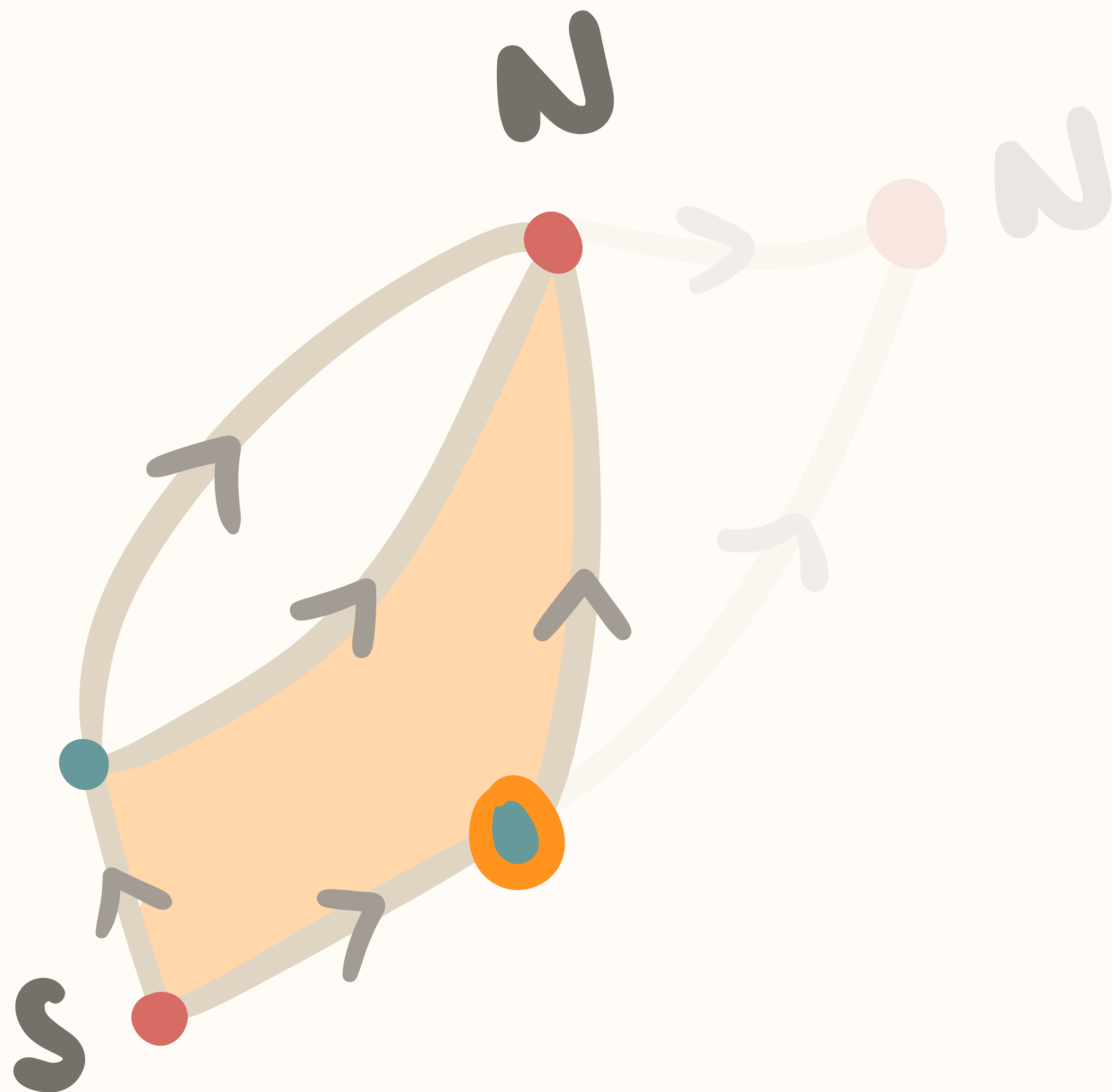
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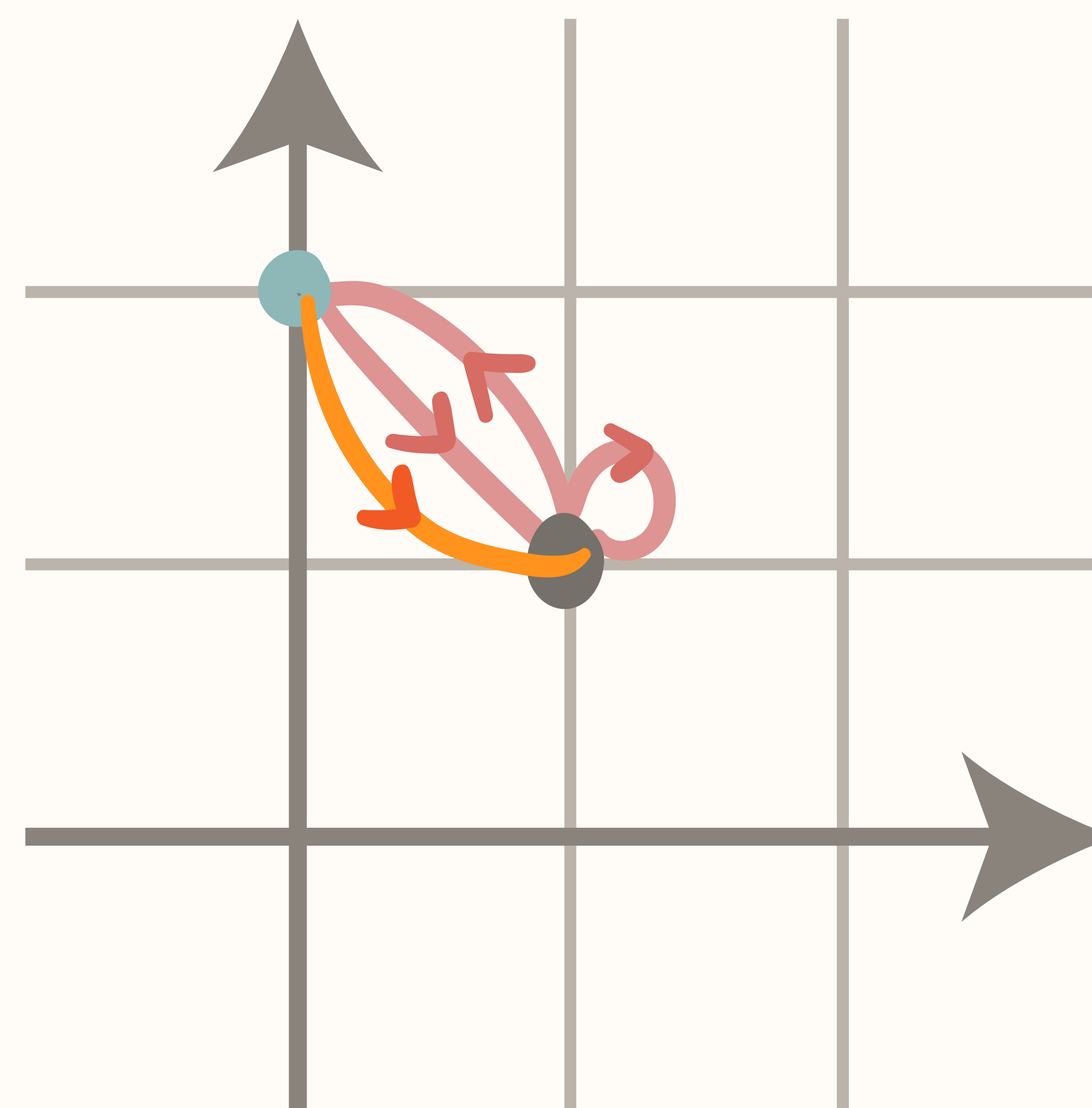
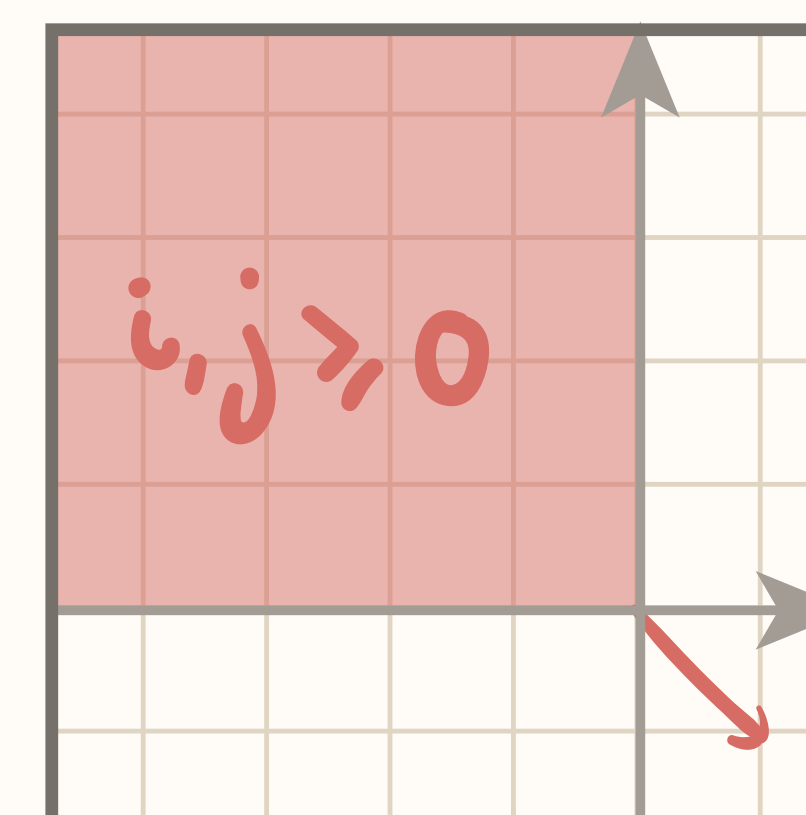
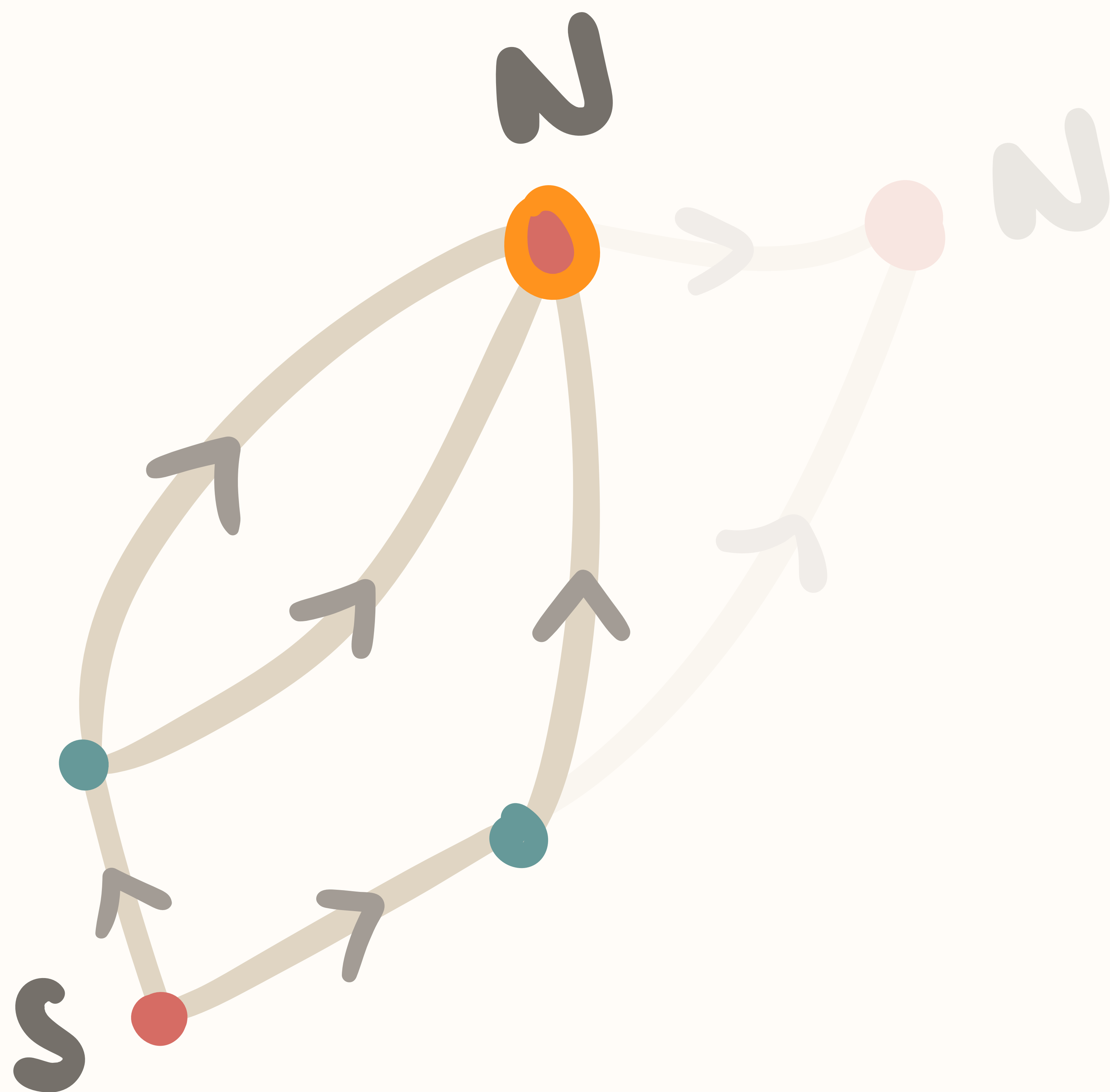
KMSW bijection example



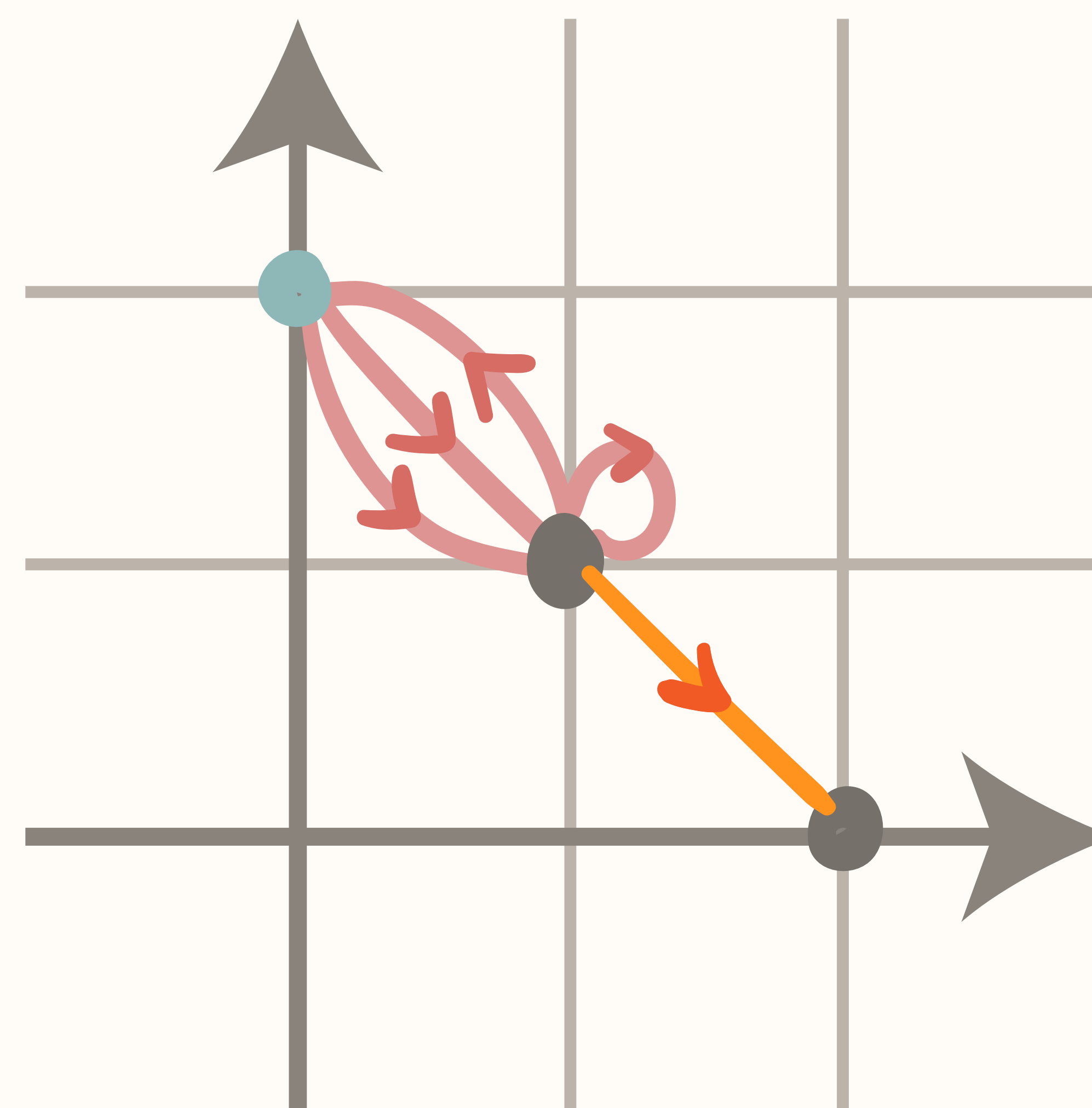
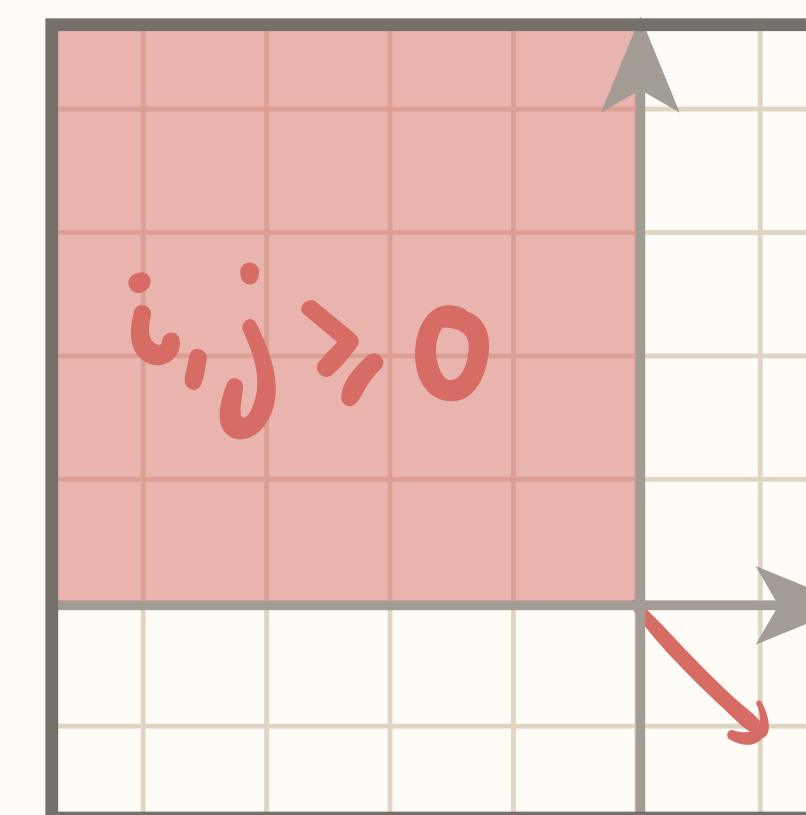
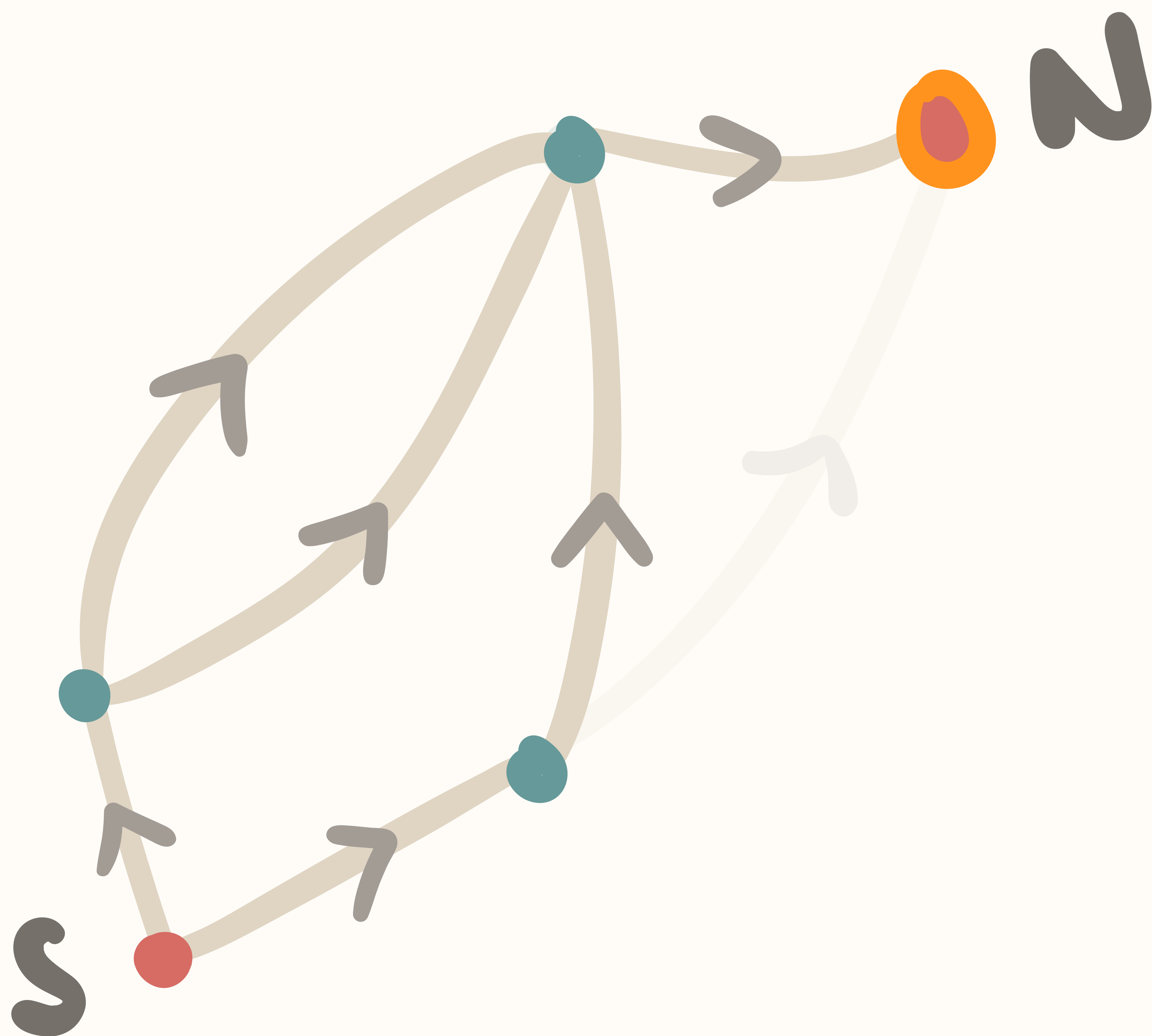
KMSW bijection example



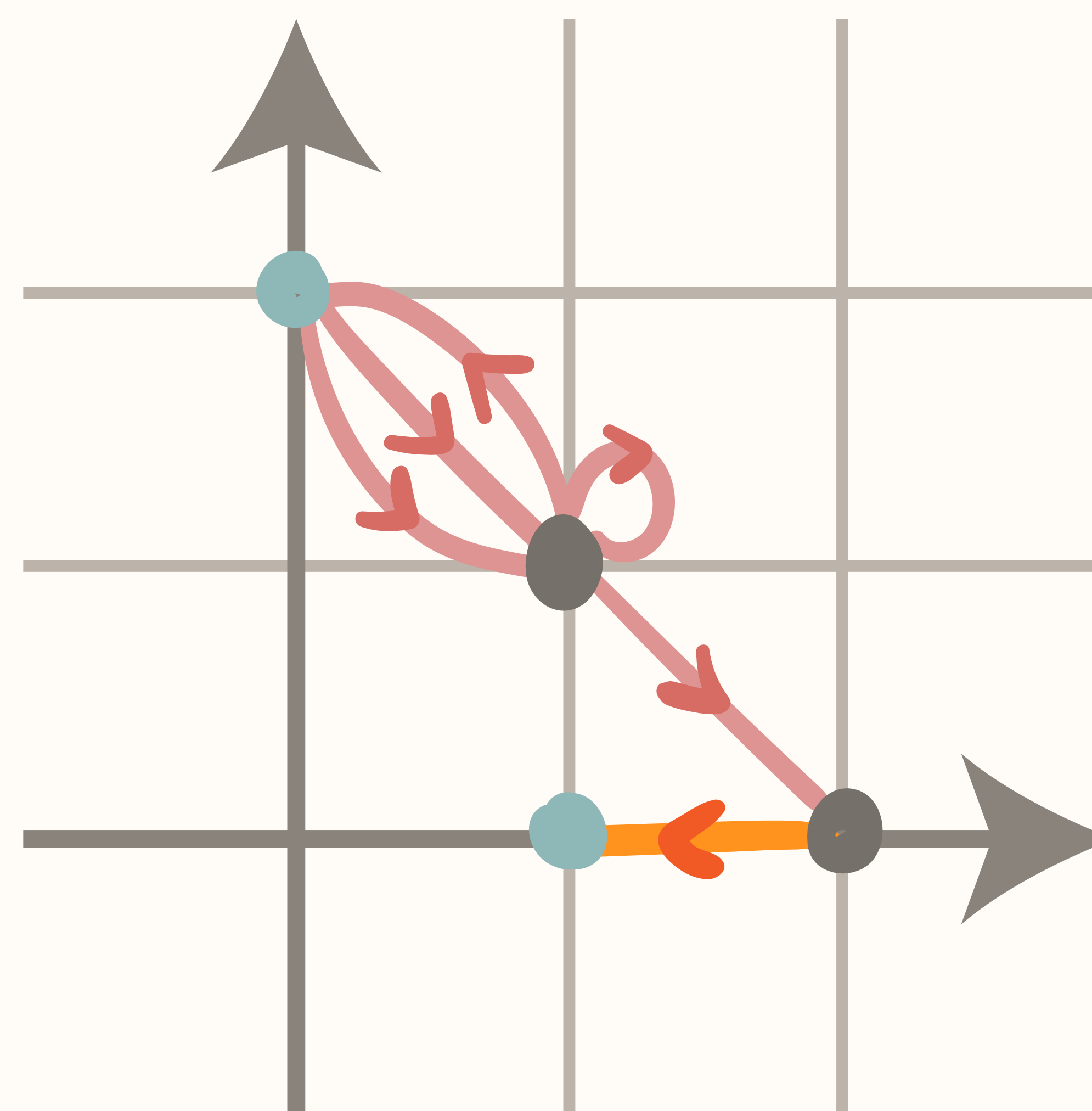
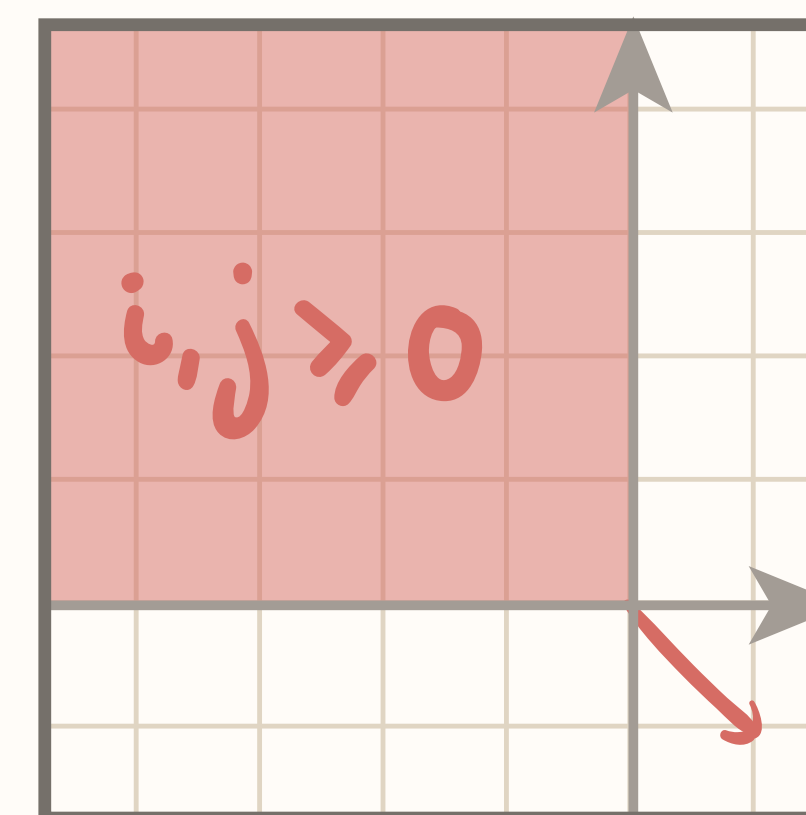
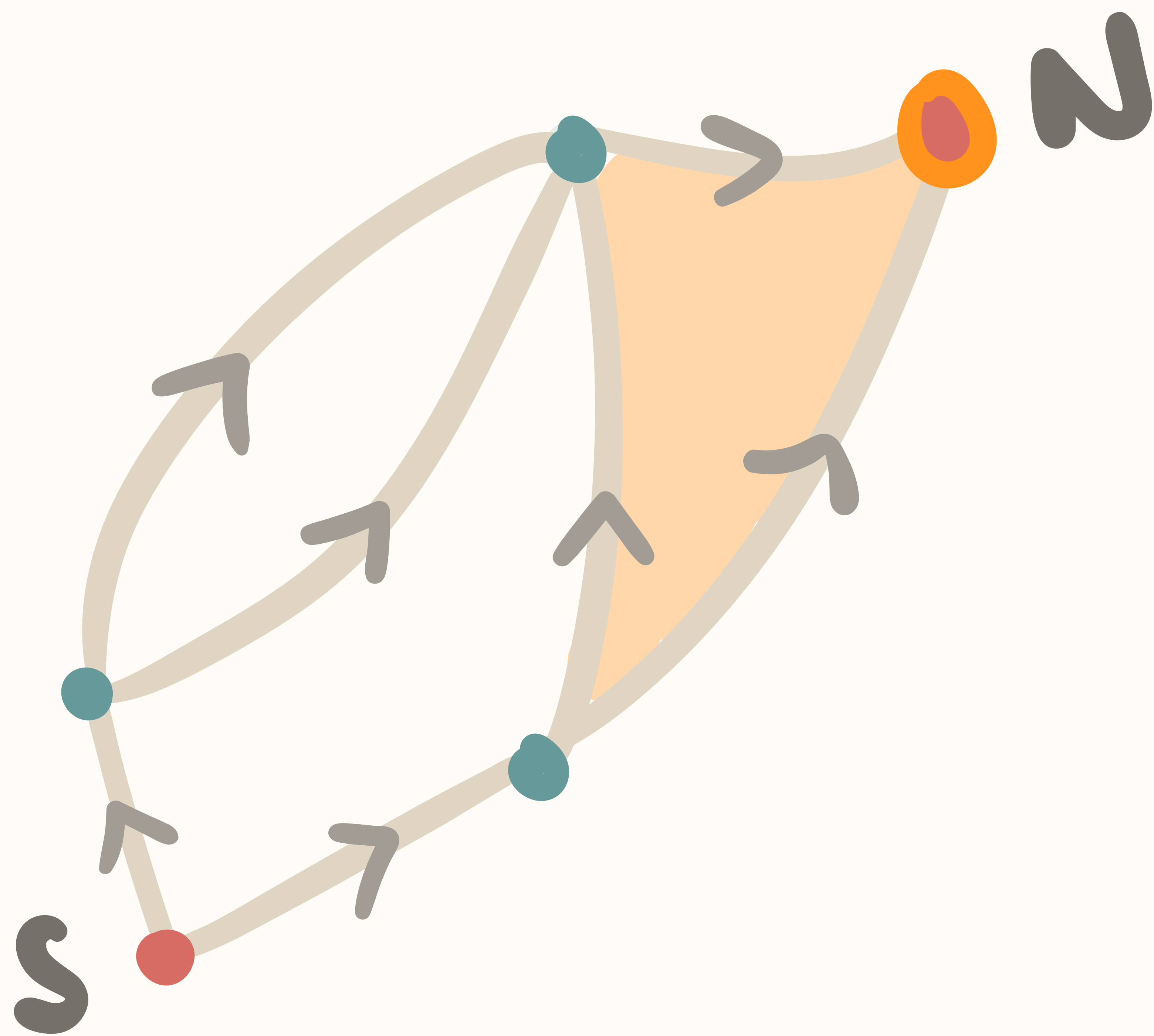
KMSW bijection example



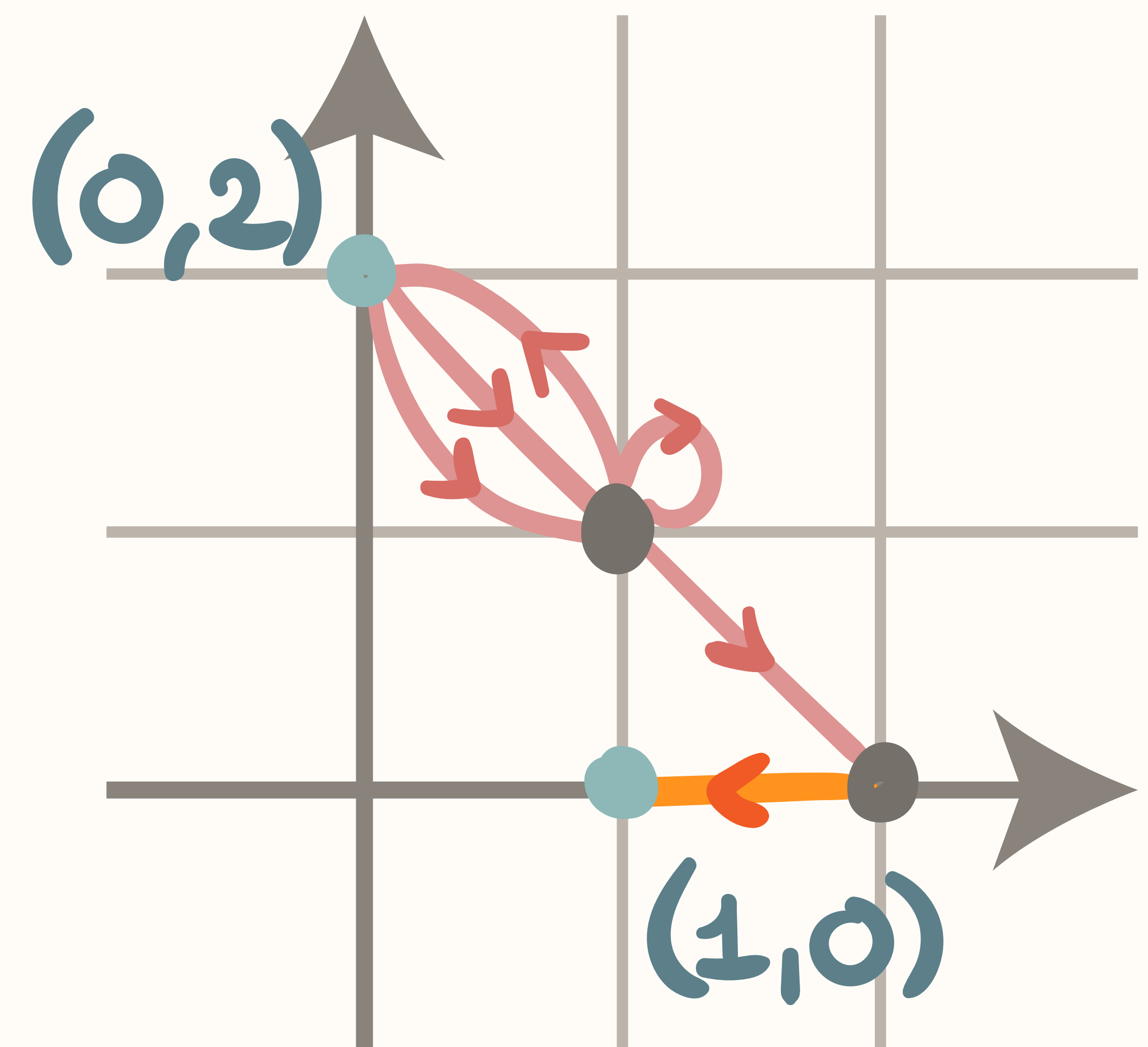
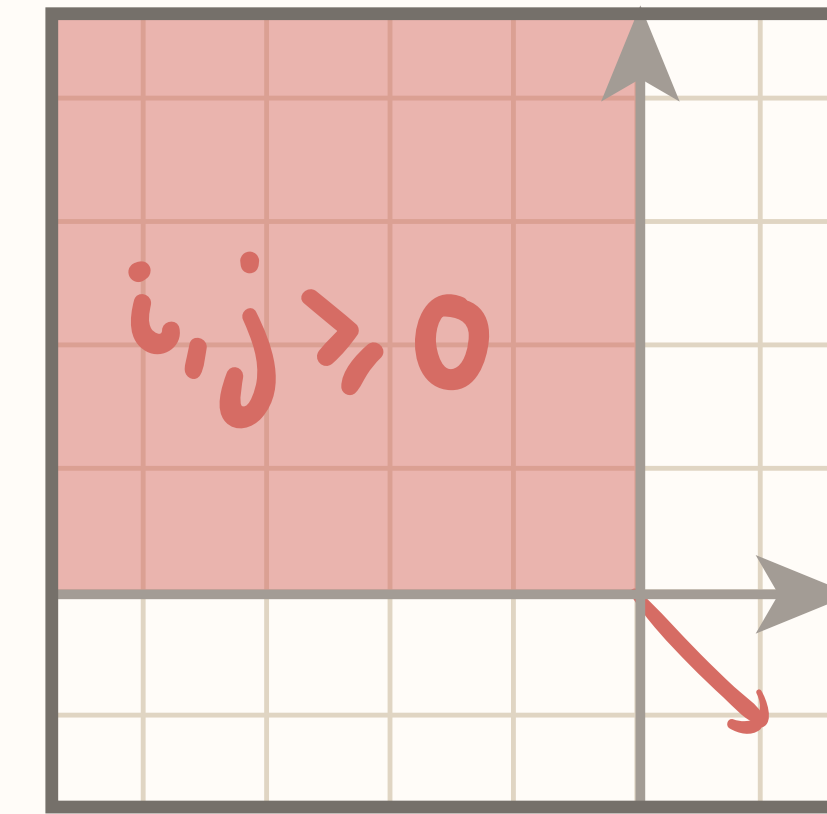
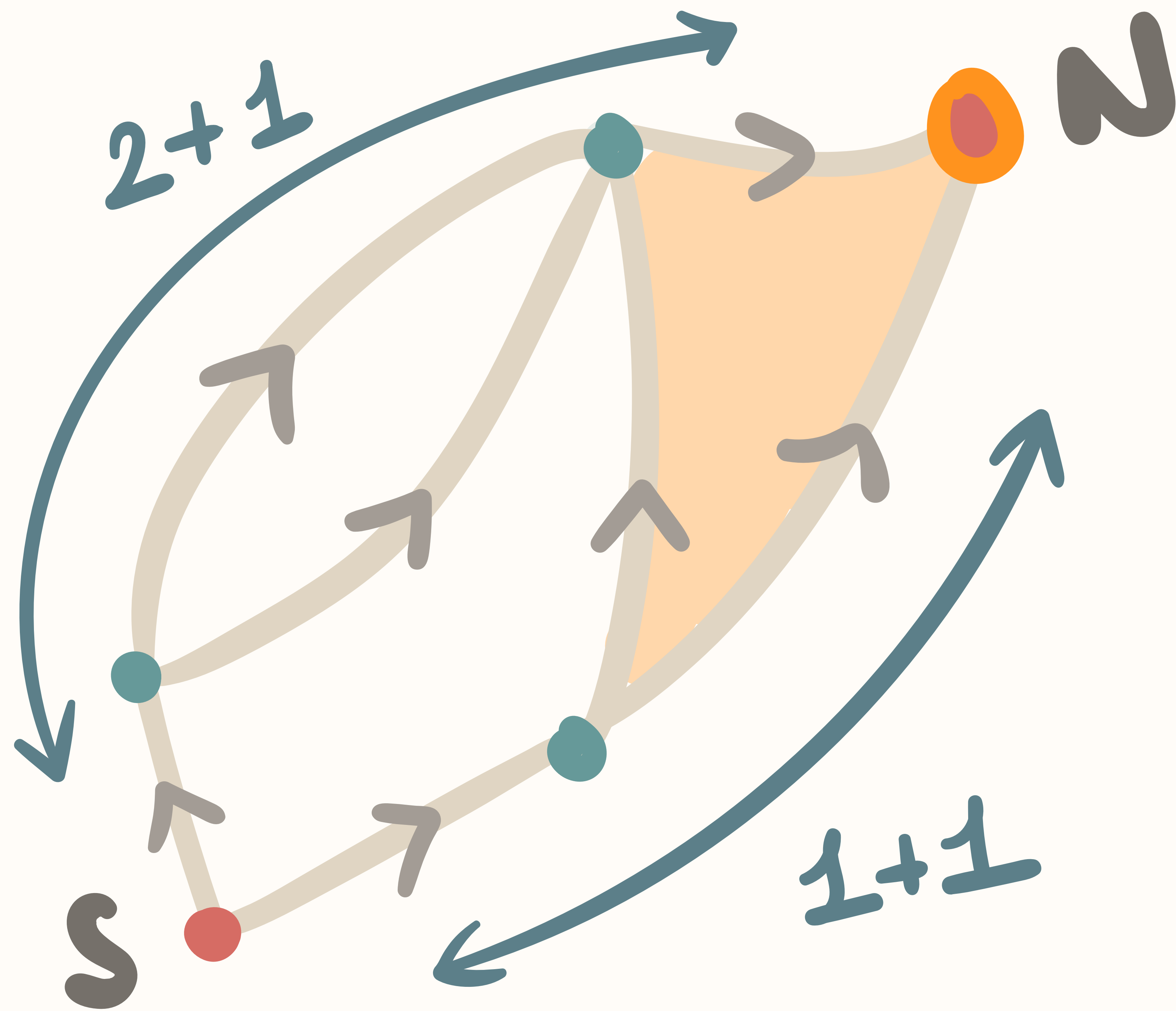
KMSW bijection example



KMSW bijection example

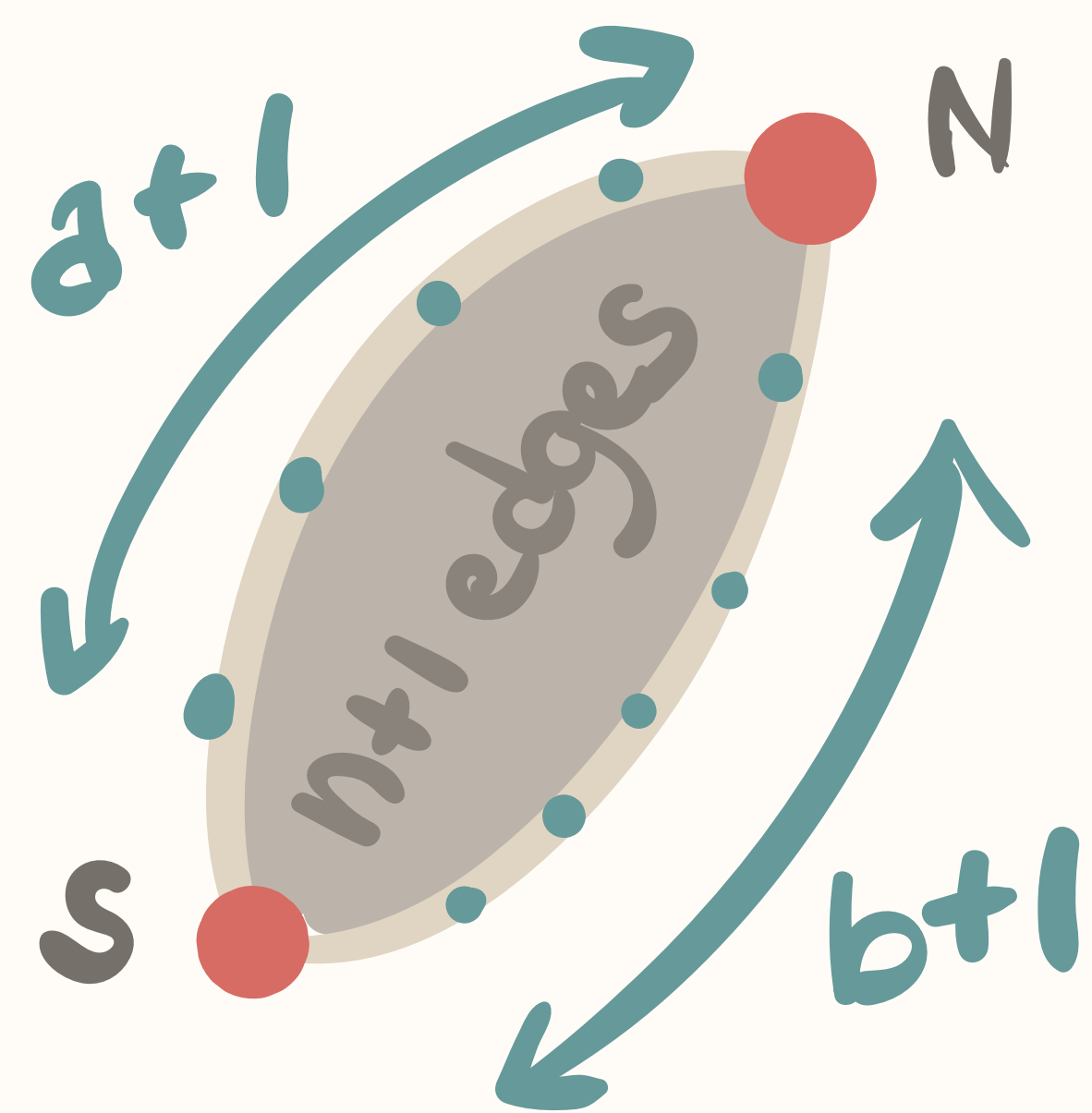


KMSW bijection example

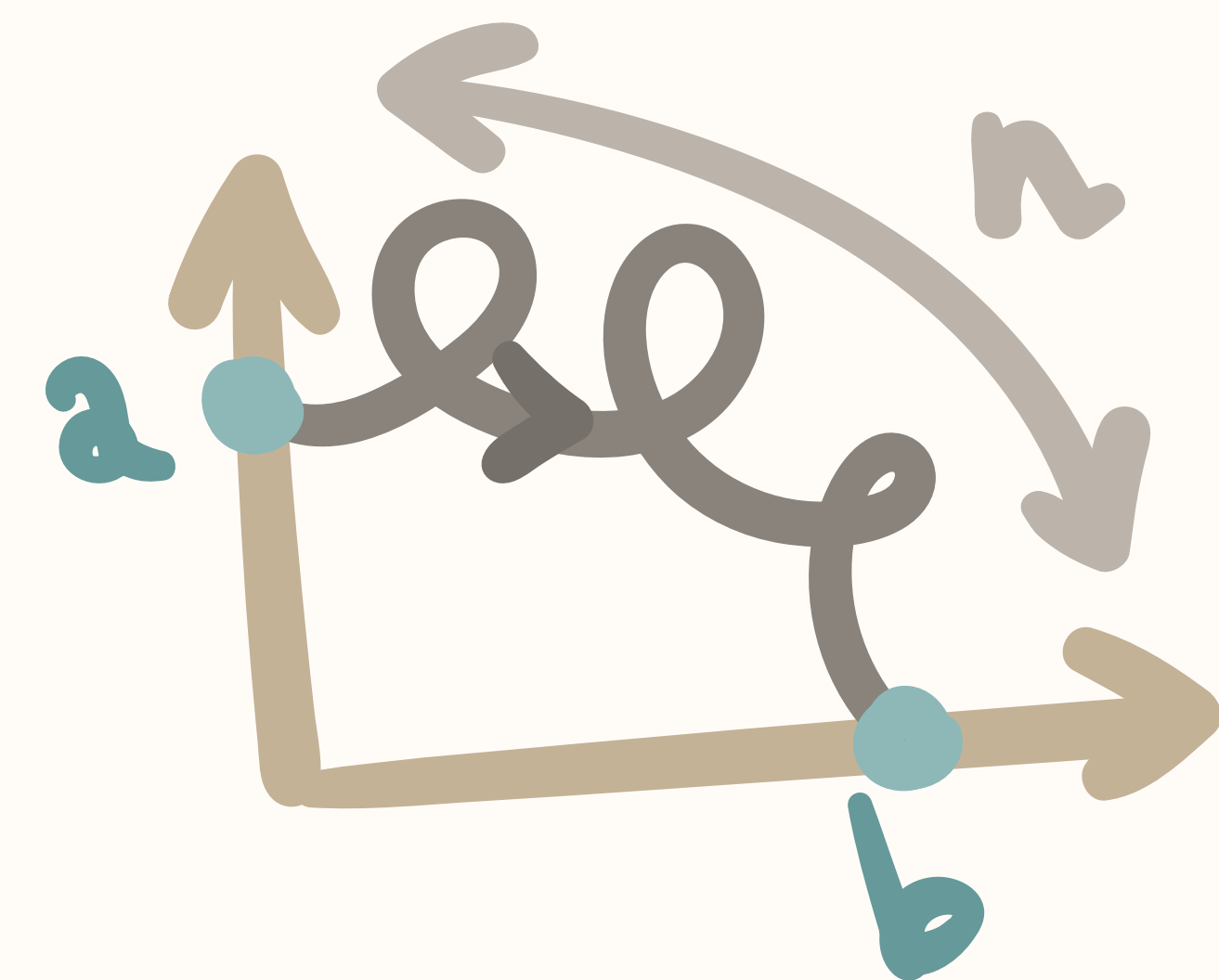
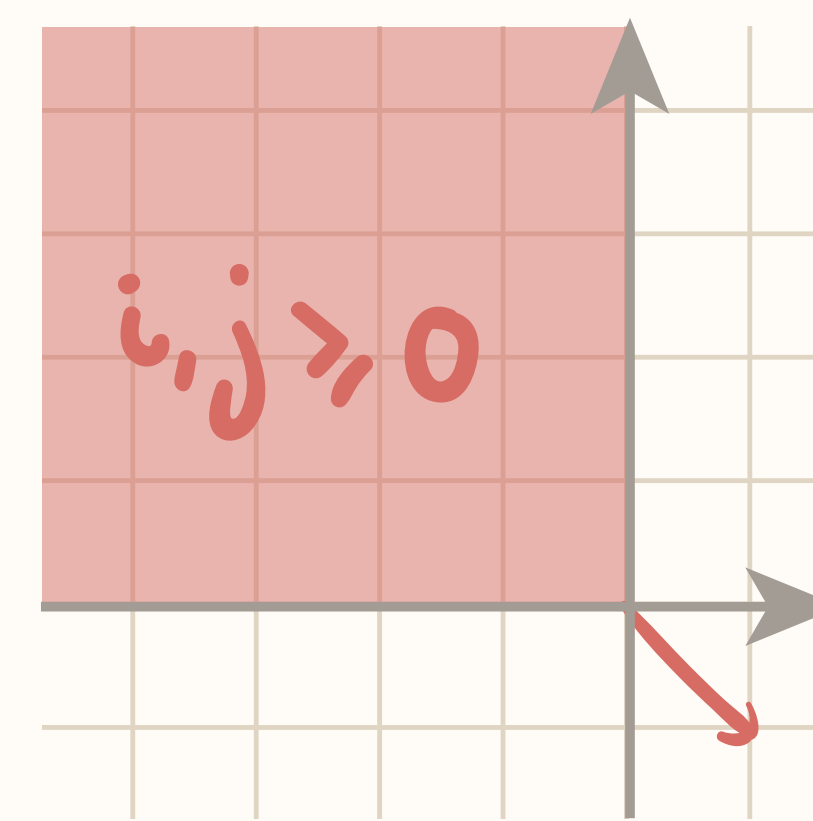


La bijection KMSW

bipolar orientations

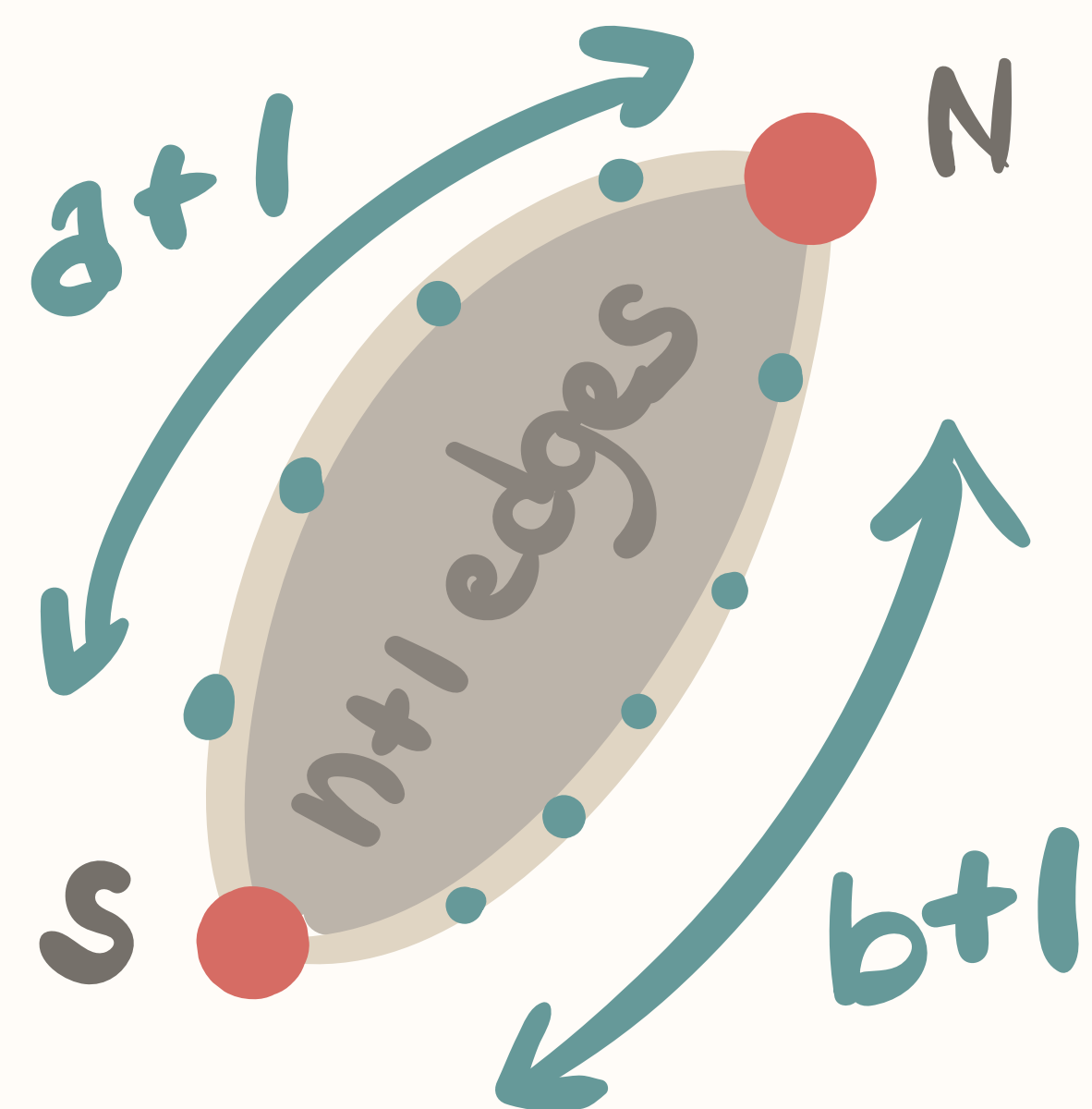


tandem walks in the quarter plane

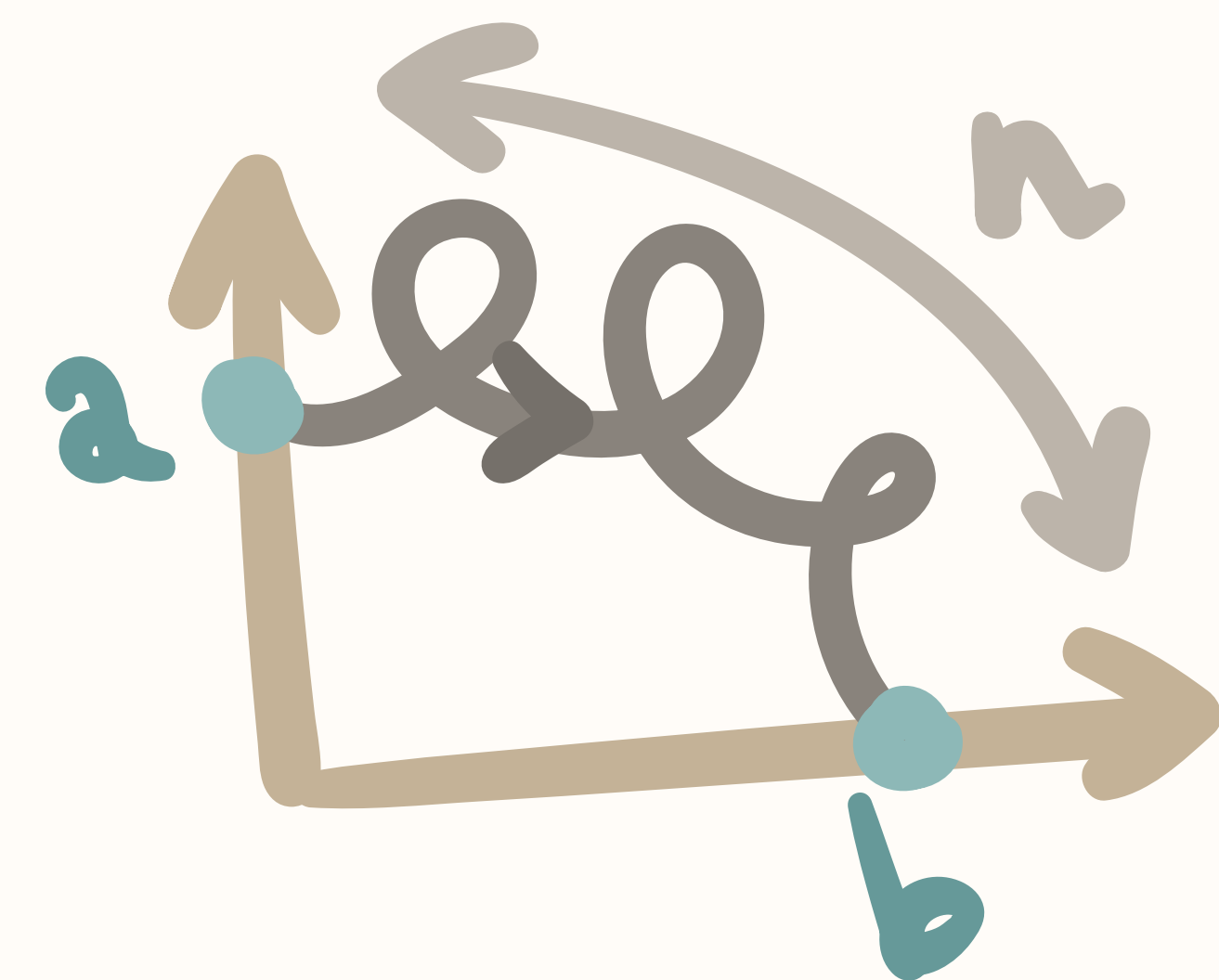
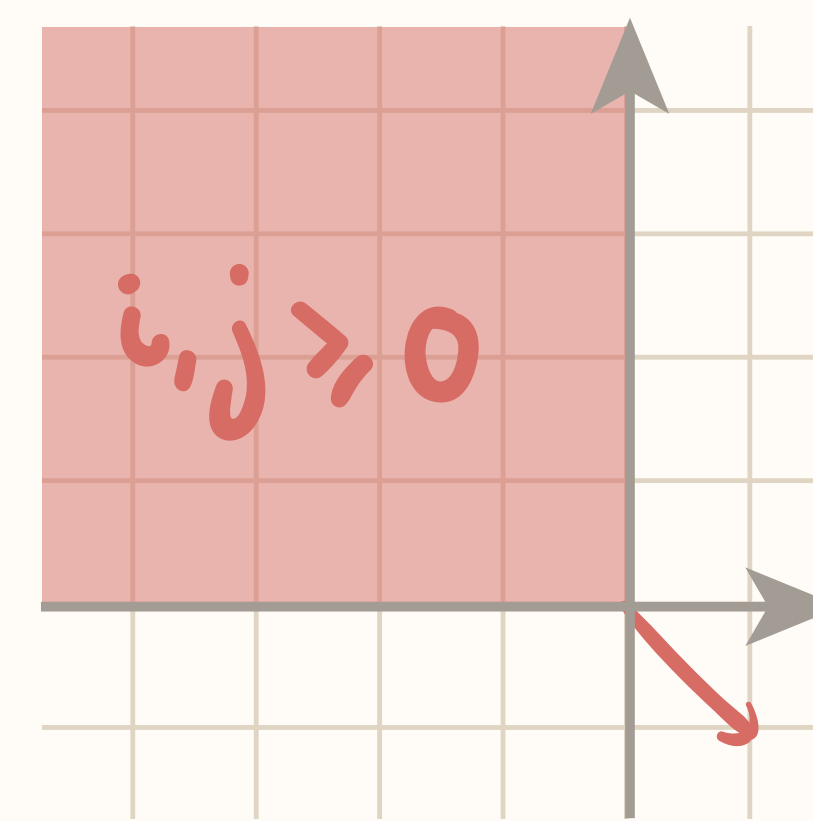


La bijection KMSW

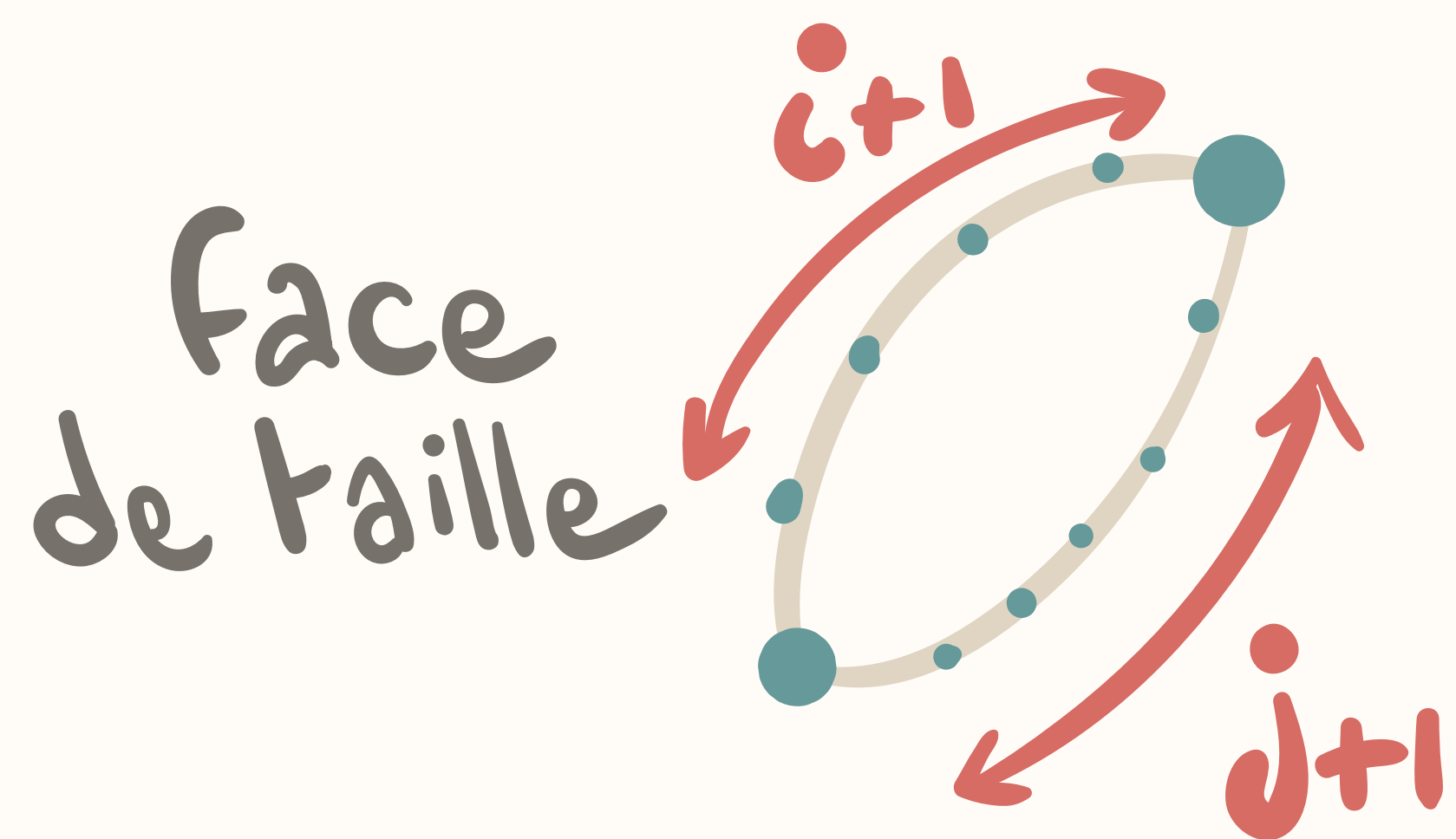
bipolar orientations



tandem walks in the quarter plane



→ *Bipolar orientations on planar maps and $SLE_{1,2}$, R. Kenyon, J. Miller, S. Sheffield and D. Wilson (2015)*



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a. Plane bipolar posets

b. Plane bipolar posets by vertices

c. Transversal structures

d. Asymptotics

2. Interlude : plane permutations

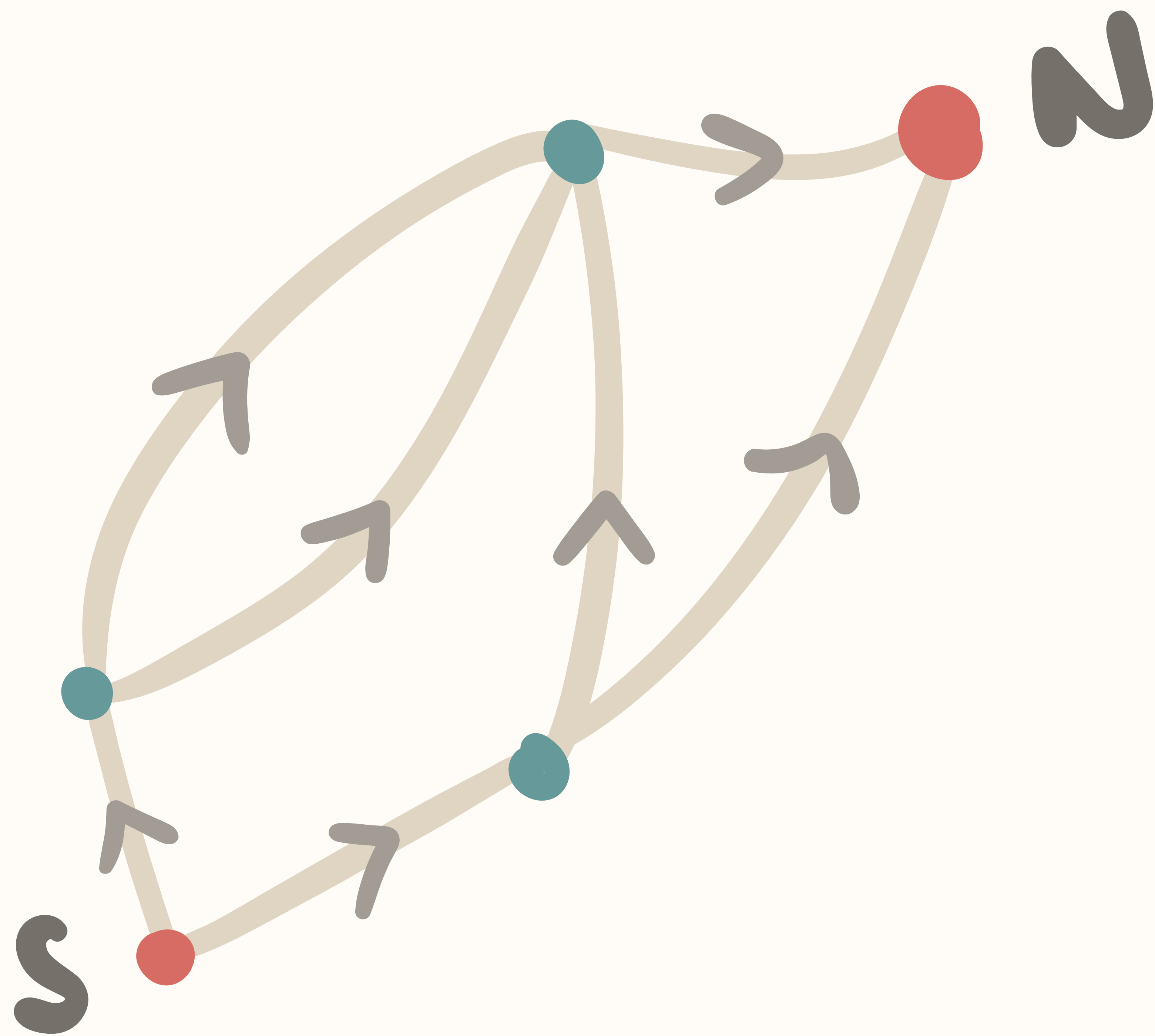
3. Application to corner polyhedra

a. Via polyhedral orientations

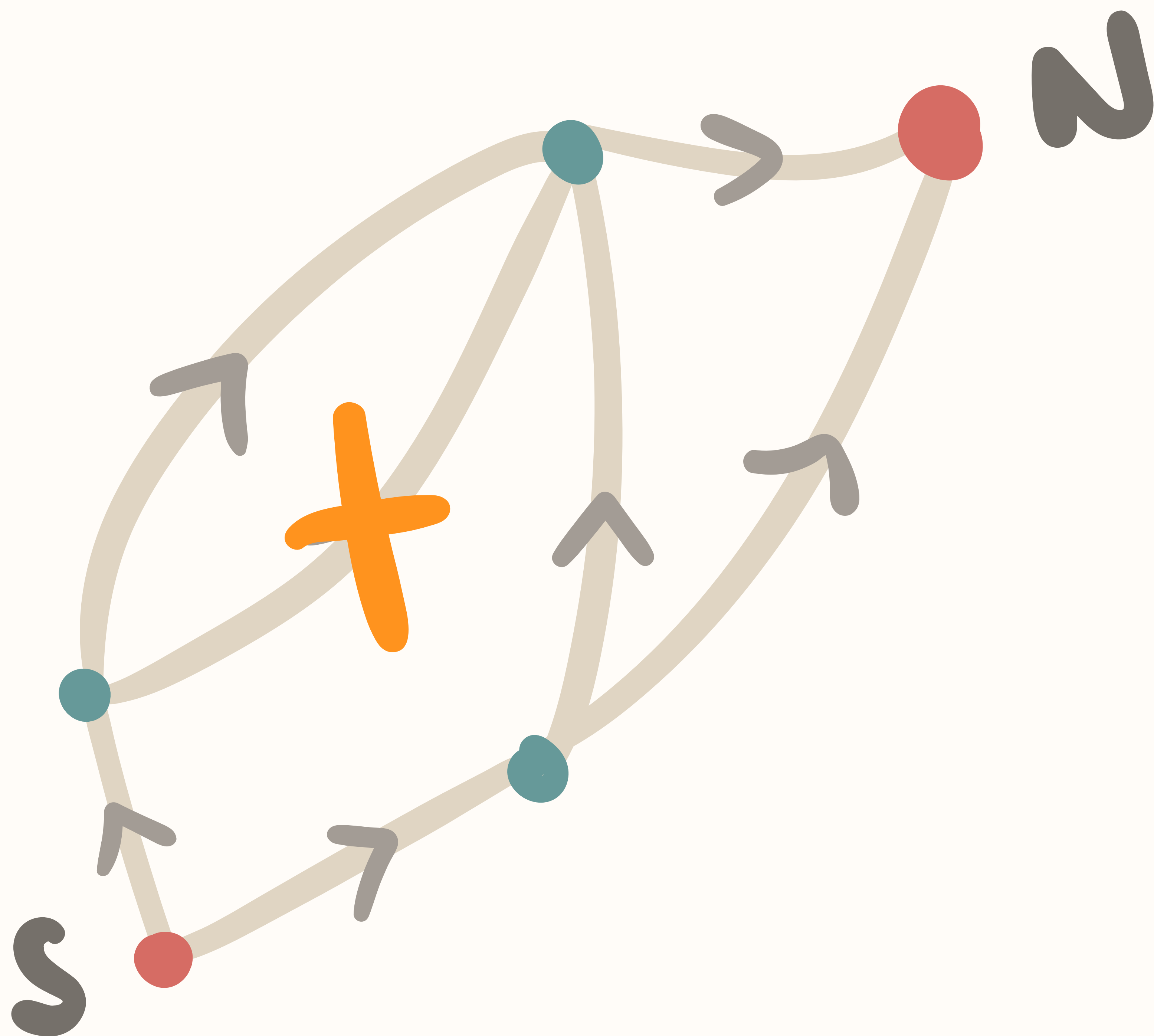
b. Via Schnyder colorings

c. Asymptotics

Plane bipolar poset



Plane bipolar poset



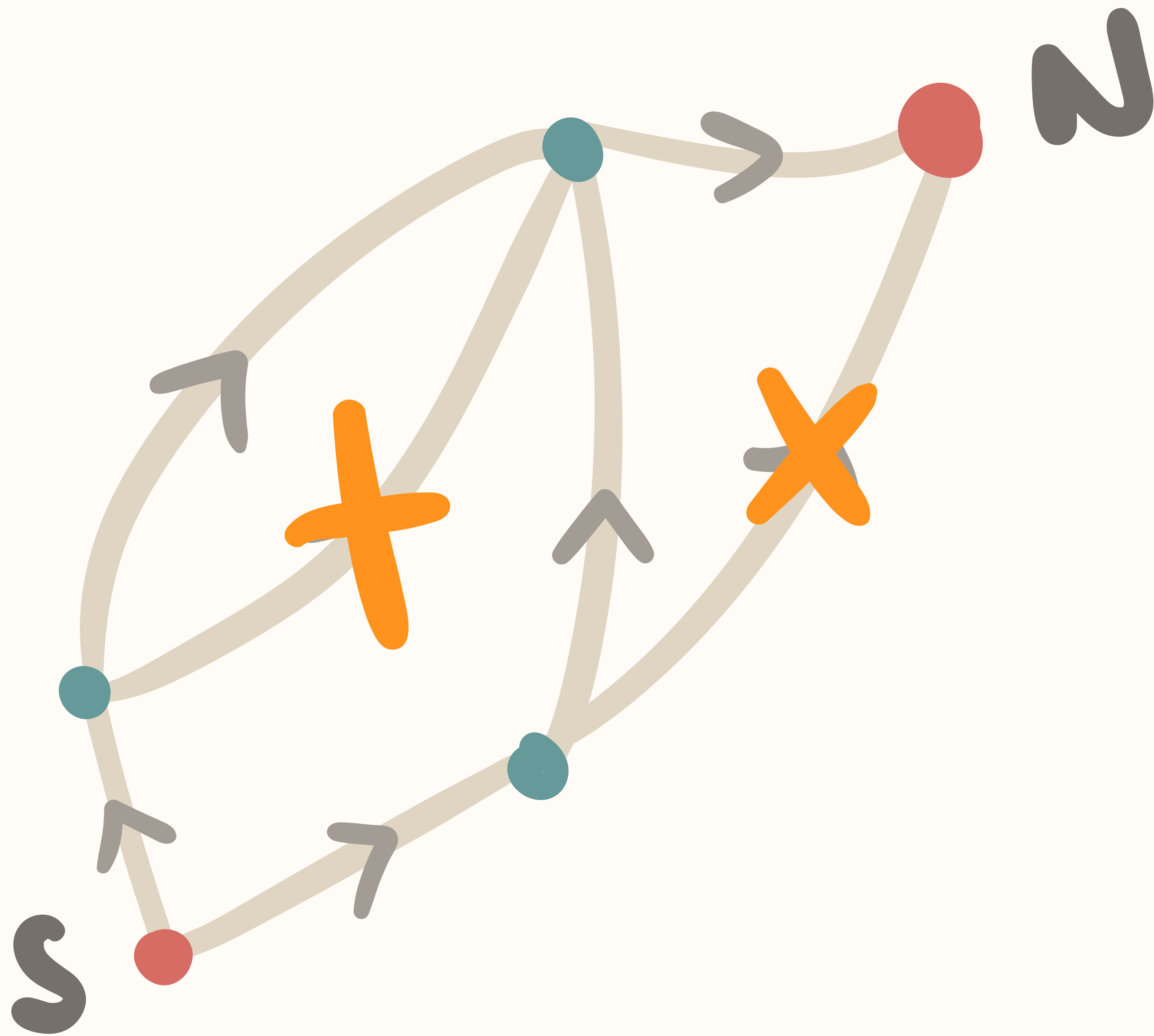
Poset

(plane bipolar poset)

= **Bipolar
orientation**

No multiple edge

Plane bipolar poset



Poset

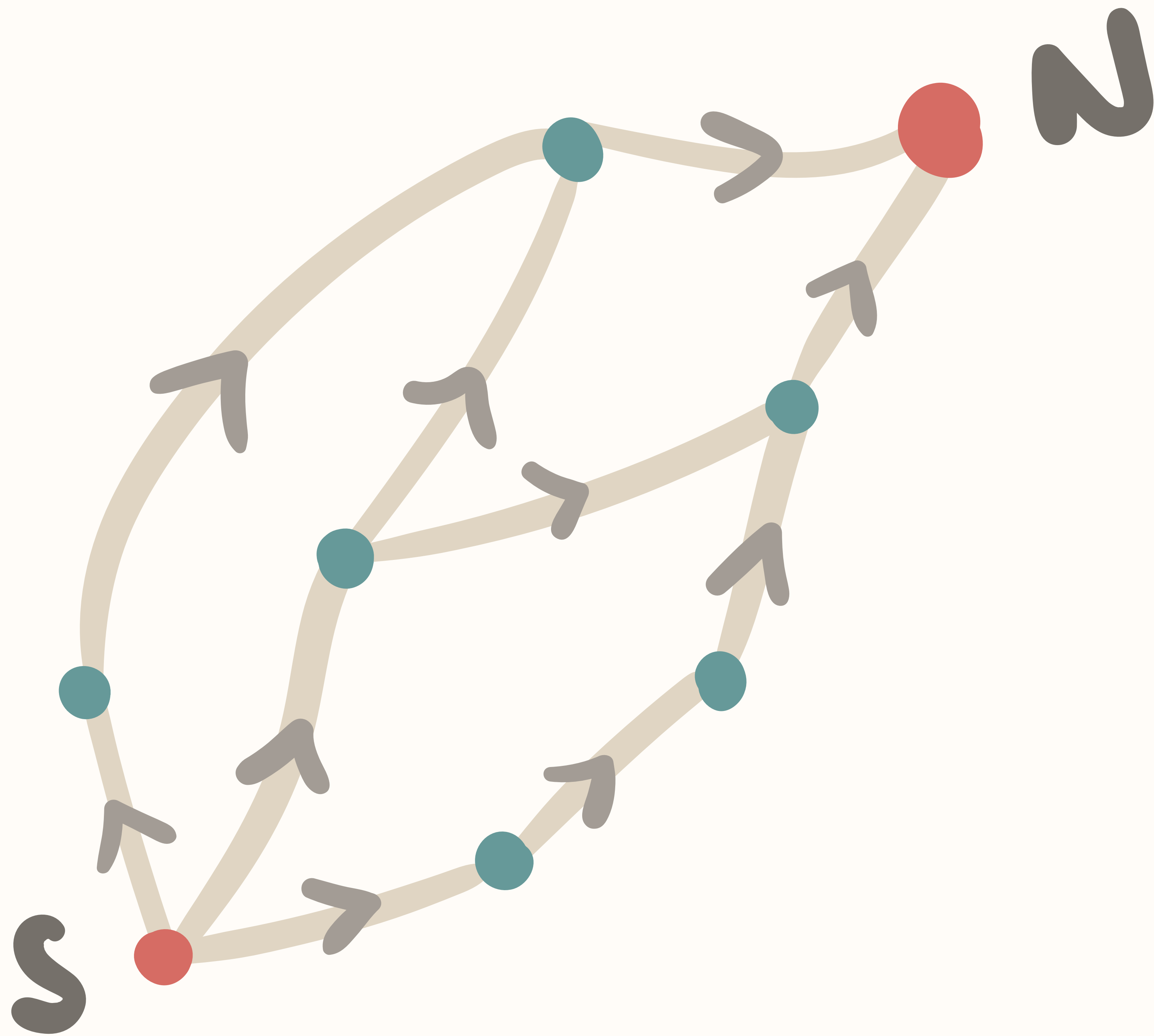
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No transitive edge

Plane bipolar poset



Poset

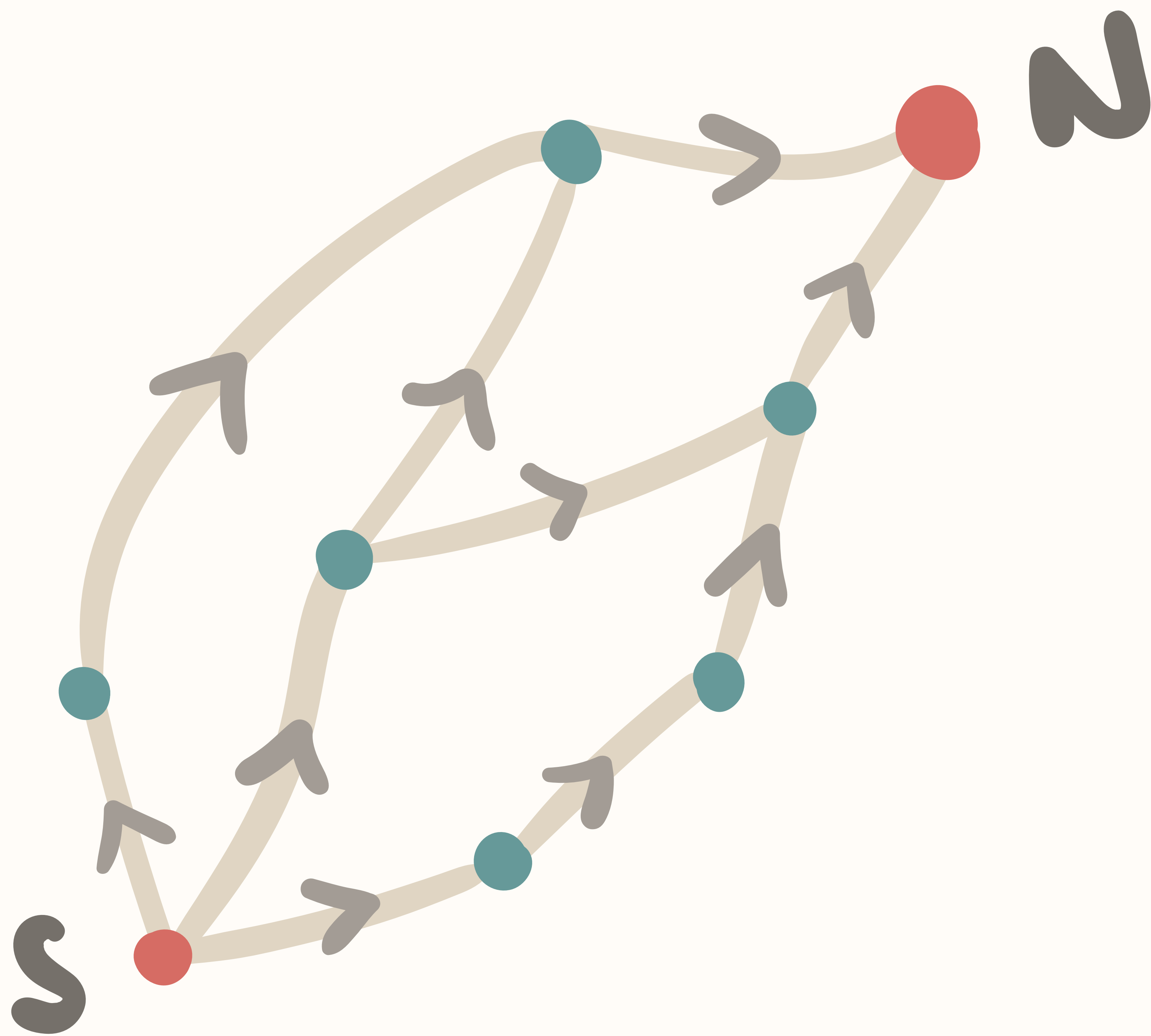
(plane bipolar poset)

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Poset

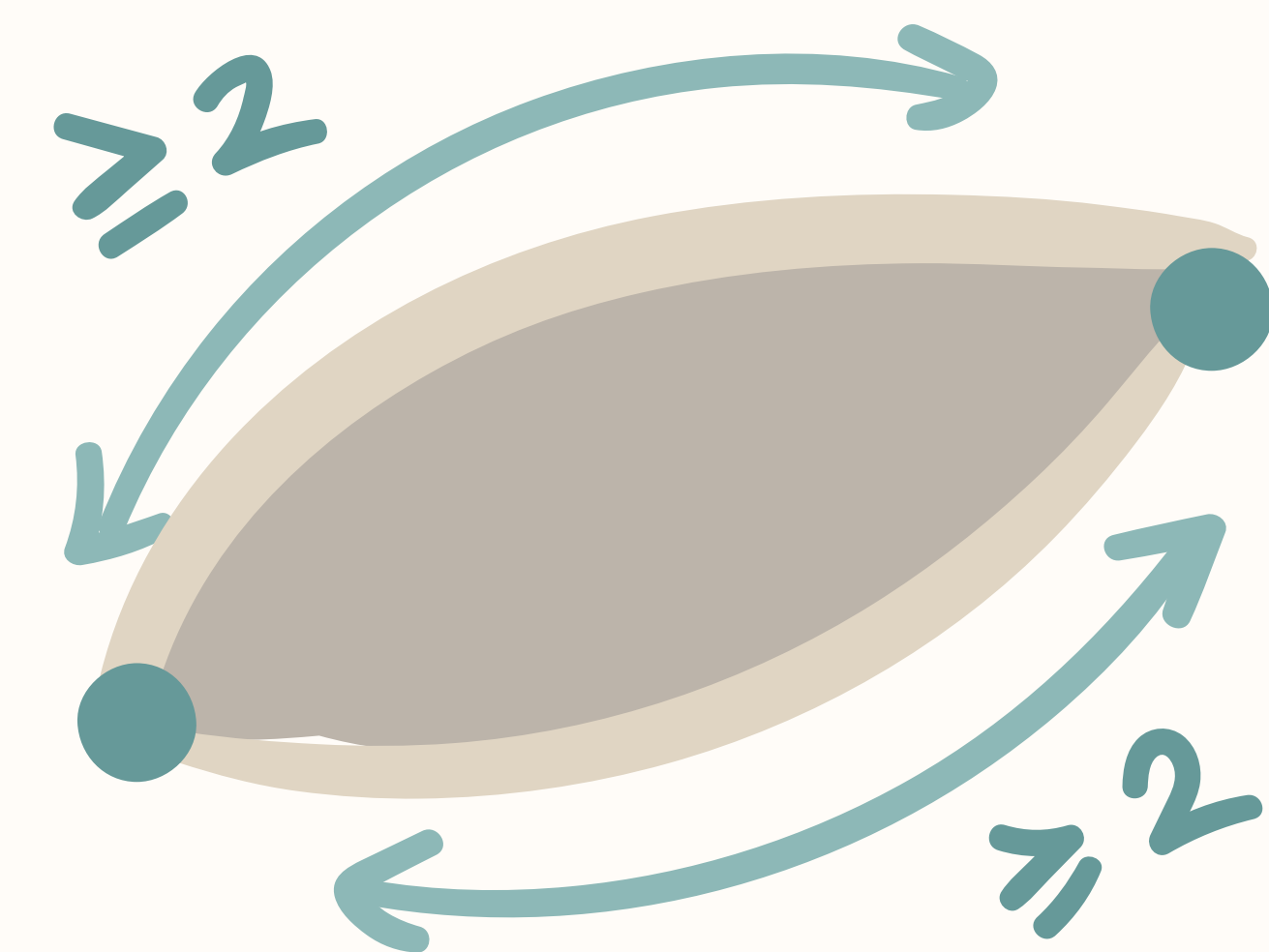
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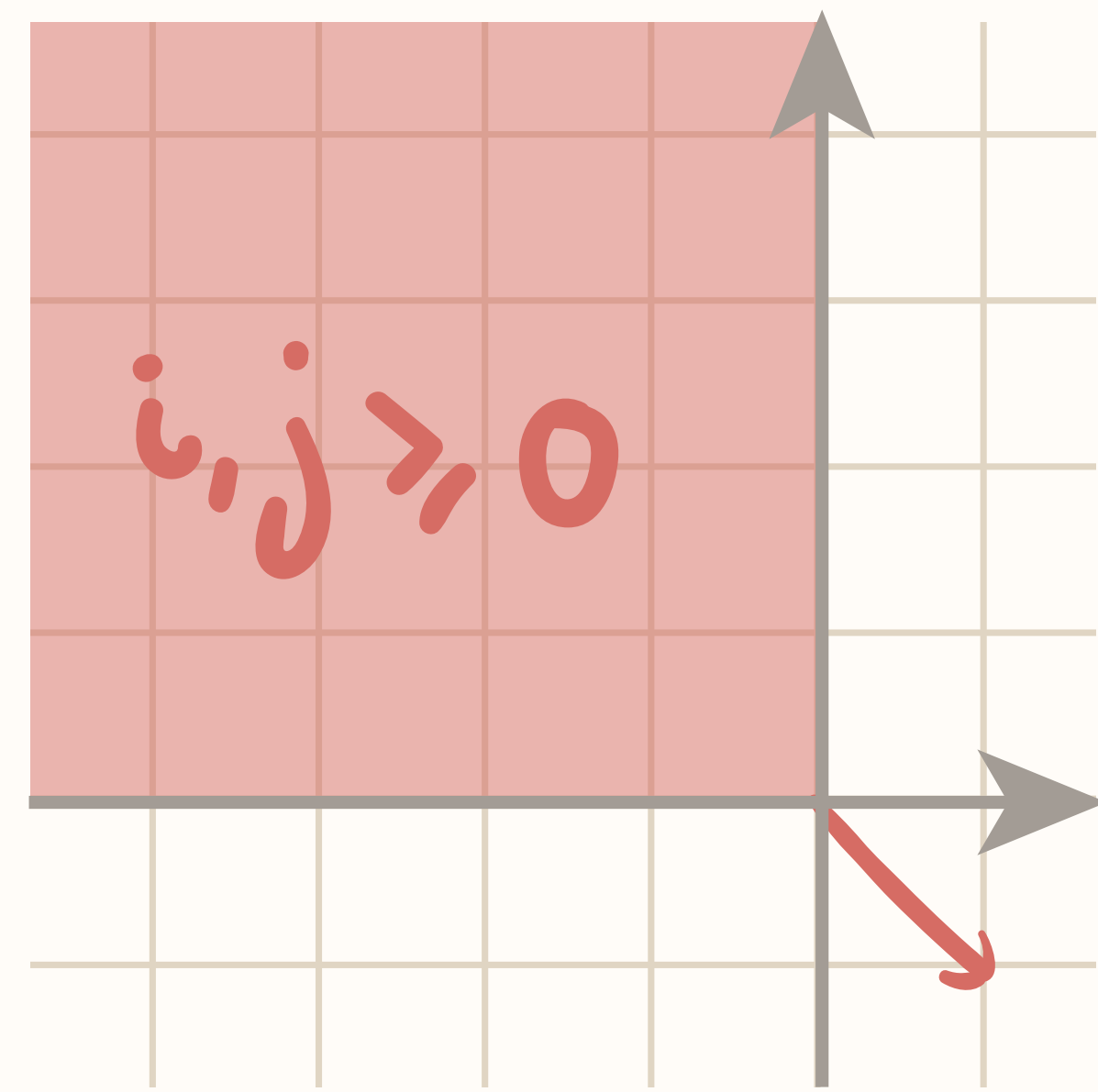


Specialization to Posets

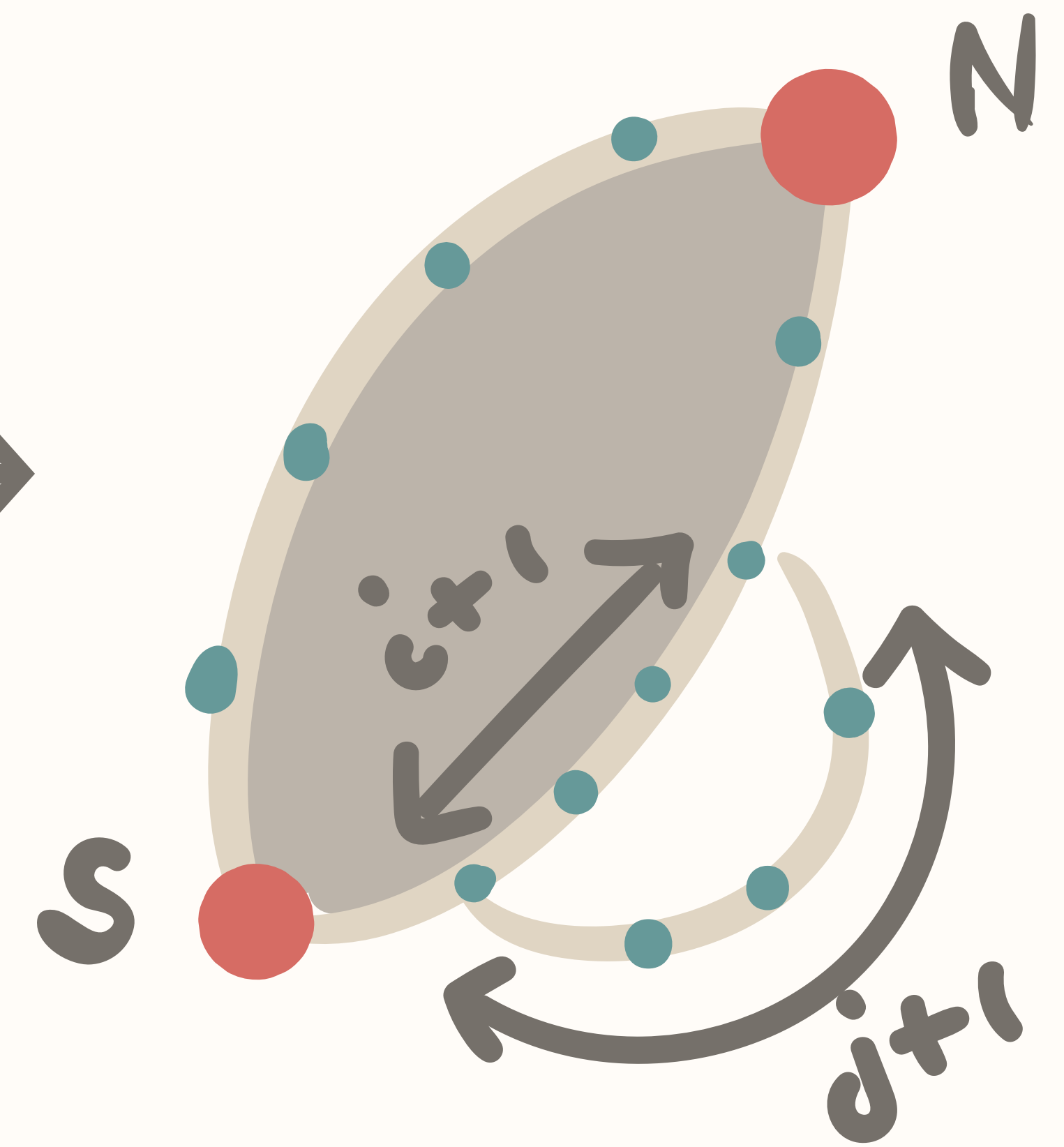
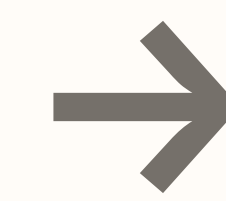
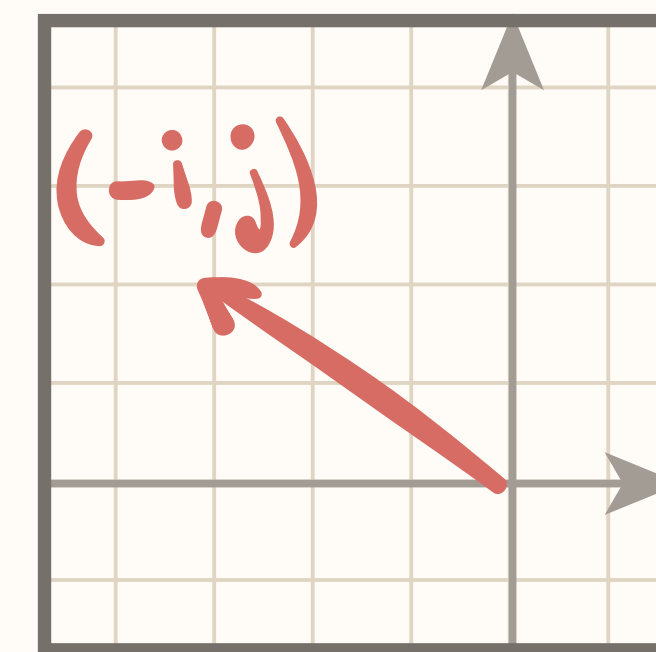
Specialization to Posets

*Bipolar
orientation*

=



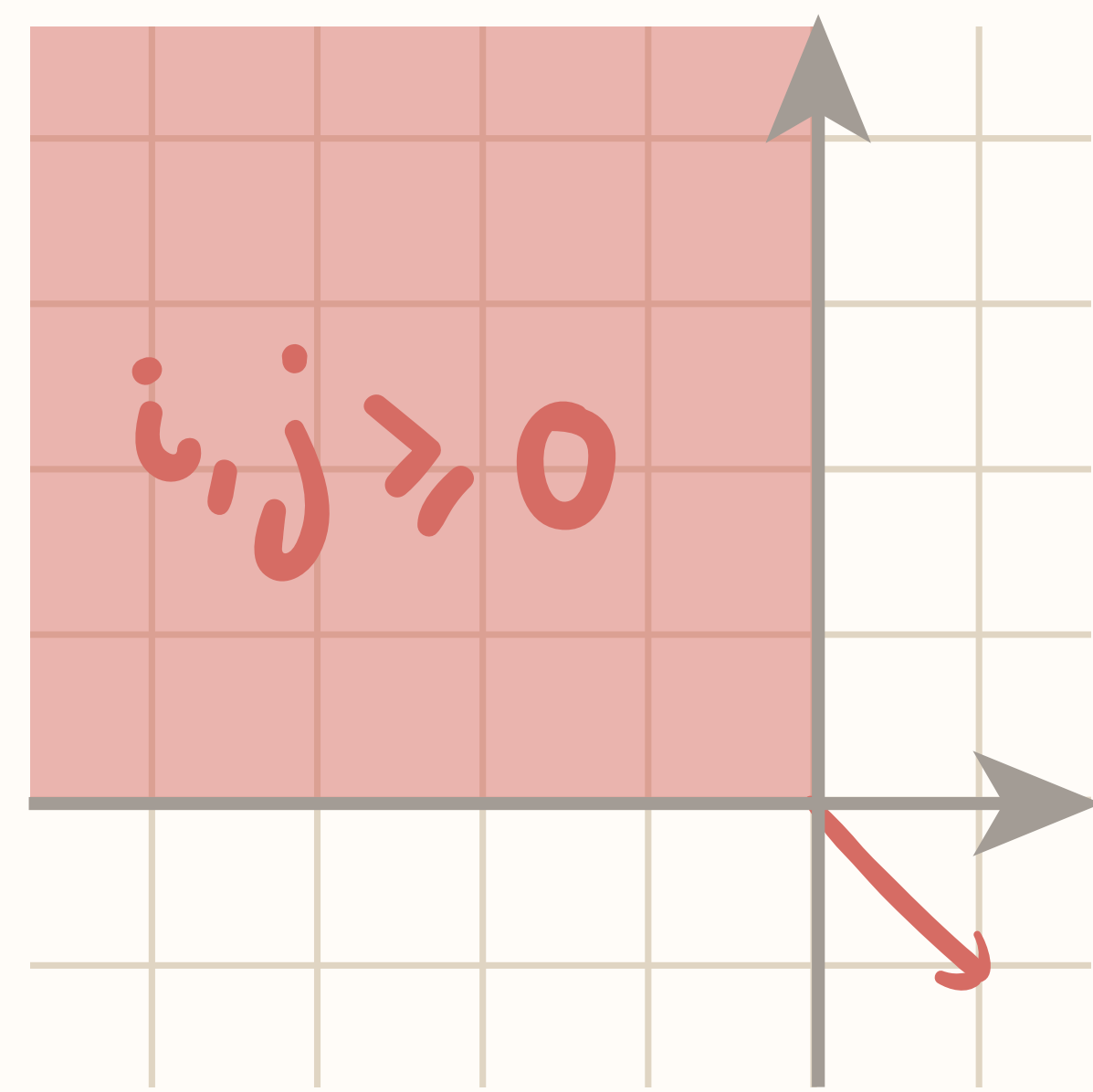
where



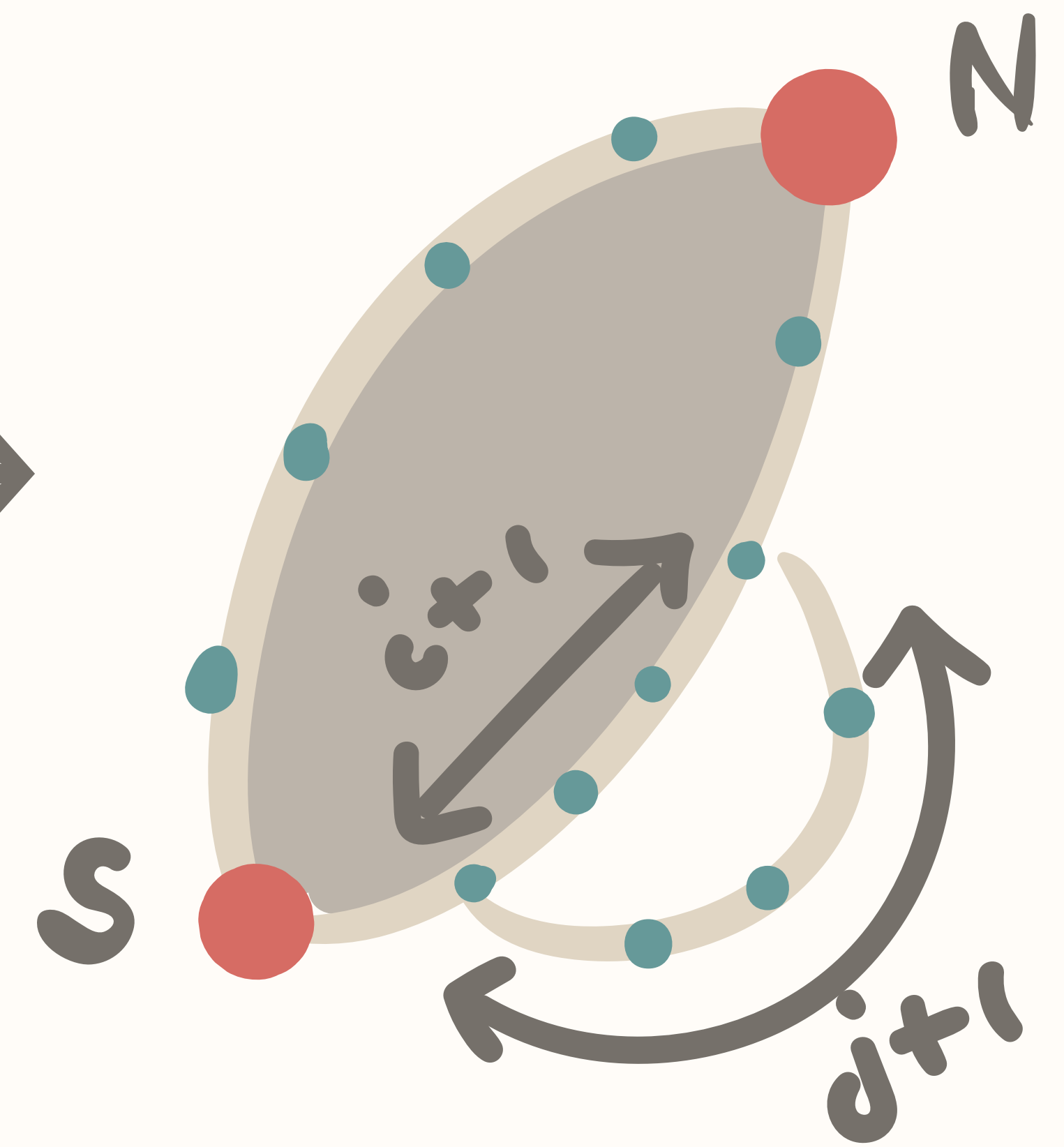
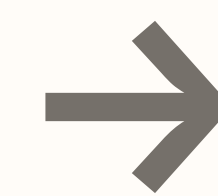
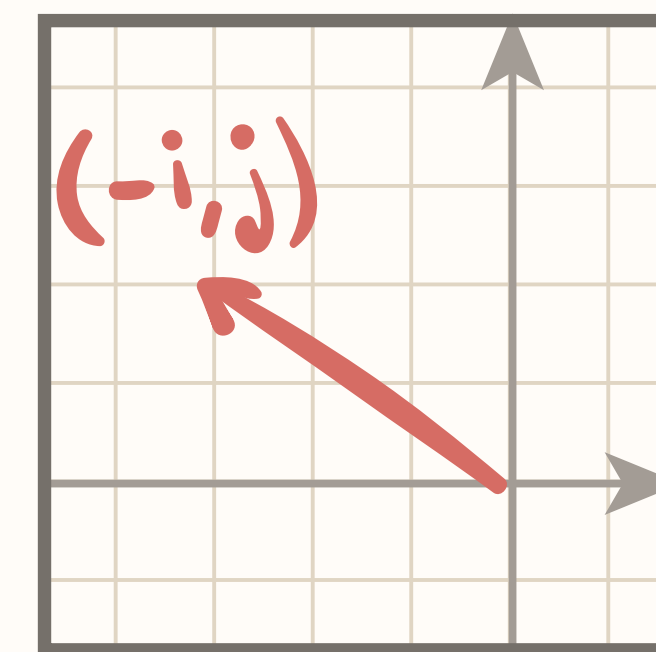
Specialization to Posets

Bipolar orientation

=

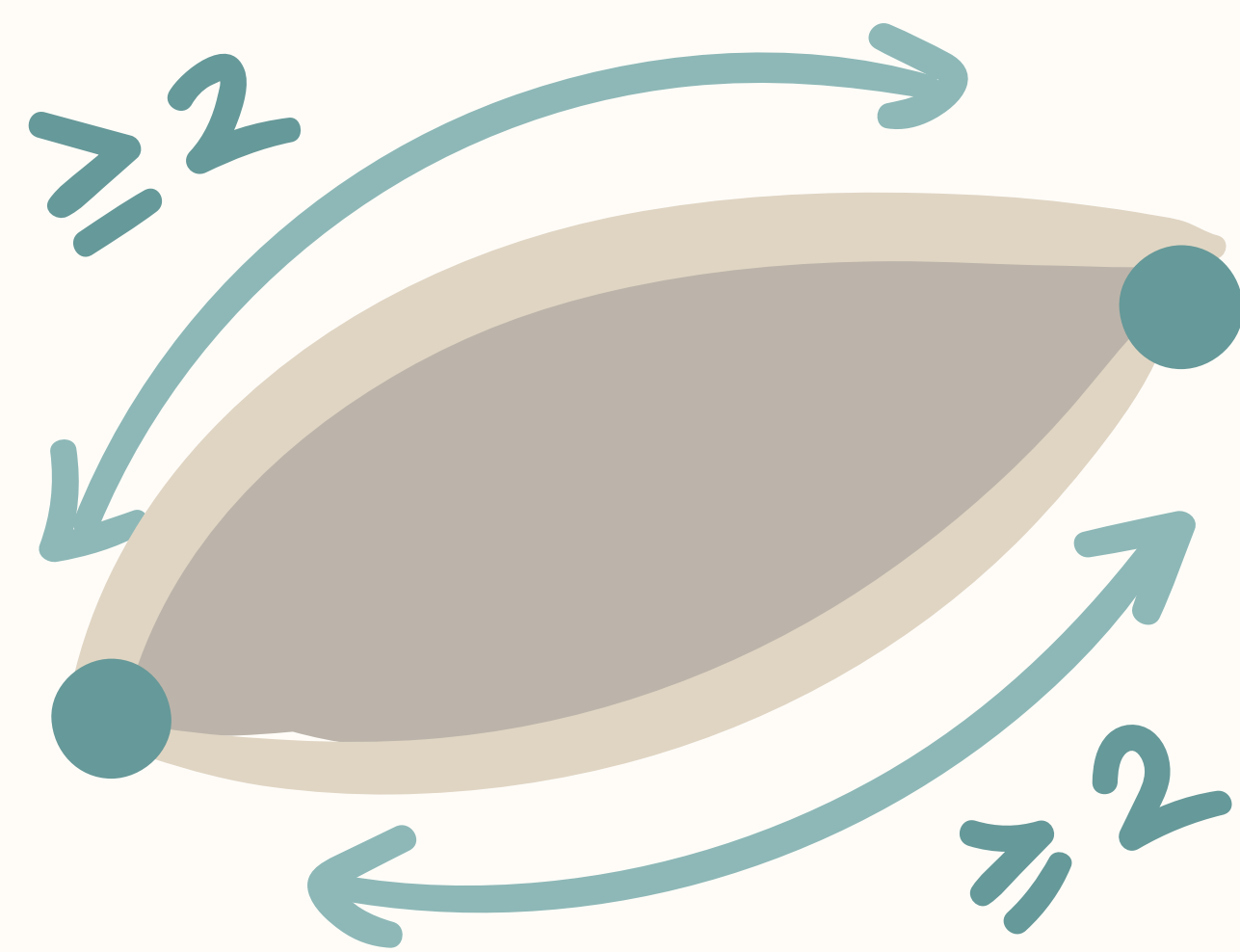


where

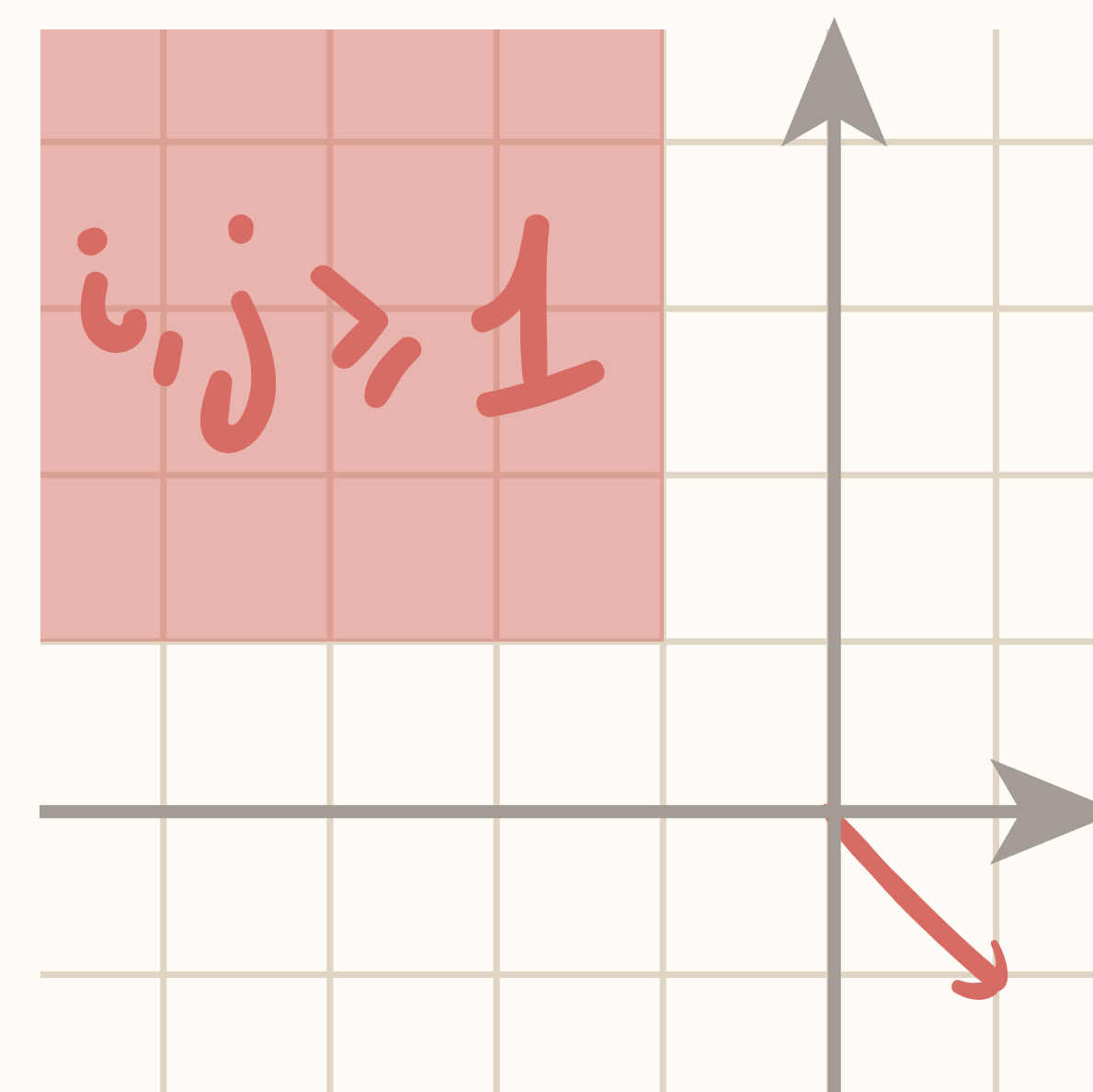


Poset

=



=



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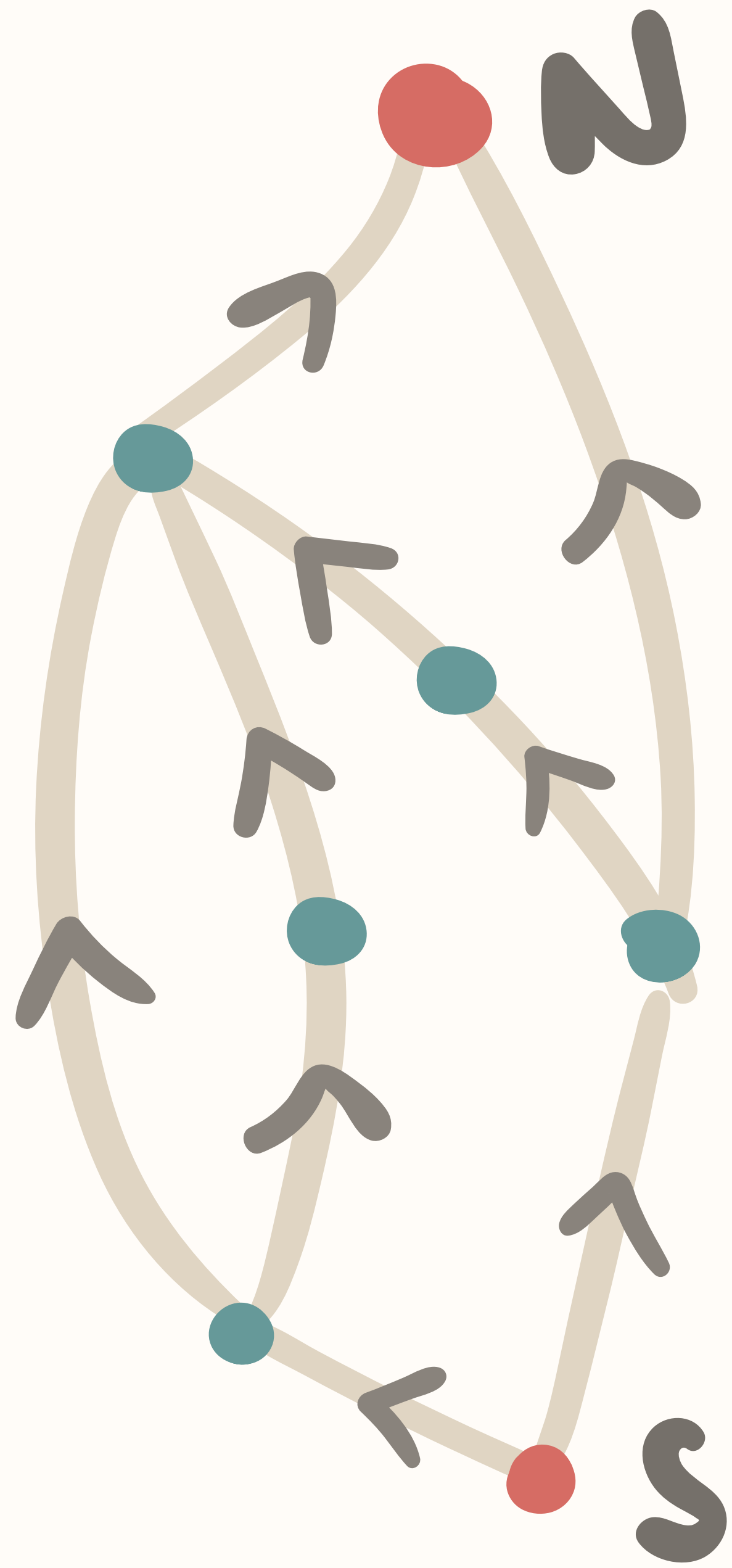
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b. Via Schnyder colorings

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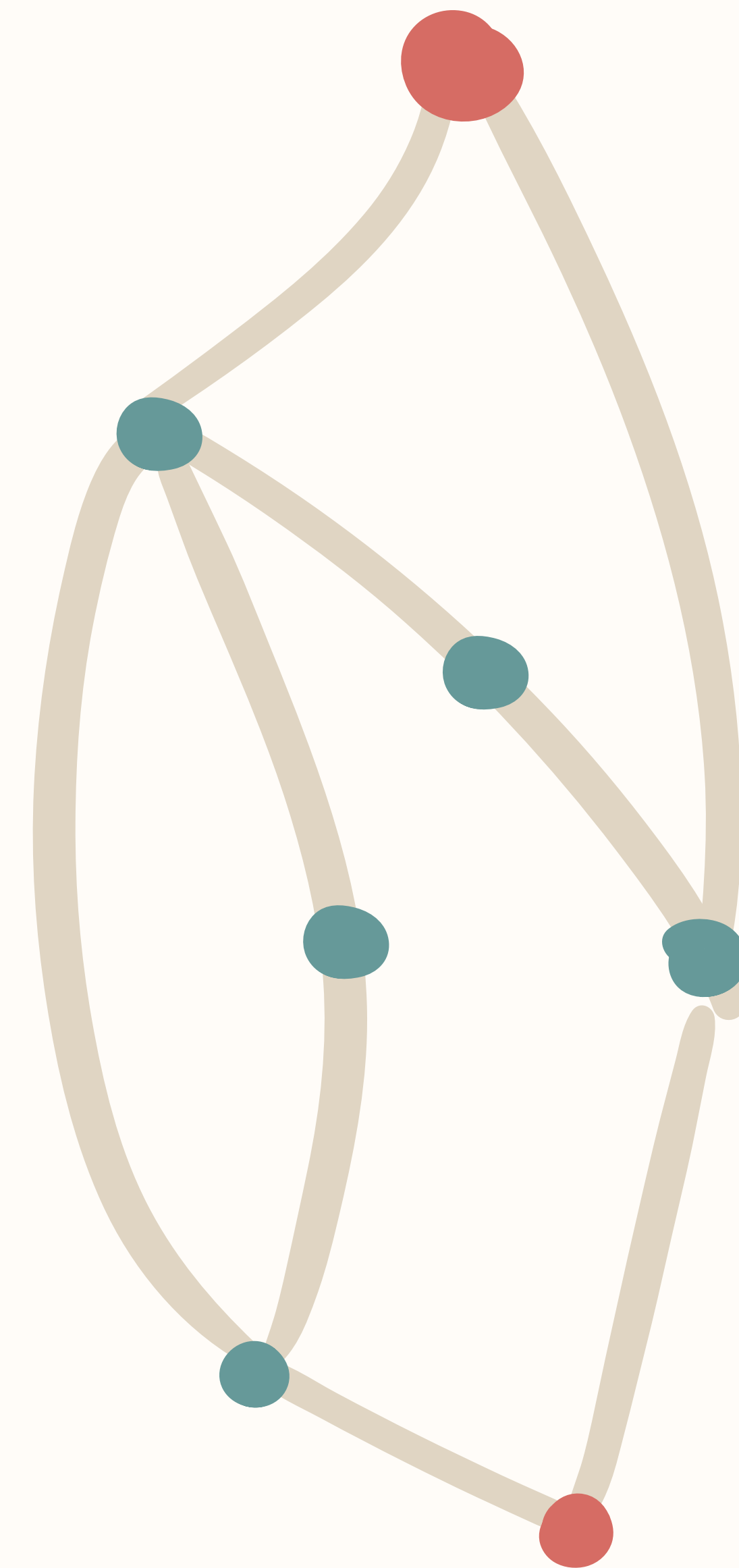
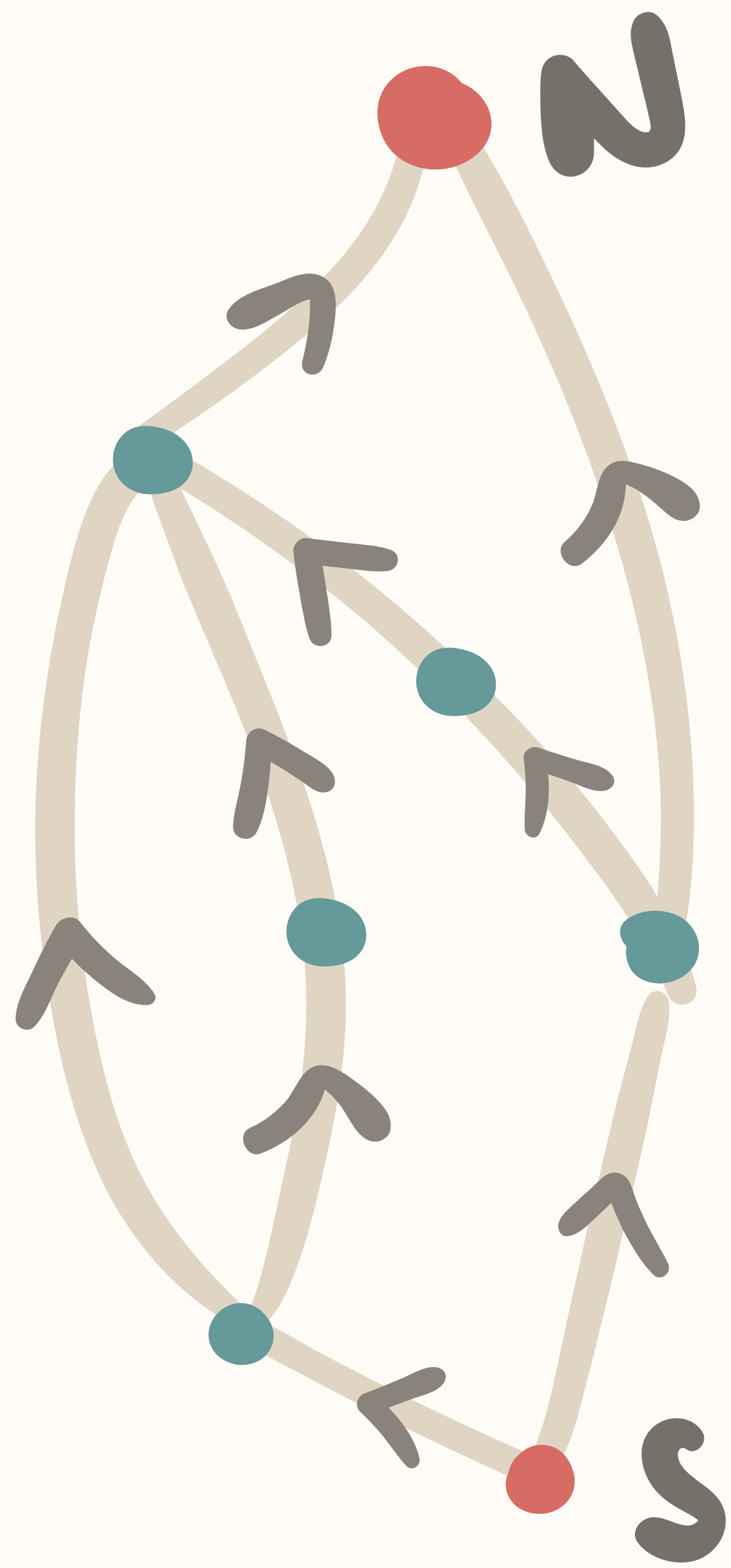
Specialization to Posets by vertices

Bipolar orientation



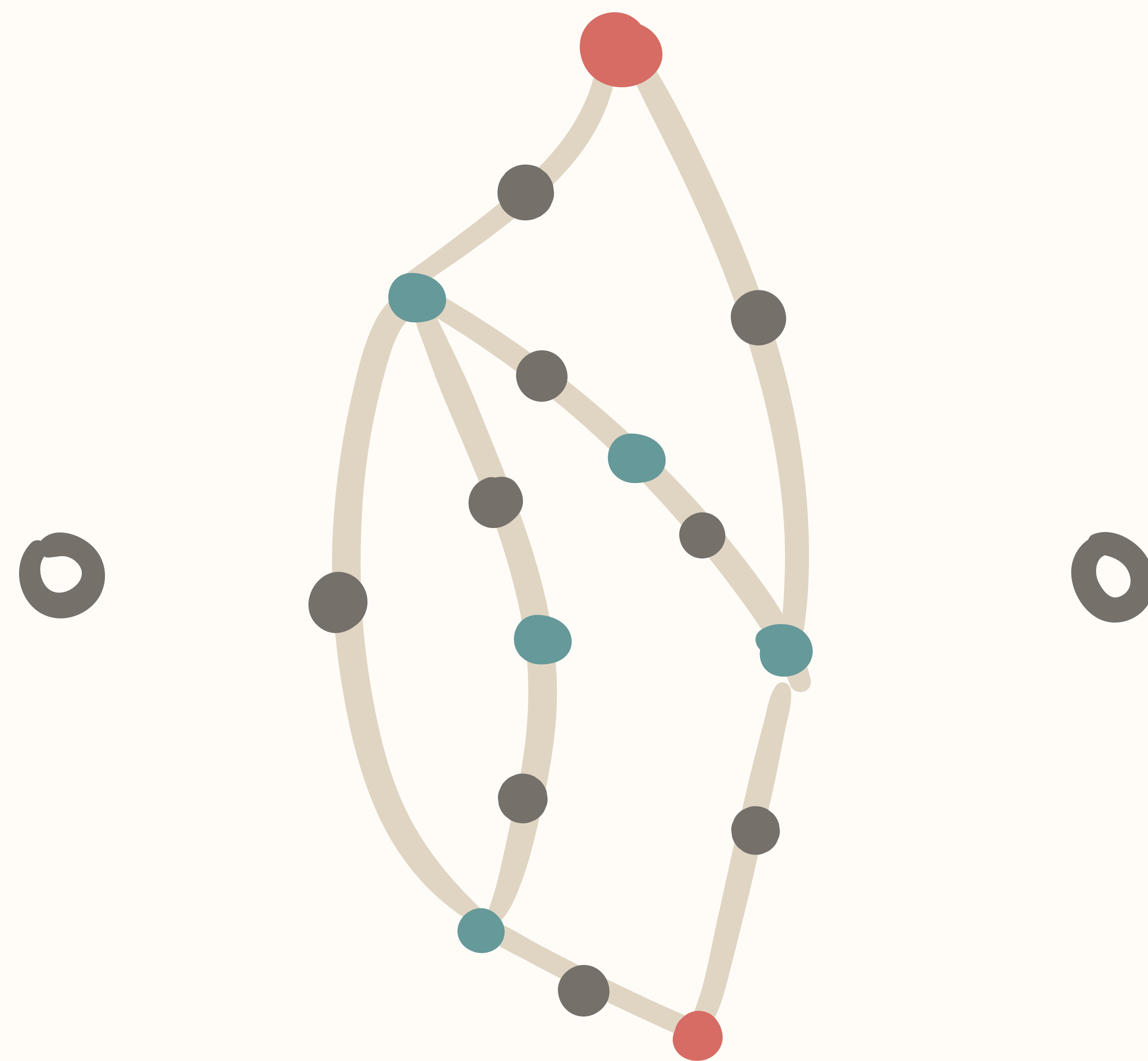
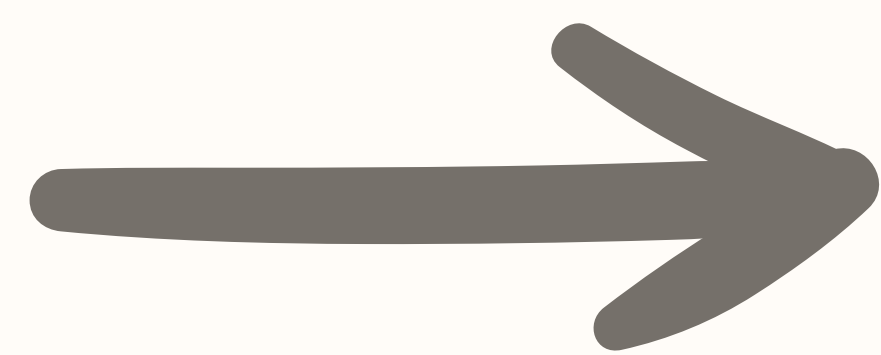
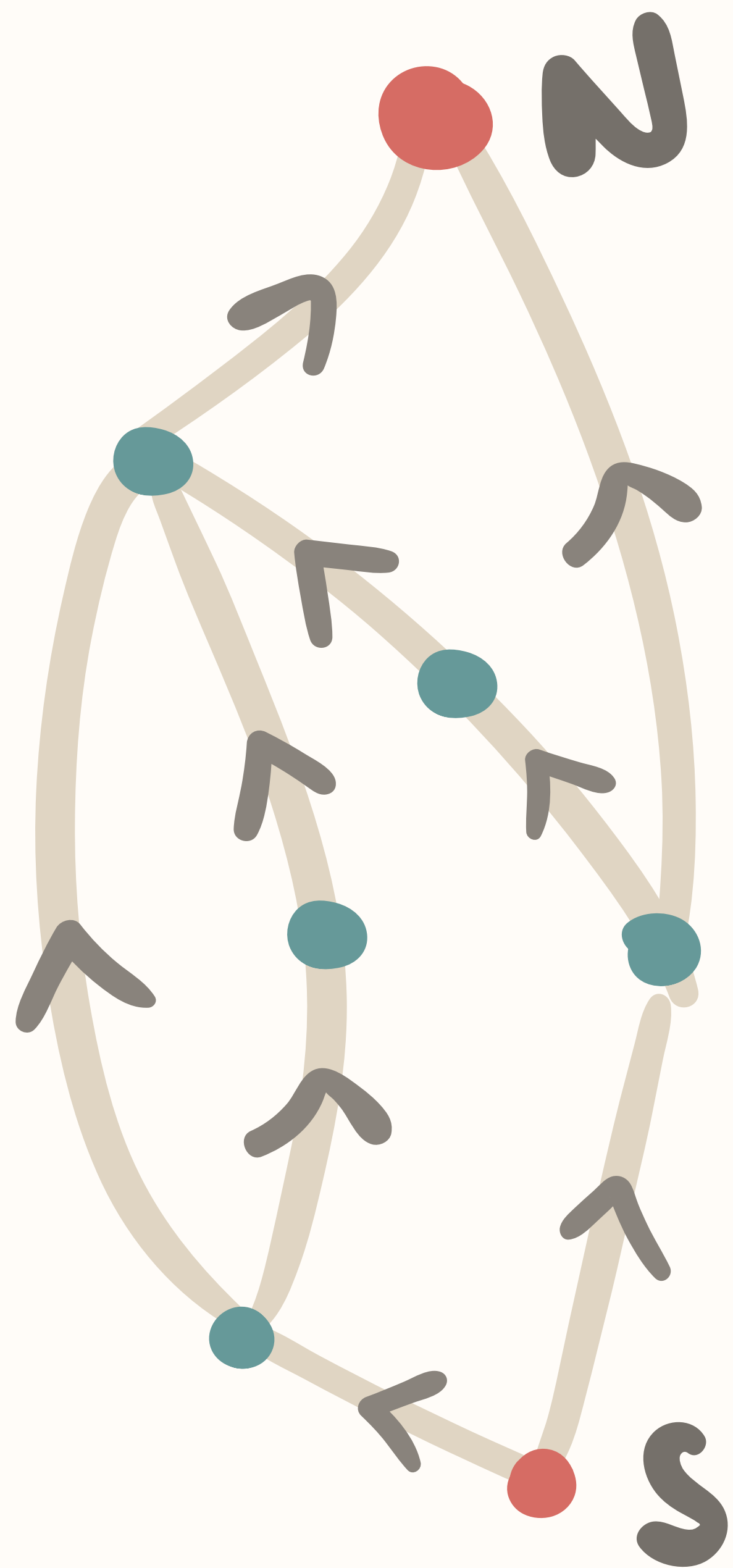
Specialization to Posets by vertices

Bipolar orientation



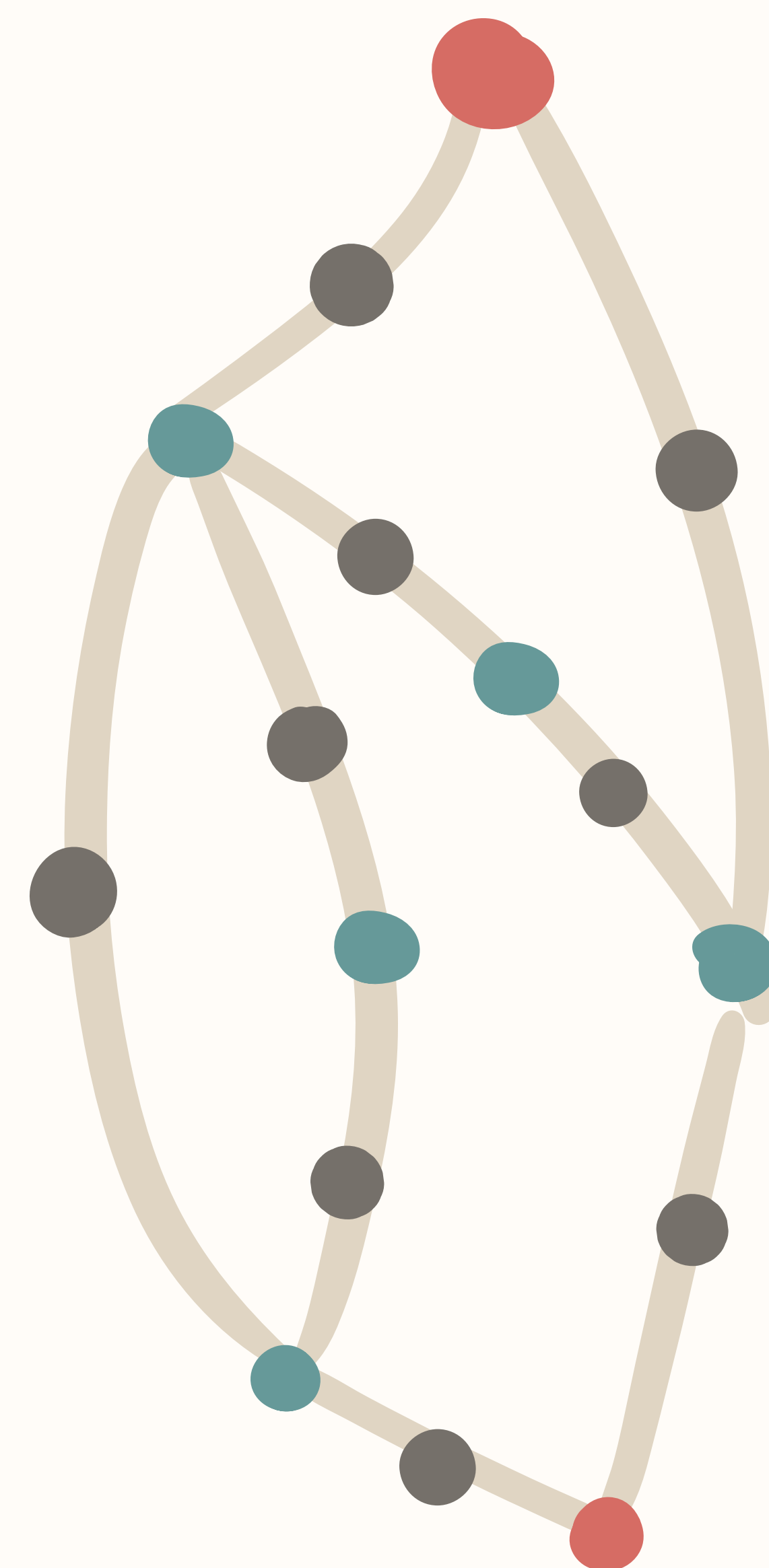
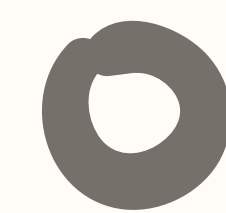
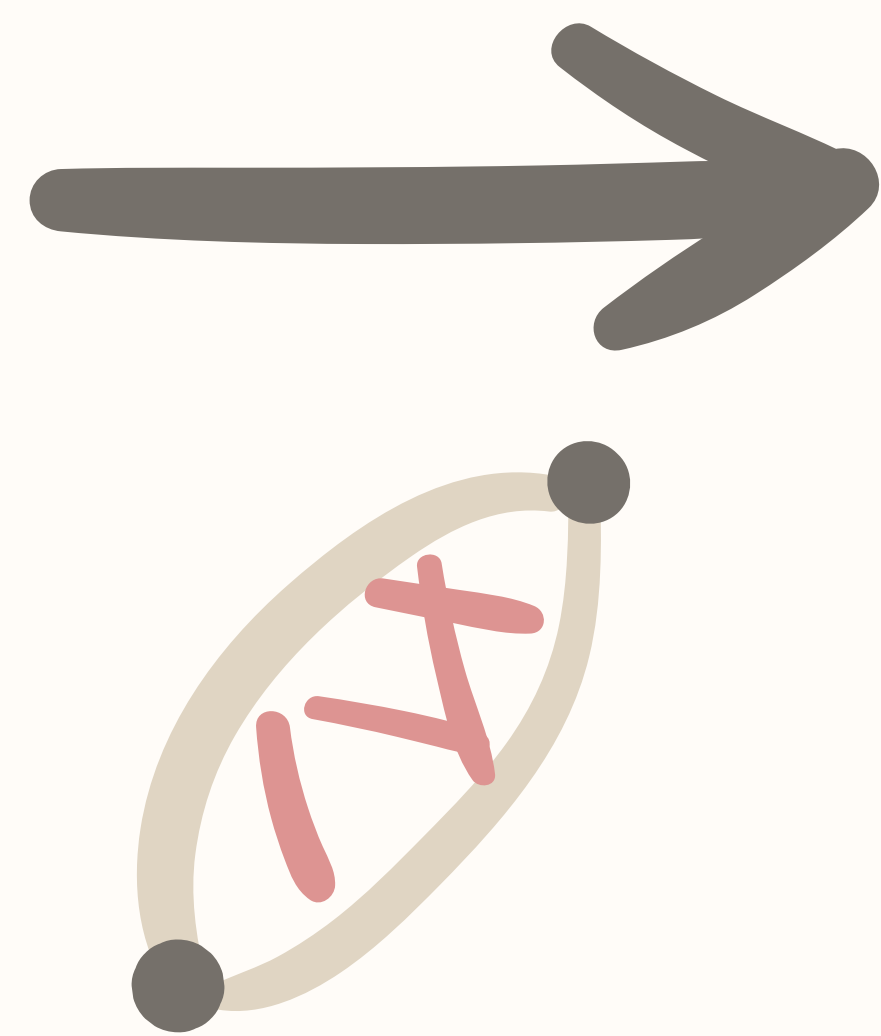
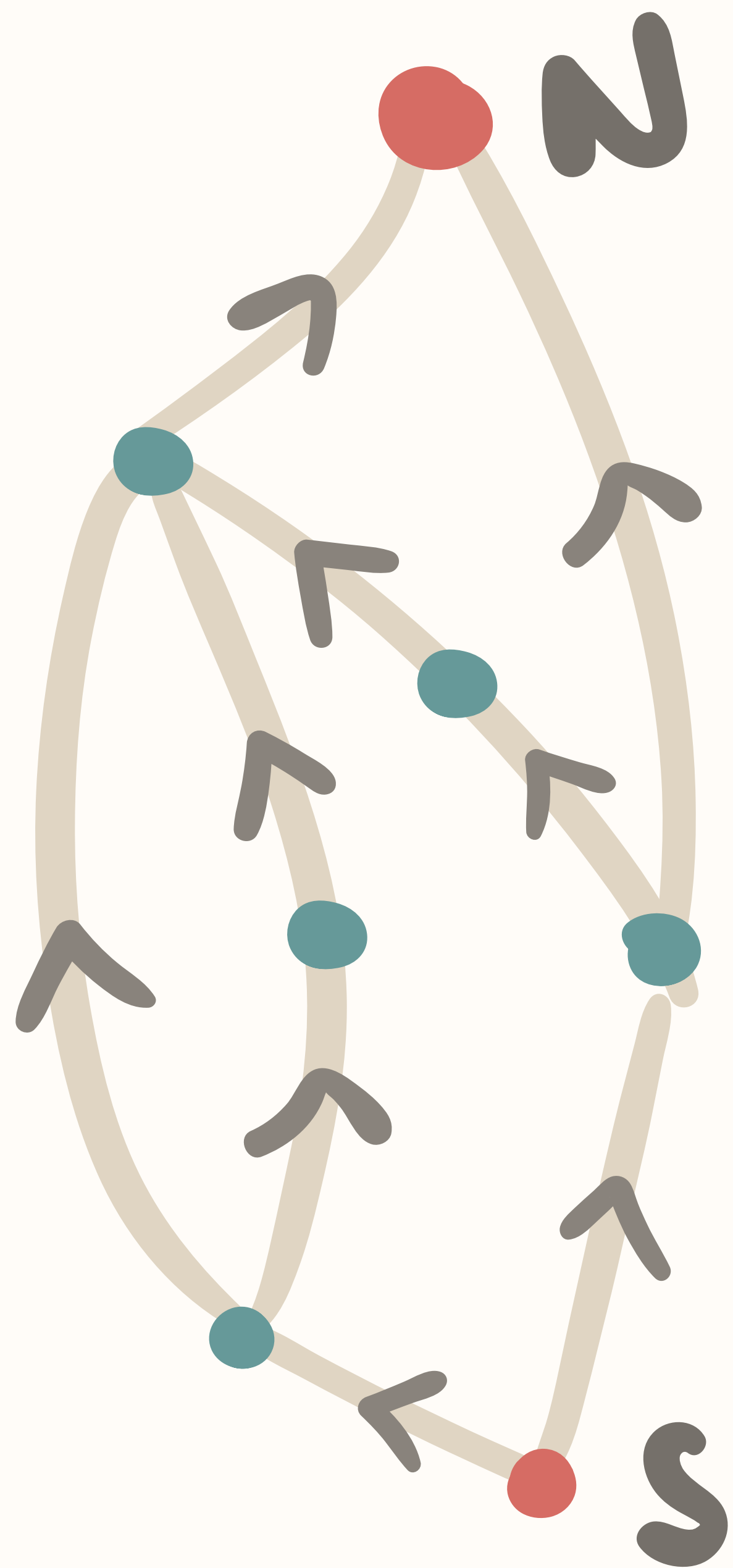
Specialization to Posets by vertices

Bipolar orientation



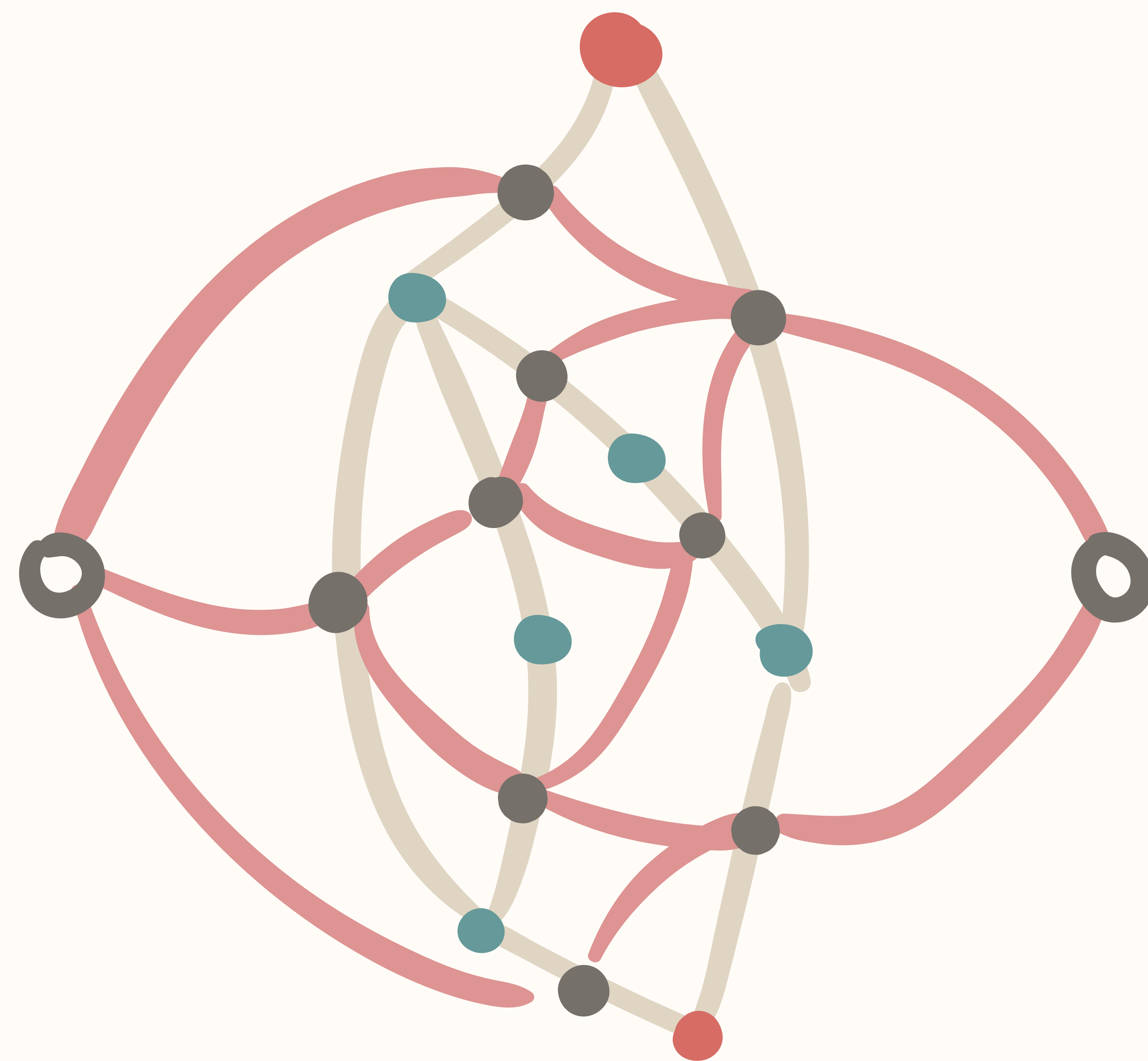
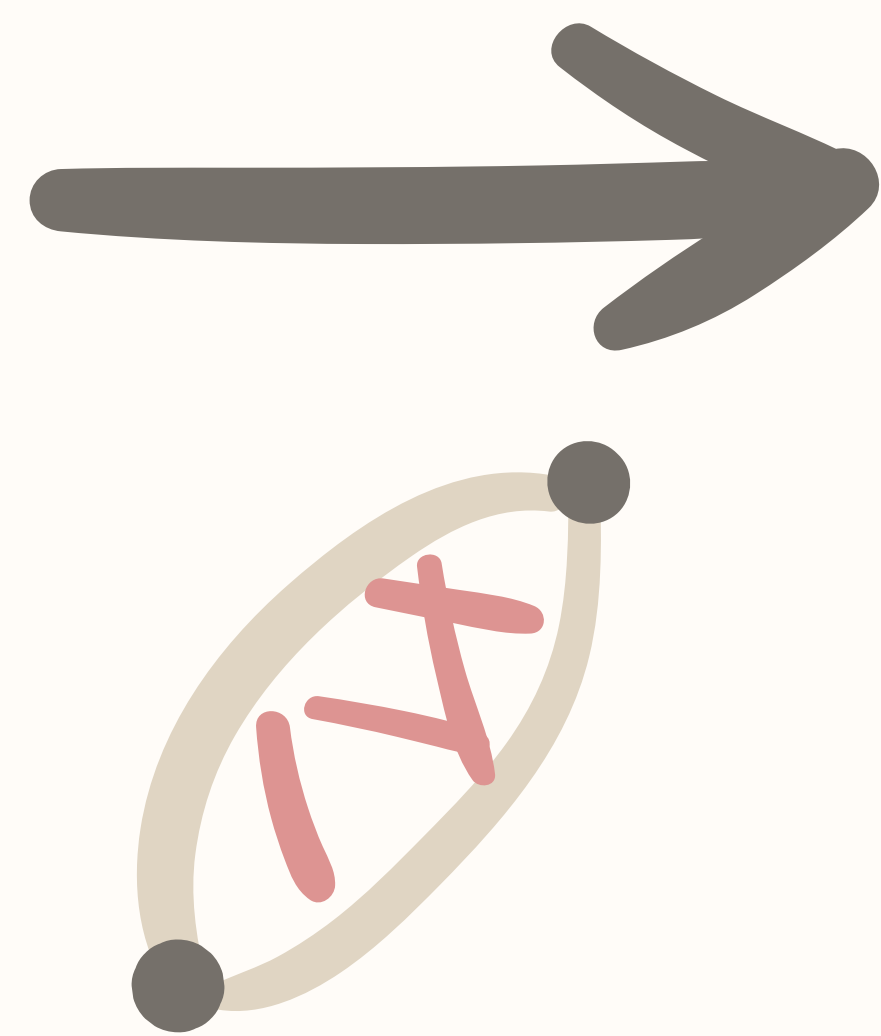
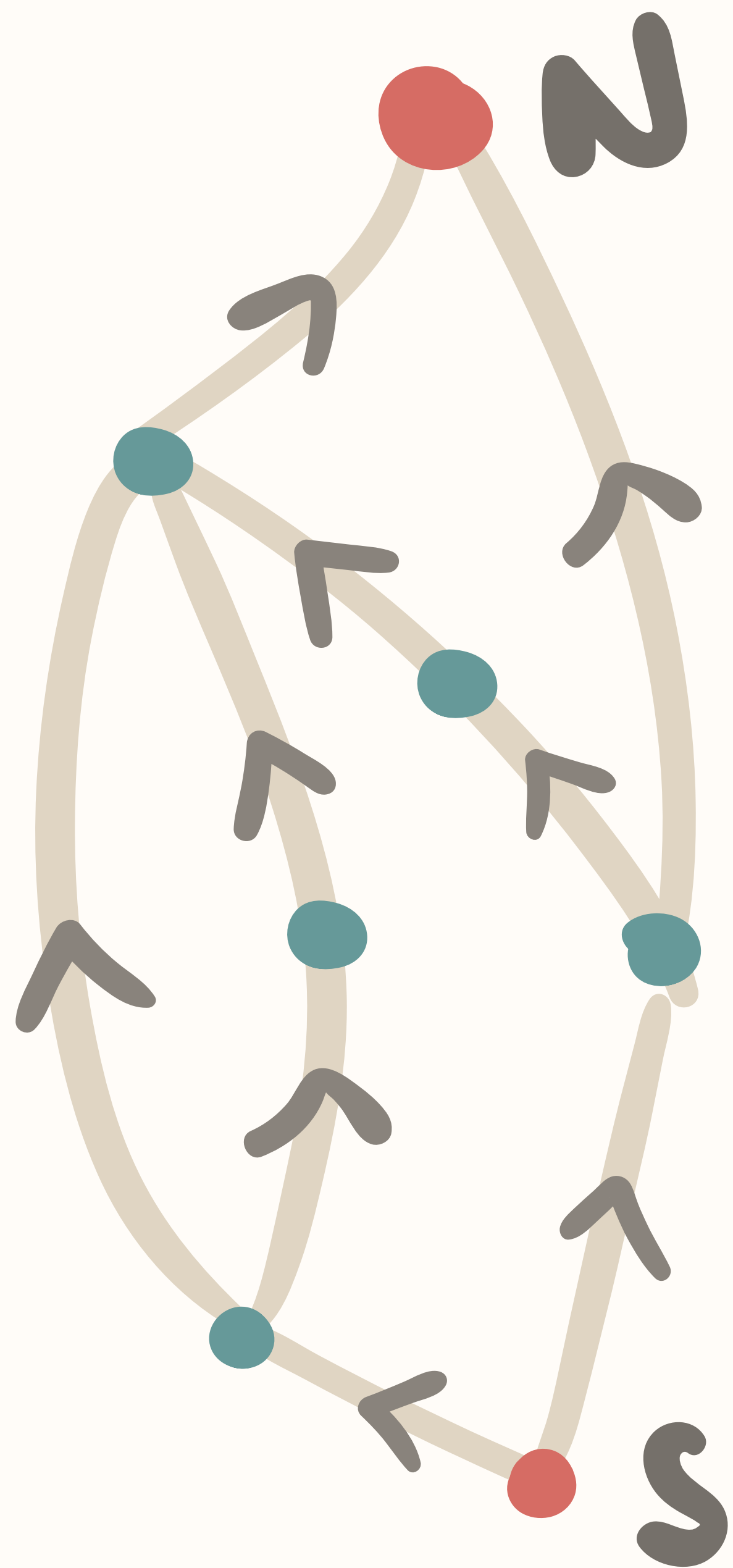
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Bipolar orientation



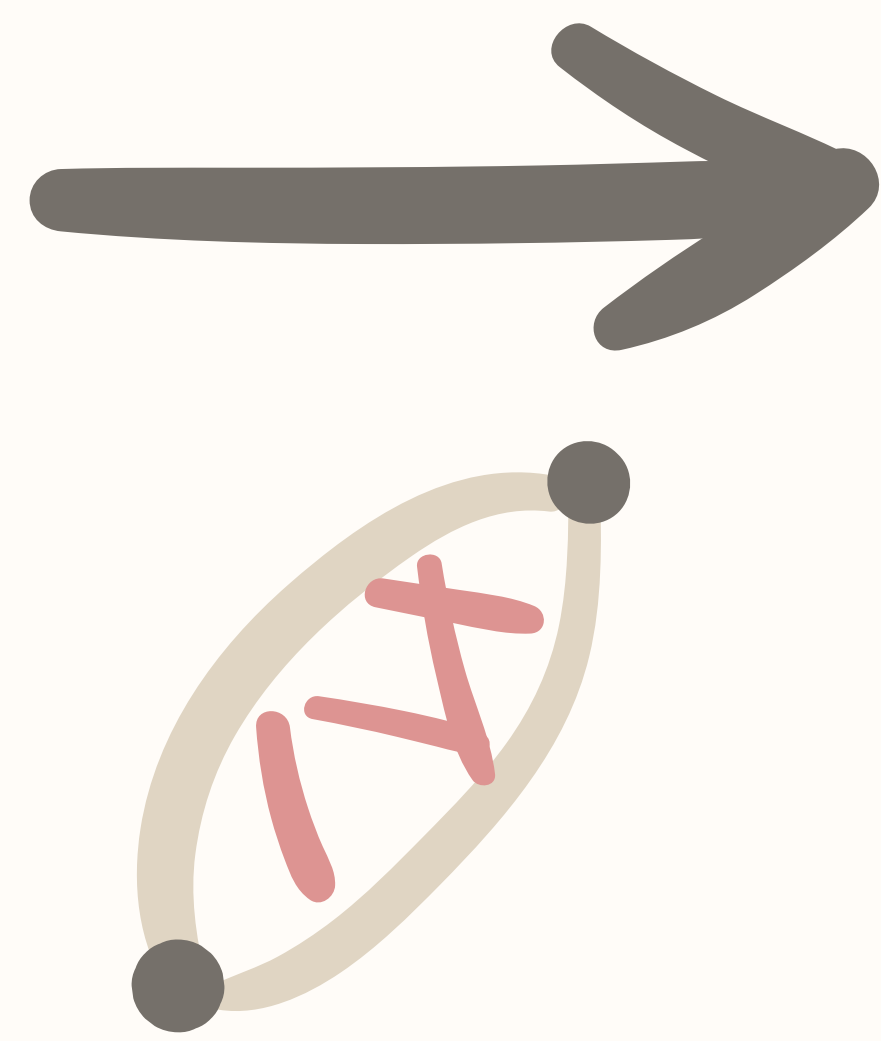
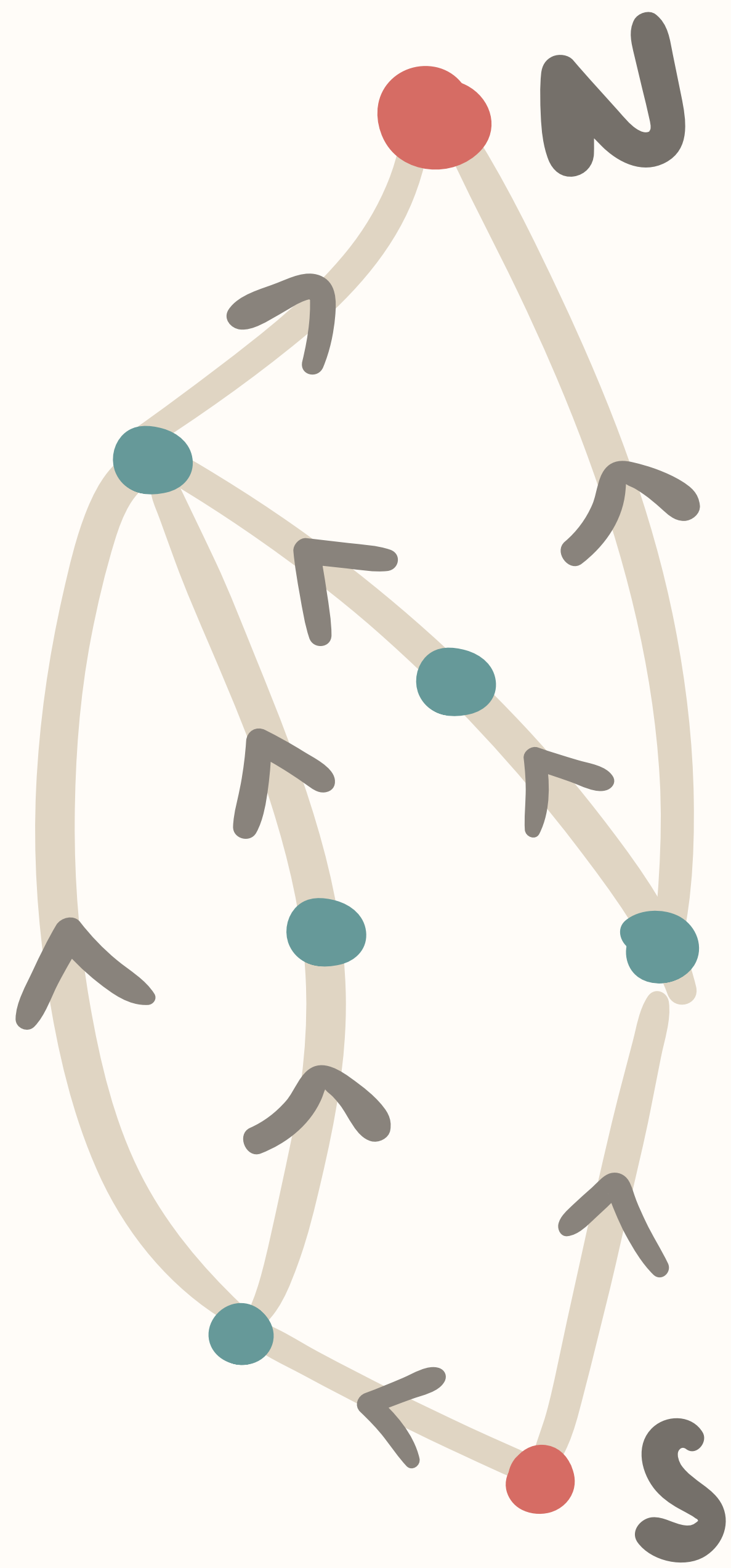
Specialization to Posets by vertices

Bipolar orientation

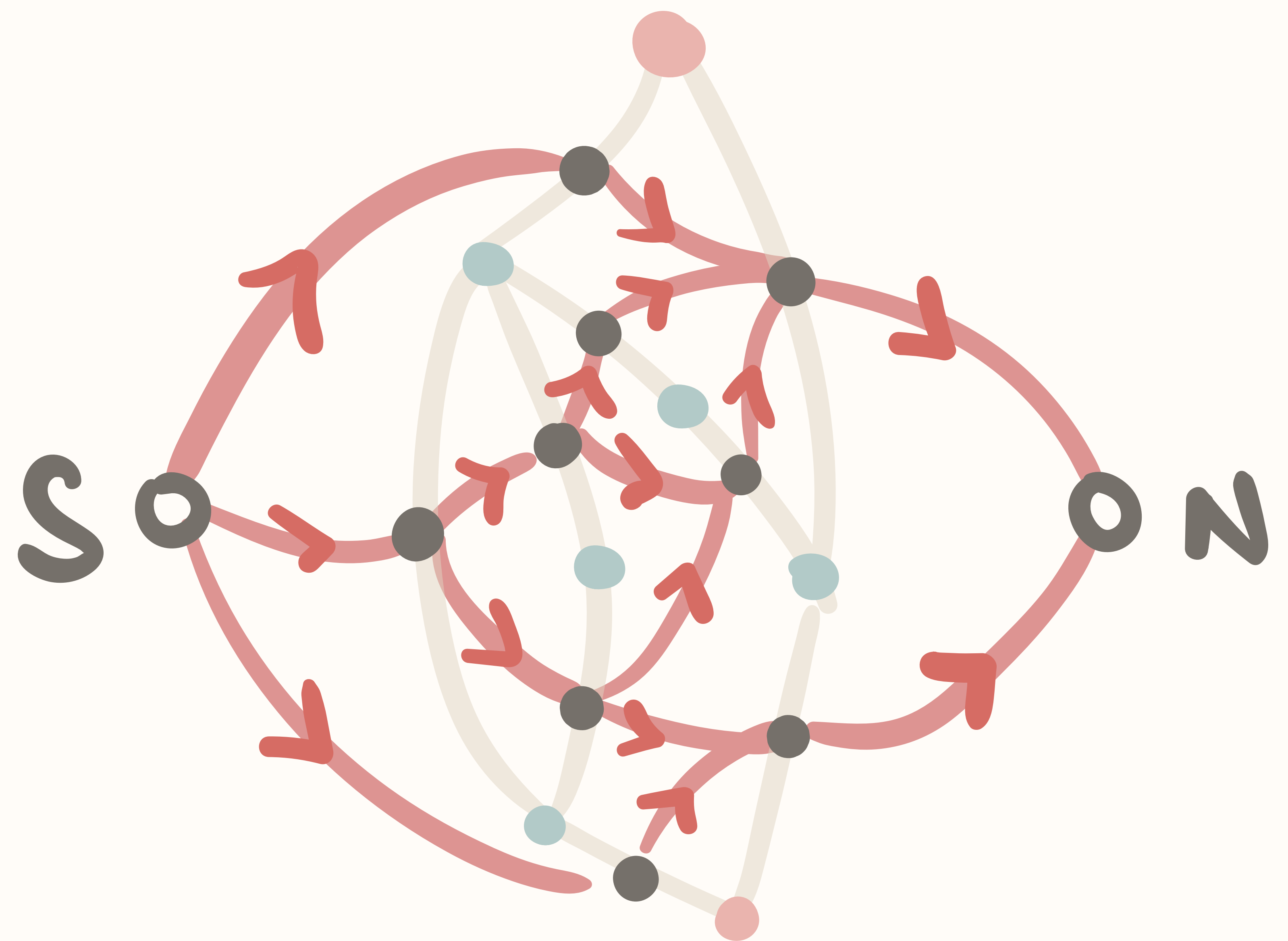


Specialization to Posets by vertices

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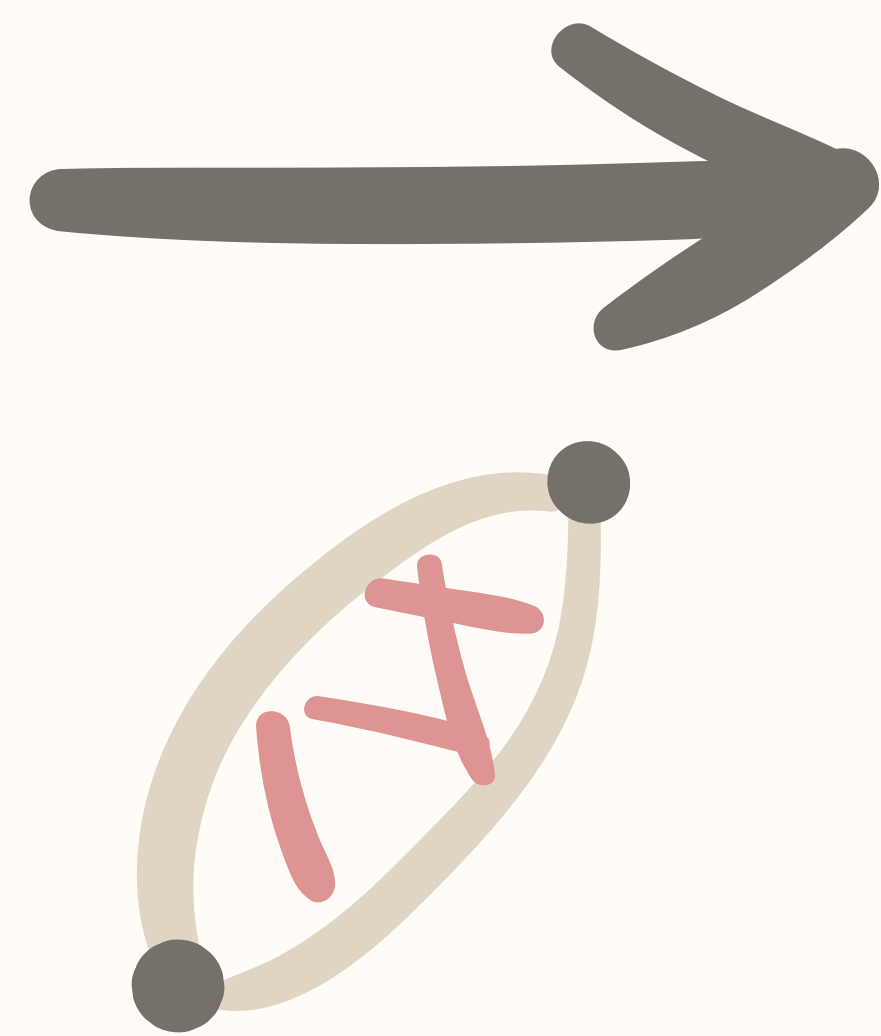
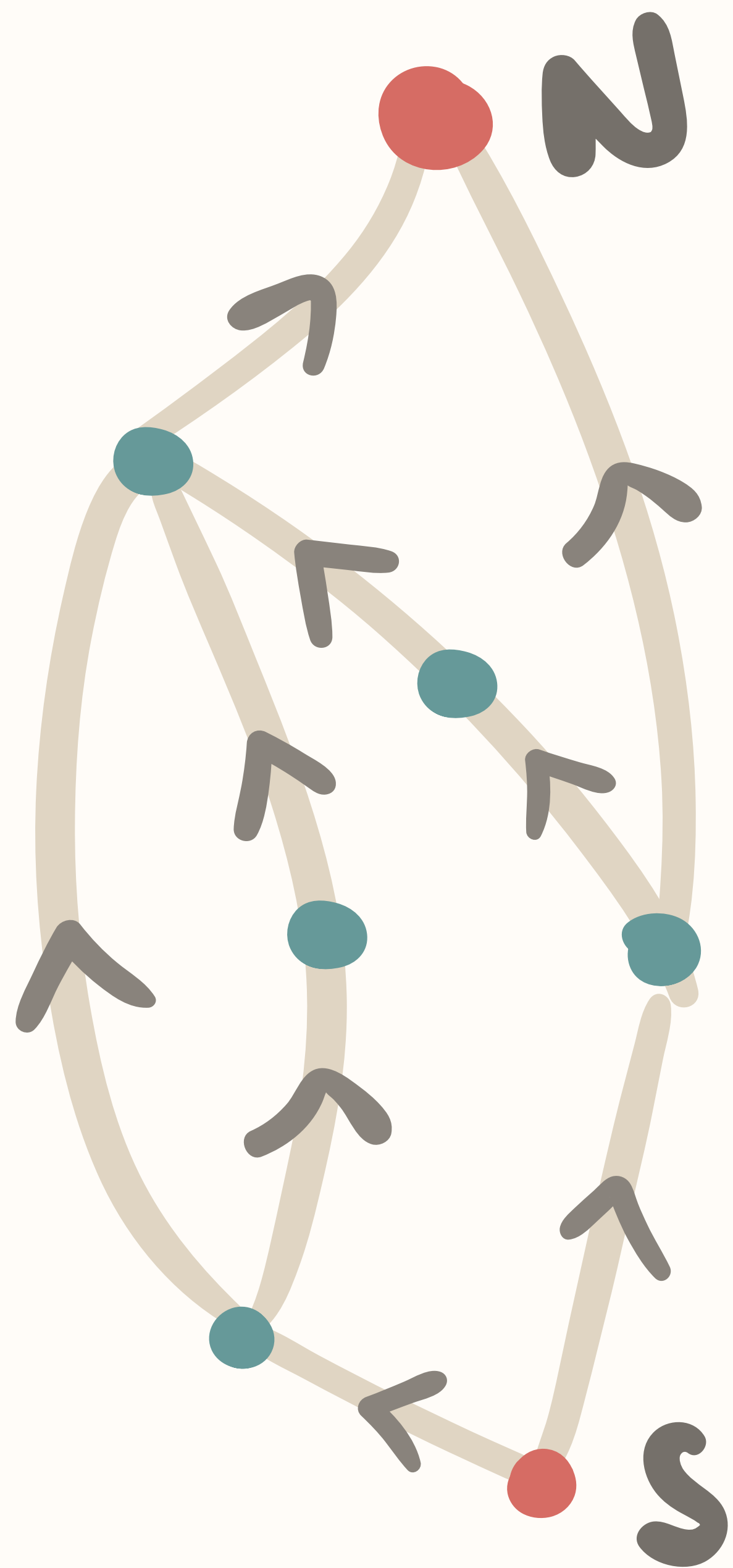


poset

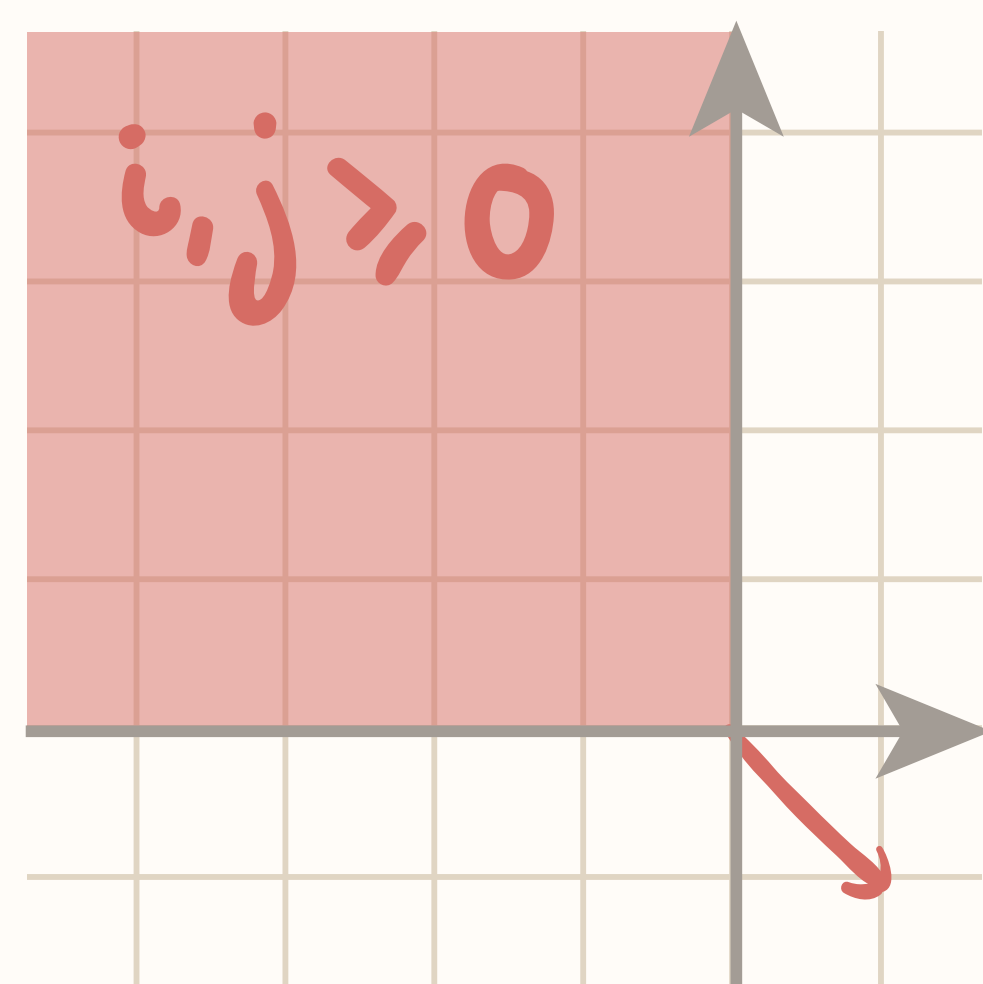
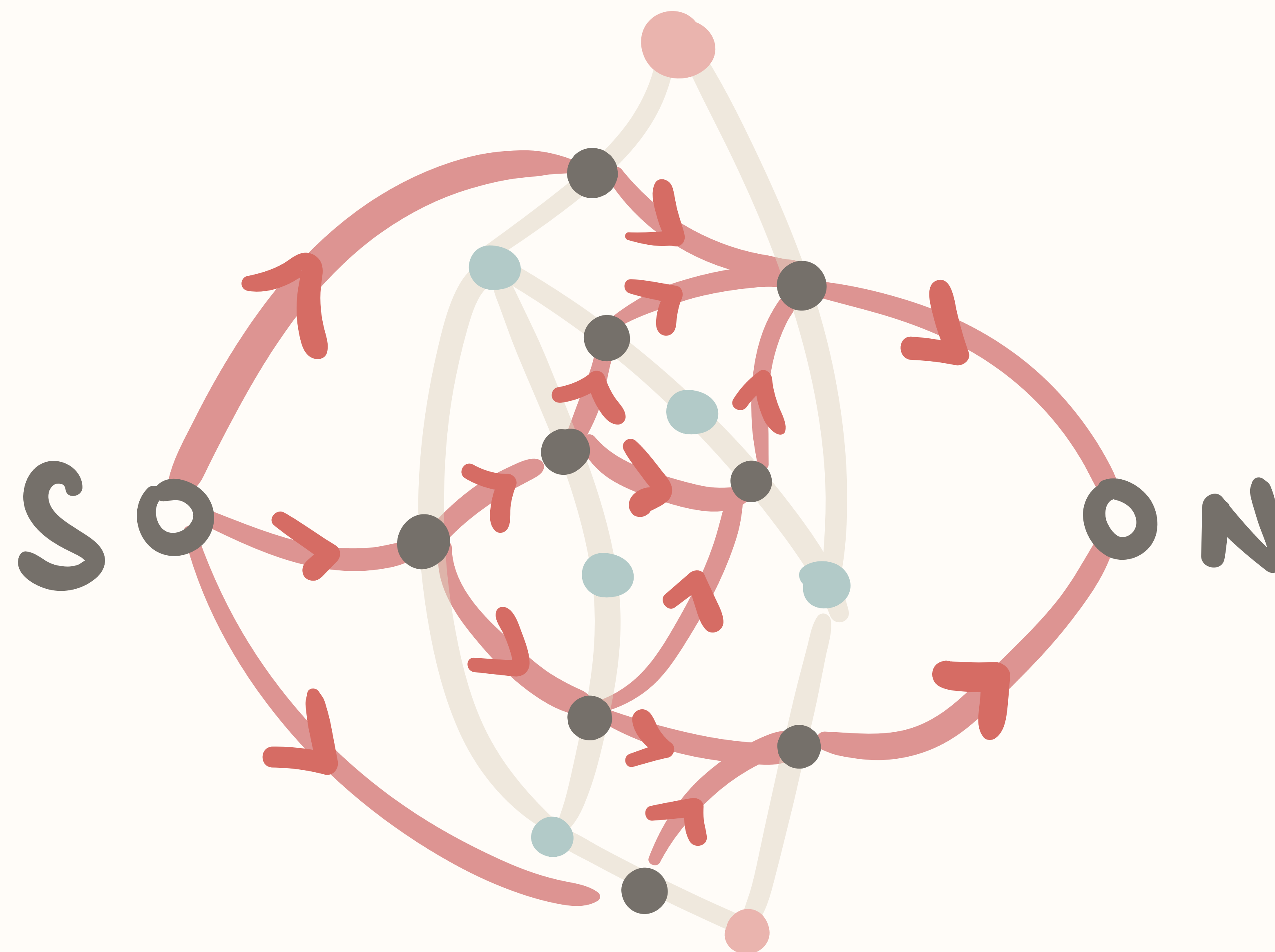


Specialization to Posets by vertices

Bipolar orientation

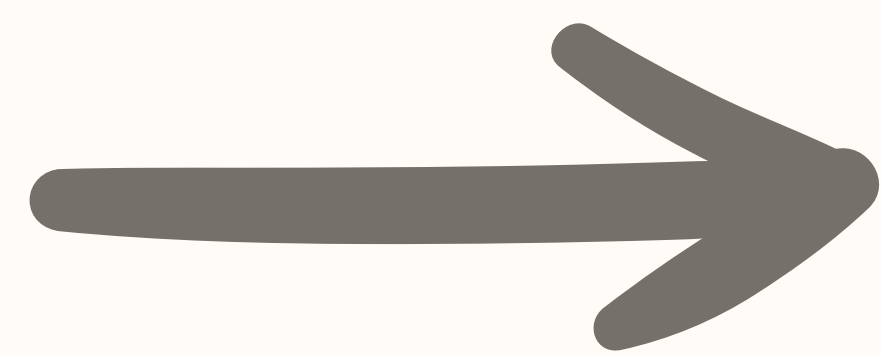
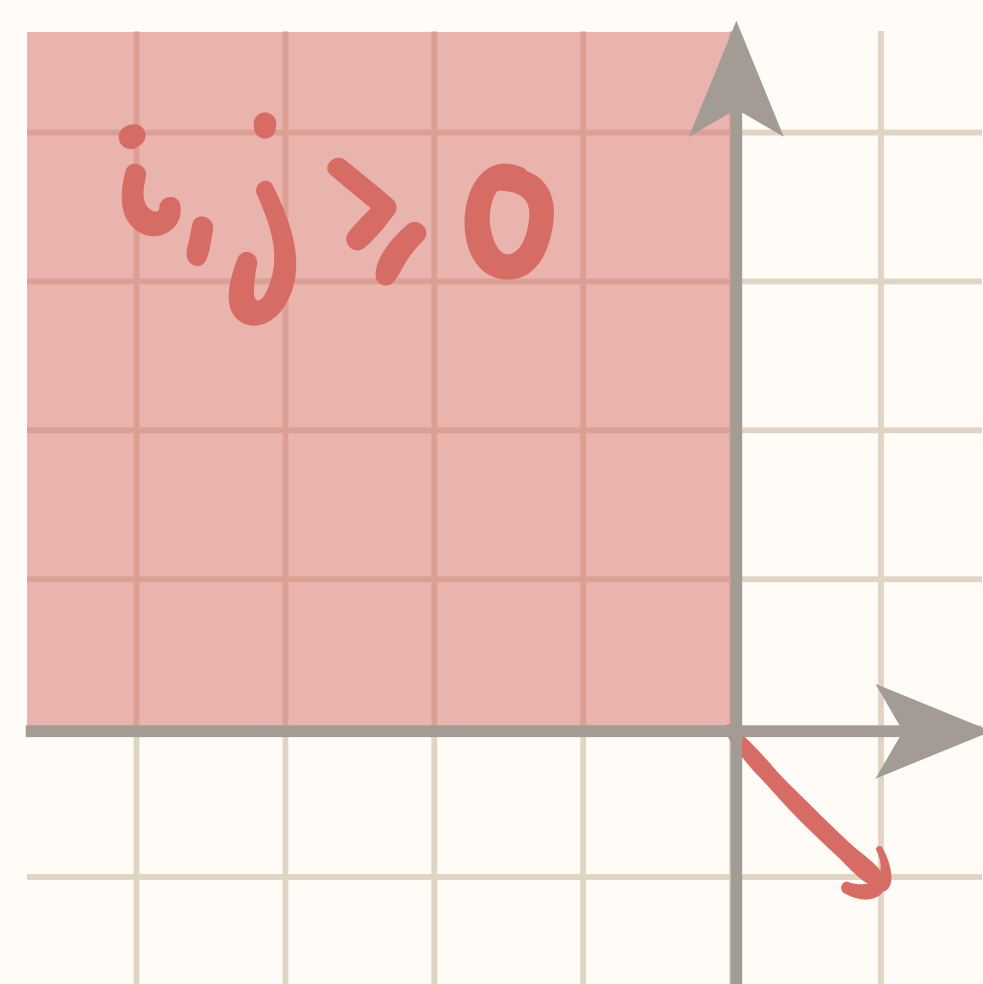
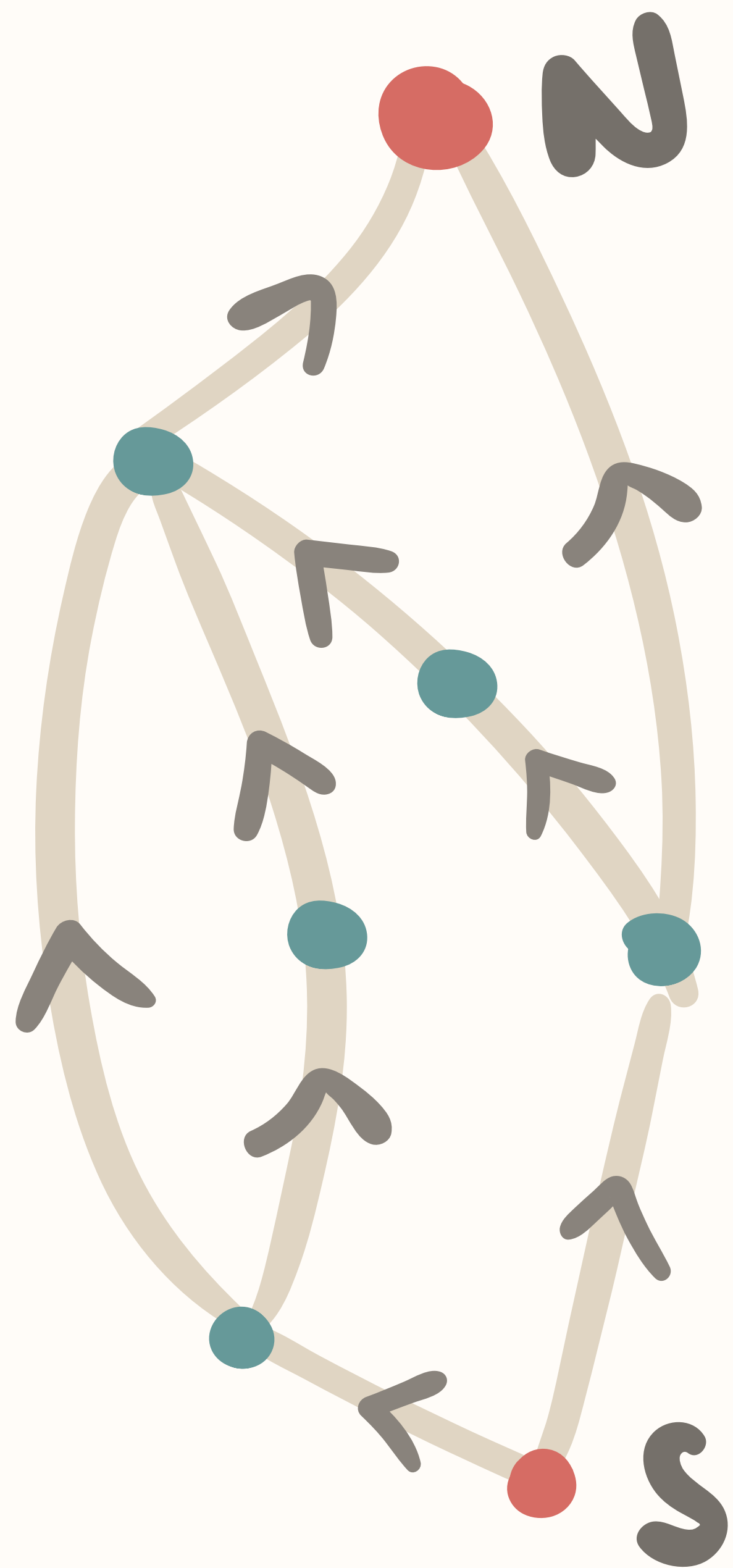


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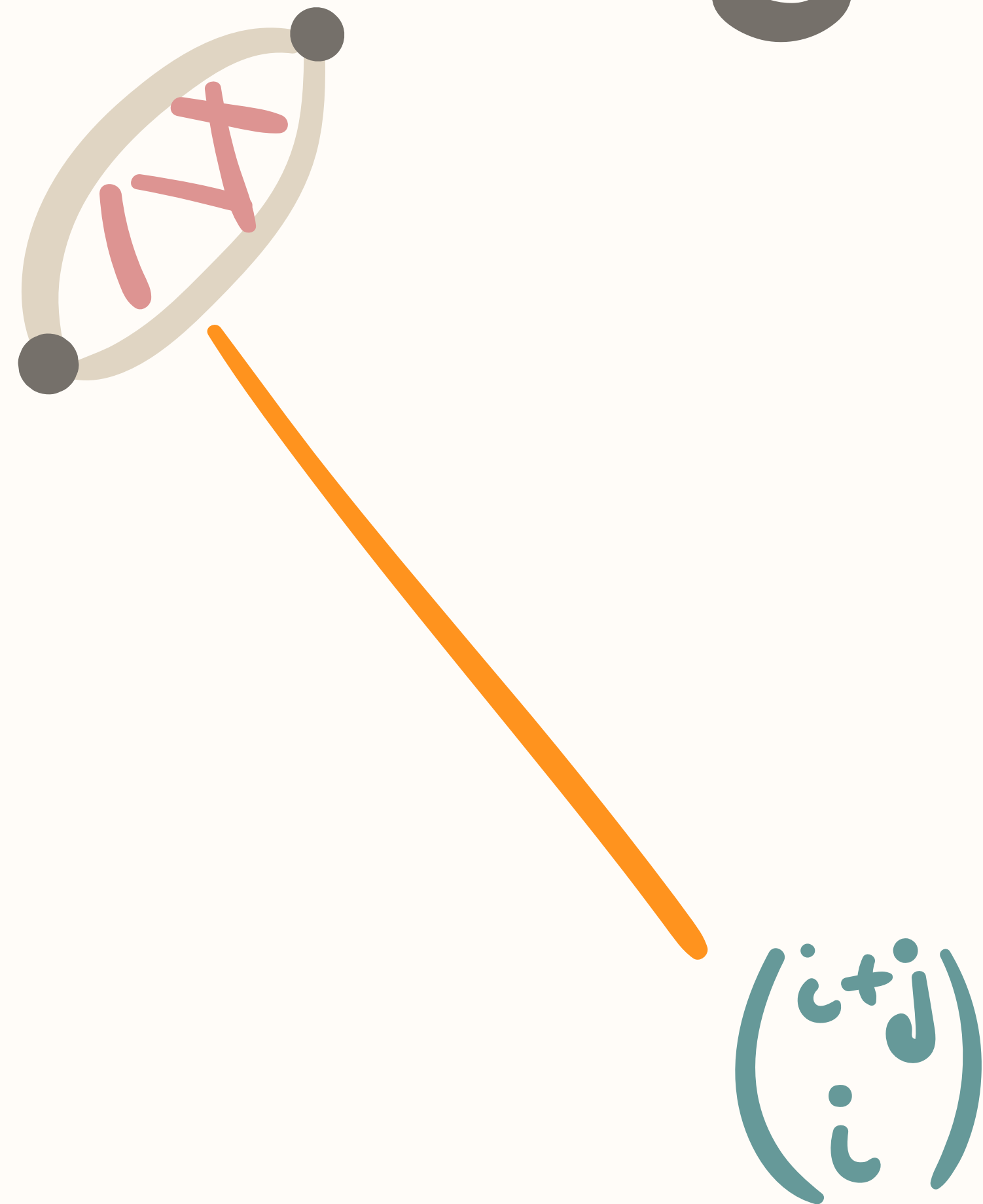
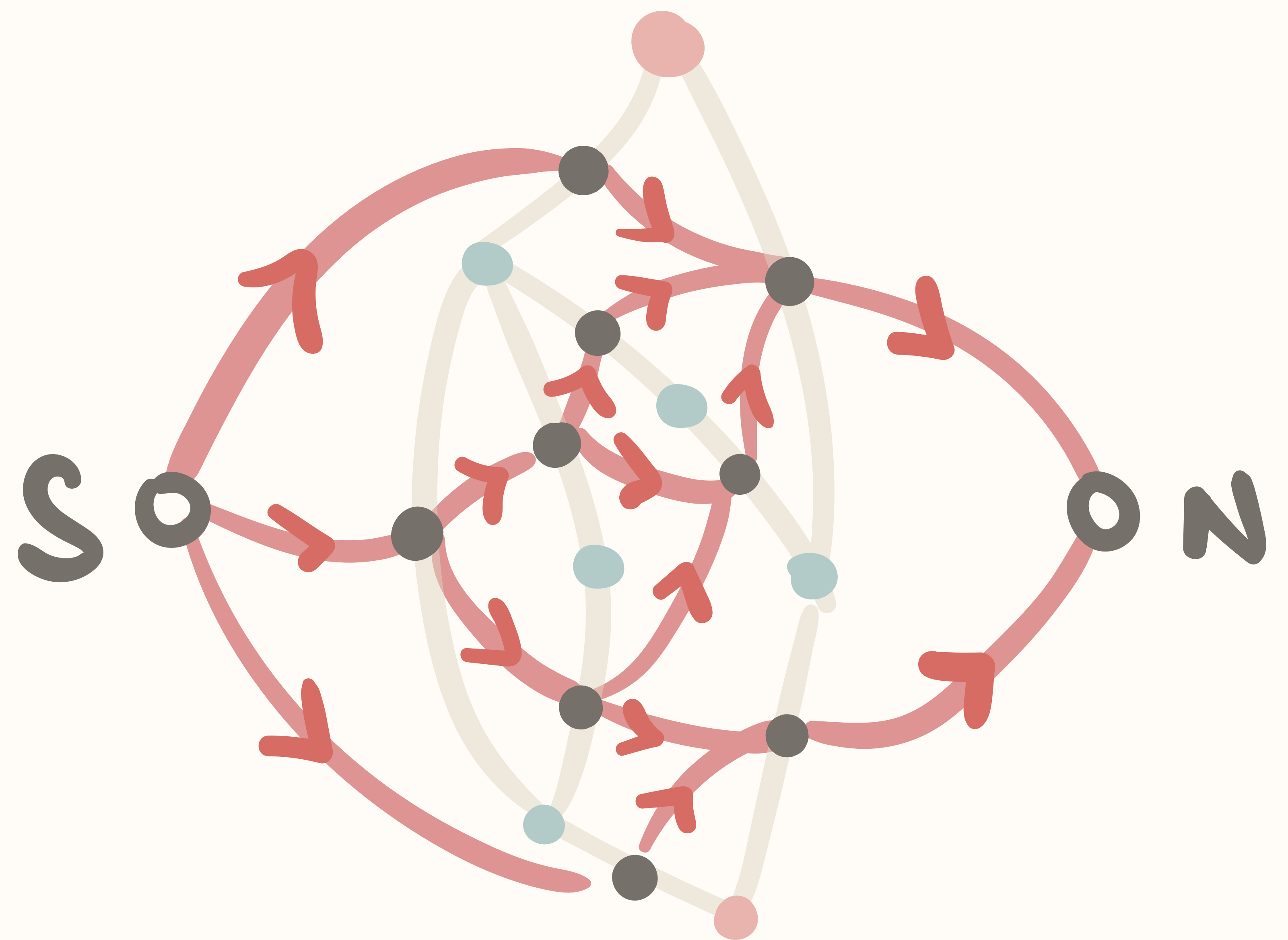


Specialization to Posets by vertices

Bipolar orientation

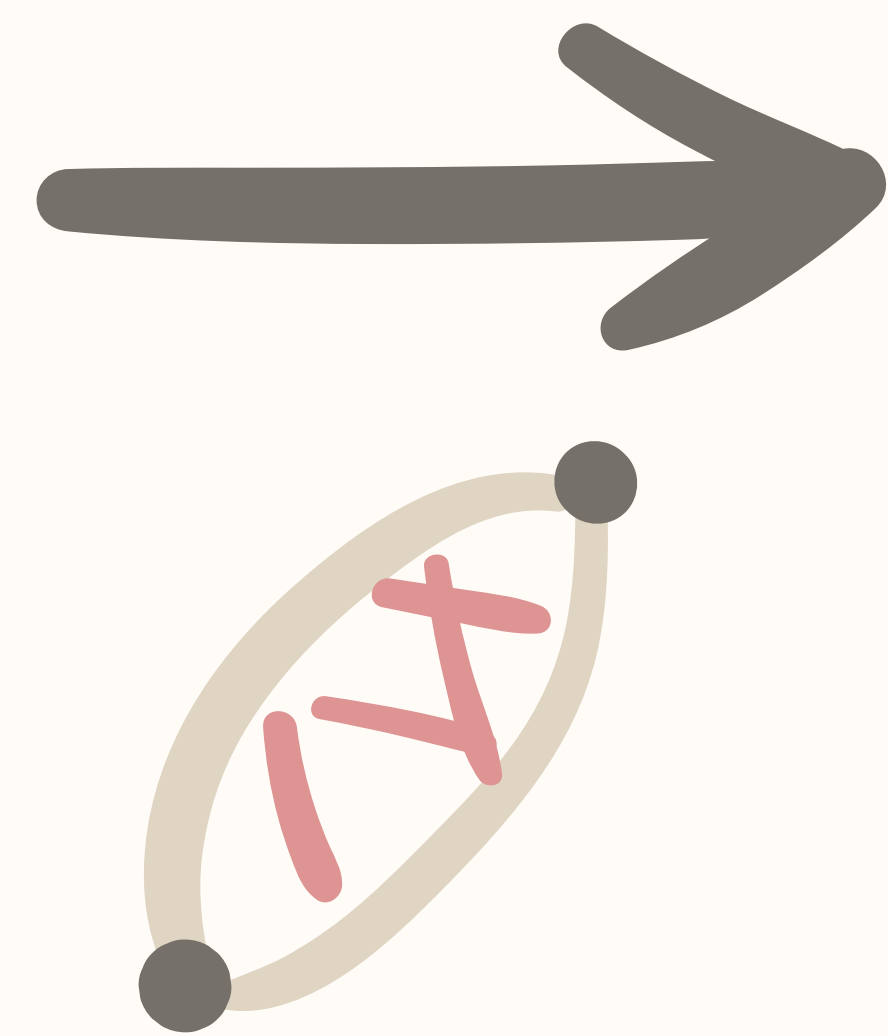
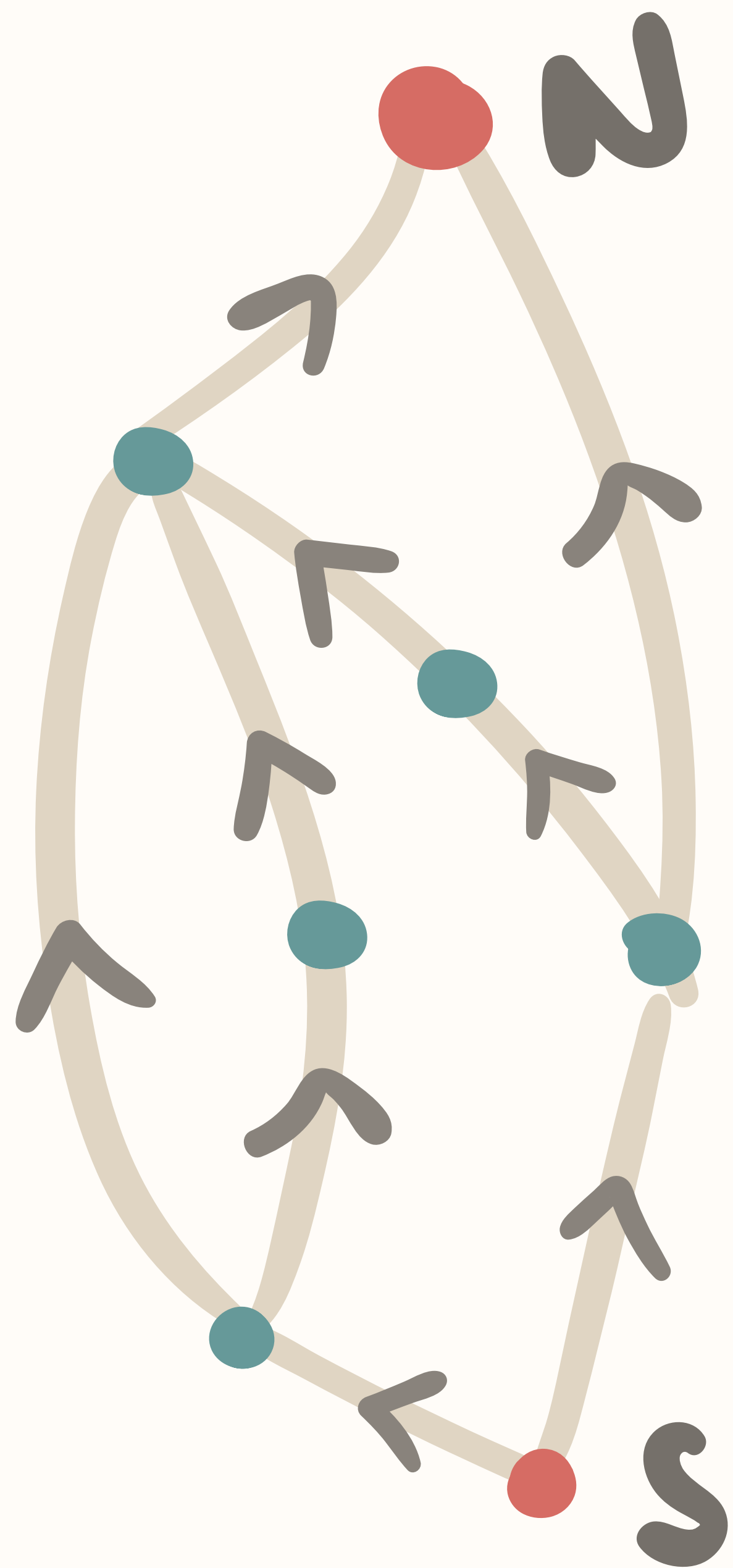


poset

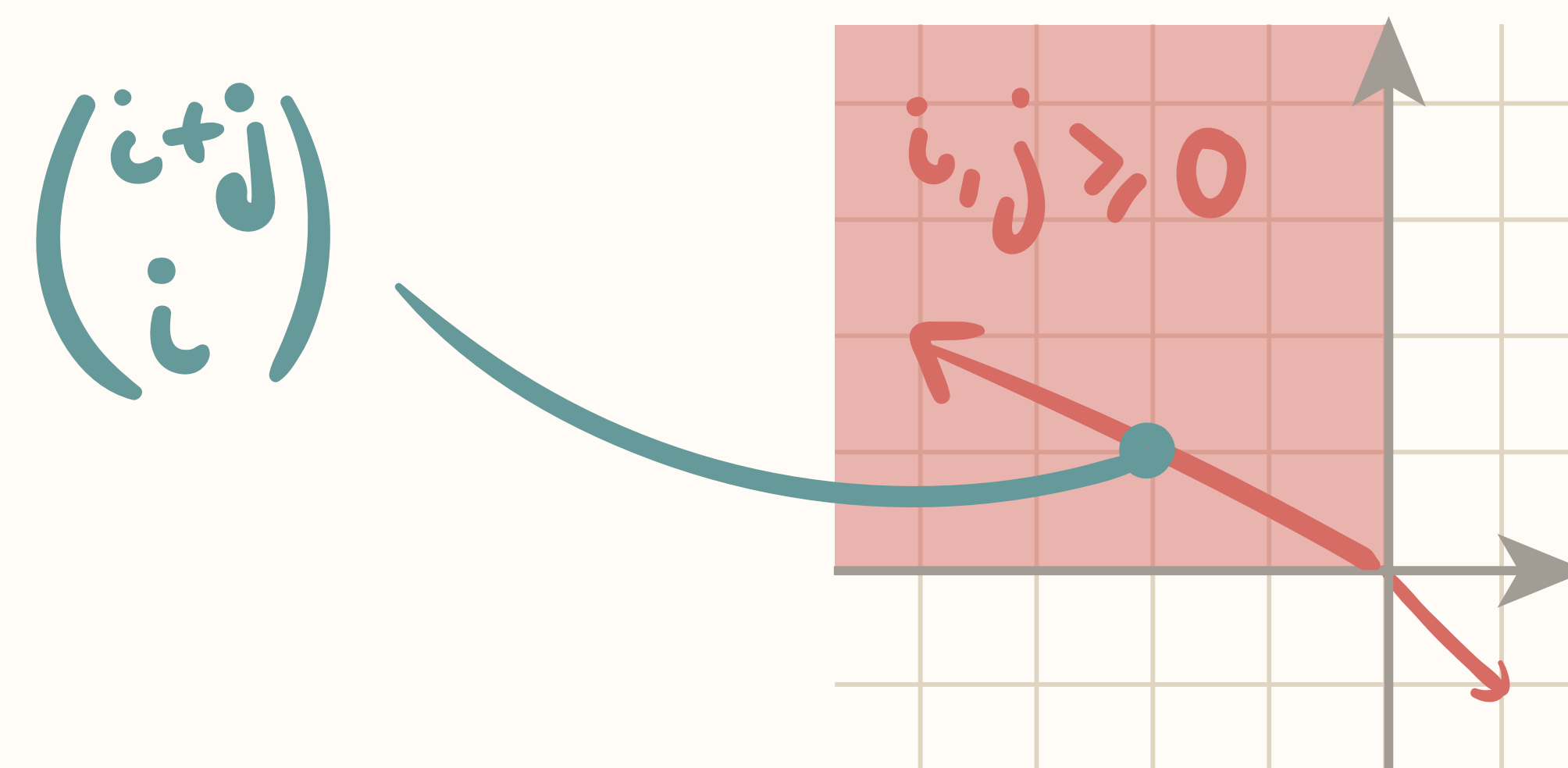
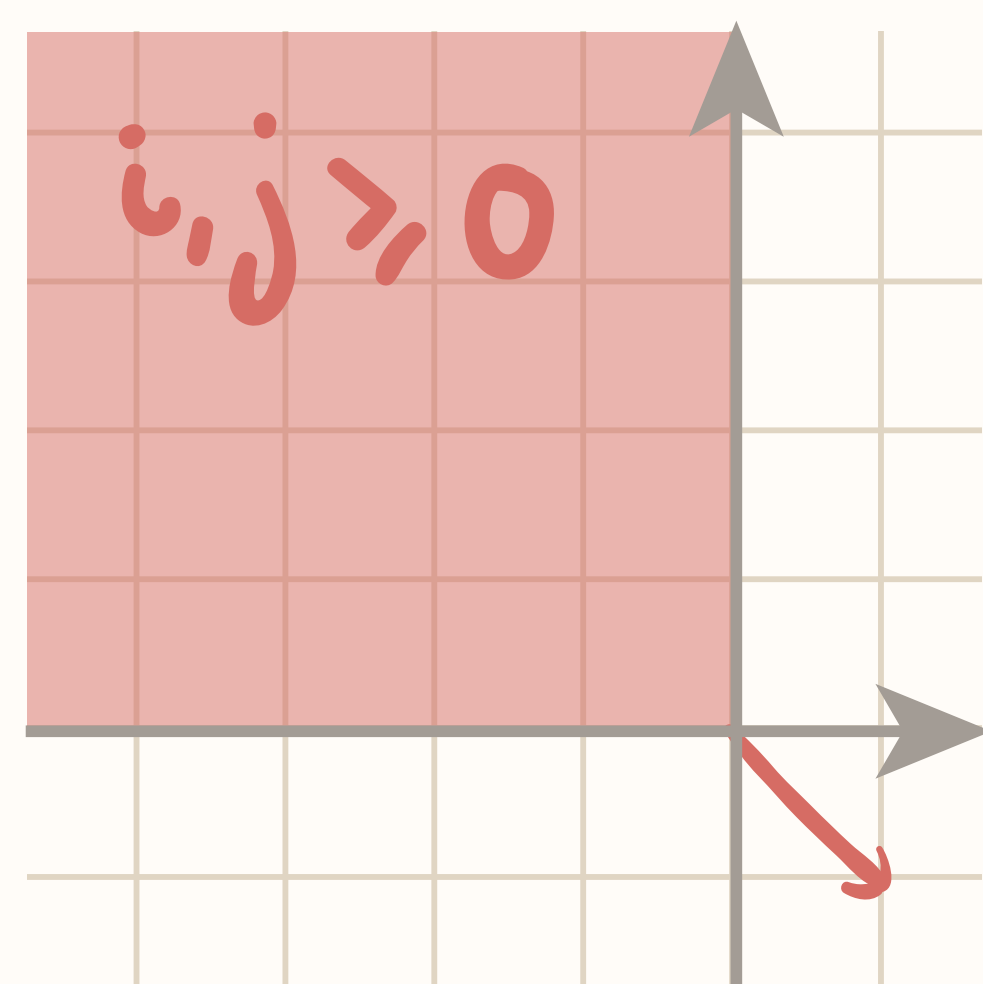
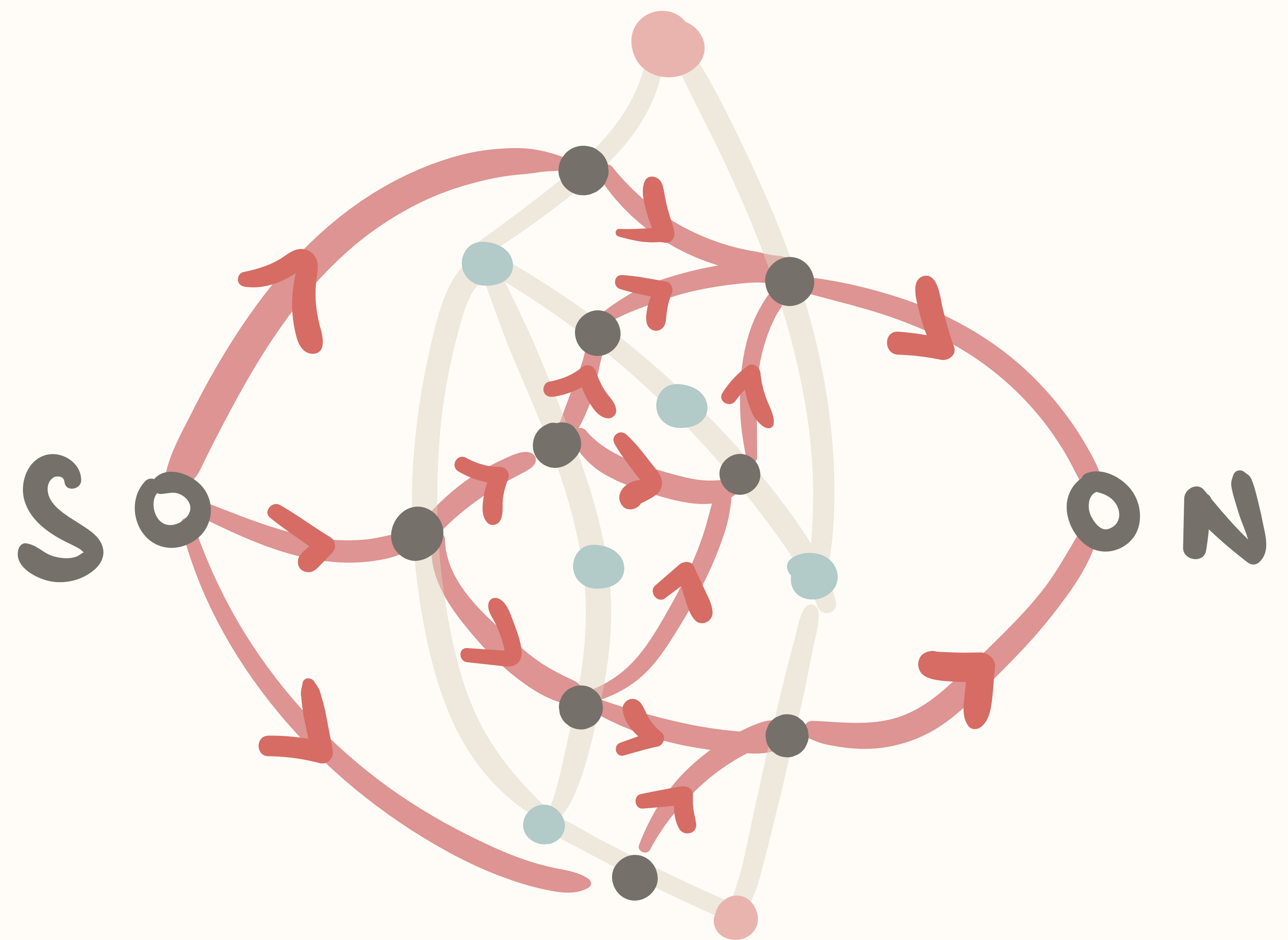


Specialization to Posets by vertices

Bipolar orientation



poset



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The KMSW Bijection

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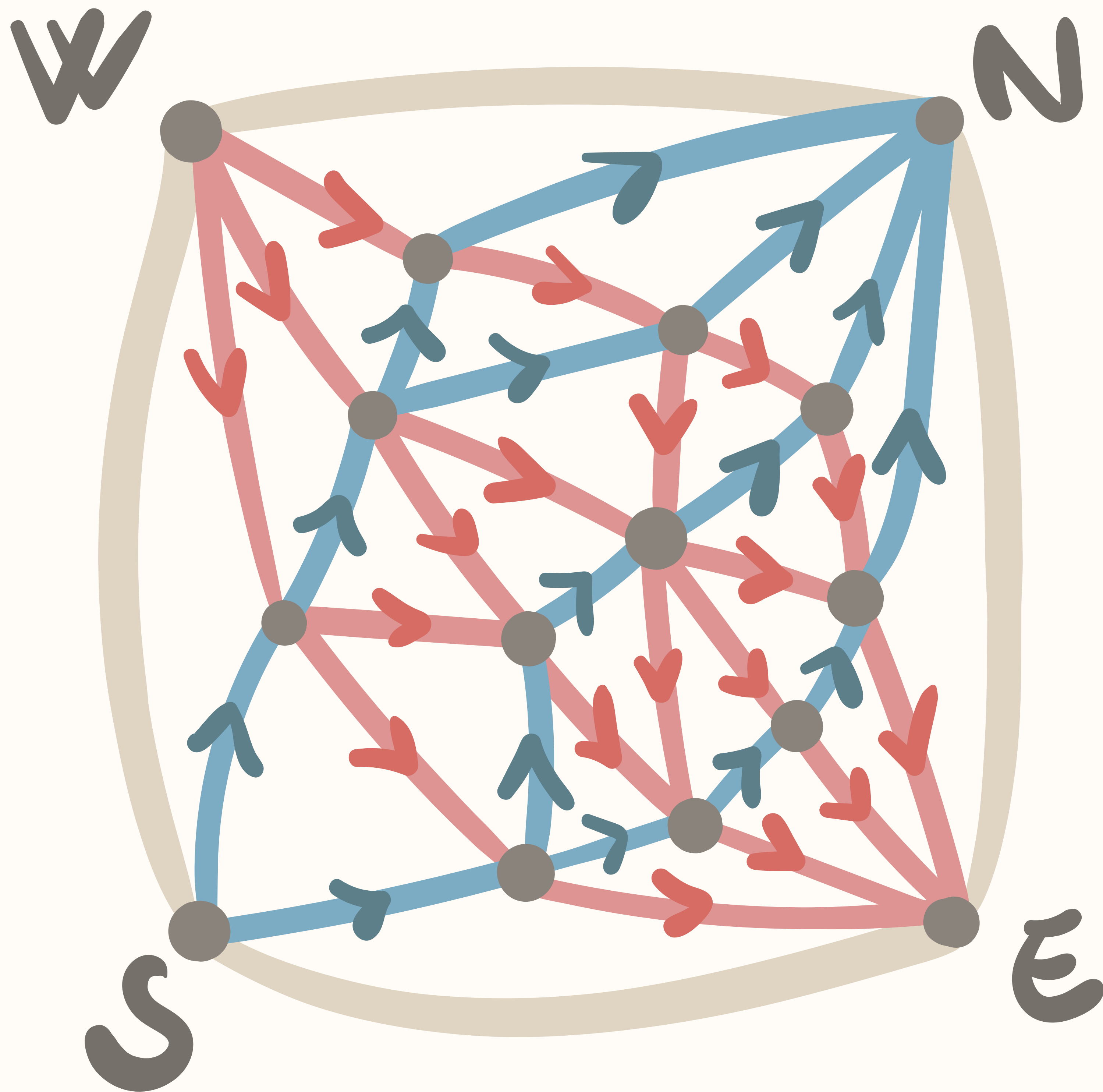
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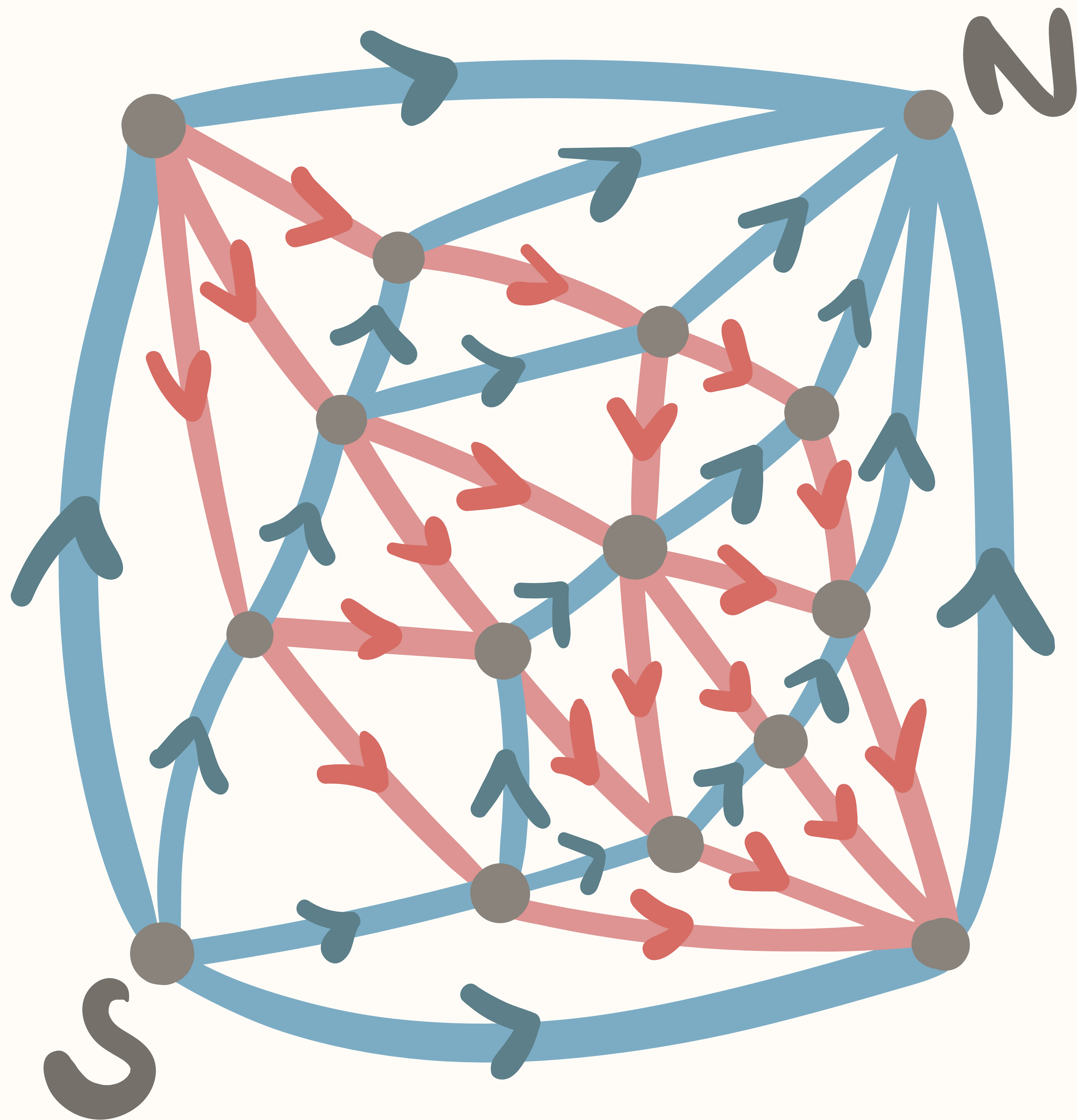
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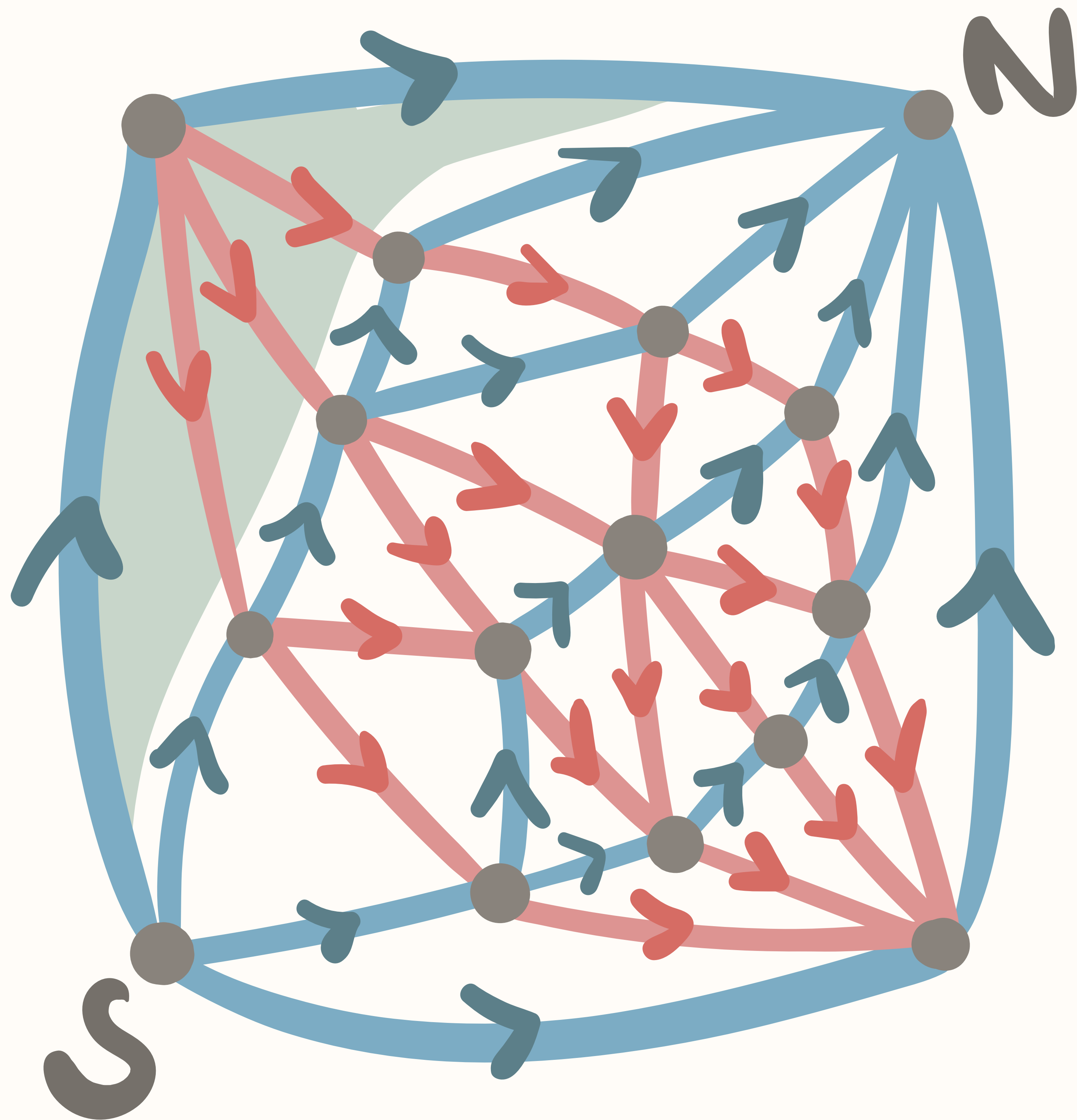
Specialization to transversal structures



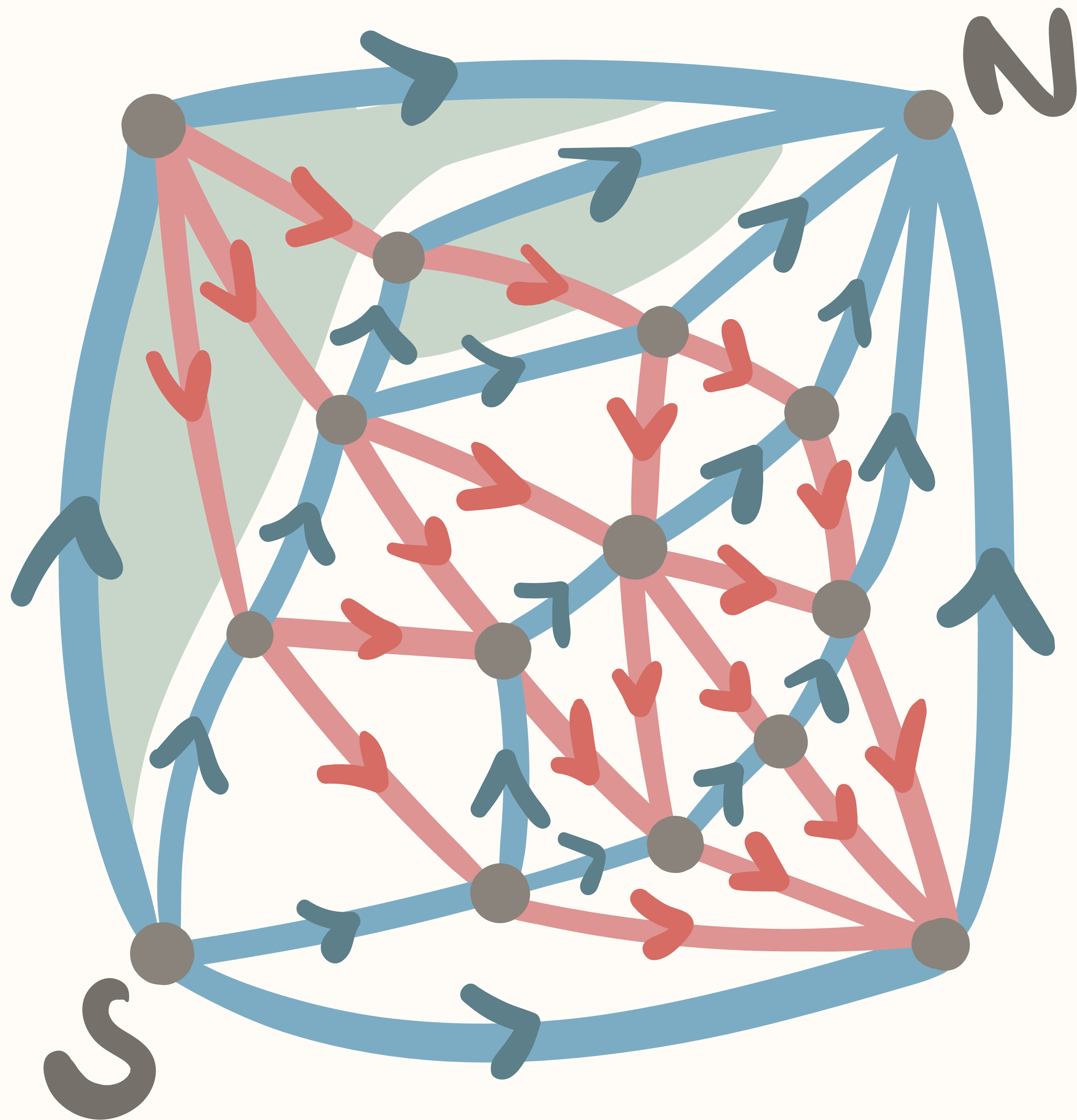
Specialization to transversal structures



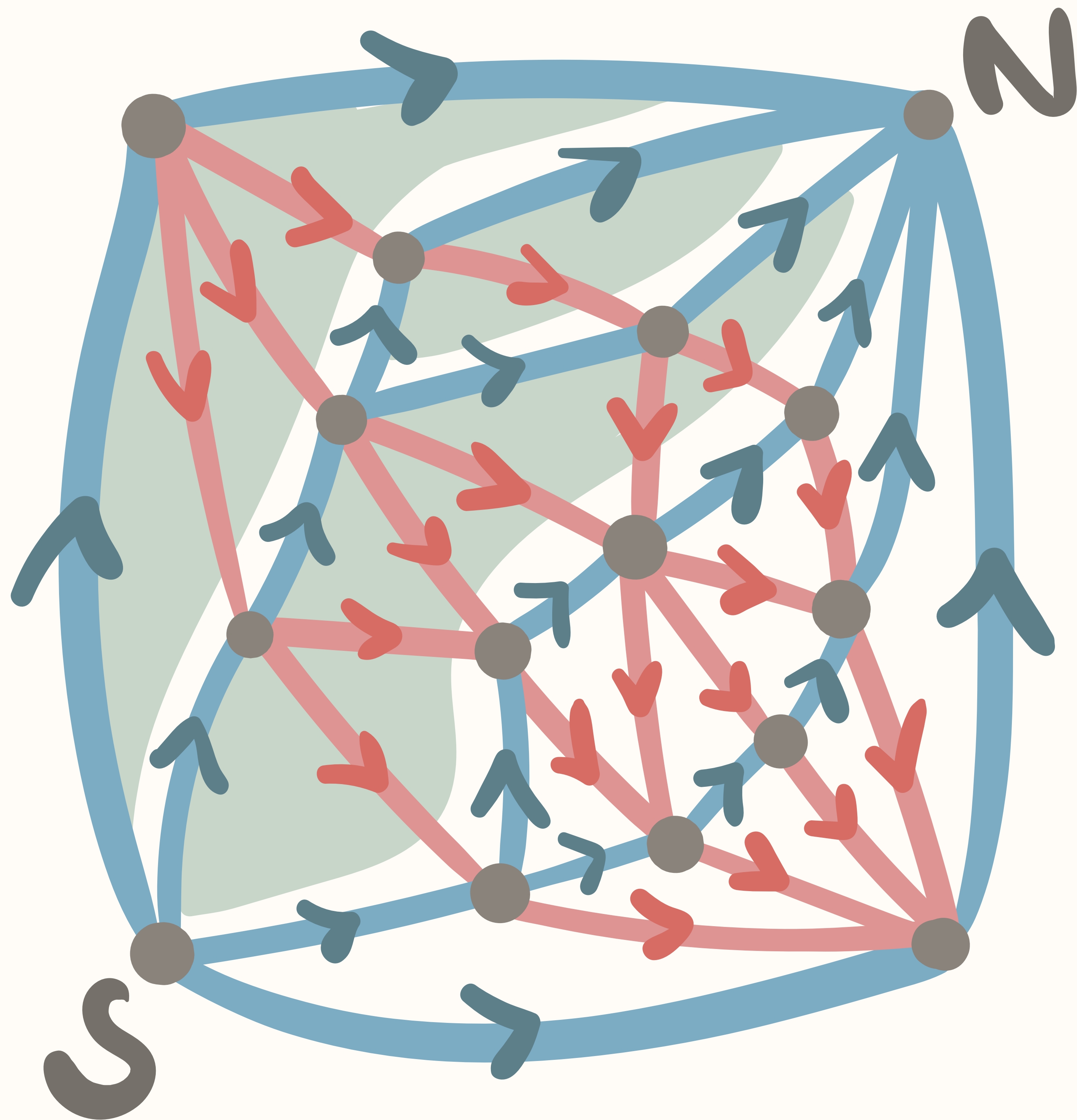
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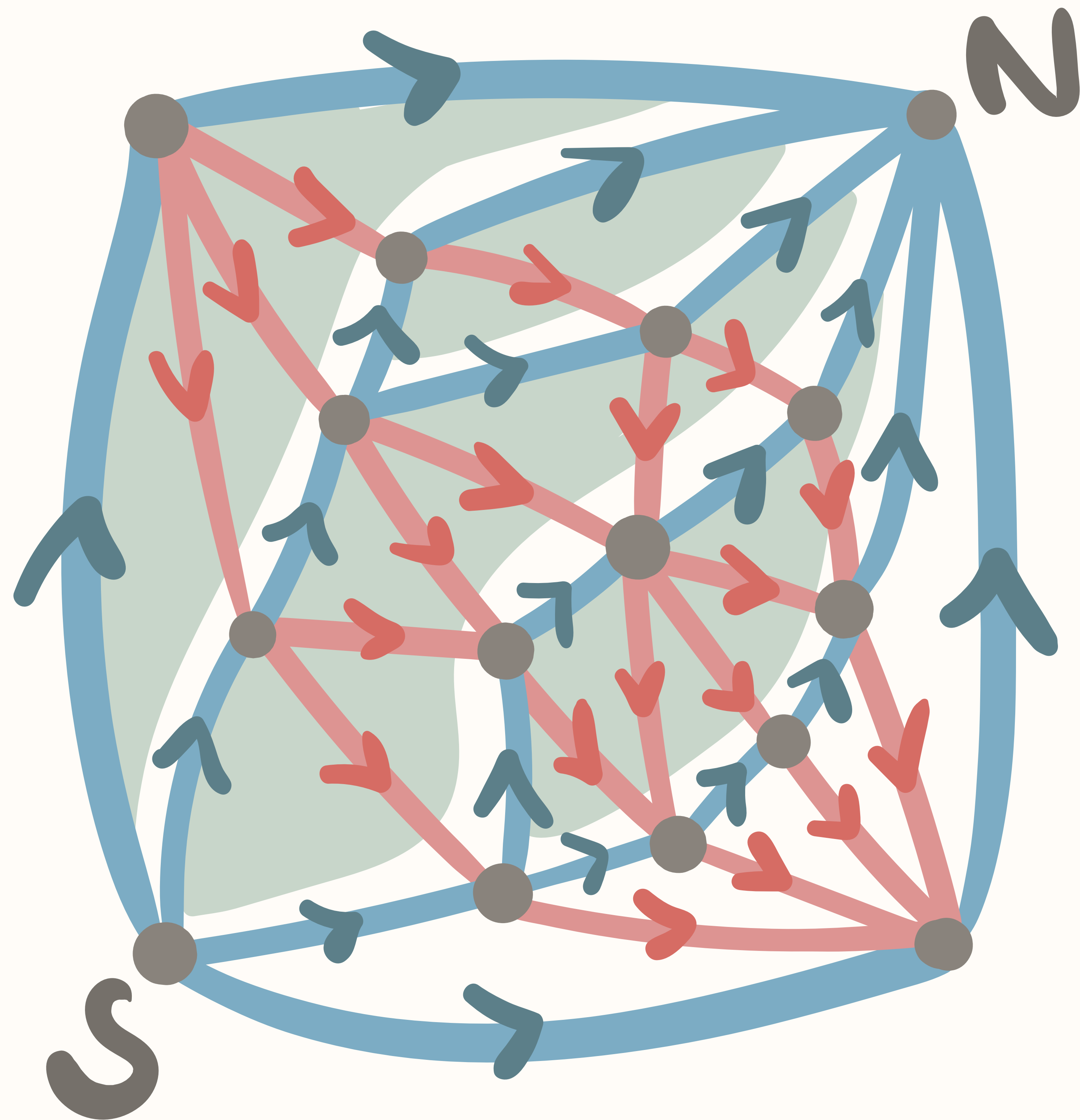
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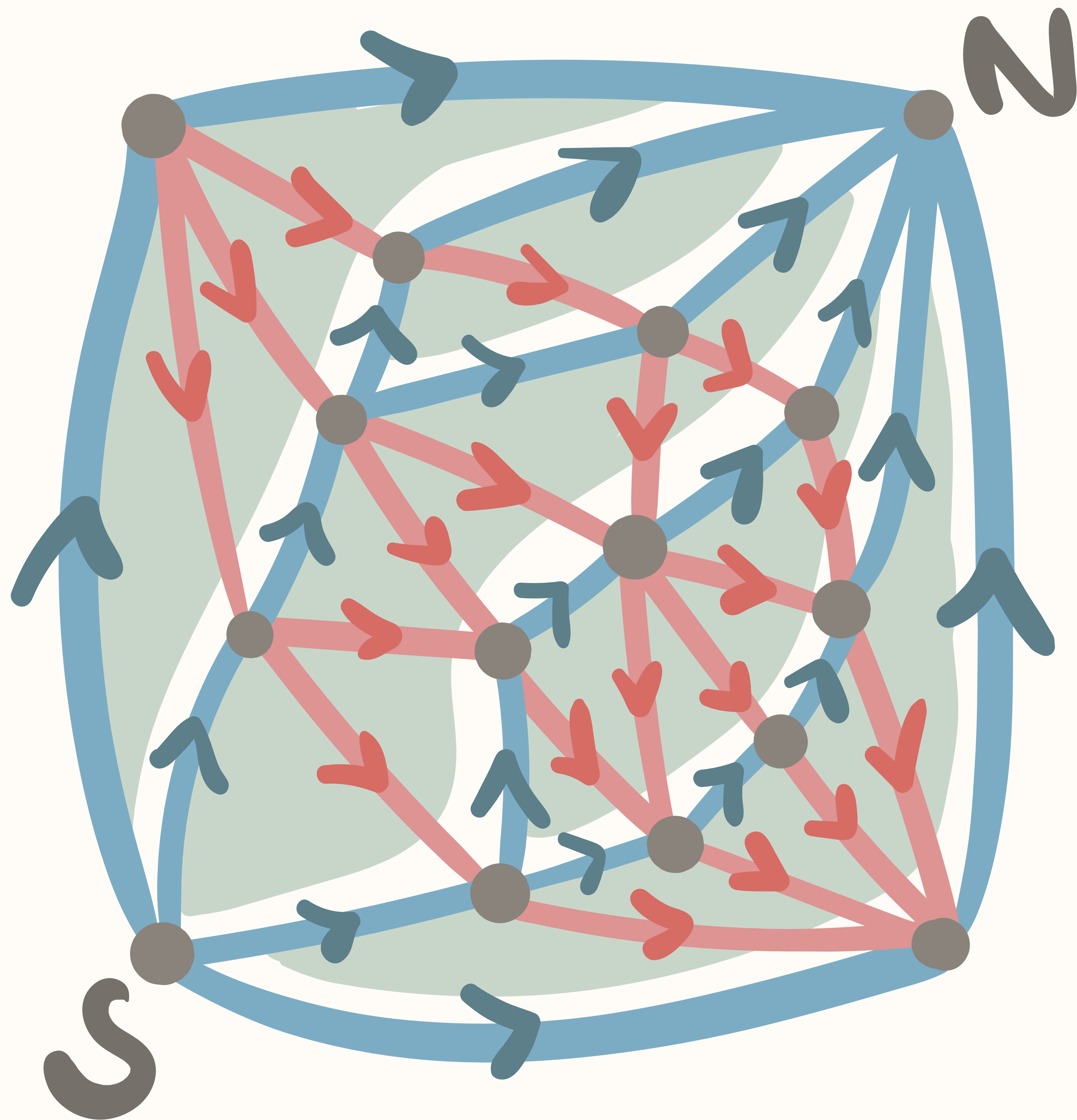
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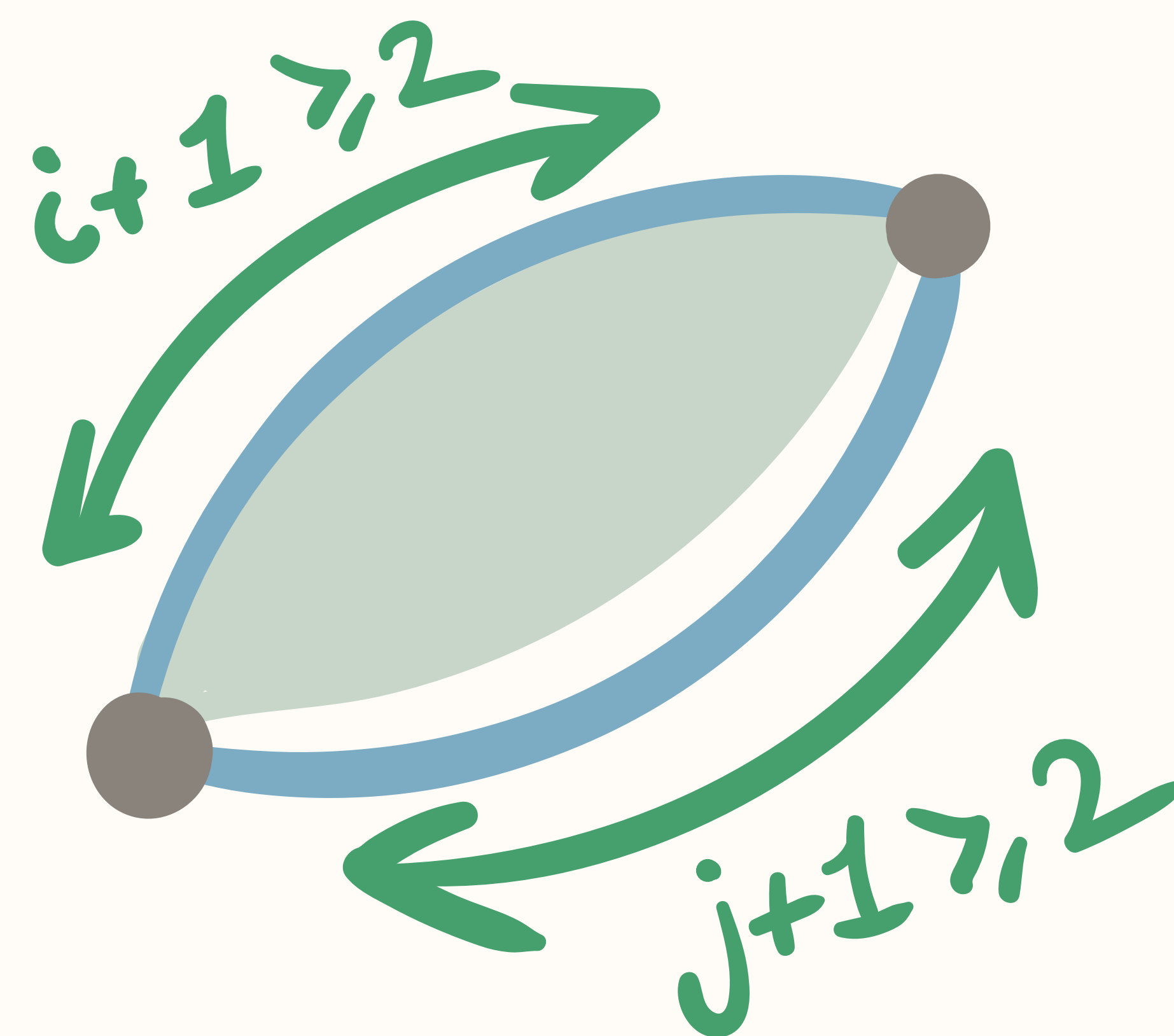
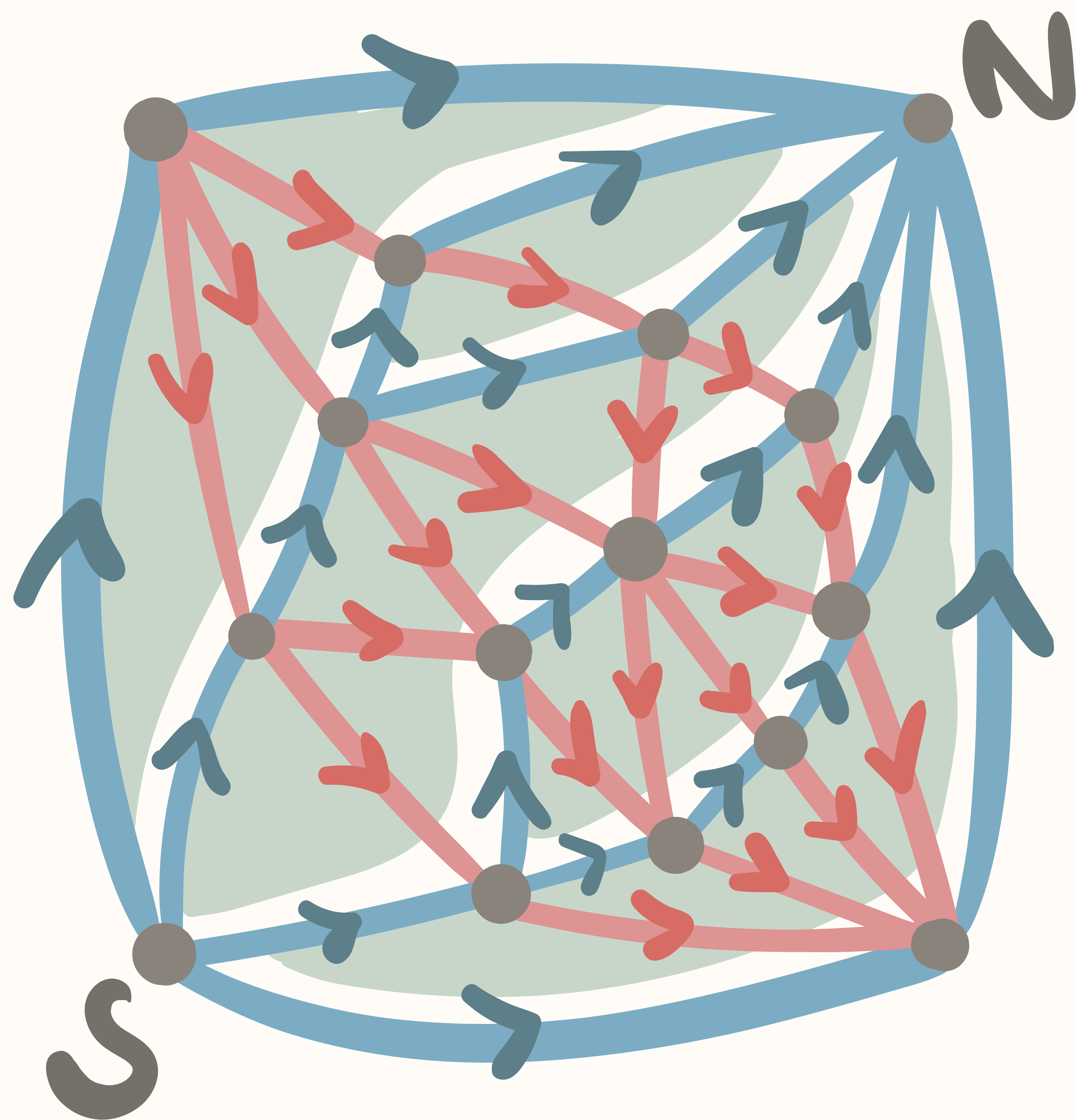
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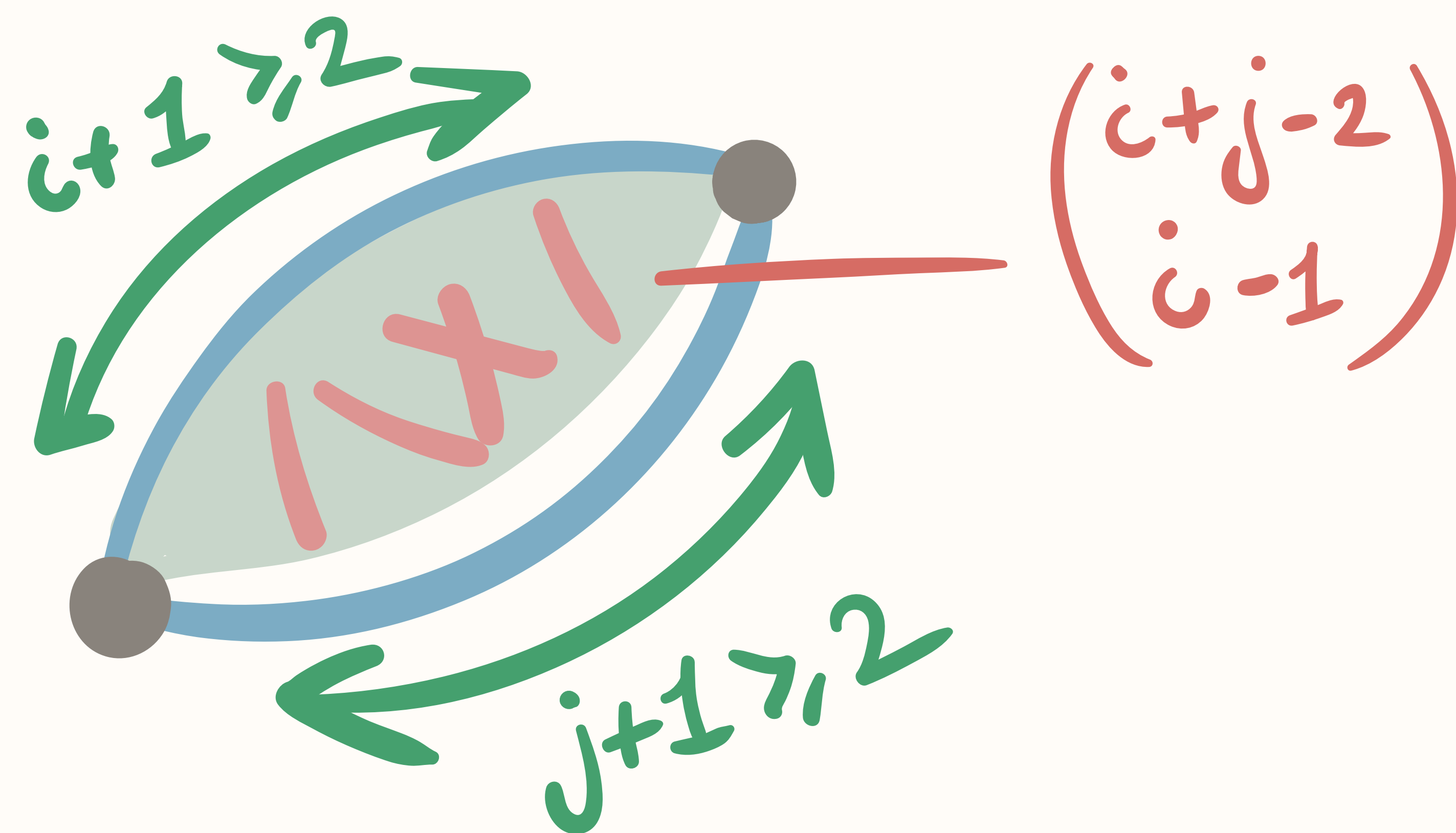
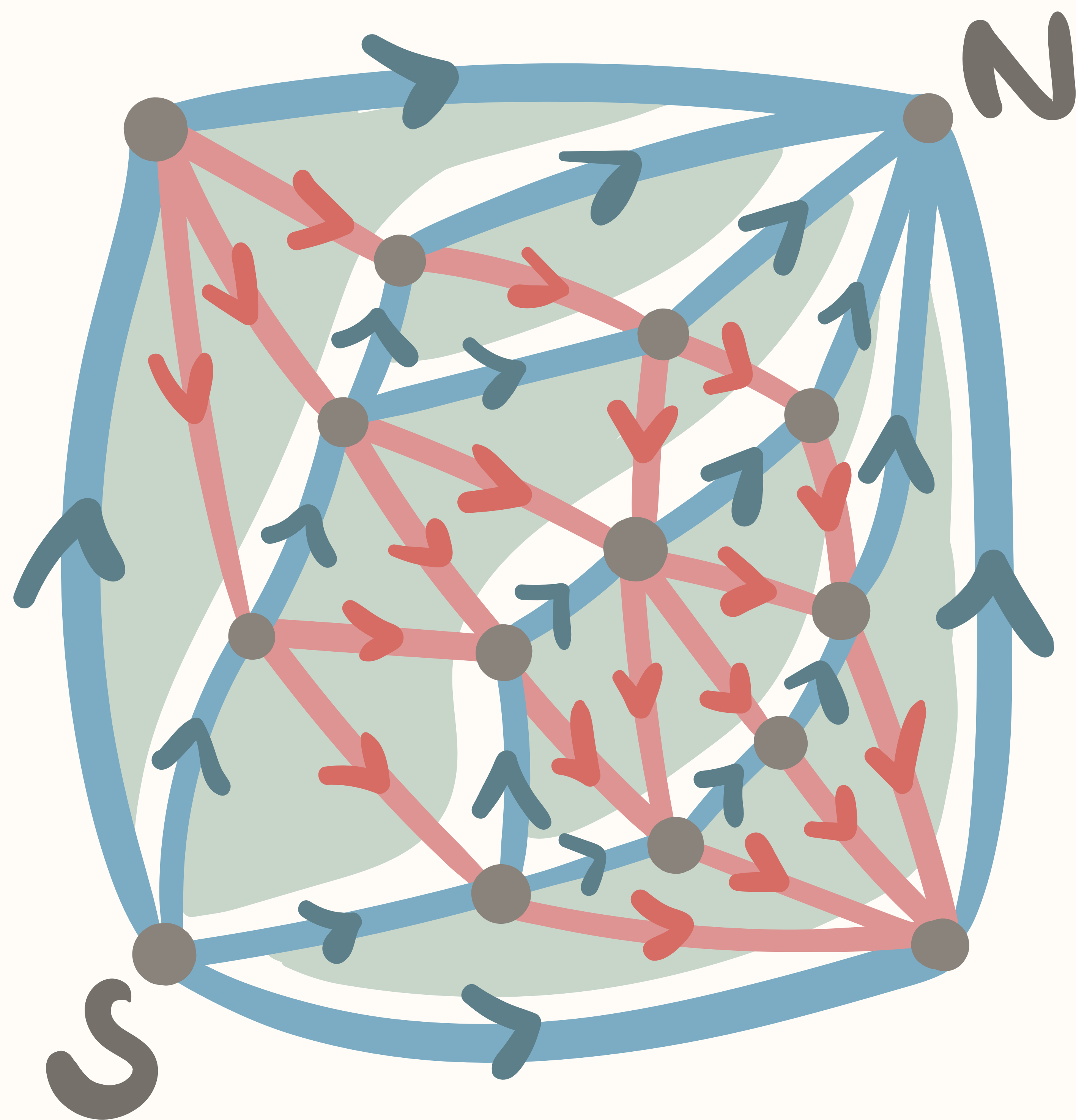
Specialization to transversal structures



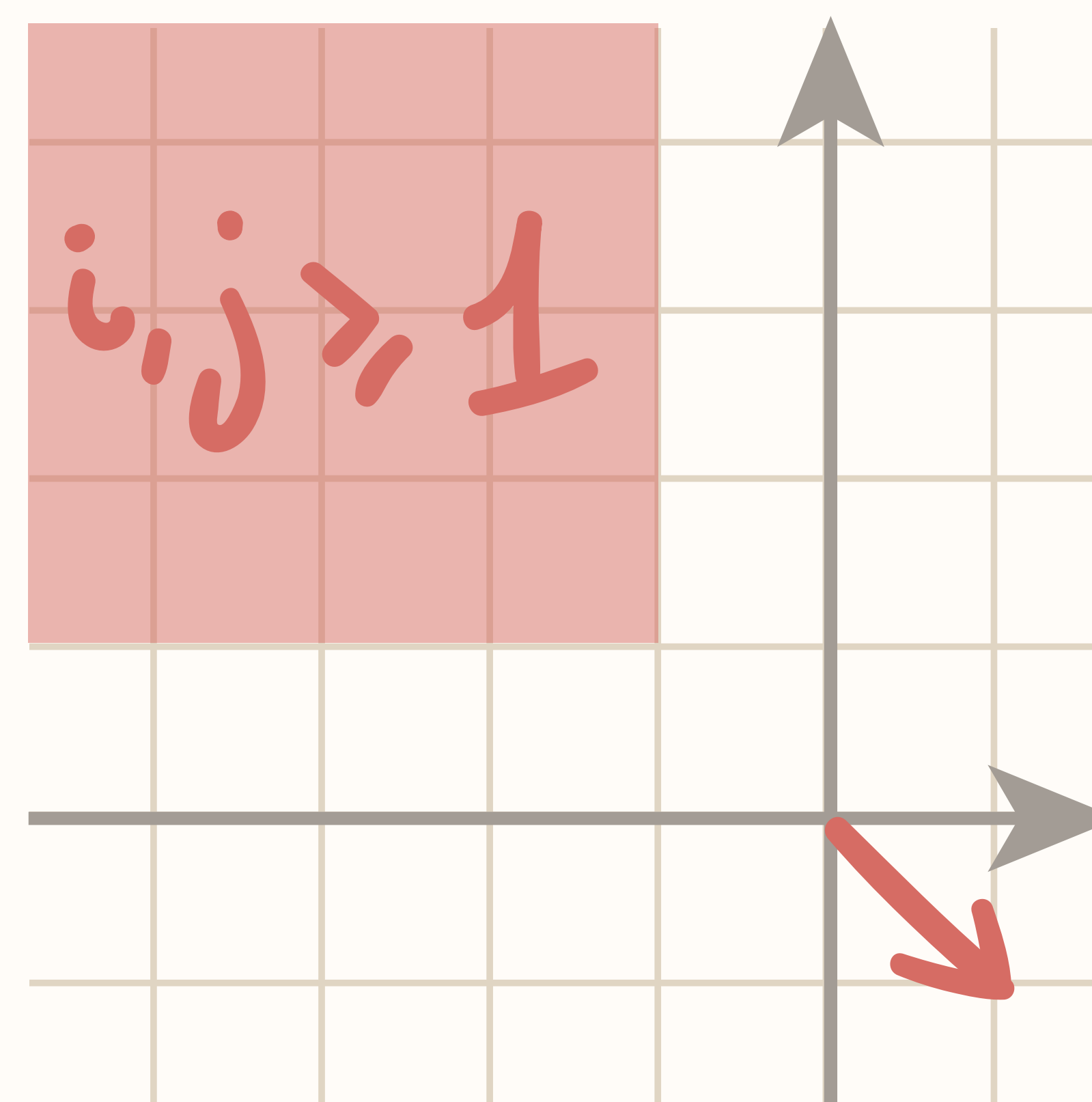
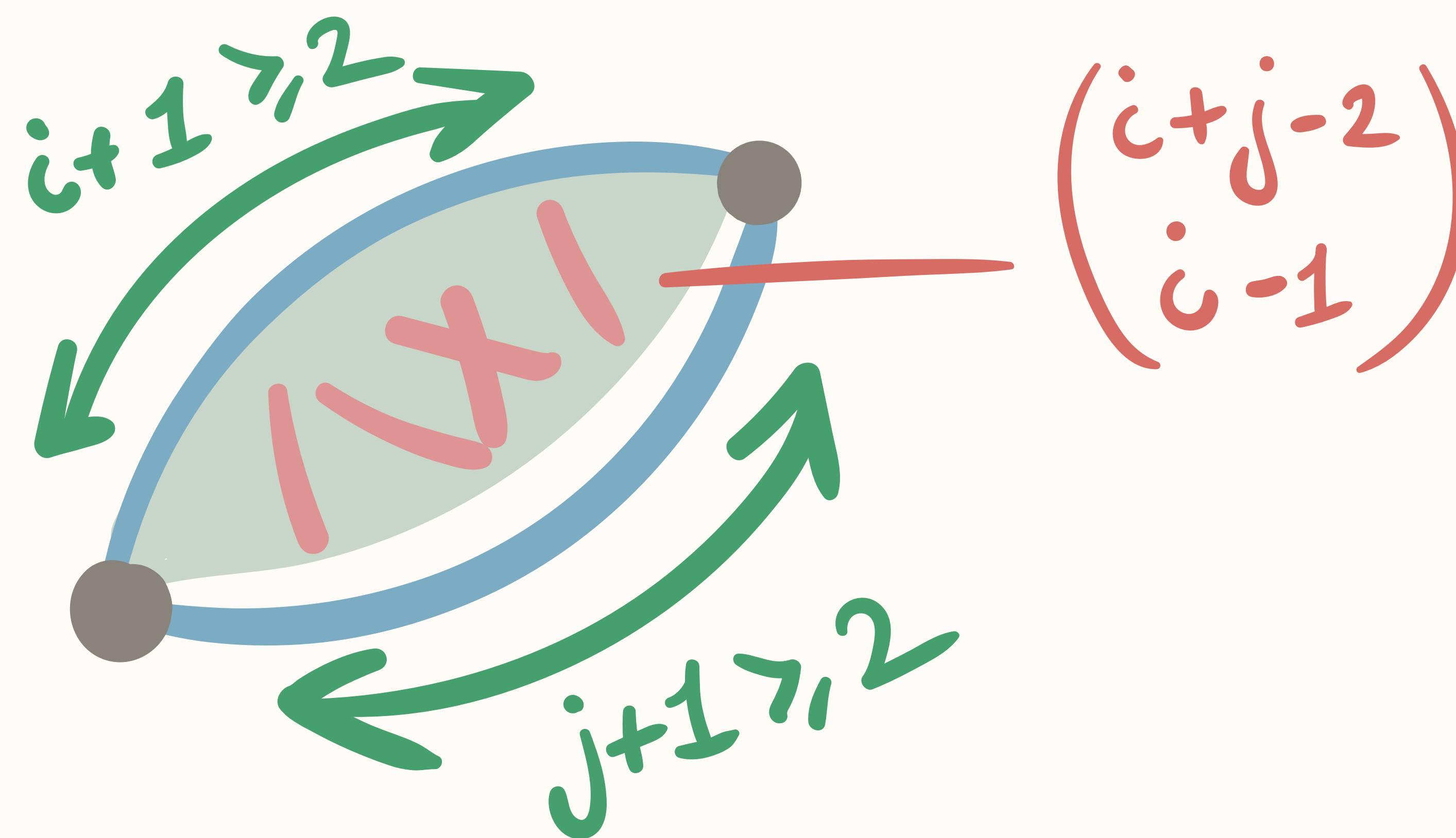
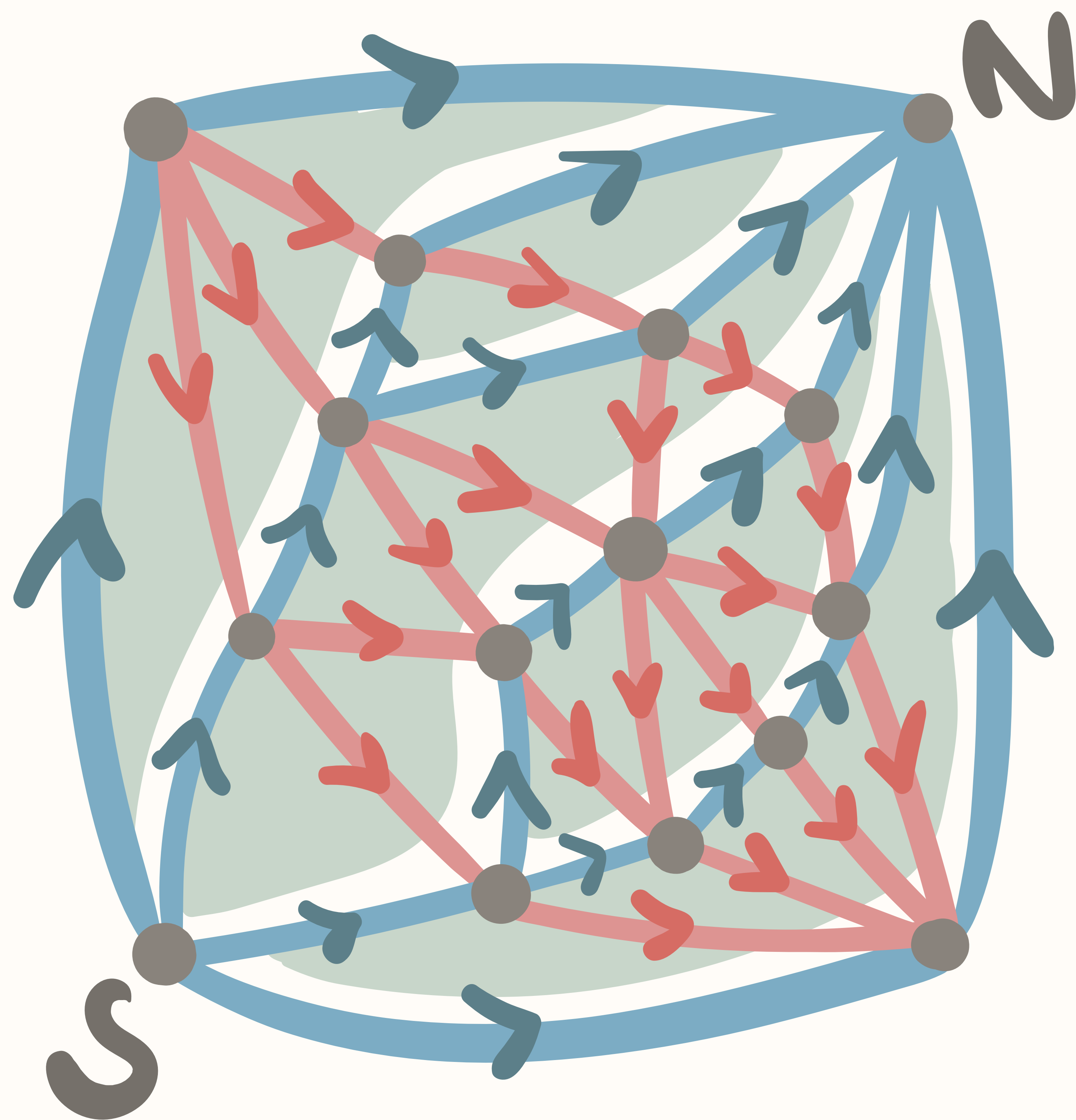
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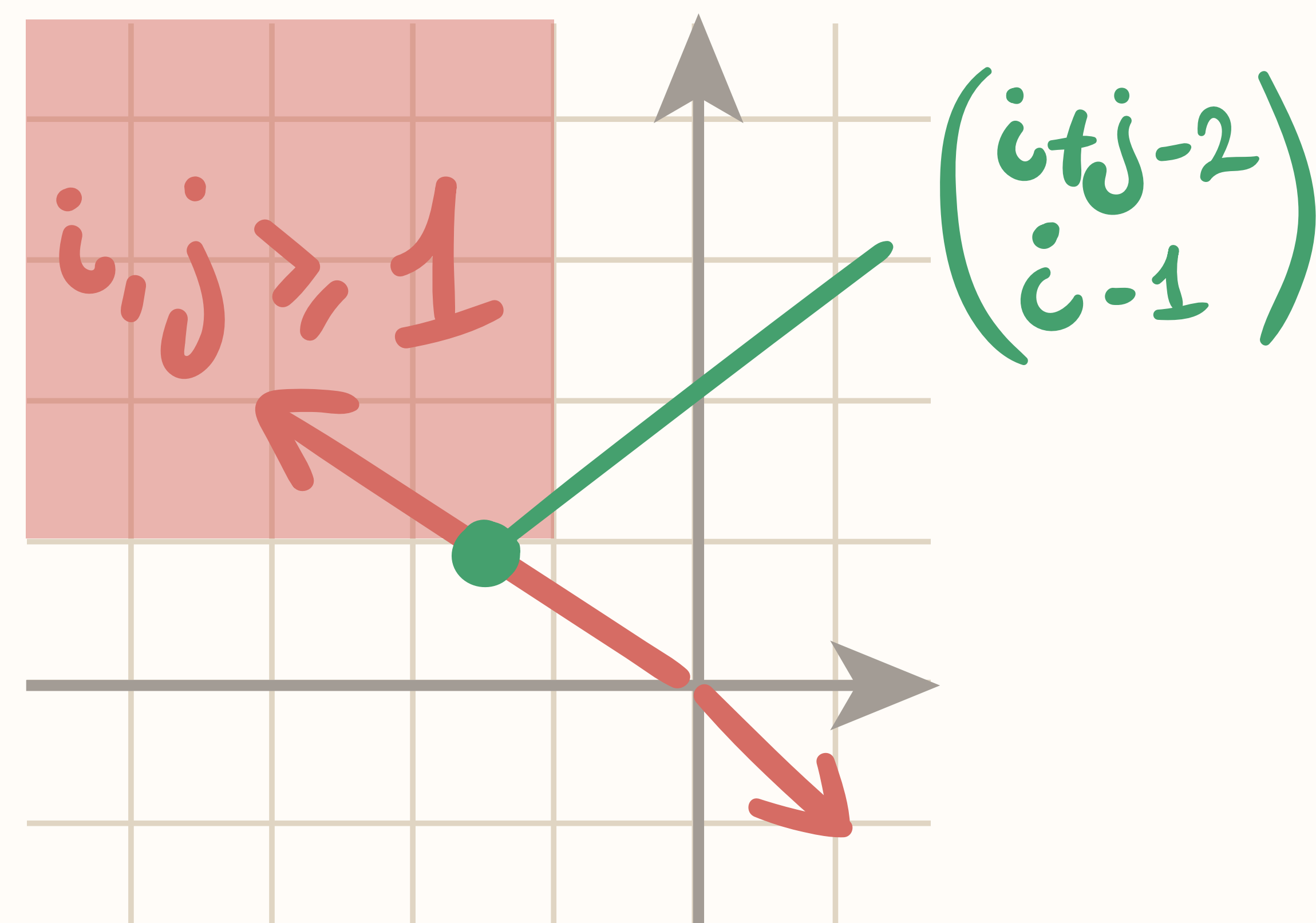
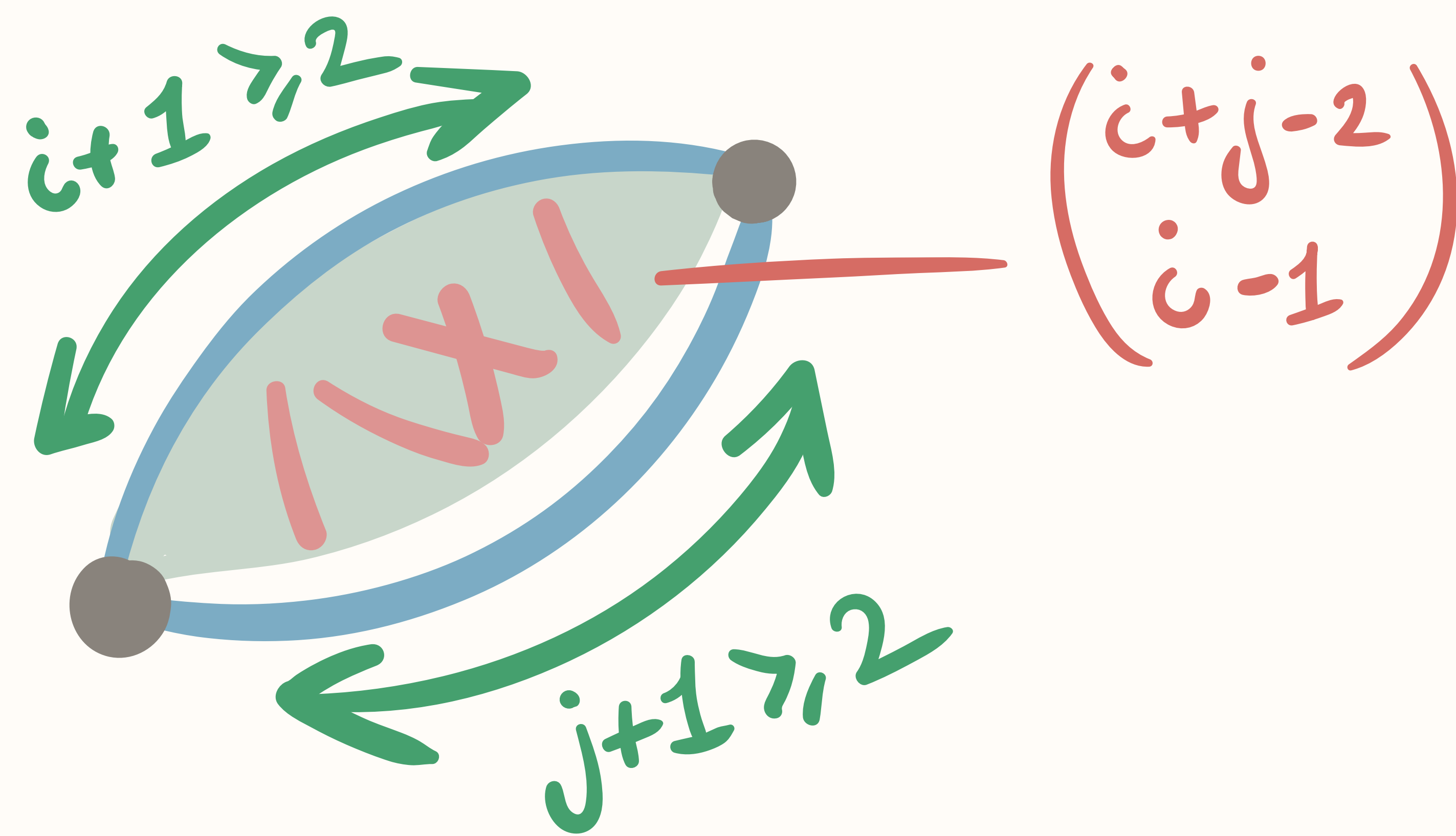
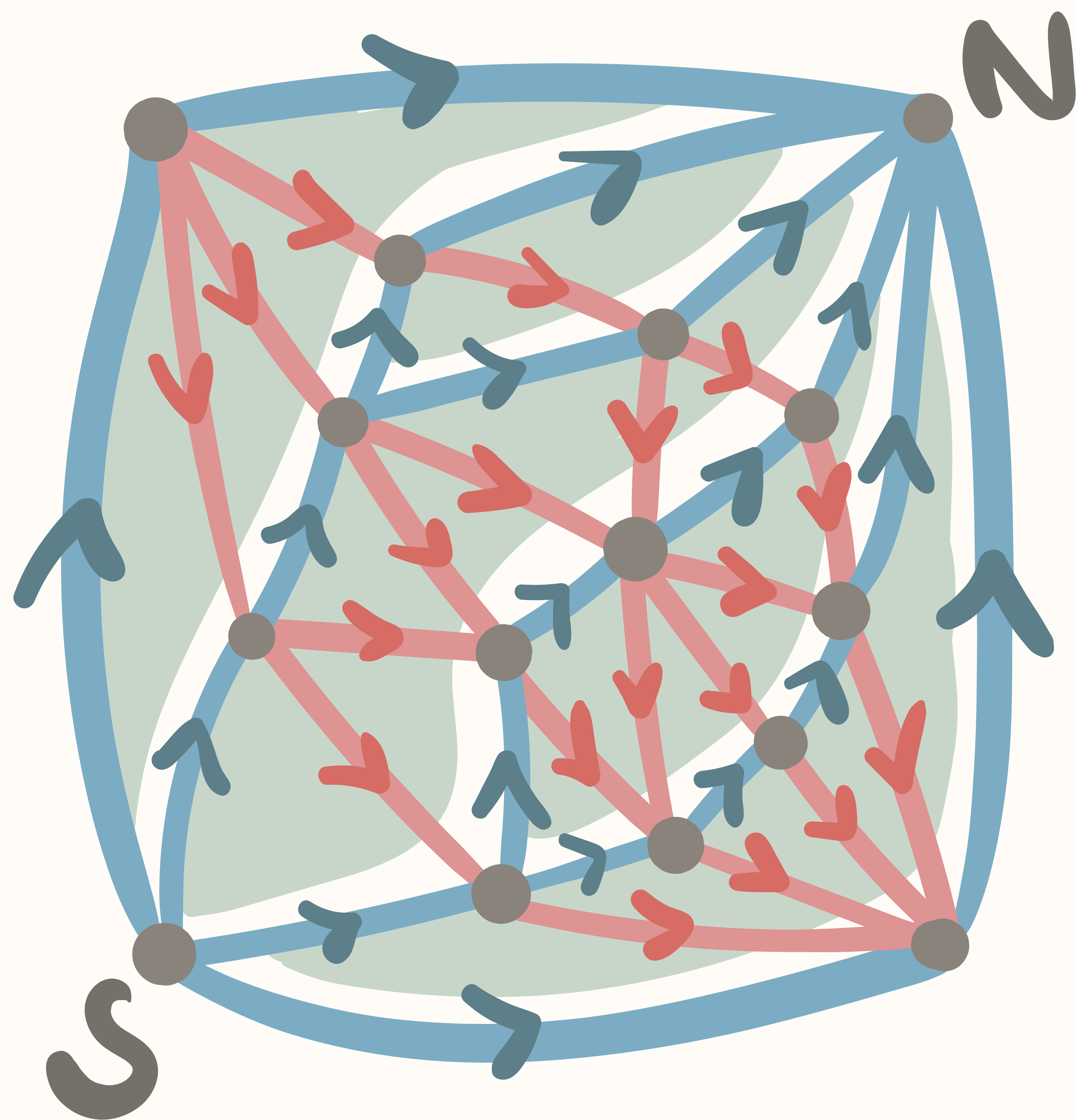
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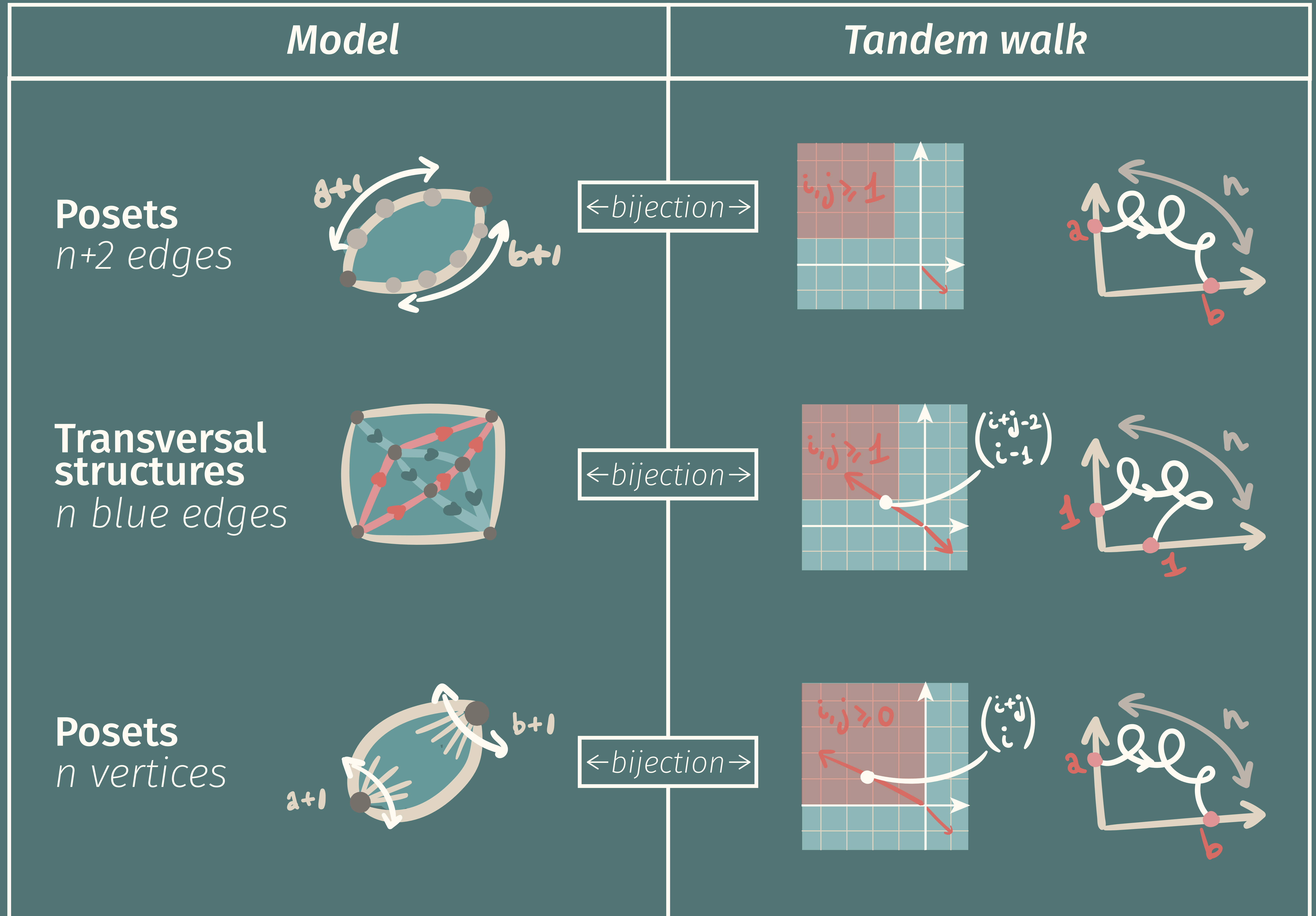
Specialization to transversal structures



Specialization to transversal structures



Specializations summary



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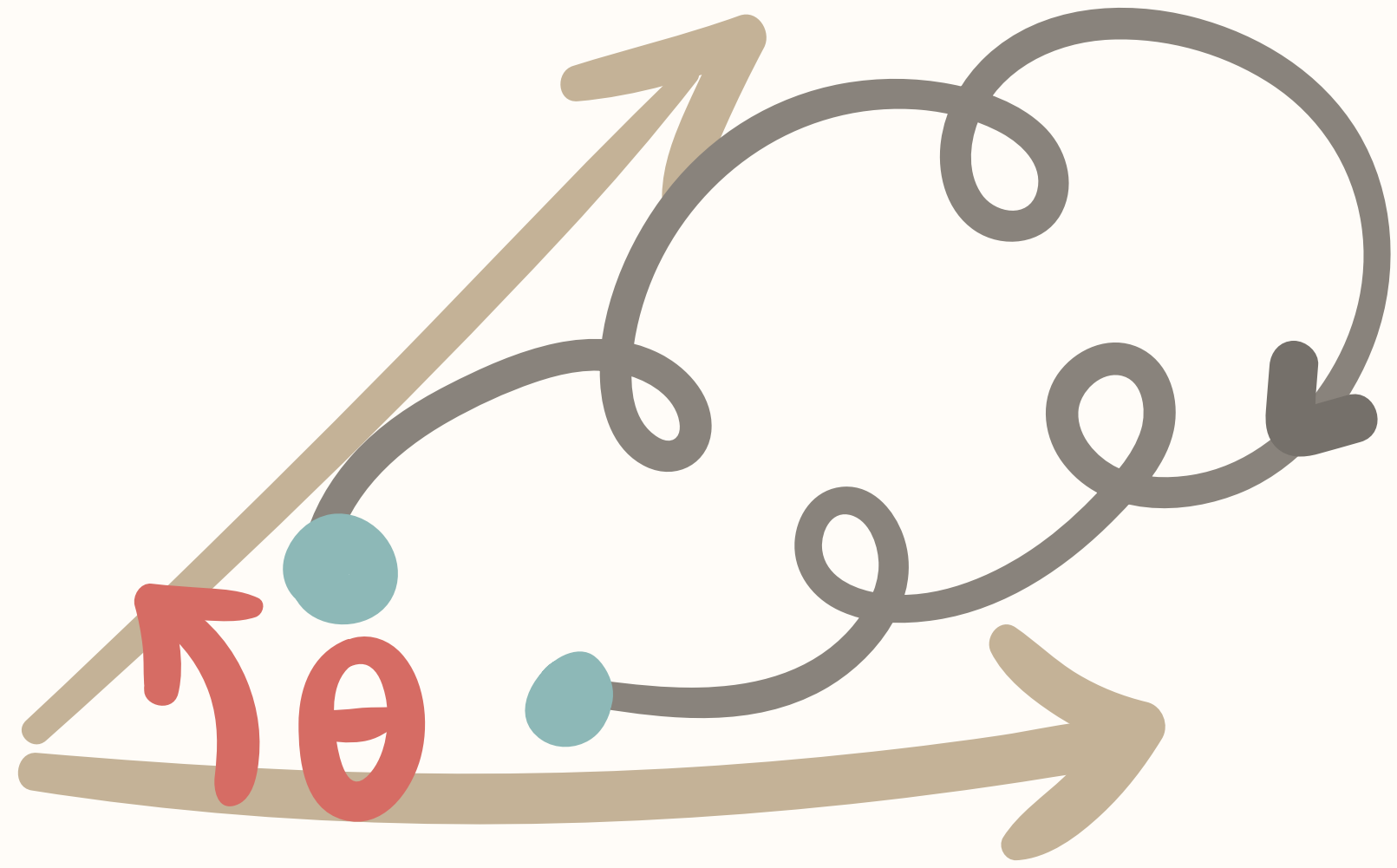
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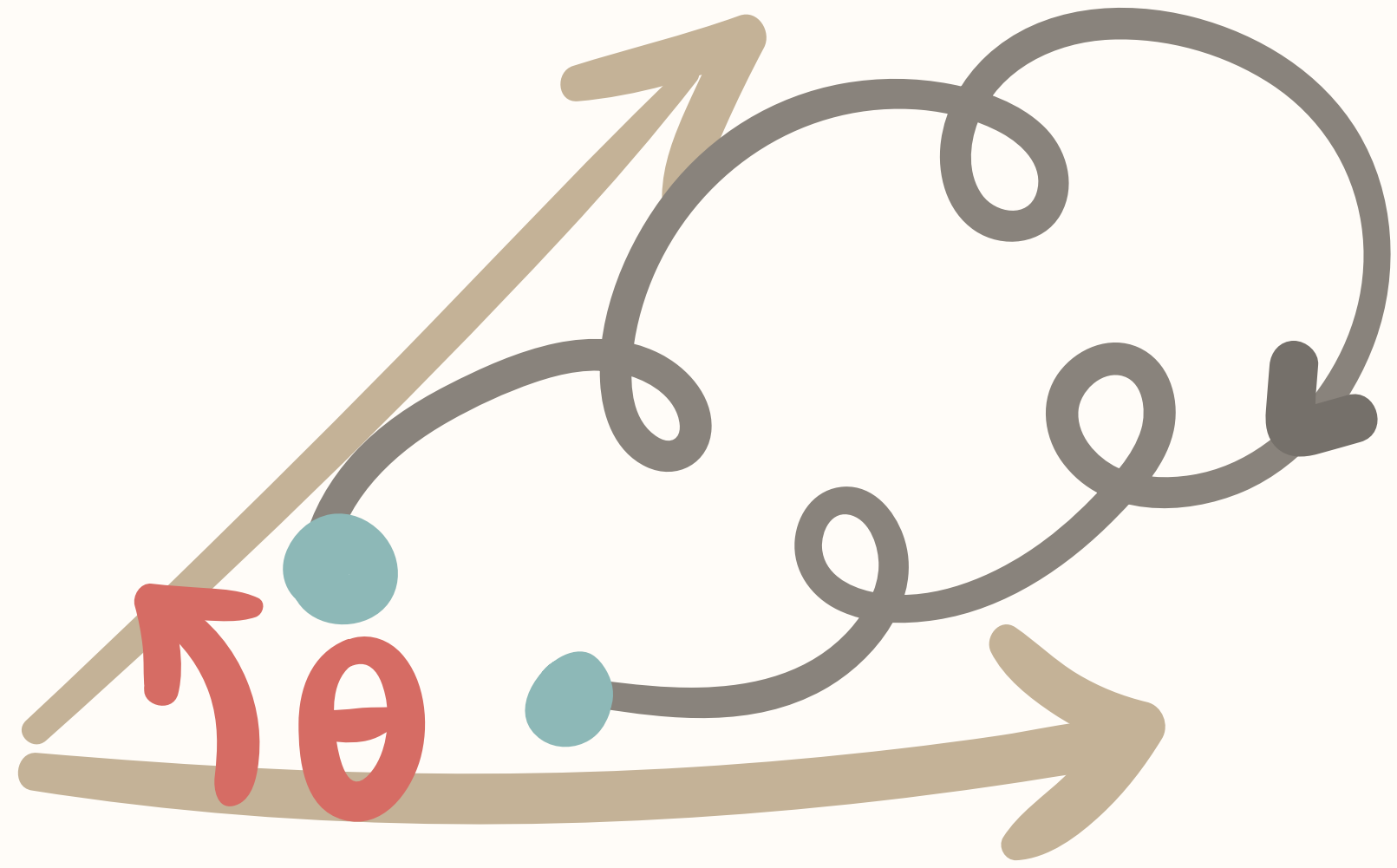
Asymptotic counting results



- ⇒ *Random walks in cone*, D. Denisov & V. Wachtel (2015)
- ⇒ *Non-D-finite excursions in the quarter plane*, D. Bostan, K. Raschel, B. Salvy (2012)

$$a_n \sim \kappa \cdot \gamma^n n^{-1-\pi/\theta}$$

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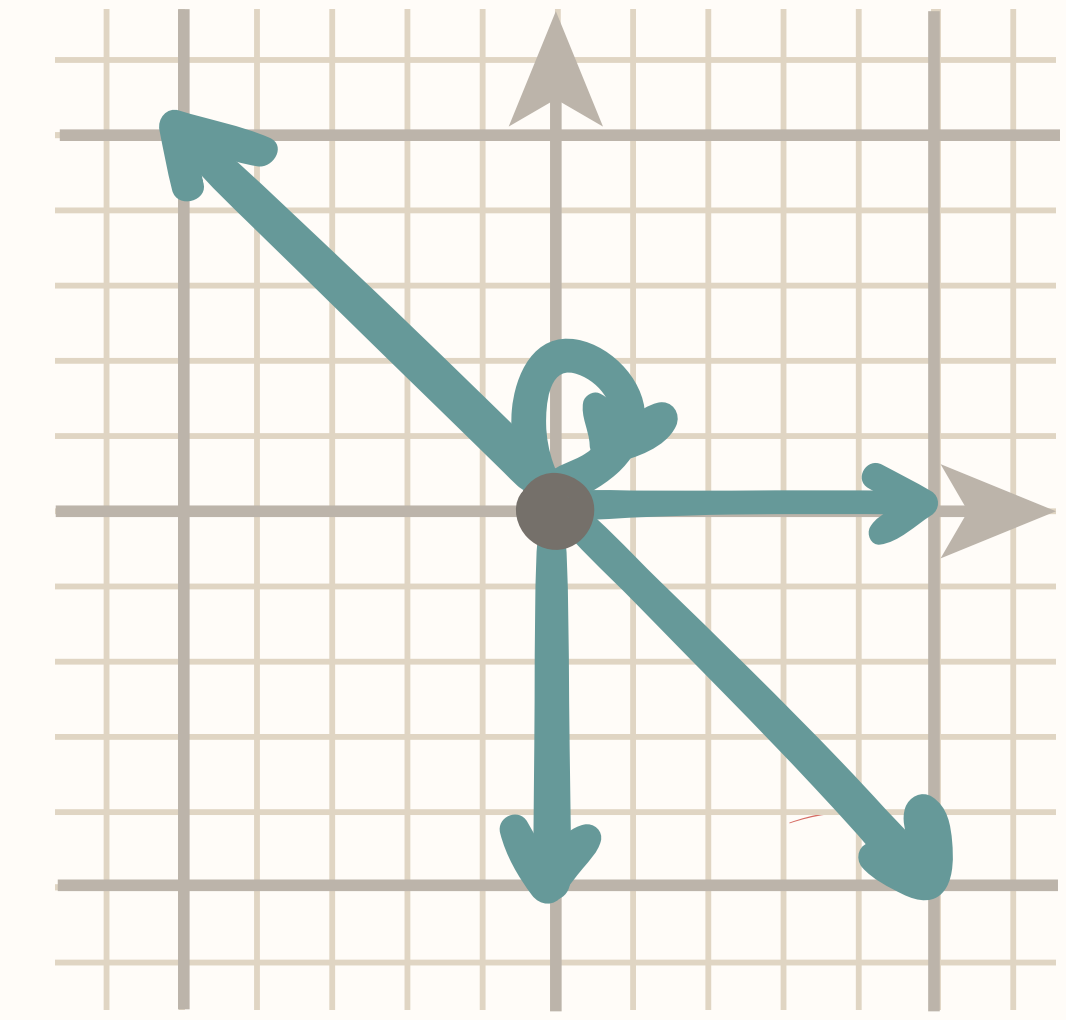
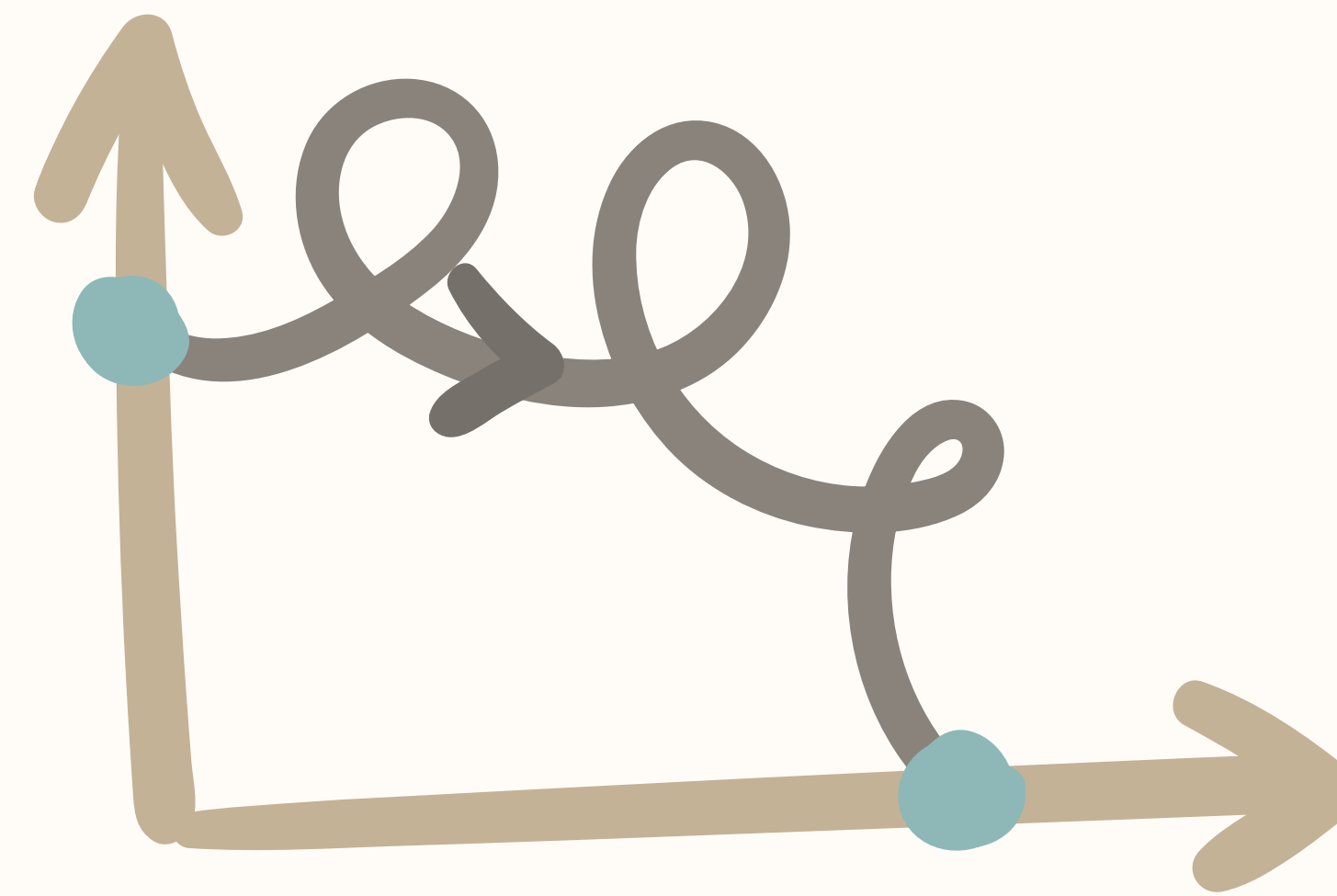
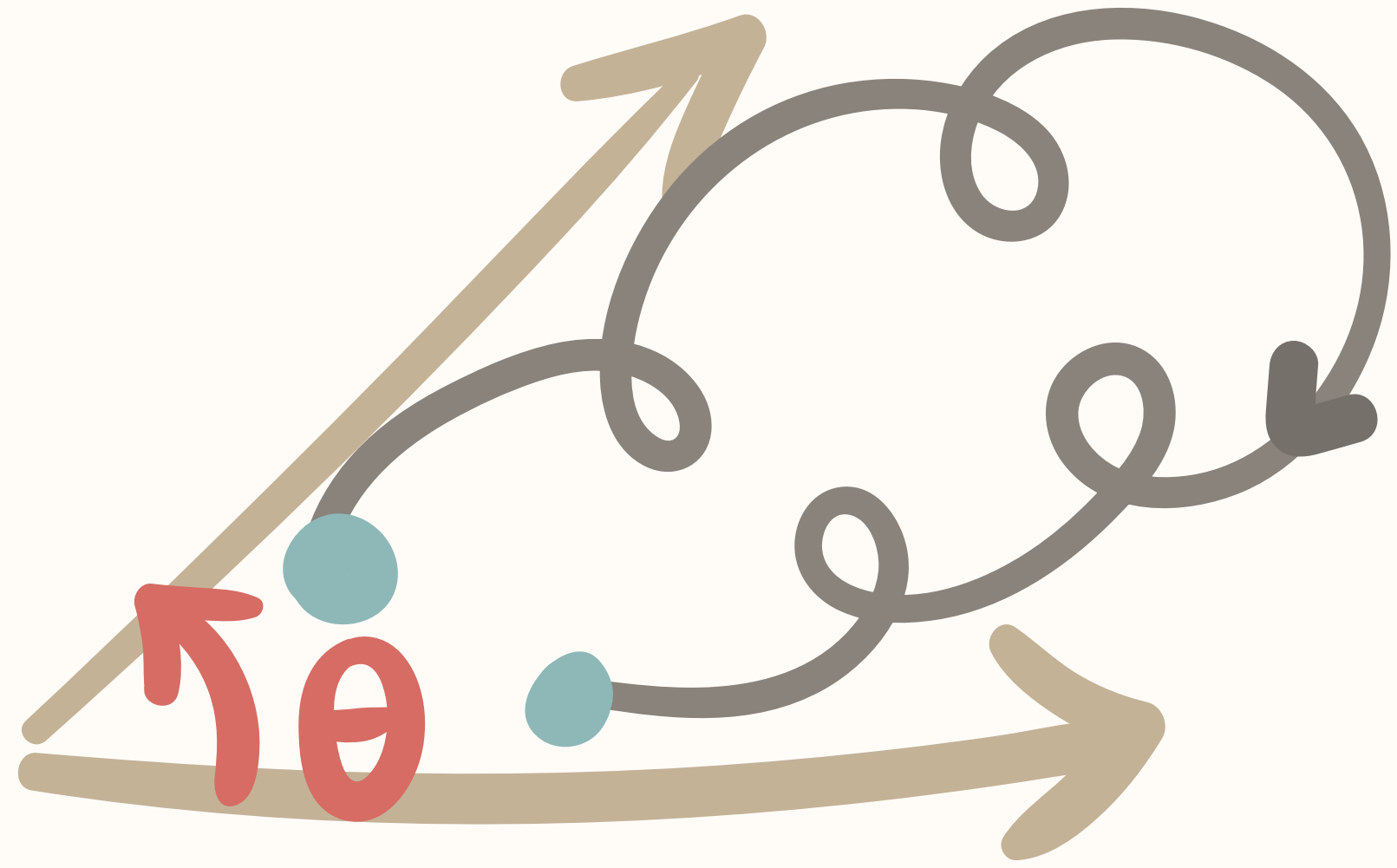
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If the drift is zero, *i.e.* :

$$\mathbf{E}[X] = \mathbf{E}[Y] = 0$$

And the covariance matrix is identity.

Asymptotic counting results



- ⇒ *Random walks in cone*, D. Denisov & V. Wachtel (2015)
- ⇒ *Non-D-finite excursions in the quarter plane*, D. Bostan, K. Raschel, B. Salvy (2012)

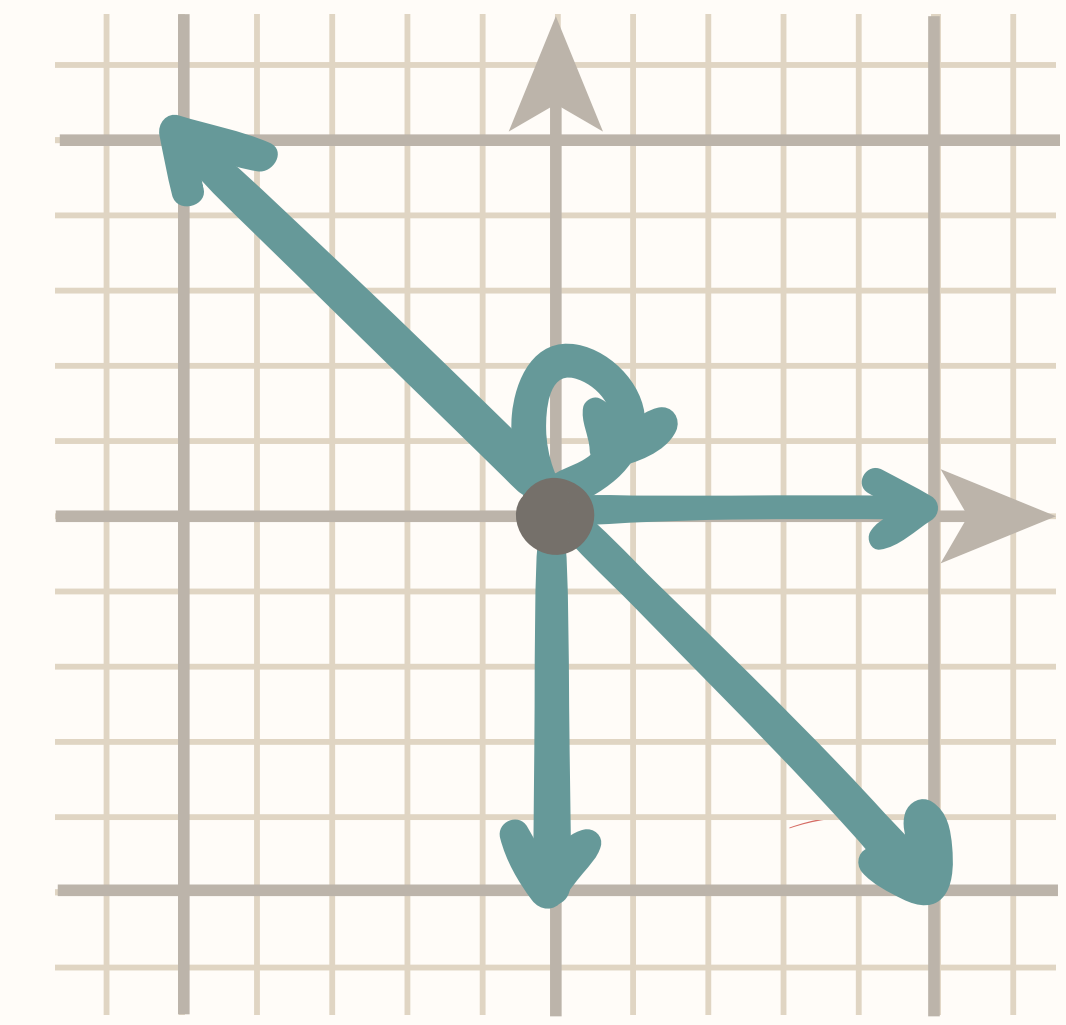
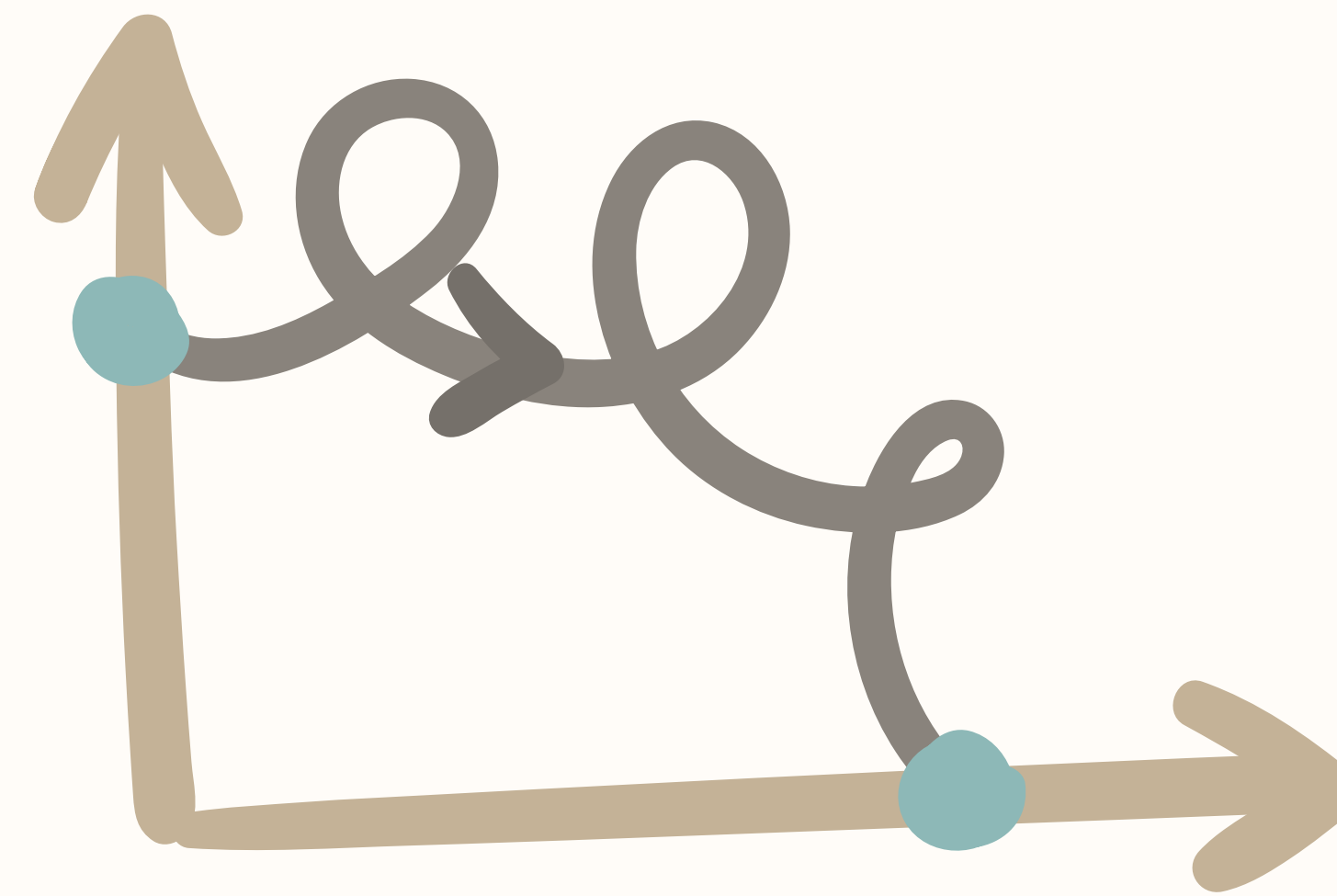
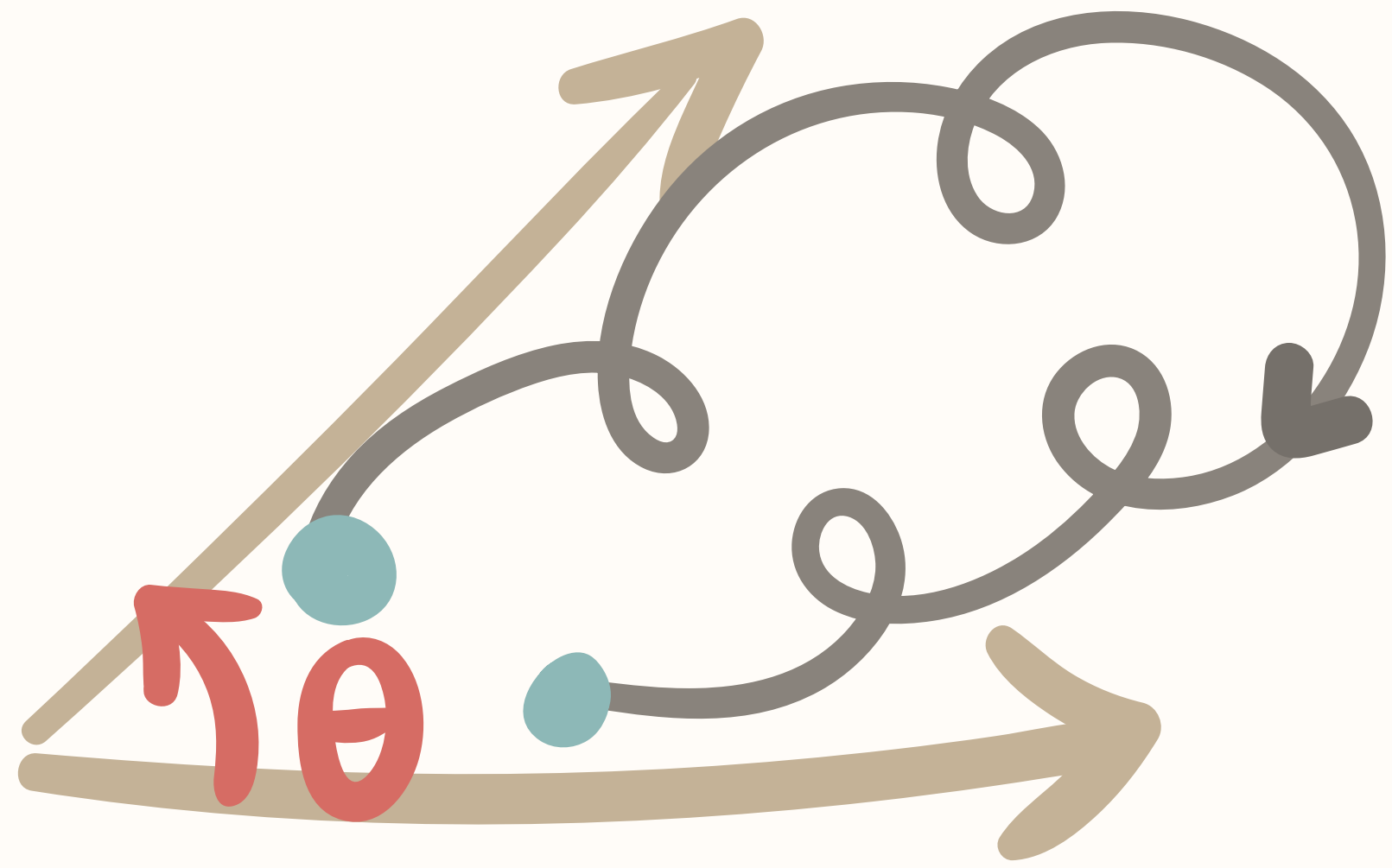
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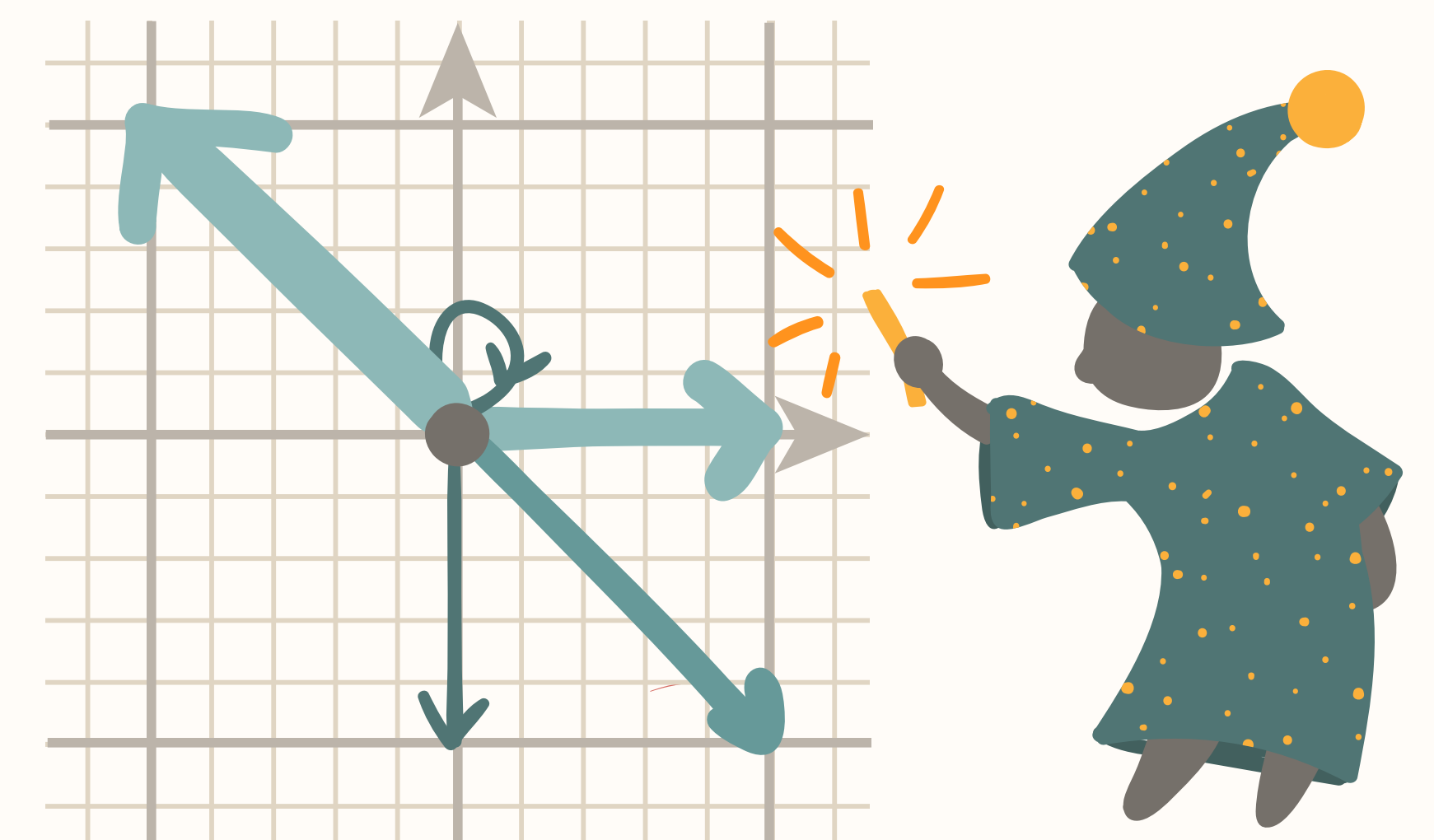
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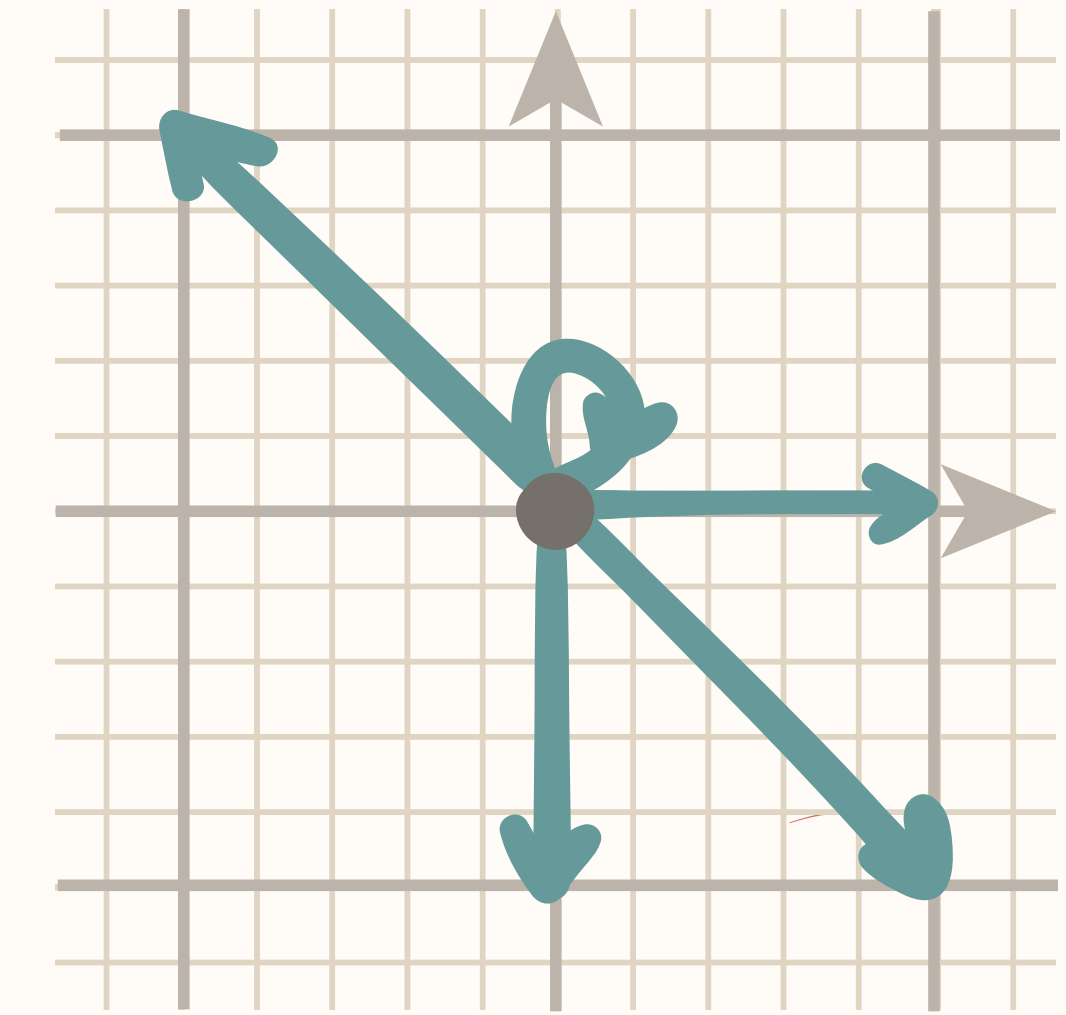
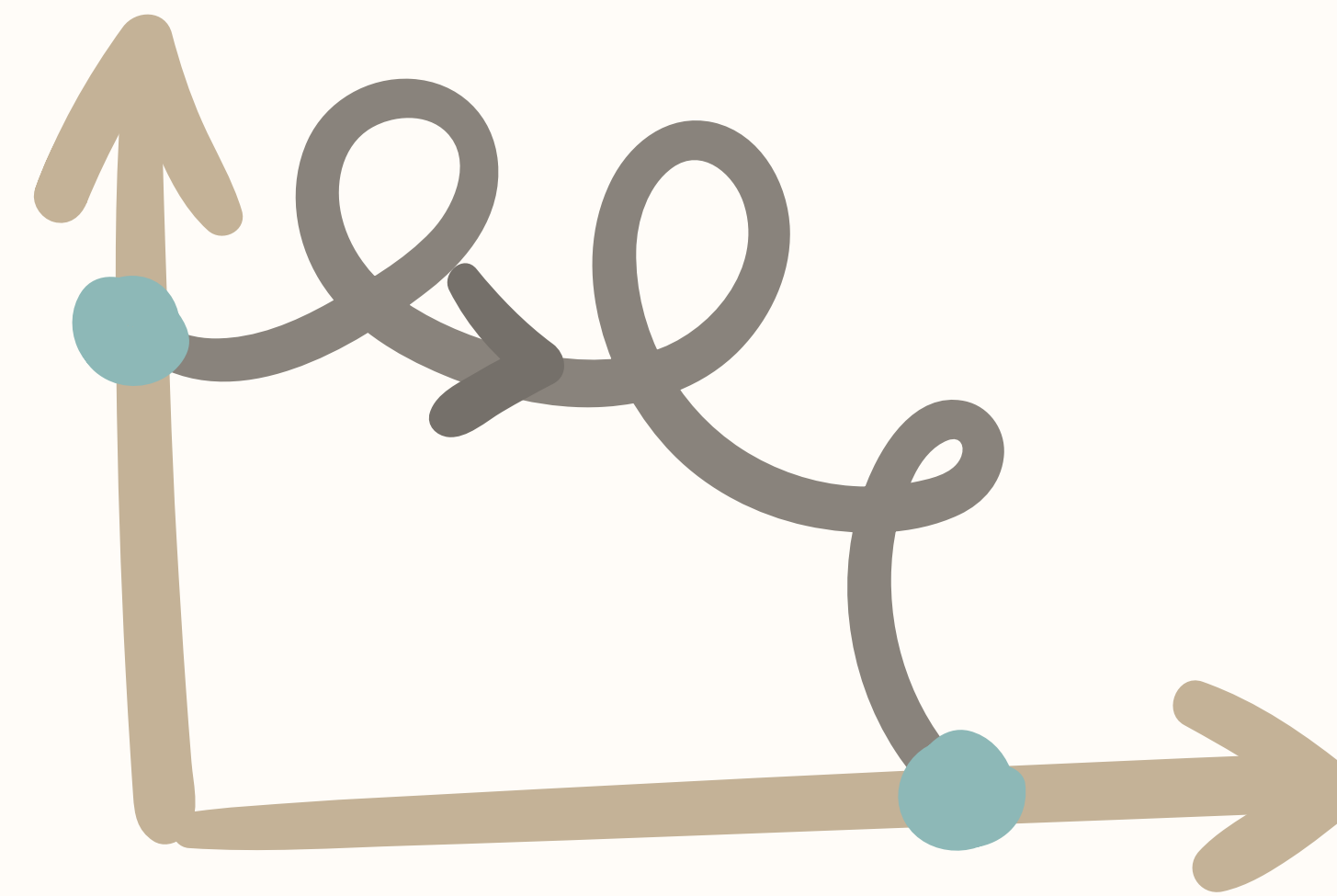
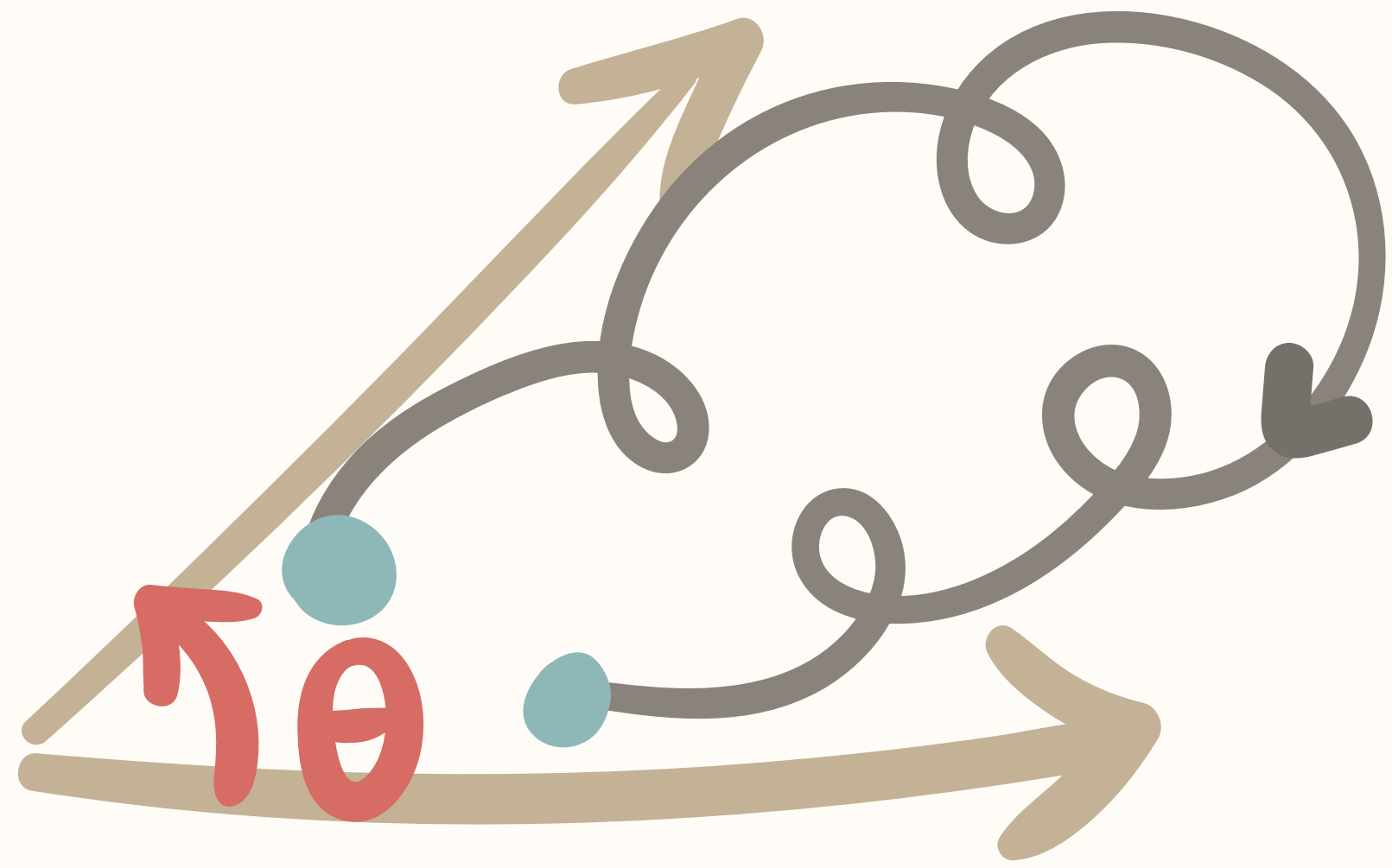
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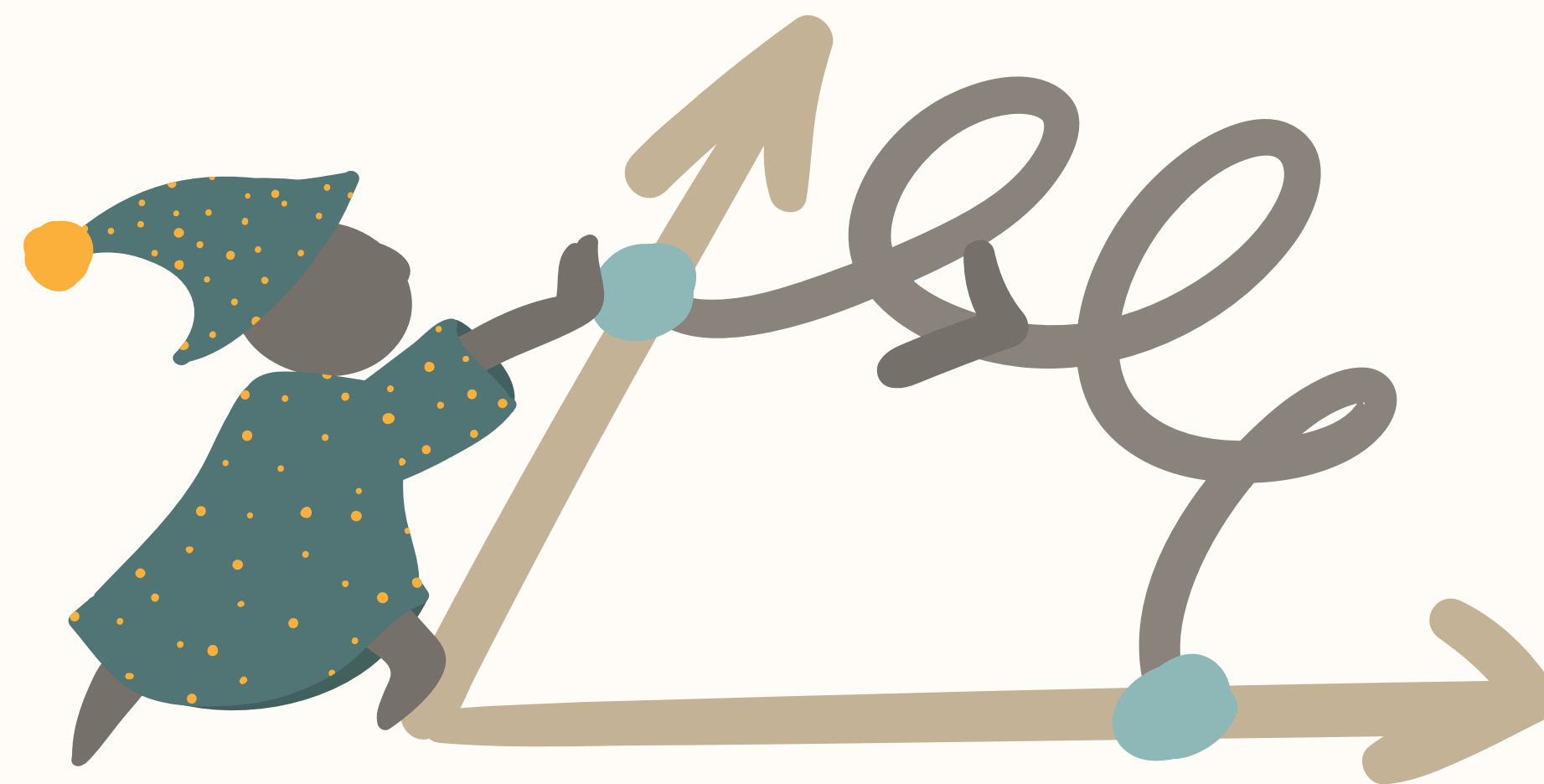
Weighted steps

Asymptotic counting results

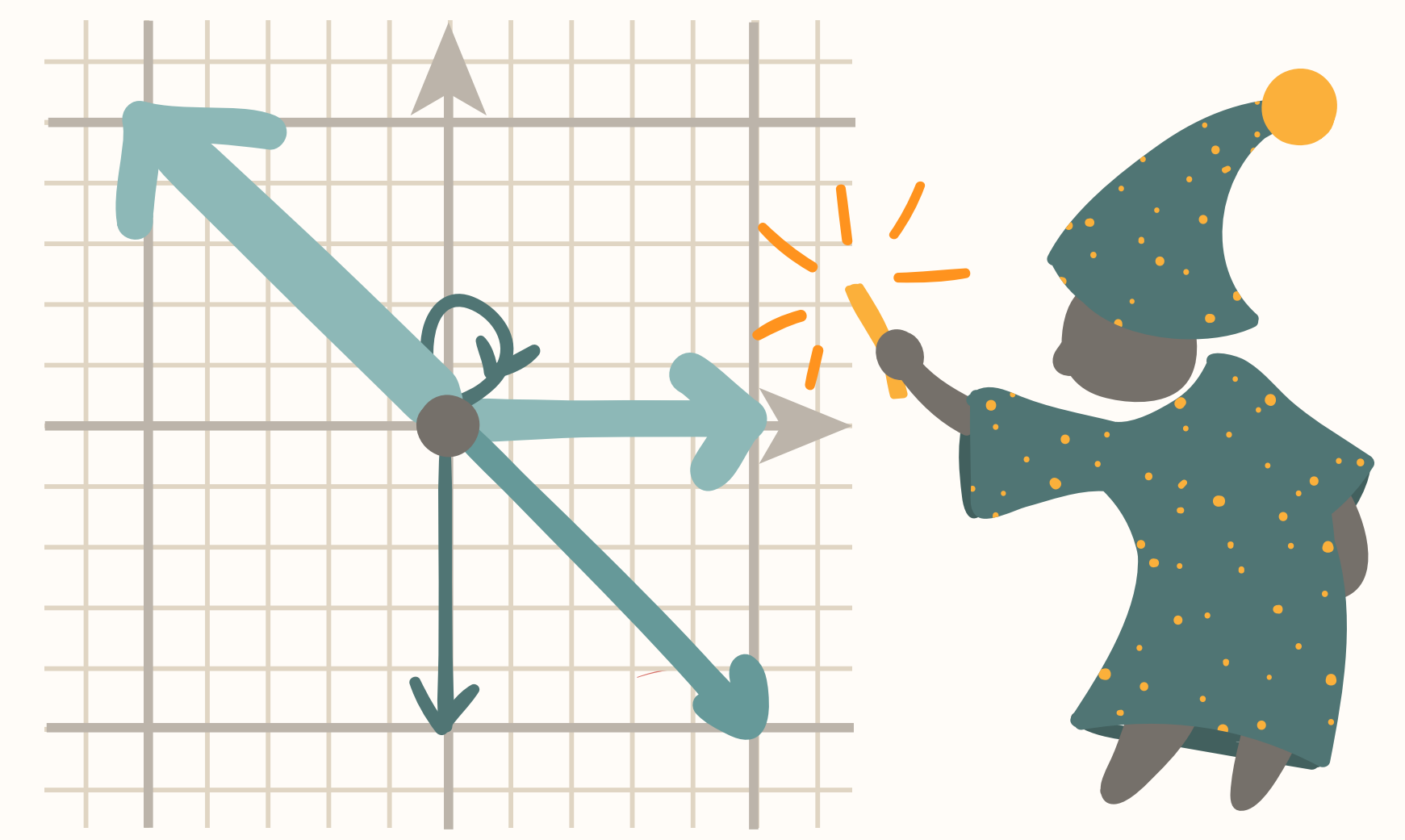


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Shear transformation



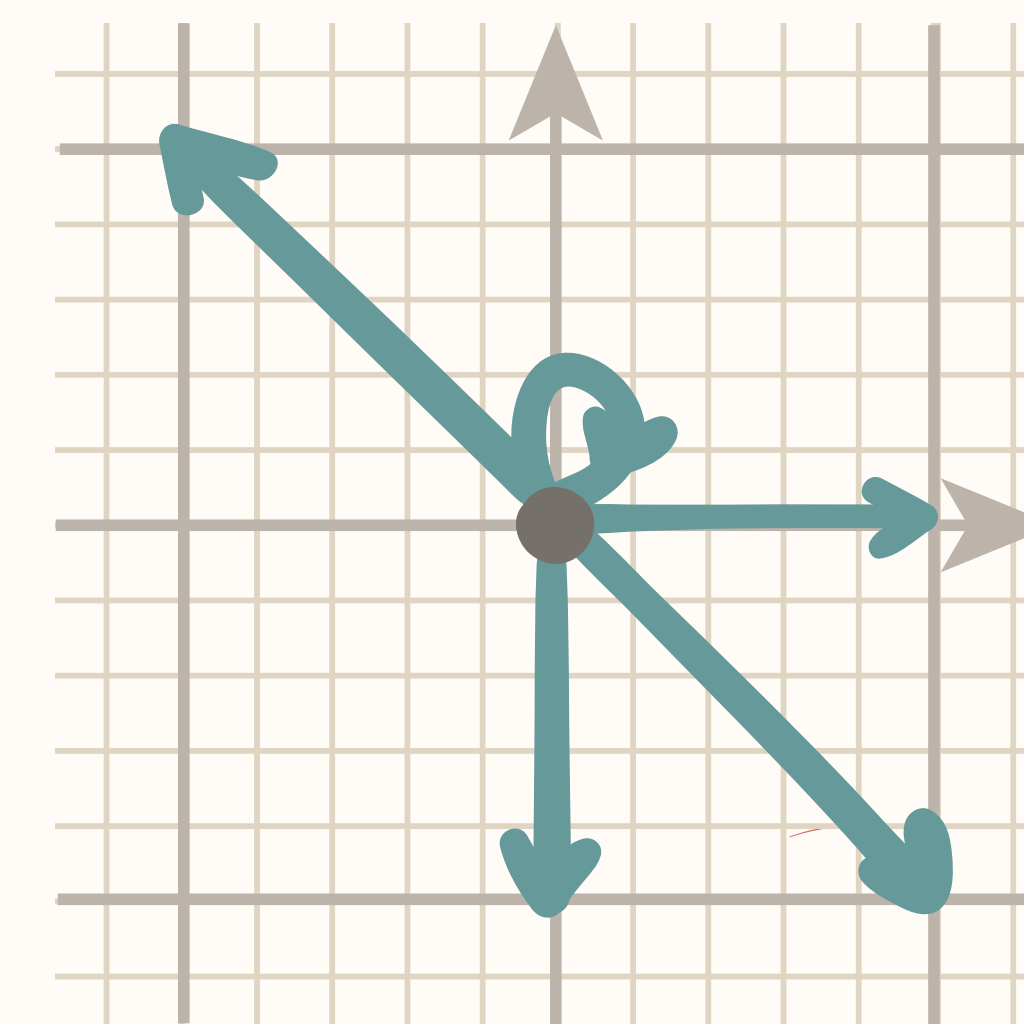
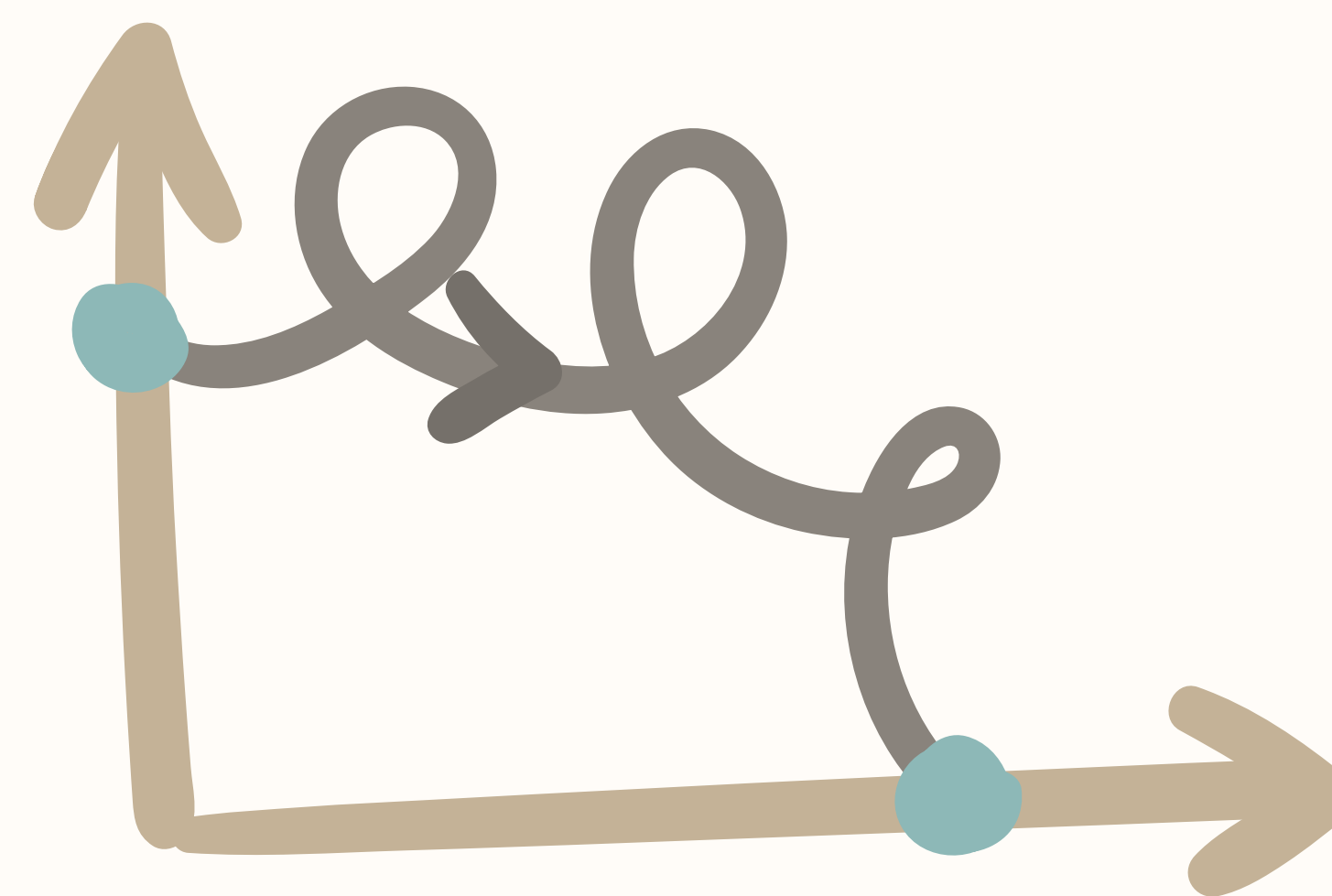
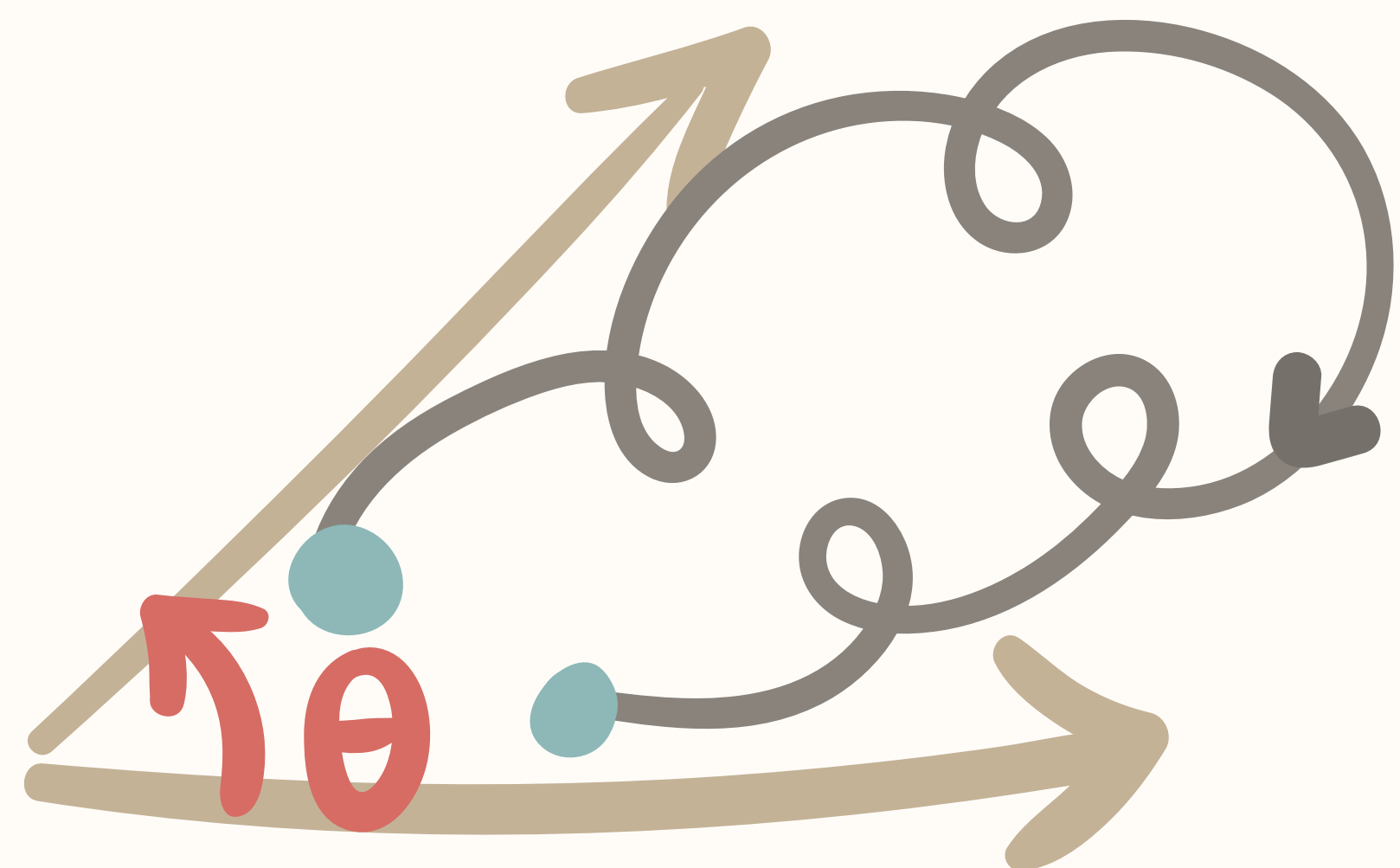
Weighted steps

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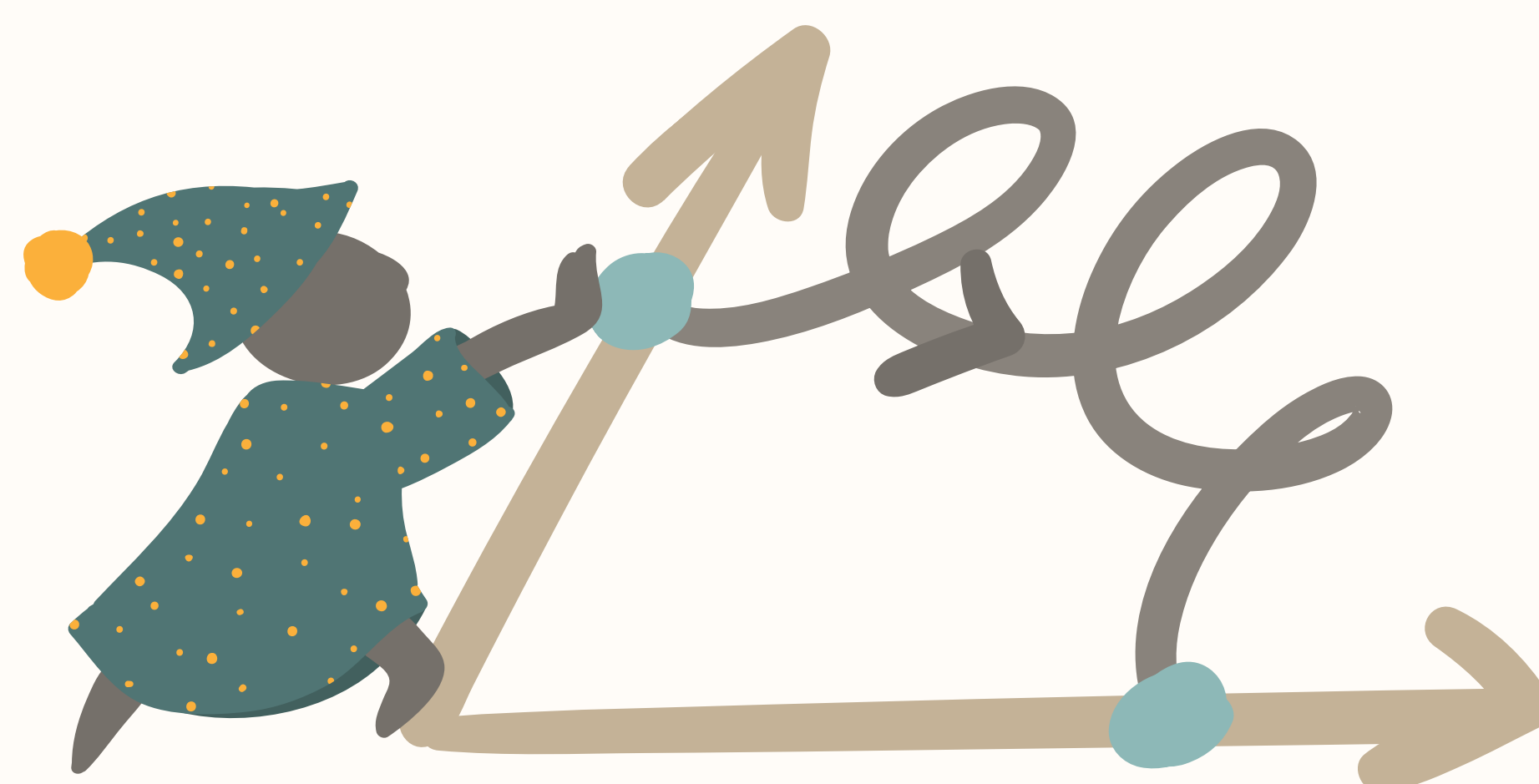
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Asymptotic counting results

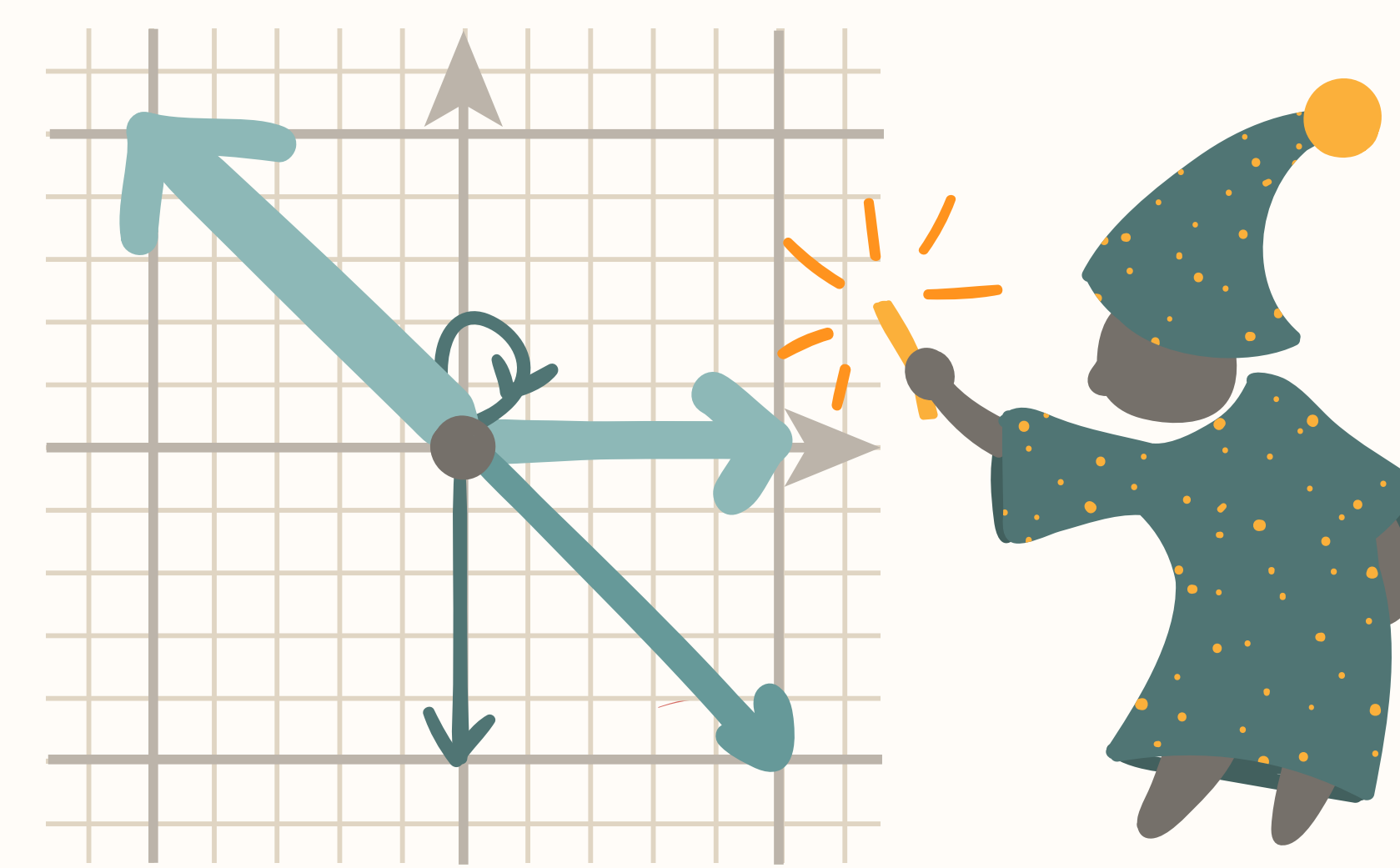


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Shear transformation

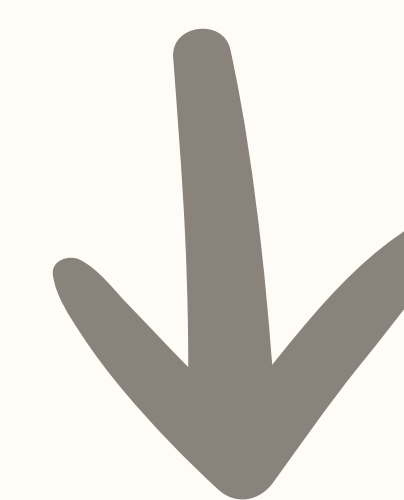


Weighted steps

If the drift is zero, *i.e.* :

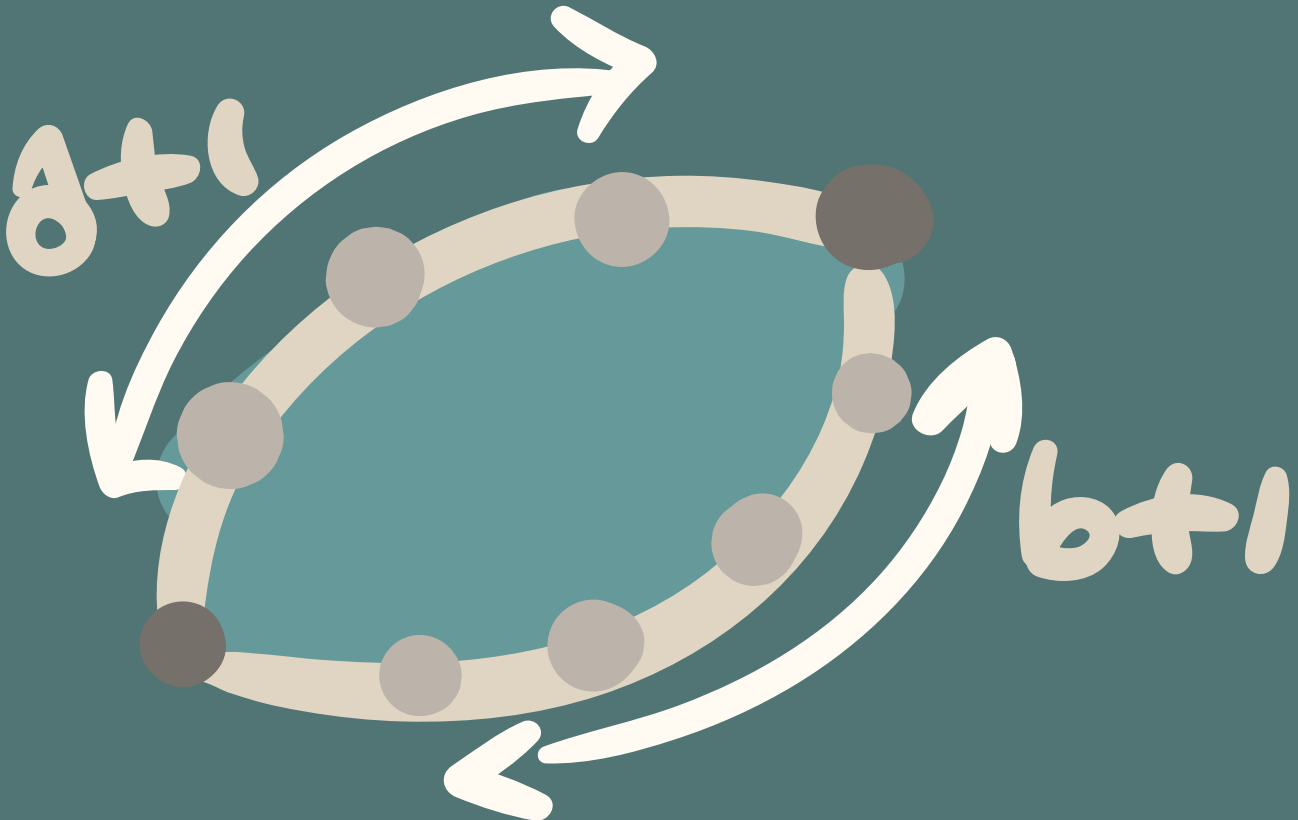
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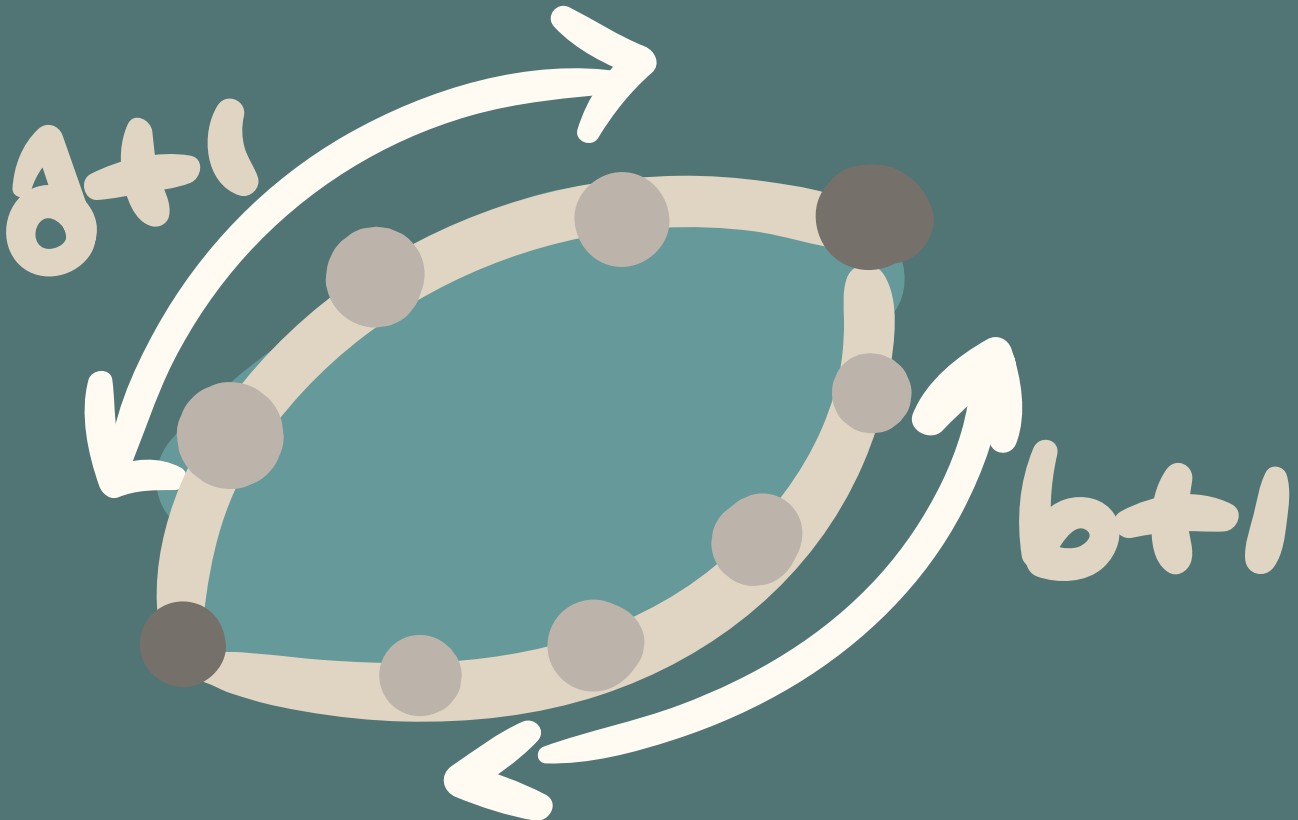
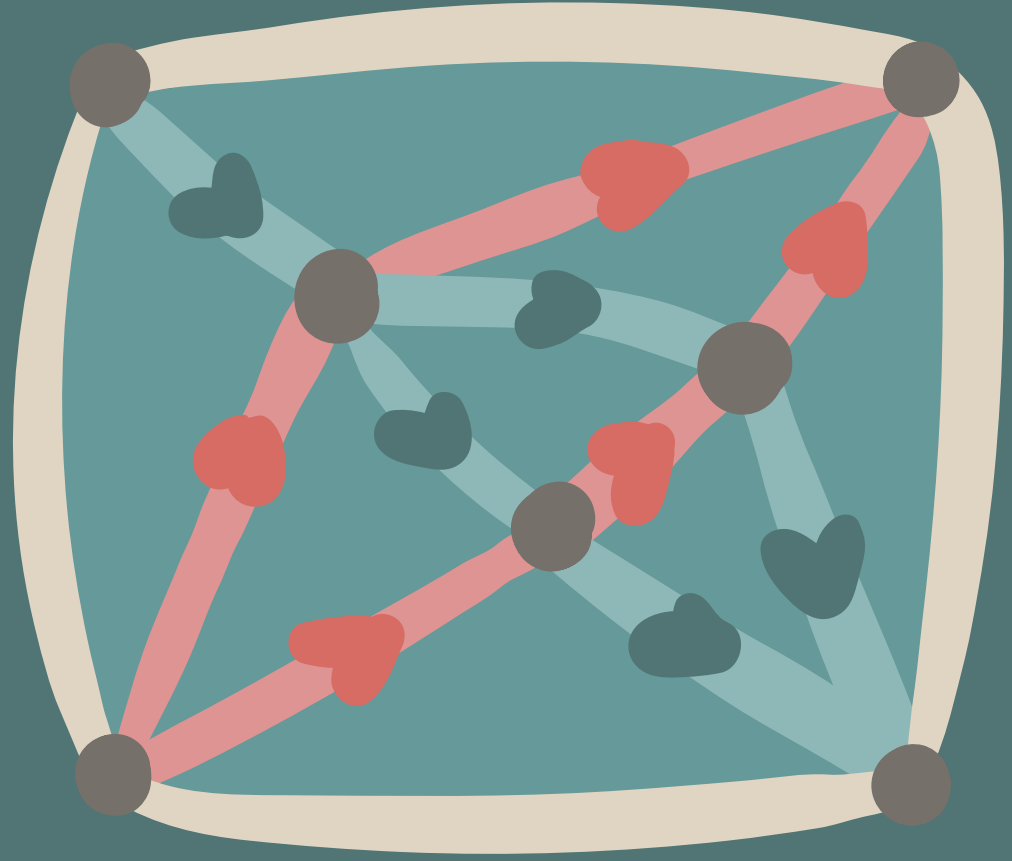


Asymptotics

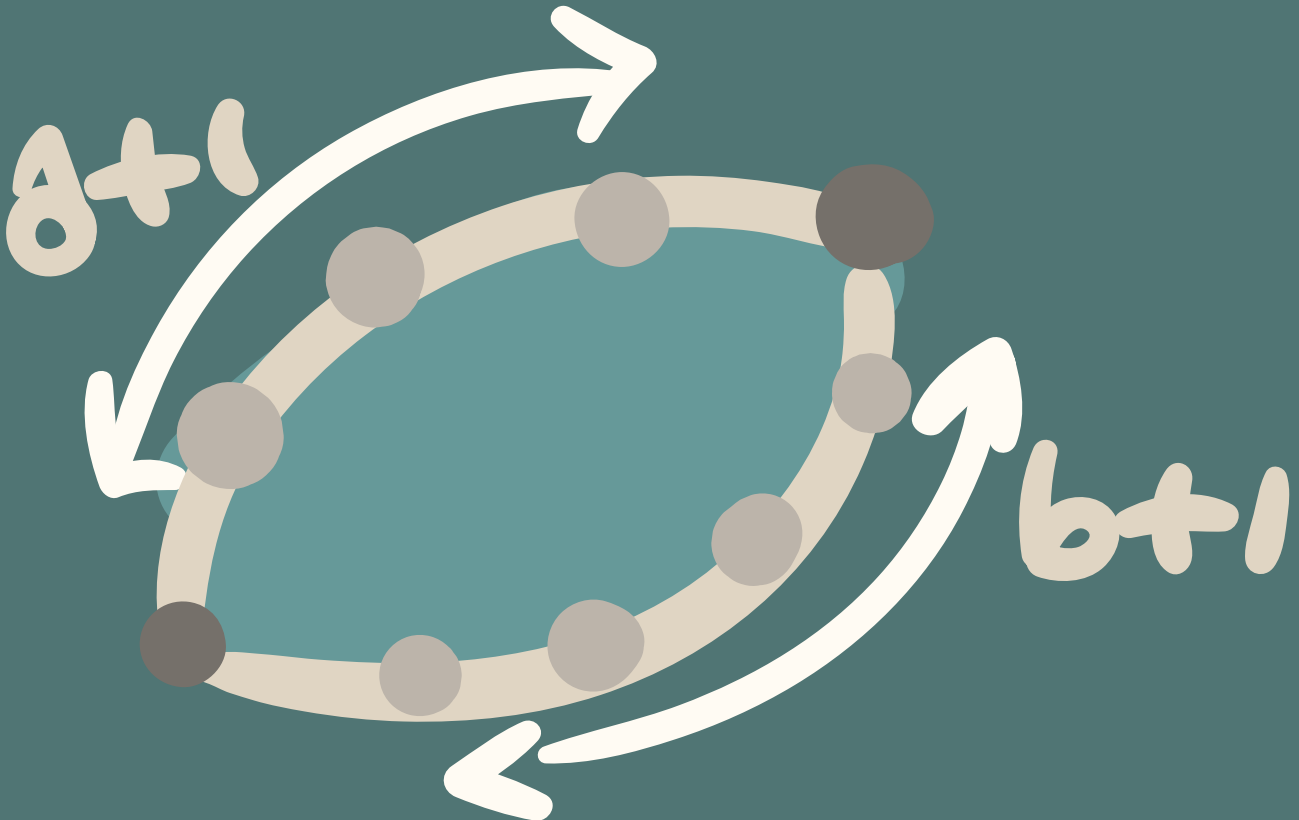
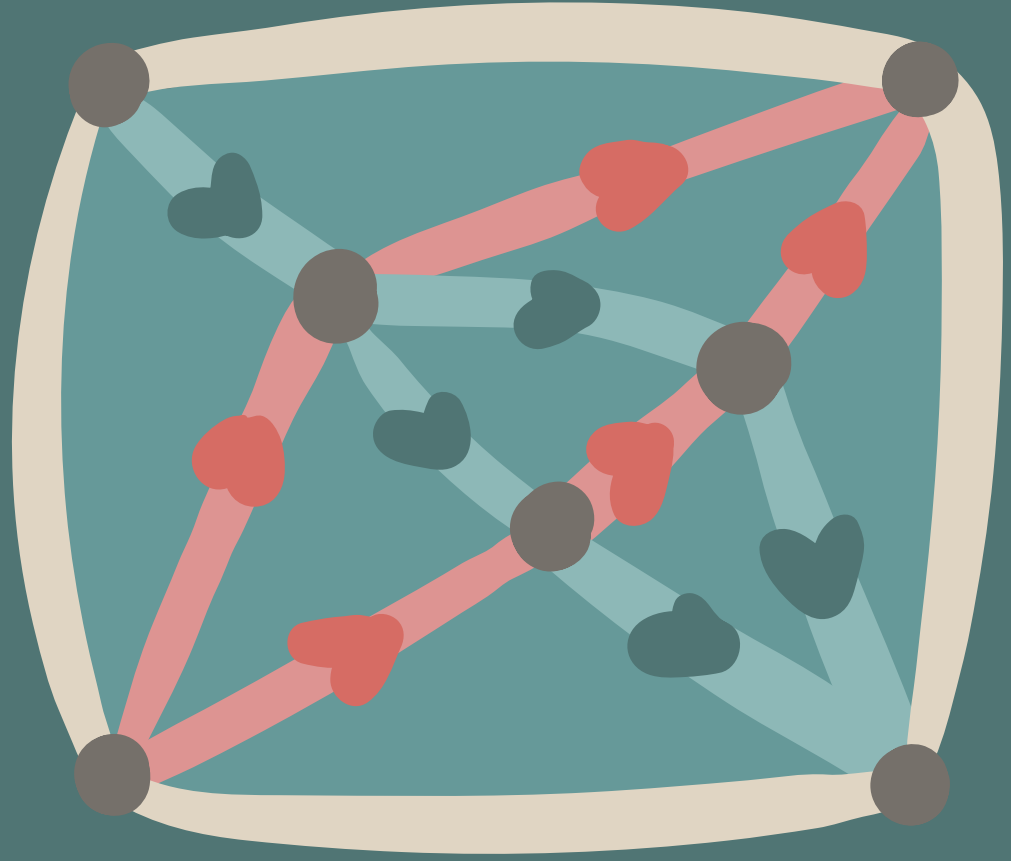
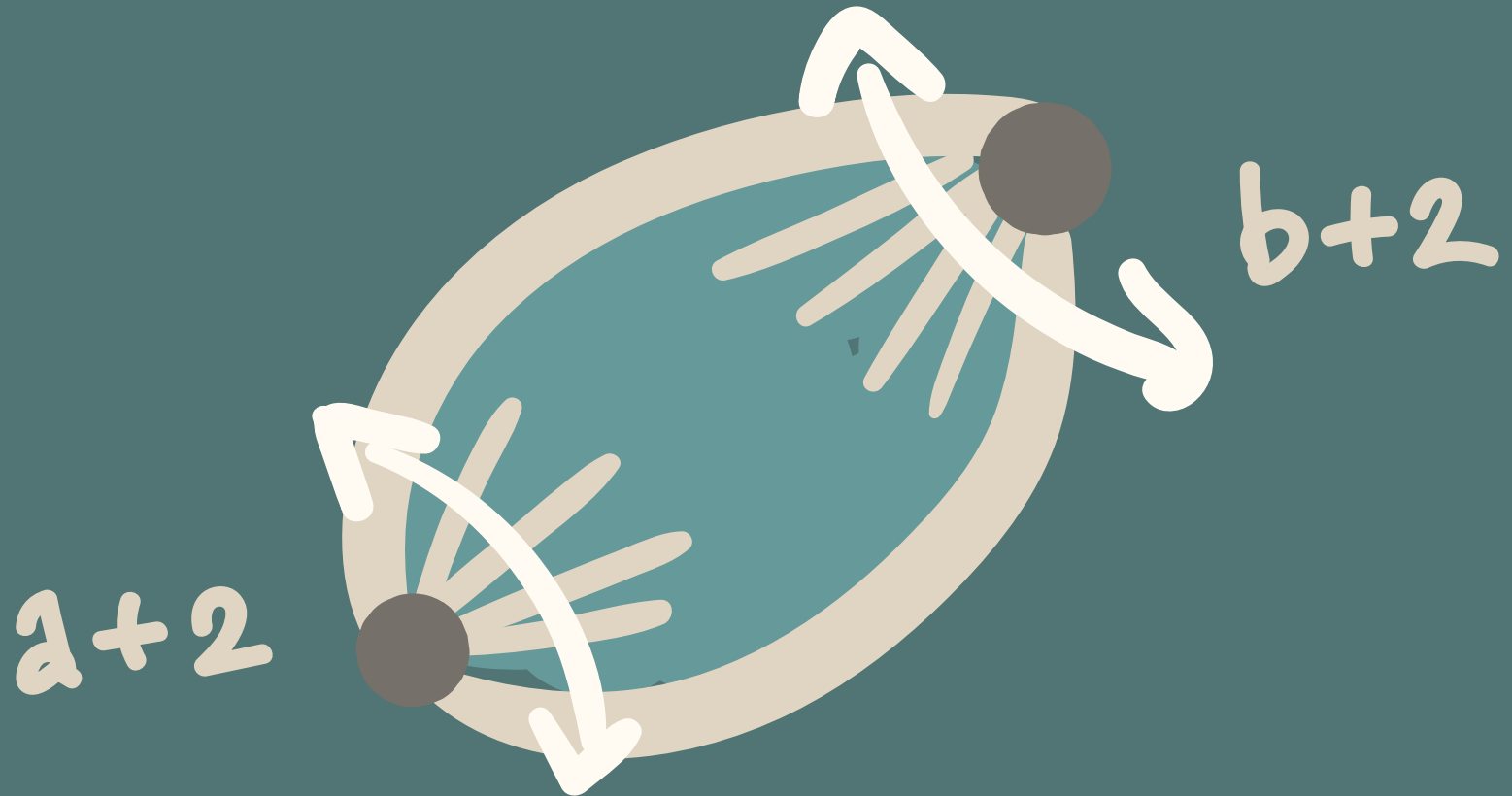
Asymptotic counting results

Model	Asymptotics
<p data-bbox="219 655 571 814">Posets $n+2$ edges</p>  <p>The diagram shows a cycle of nodes connected by edges. Two specific edges are highlighted with arrows and labels: one labeled $a+1$ and another labeled $b+1$. The cycle contains several other nodes and edges, forming a closed loop.</p>	$e_n \sim \kappa \gamma^n n^{-\alpha}$ <p data-bbox="1470 698 2070 784">γ and ξ are algebraic</p> $\gamma \approx 4.80 \dots \quad \alpha \approx 5.14 \dots$ $\alpha = -1 - \frac{\pi}{\arccos(\xi)}$

Asymptotic counting results

Model	Asymptotics
<p>Posets <i>n+2 edges</i></p> 	$e_n \sim \kappa \gamma^n n^{-\alpha} \quad \alpha = -1 - \frac{\pi}{\arccos(\xi)}$ <p>γ and ξ are algebraic $\gamma \approx 4.80 \dots \quad \alpha \approx 5.14 \dots$</p>
<p>Transversal structures <i>n vertices</i></p> 	$t_n \sim \kappa \left(\frac{27}{2} \right)^n n^{-1 - \frac{\pi}{\arccos(7/8)}}$ <p>⇒ Counting rectangular drawings, Y. Inoue, T. Takahashi & R. Fujimaki (2009)</p>

Asymptotic counting results

Model	Asymptotics
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Asymptotic counti

Mod

POSETS PER VERTEX

$$b_n \sim \kappa \left(\frac{1 + \sqrt{5}}{2} \right)^n \cdot n^{-6}$$

$$\frac{\pi}{\cos(\xi)}$$

Pose
n ver

Asymptotic counti

Mod

POSETS PER VERTEX

$$b_n \sim \kappa \left(\frac{11 + \sqrt{51}}{2} \right)^n \cdot n^{-6}$$

PLANE PERMUTATIONS

$$p_n \sim \kappa \left(\frac{11 + \sqrt{51}}{2} \right)^n \cdot n^{-6}$$

⇒ Semi-Baxter and strong-Baxter :
two relatives of Baxter Sequences,
M. Bouvel, V. Guerrini, A. Rechnitzer
& S. Rinaldi (2018)

$\frac{\pi}{\cos(\xi)}$

Pose
n ver

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$$b_n \sim \kappa \left(\frac{1 + \sqrt{5}}{2} \right)^n \cdot n^{-6}$$

1, 1, 2, 6, 23, 104, 530, 2958,
17734, 112657, ...

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⇒ <http://oeis.org/A117106>

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Asymptotic counti

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TODO

Bijection

poset
per
vertices



plane
permutations

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Poset
n ver

Sommaire

The KMSW Bijection

1. Application to three maps model

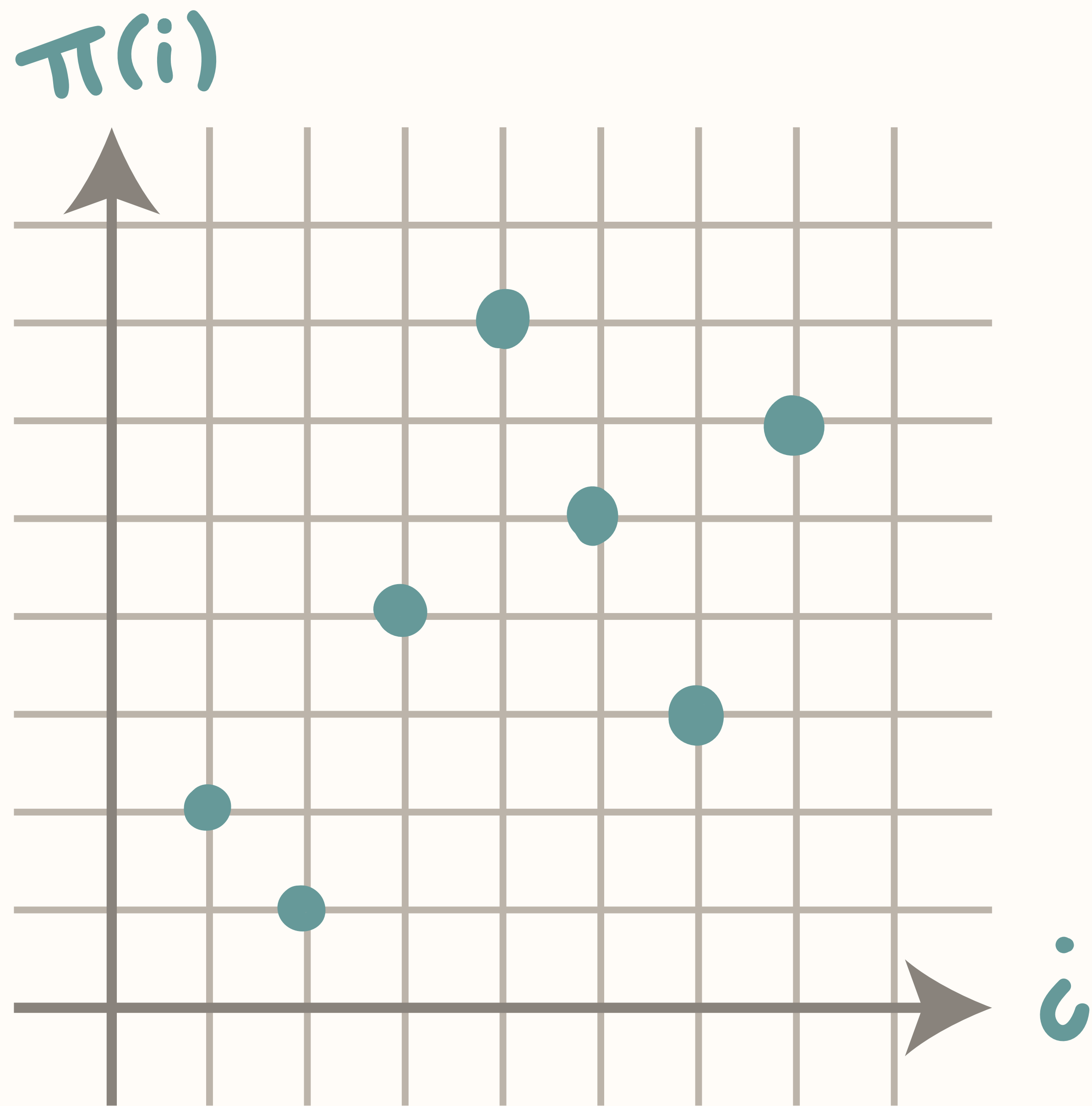
- a. Plane bipolar posets*
- b. Plane bipolar posets by vertices*
- c. Transversal structures*
- d. Asymtotoxic*

2. Interlude : plane permutations

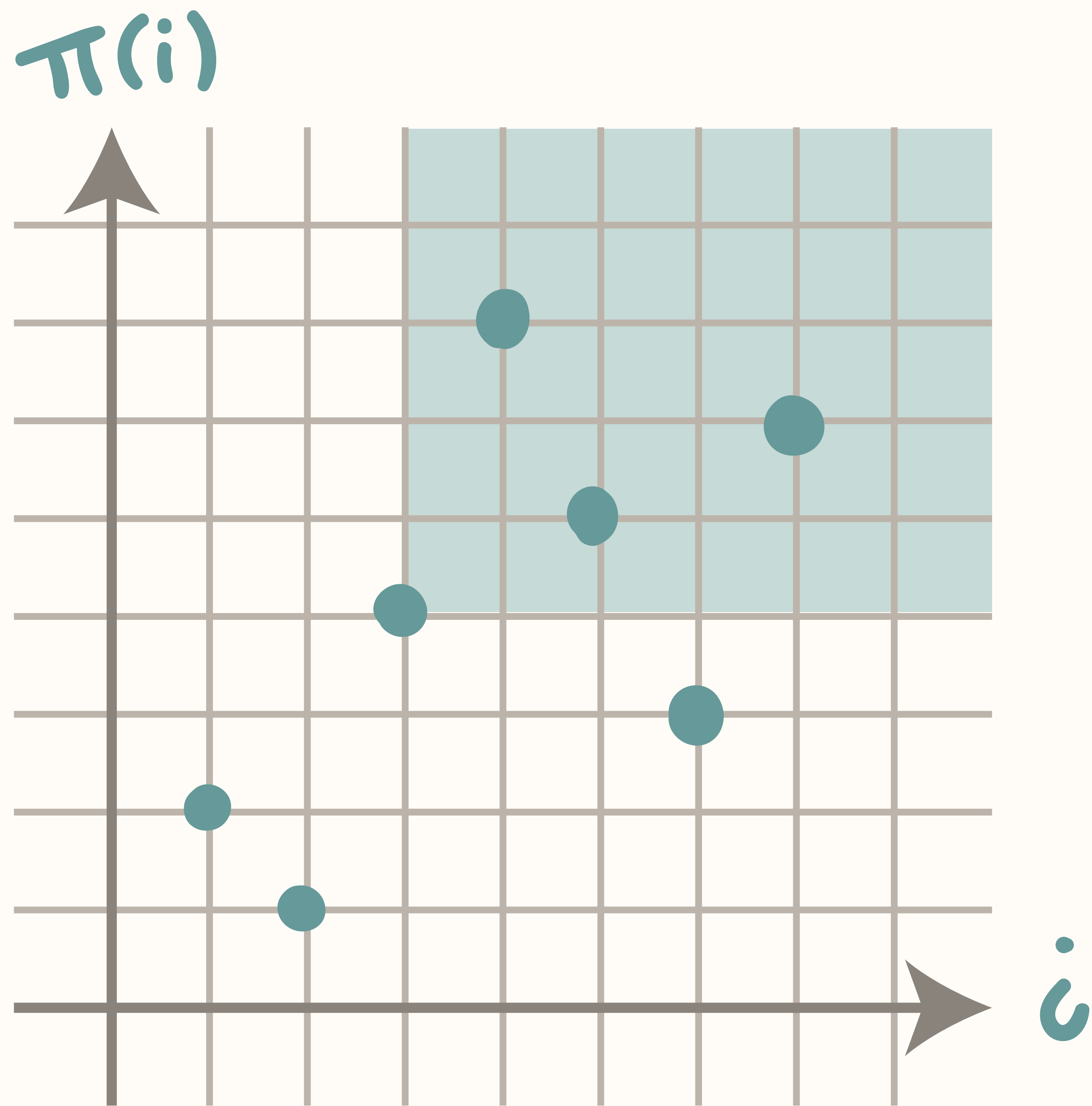
3. Application to corner polyhedra

- a. Via polyhedral orientations*
- b. Via Schnyder colorings*
- c. Asymtotoxic*

Plane permutations

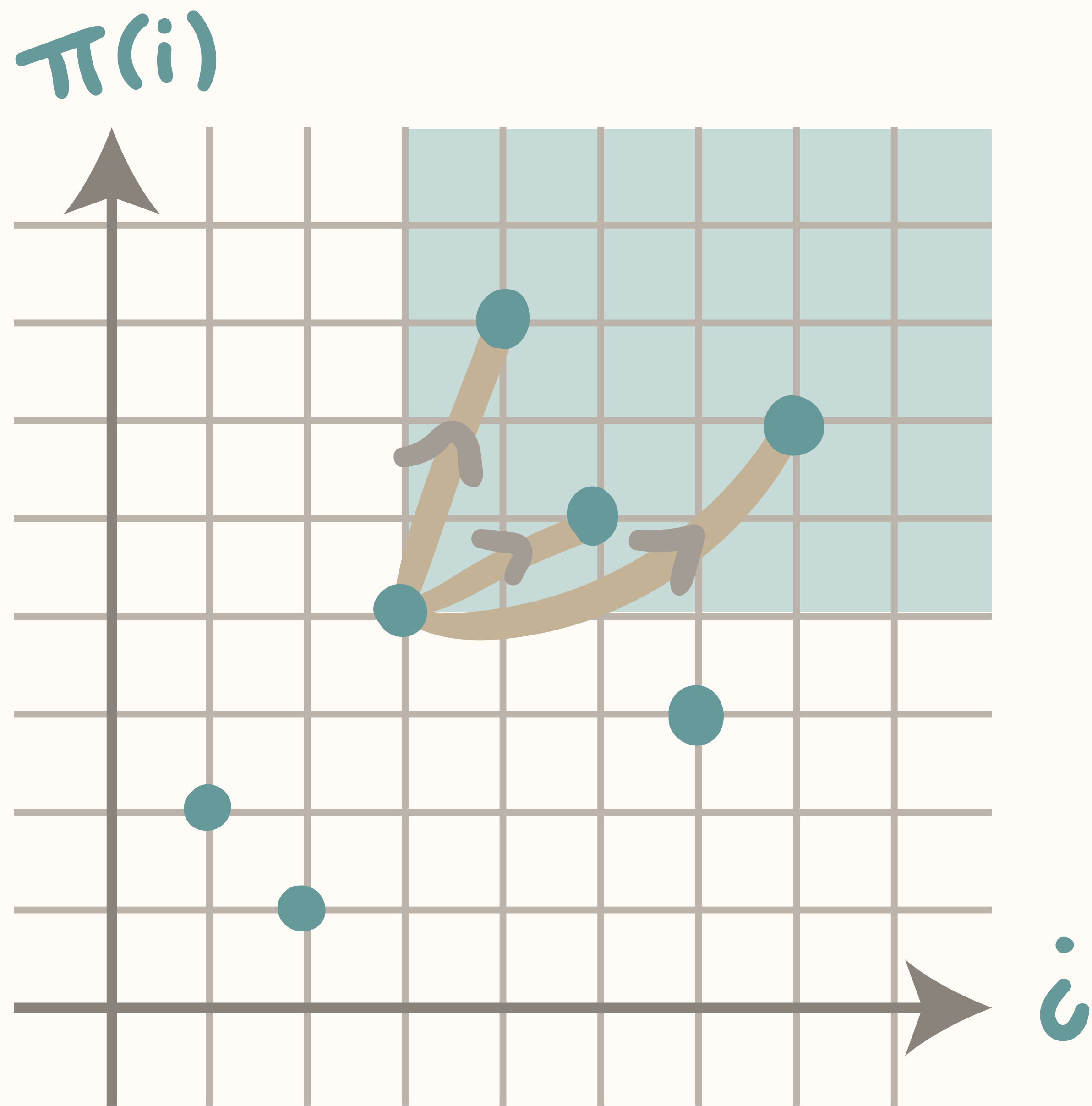


Plane permutations



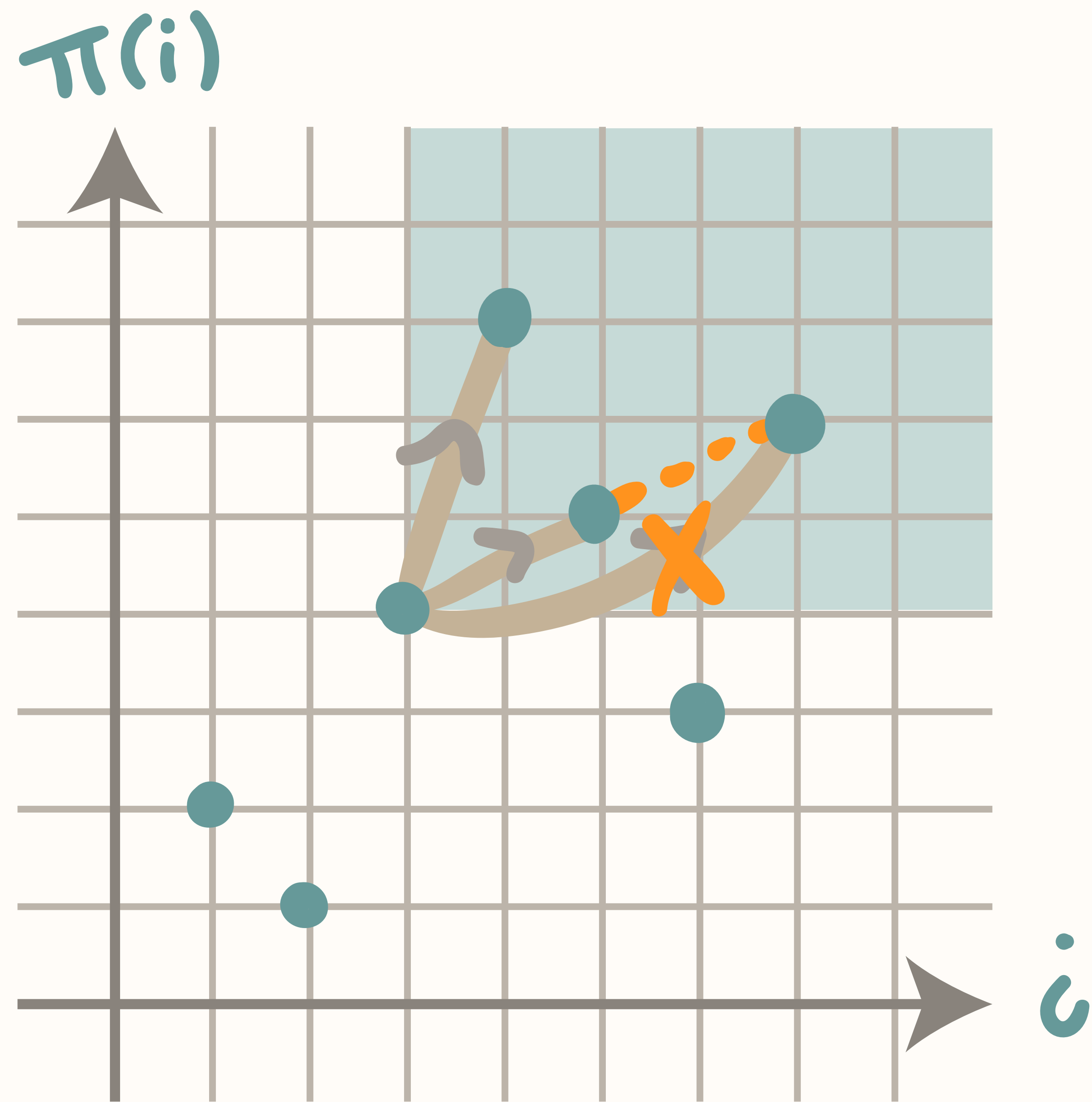
Dominance relation

Plane permutations



Dominance relation

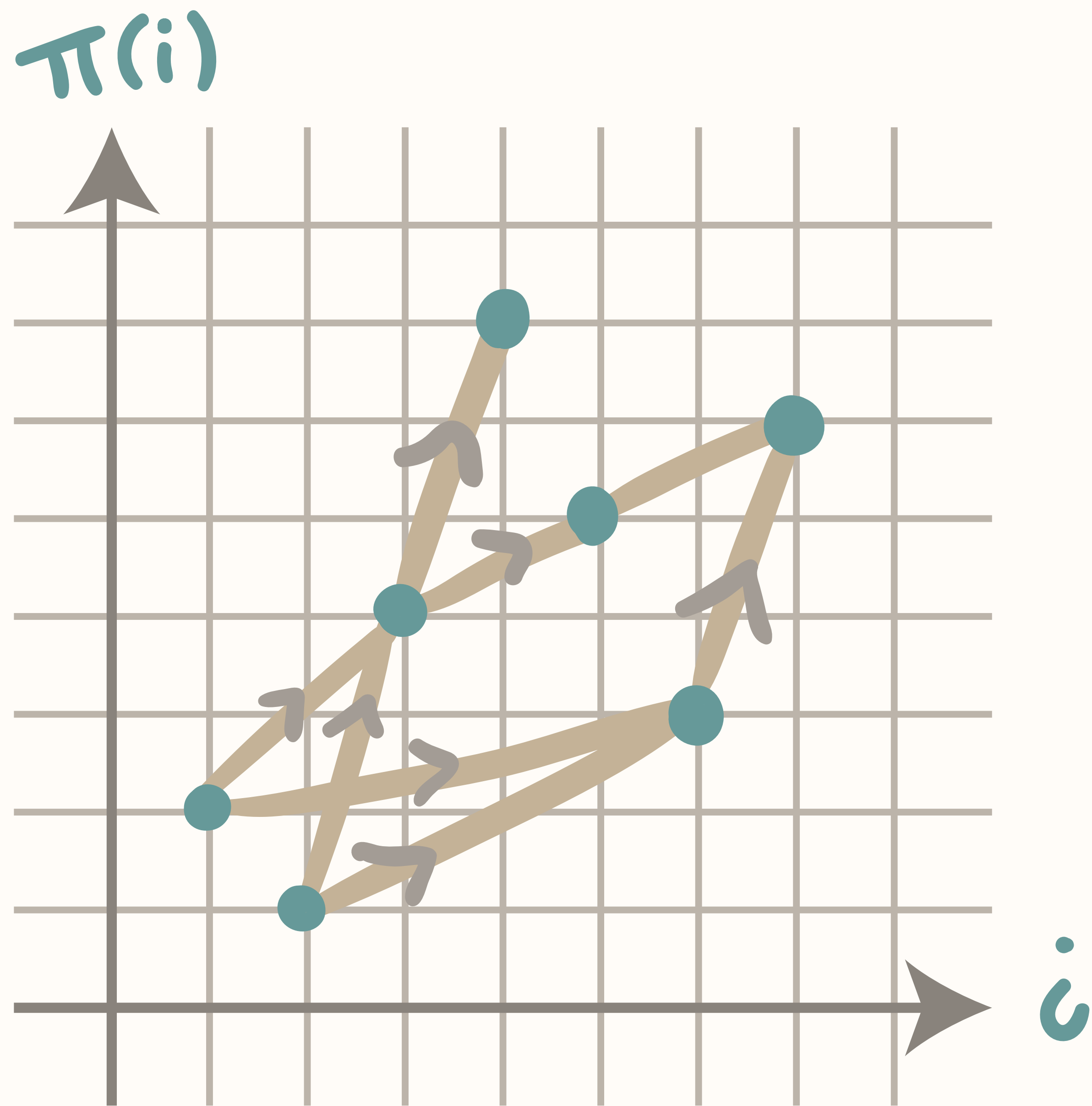
Plane permutations



***Dominance
diagram***

***= Dominance relation
with no transitive edges***

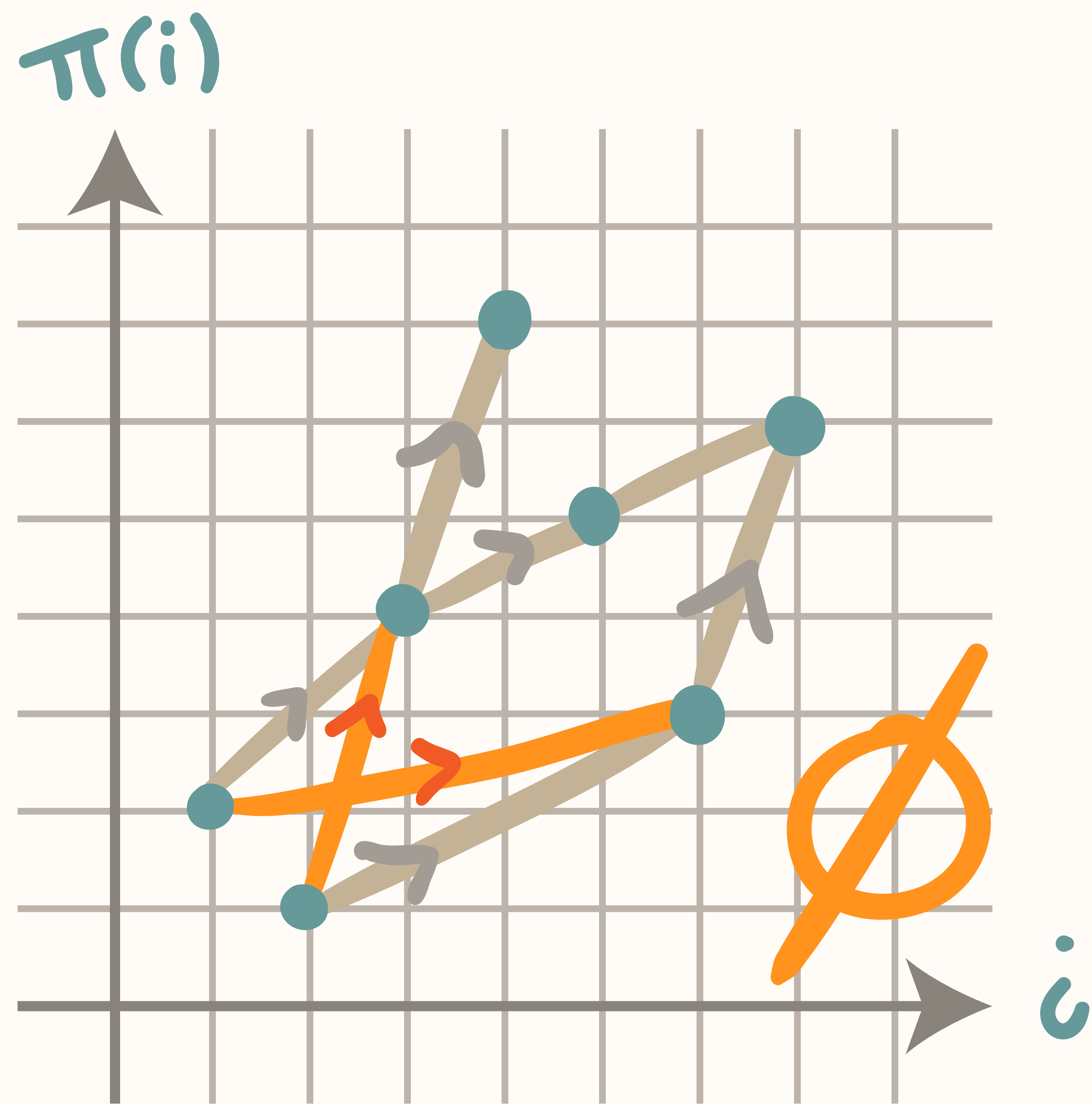
Plane permutations



***Dominance
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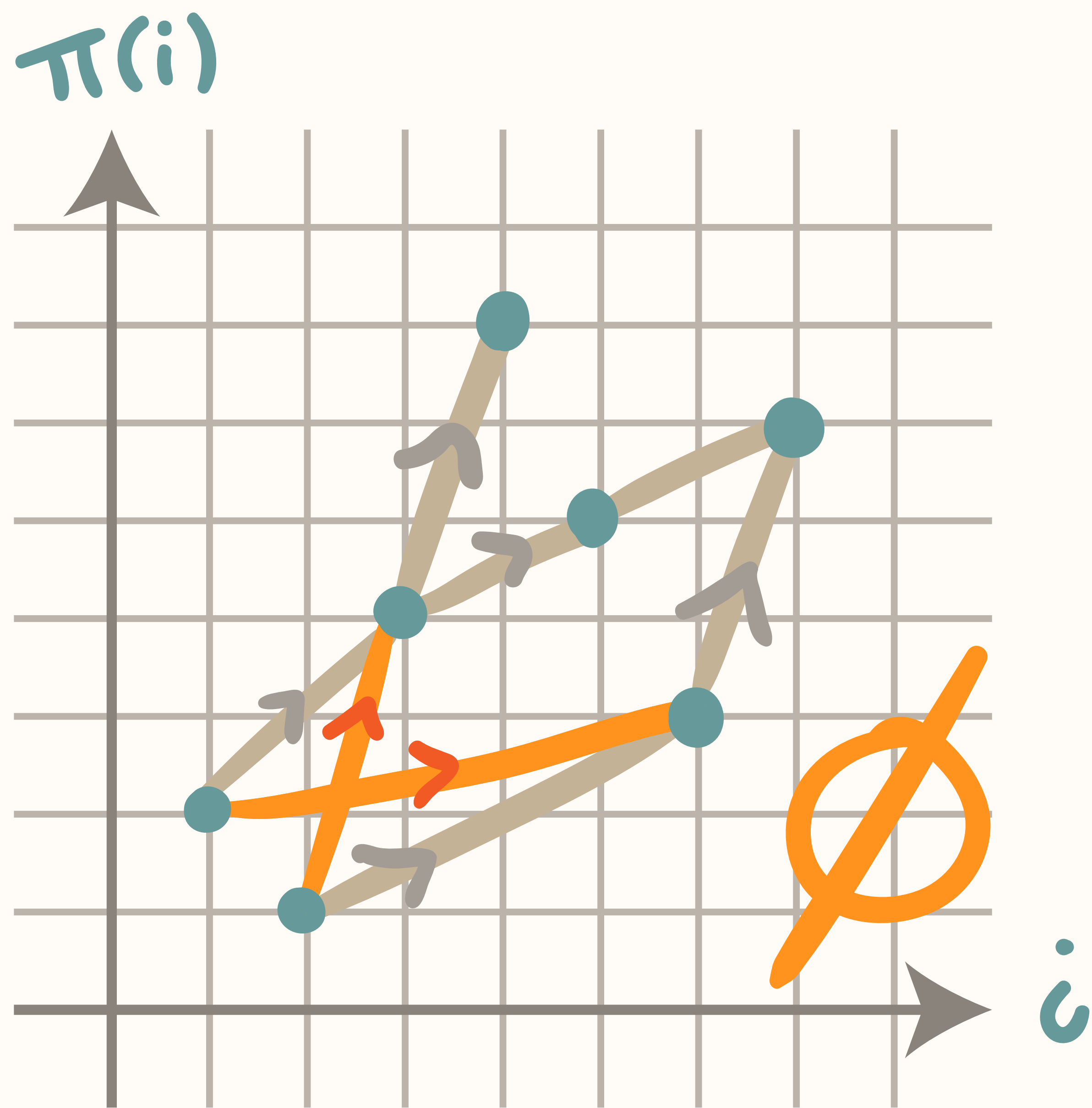
***= Dominance relation
with no transitive edges***

Plane permutations



Plane permutation
= No edge crossing in the dominance diagram

Plane permutations

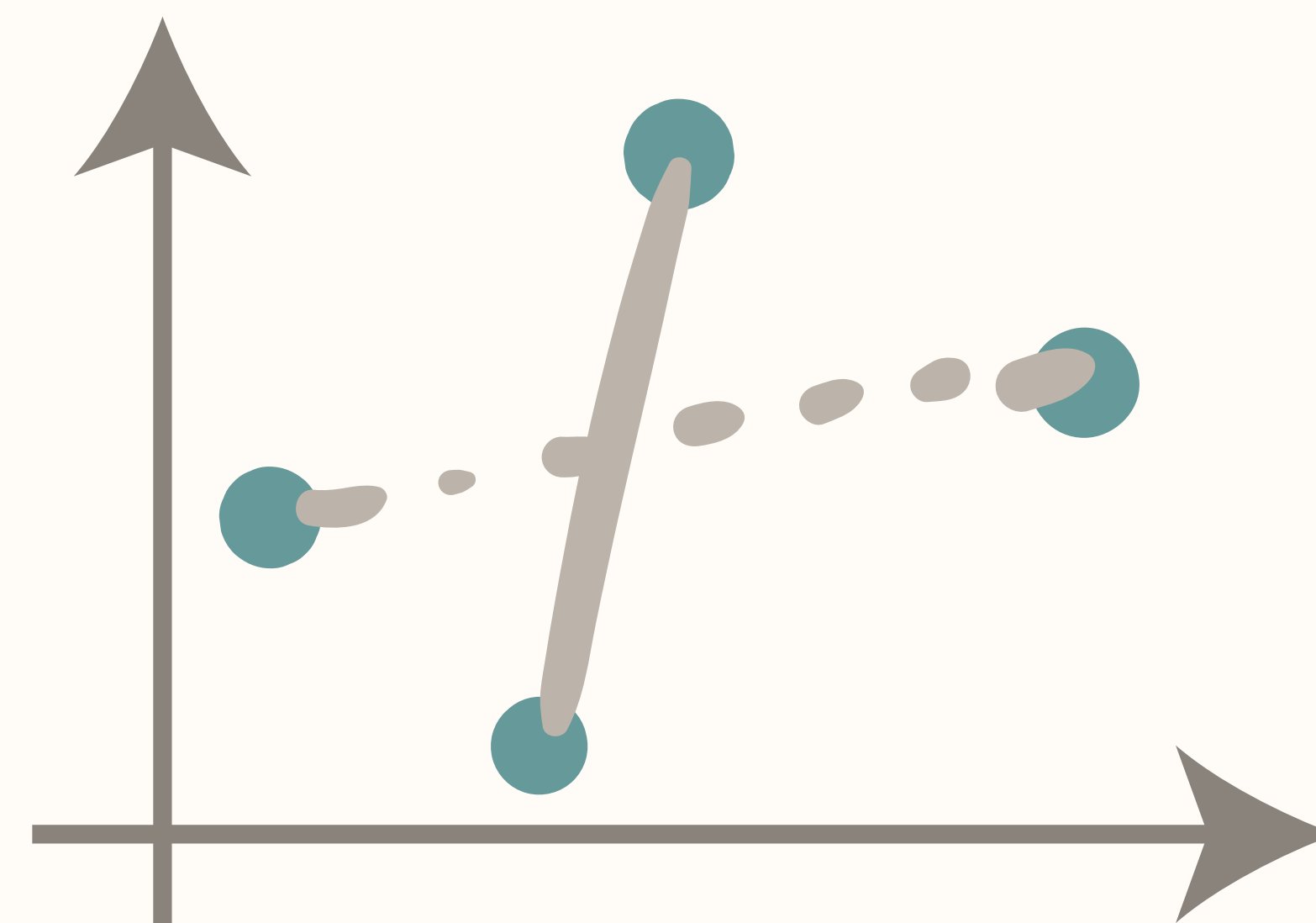


Plane permutation

= No edge crossing in the dominance diagram

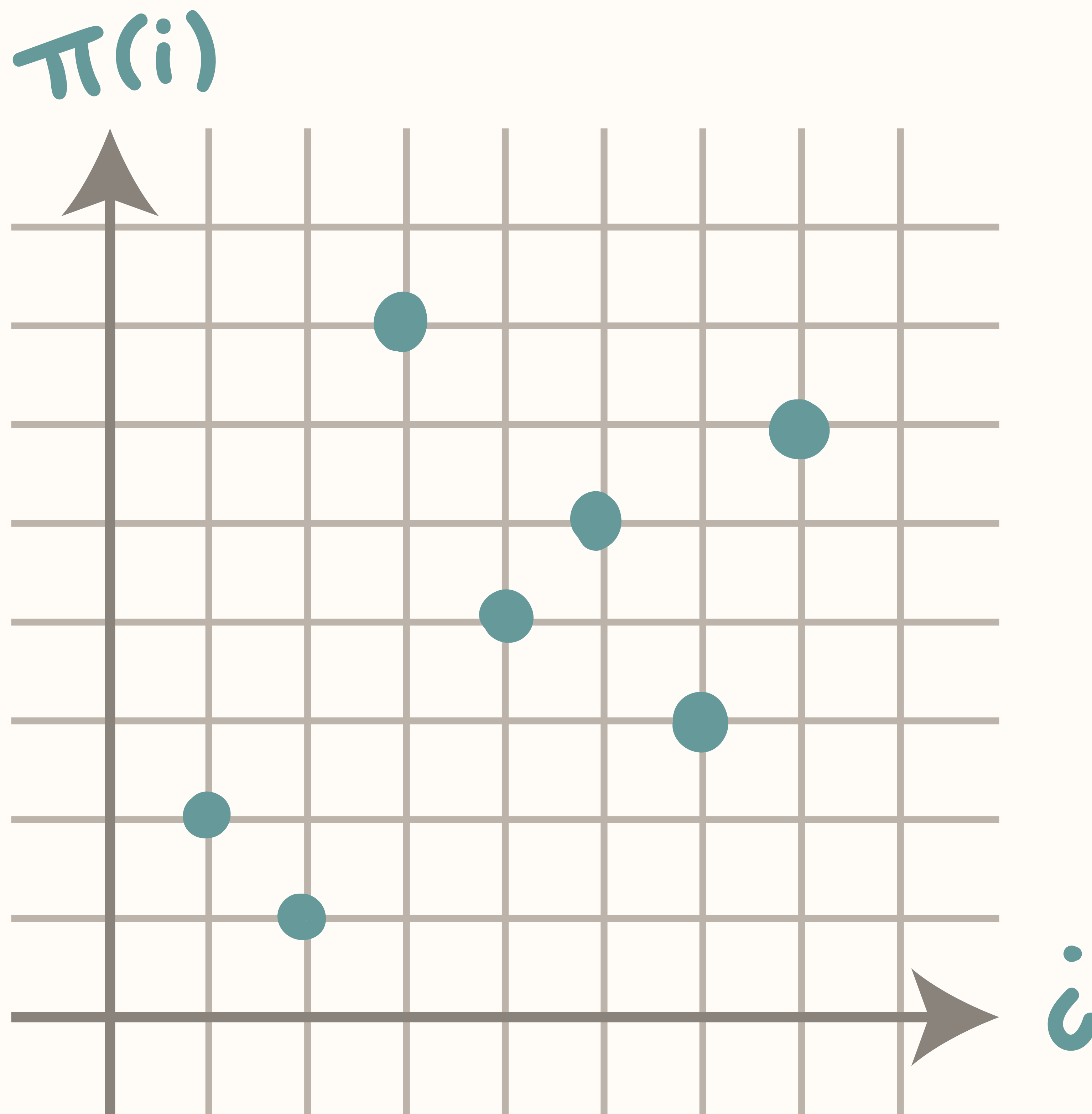
= Avoid the vincular pattern :

2 1 4 3



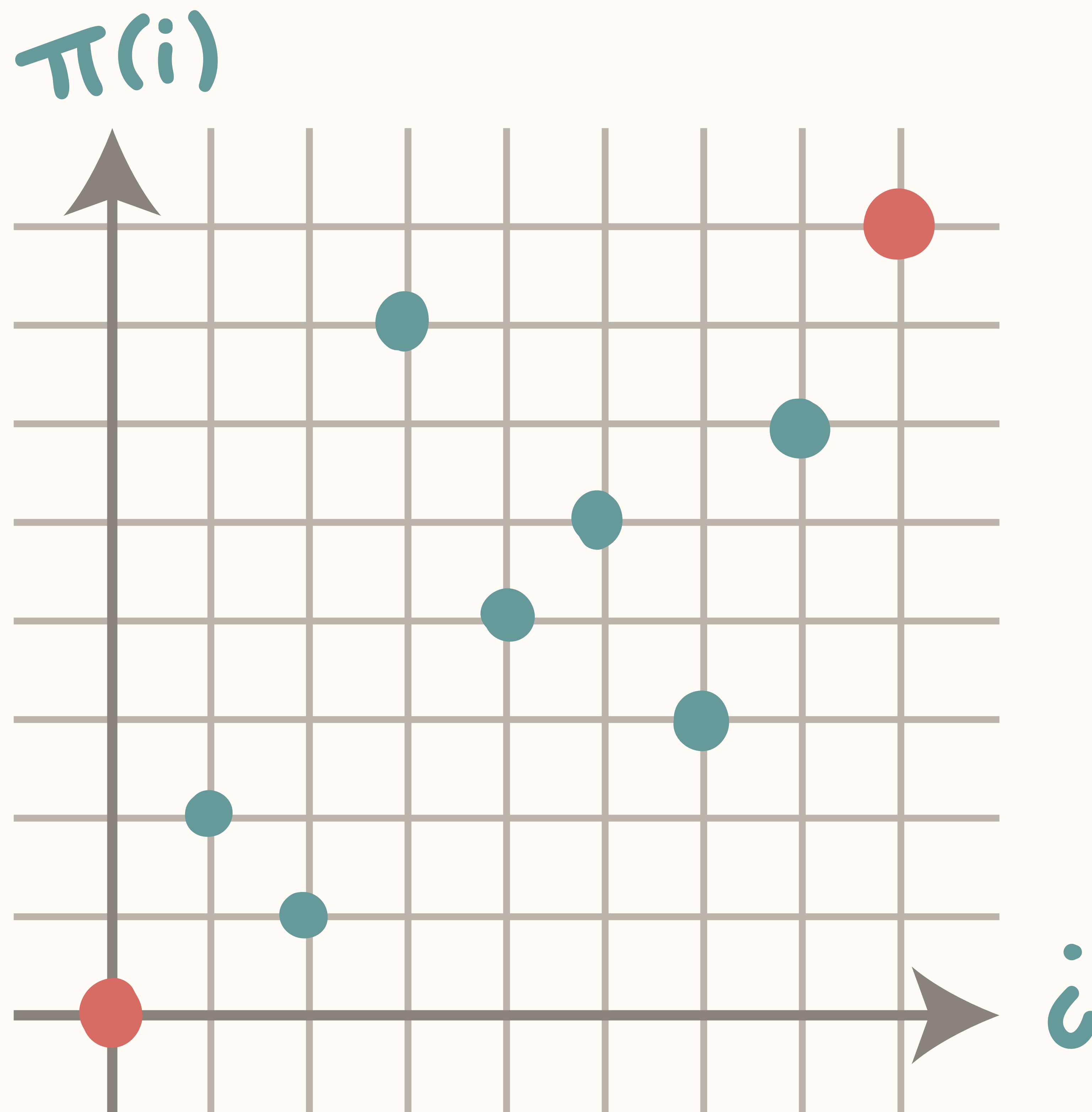
Link with plane permutations

Plane permutation \longrightarrow Poset



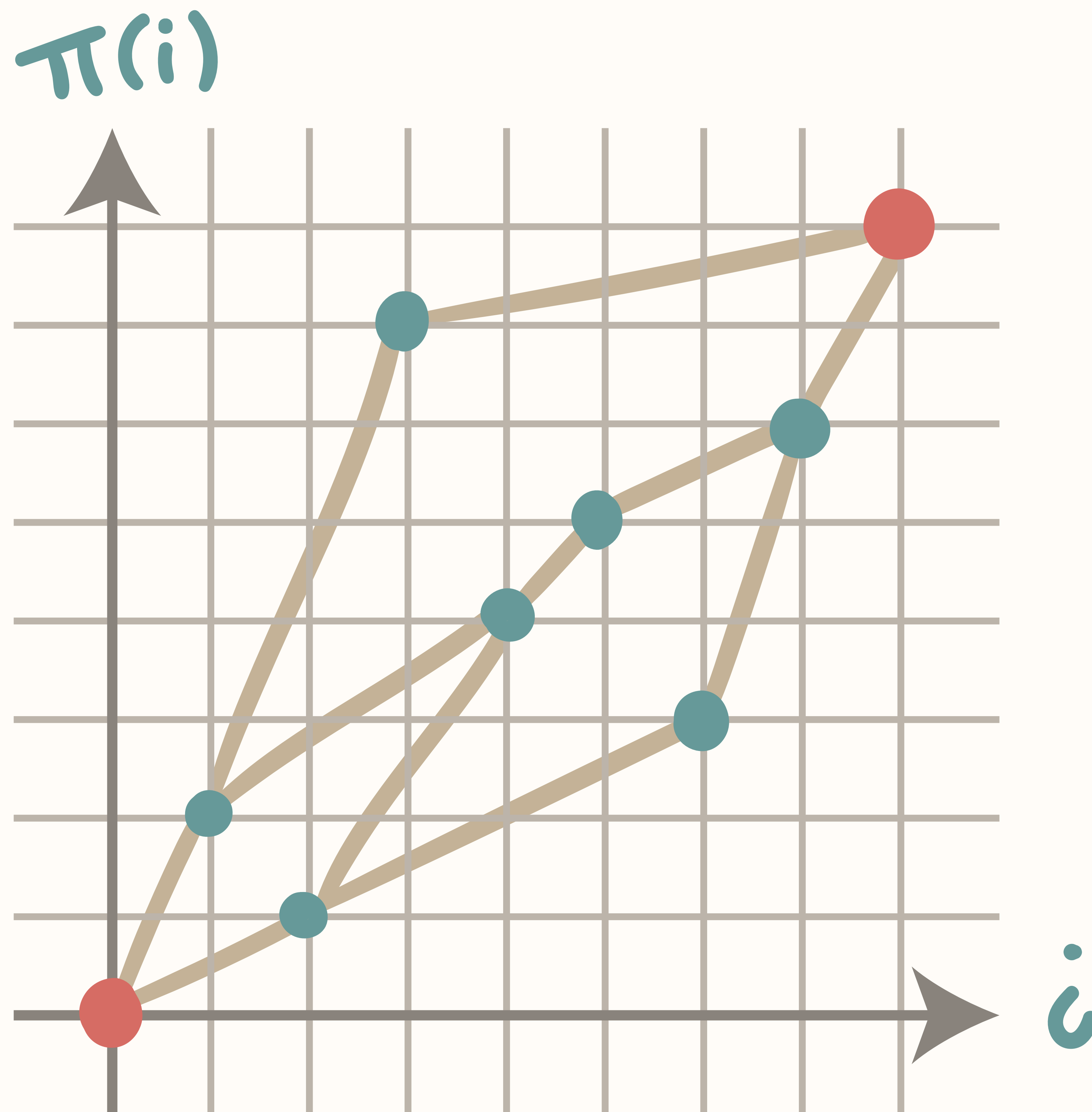
Link with plane permutations

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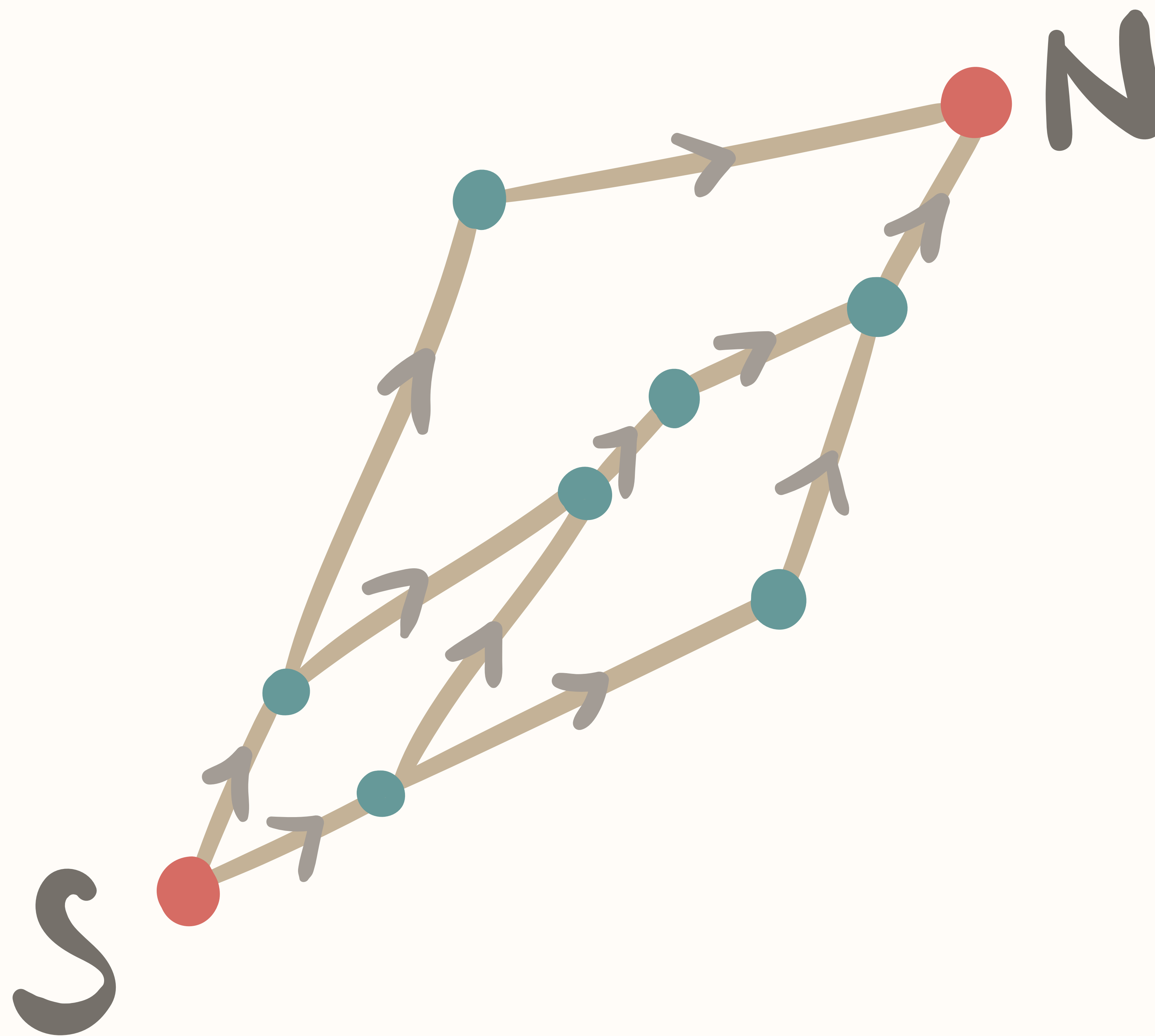
Link with plane permutations

Plane permutation \longrightarrow Poset



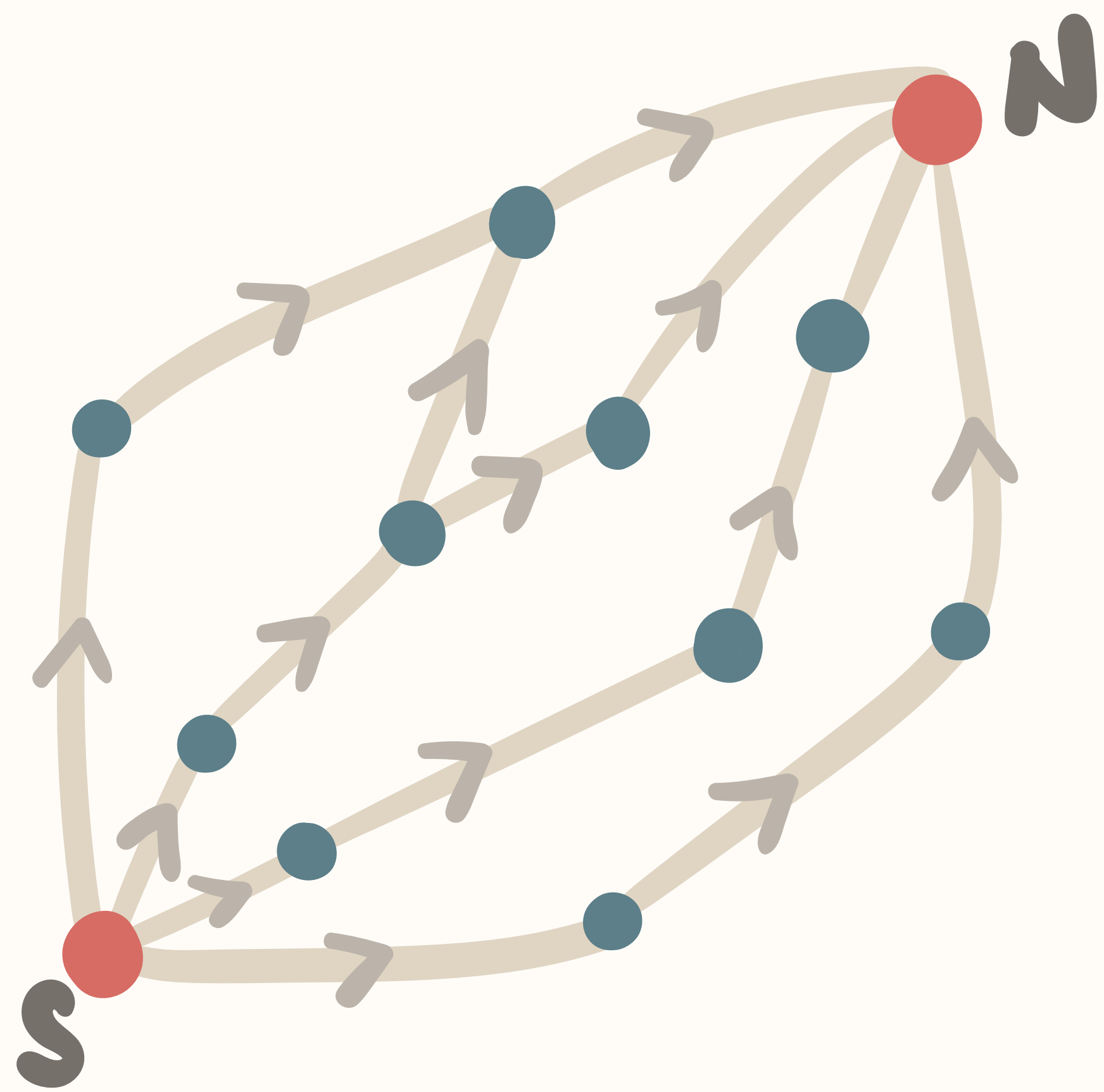
Link with plane permutations

Plane permutation \longrightarrow *Poset*



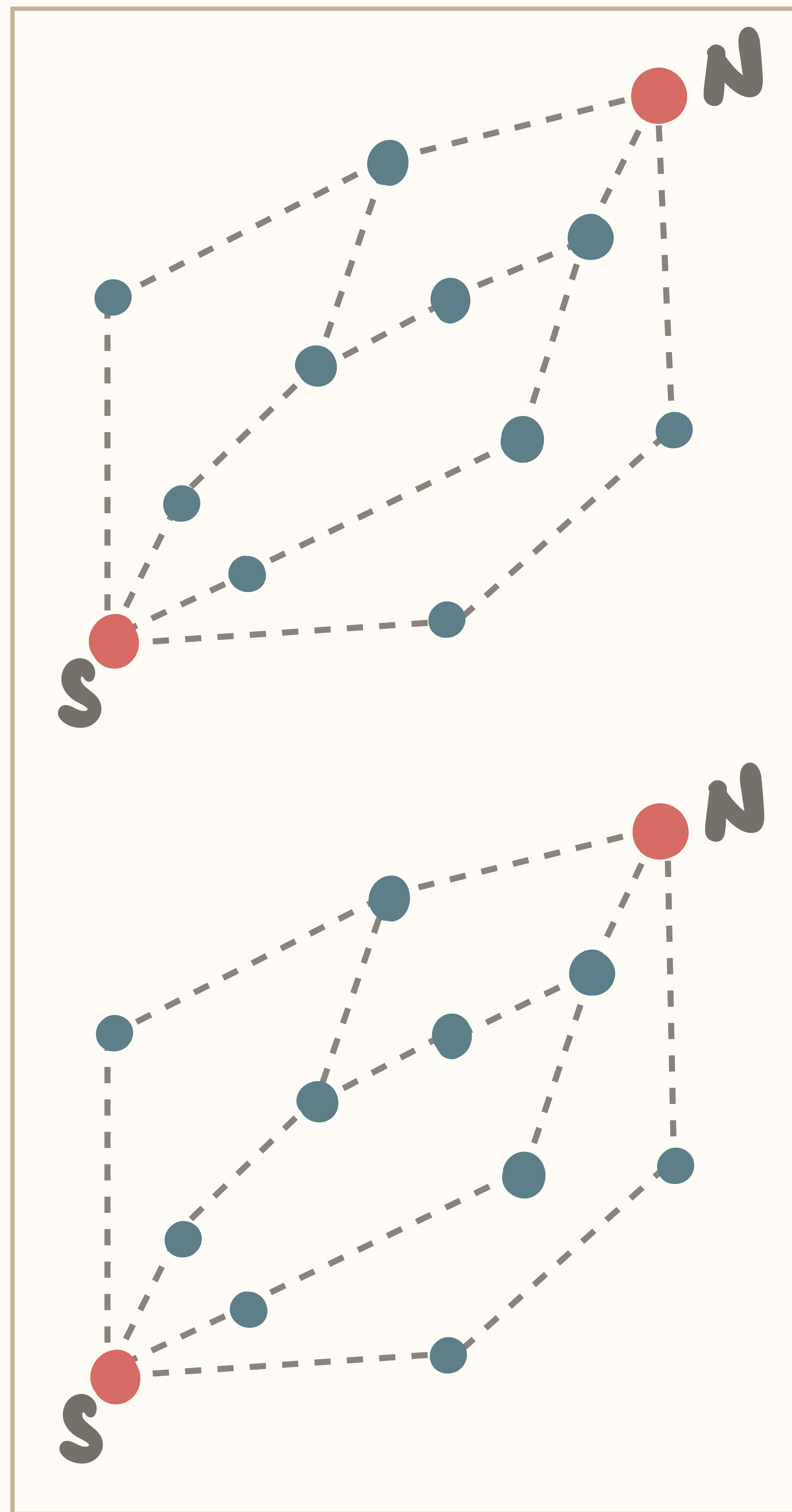
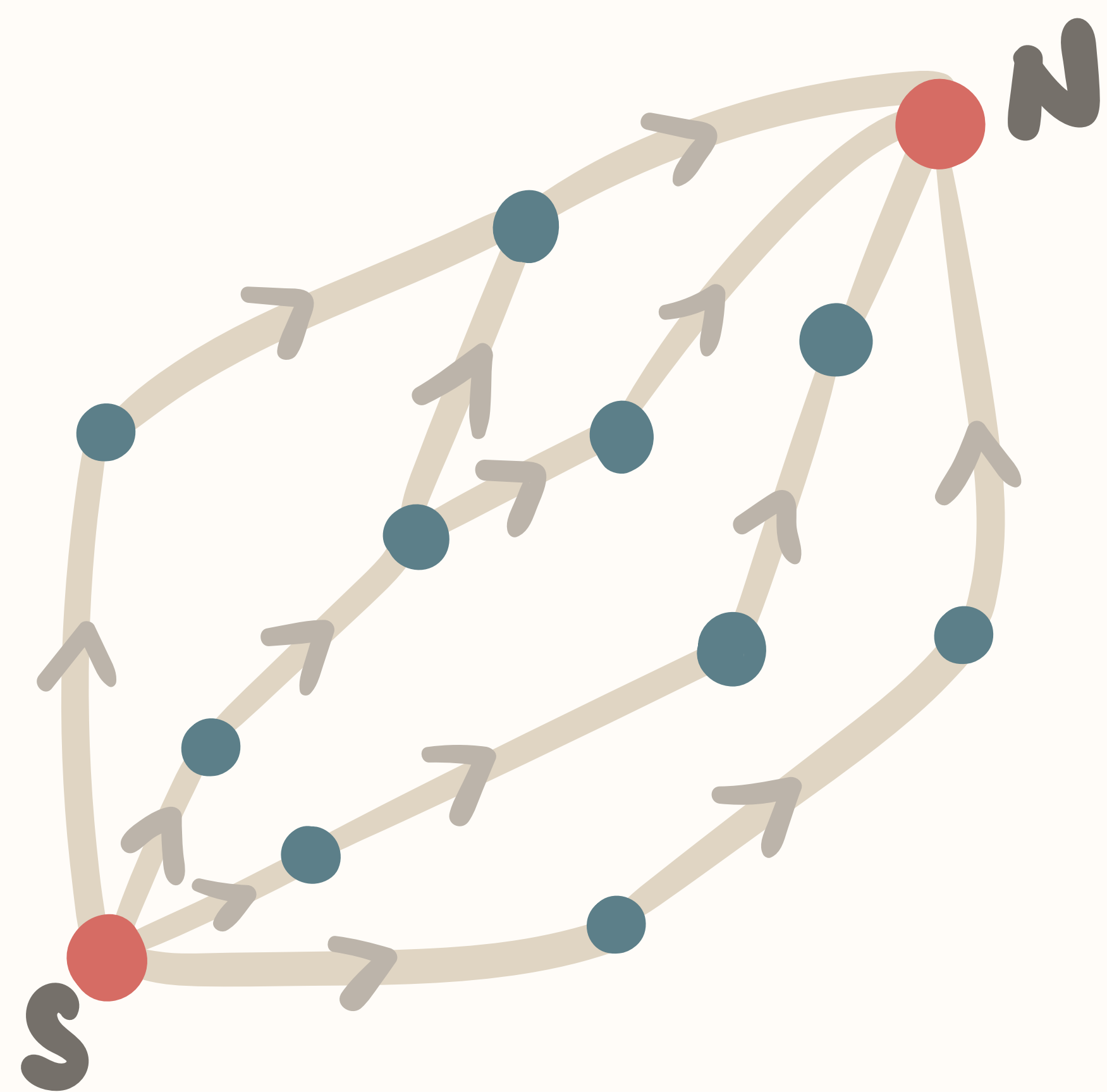
Link with plane permutations

Poset \longrightarrow *Plane permutation*



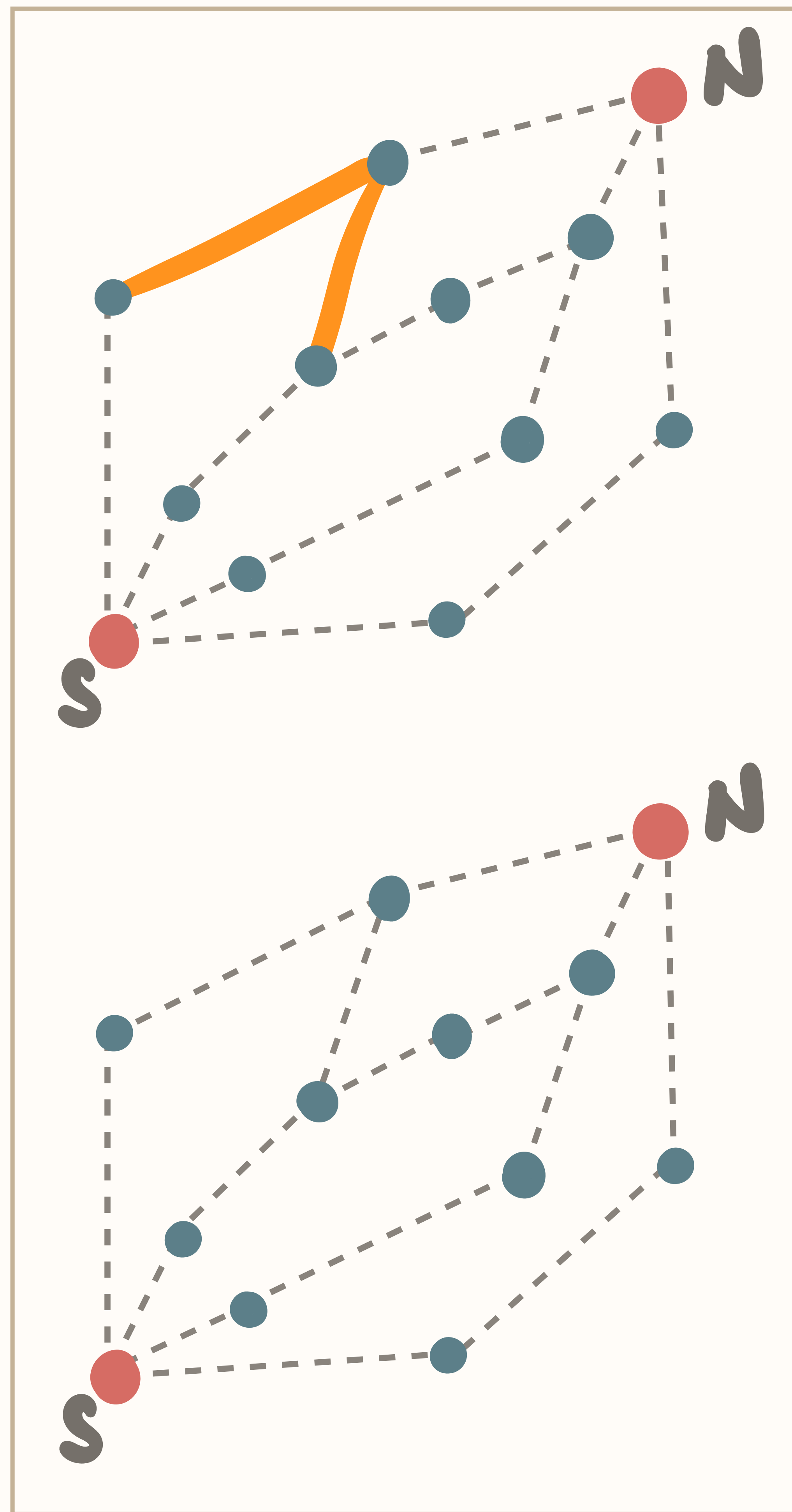
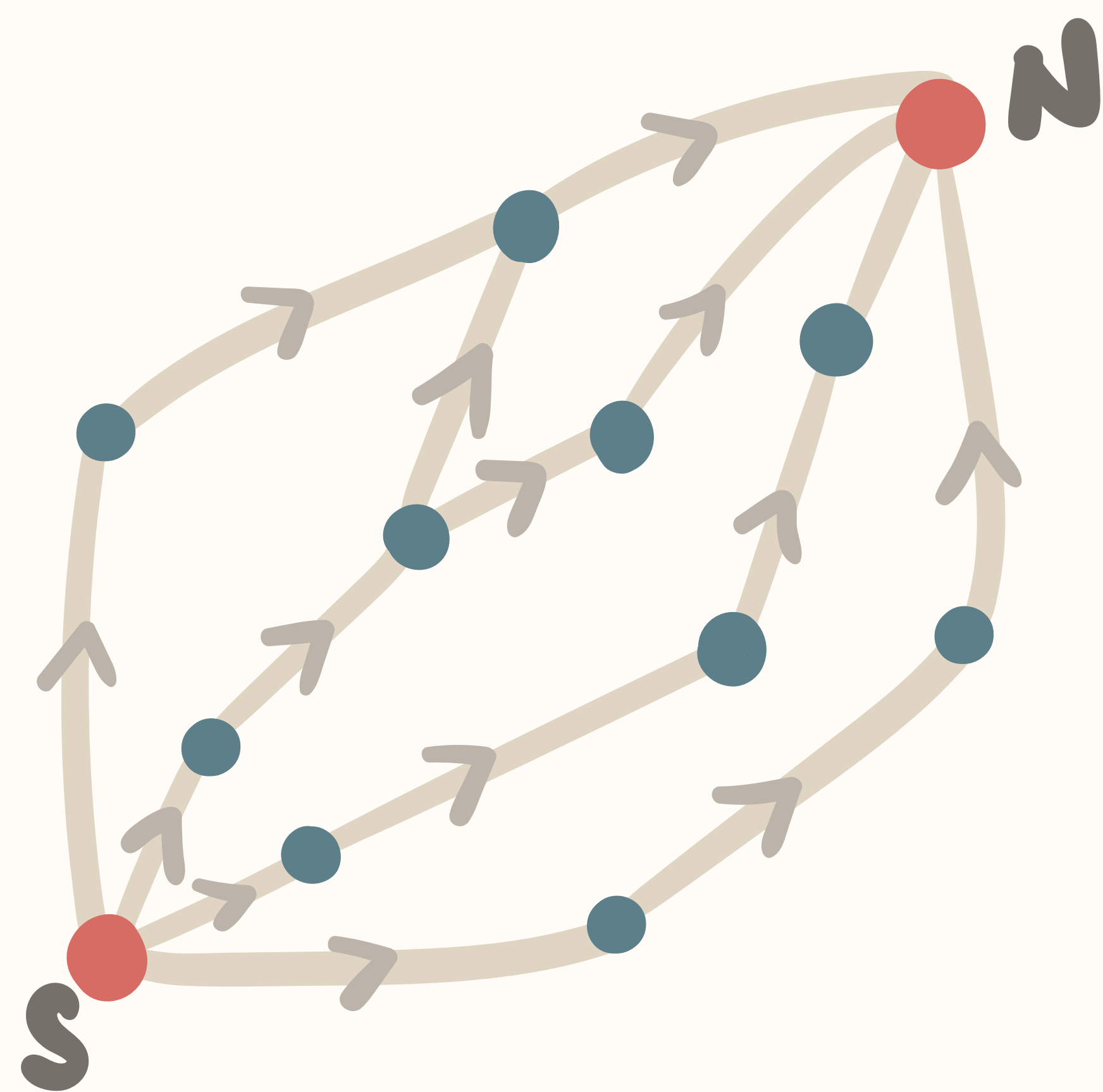
Link with plane permutations

Poset \longrightarrow *Plane permutation*



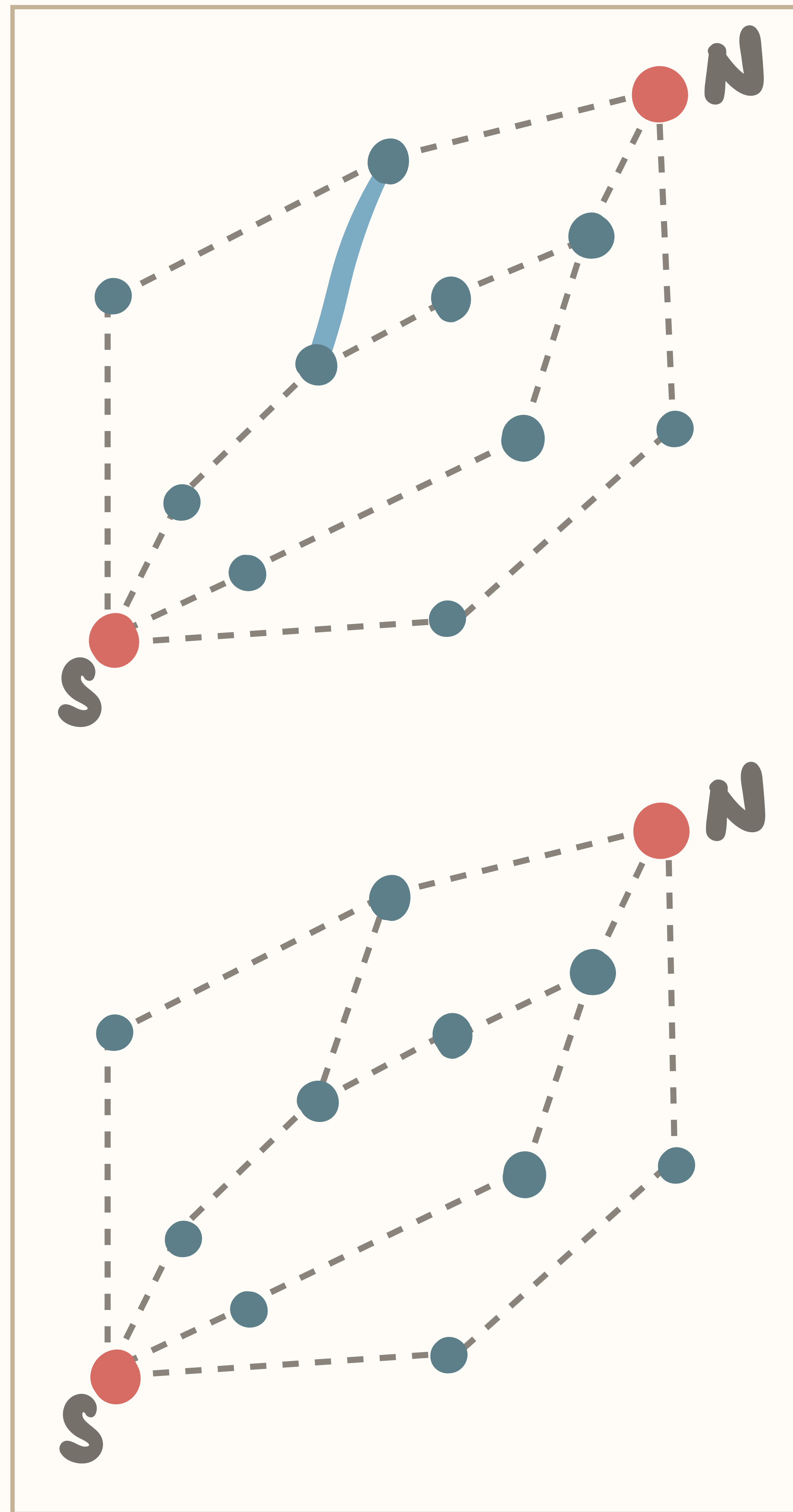
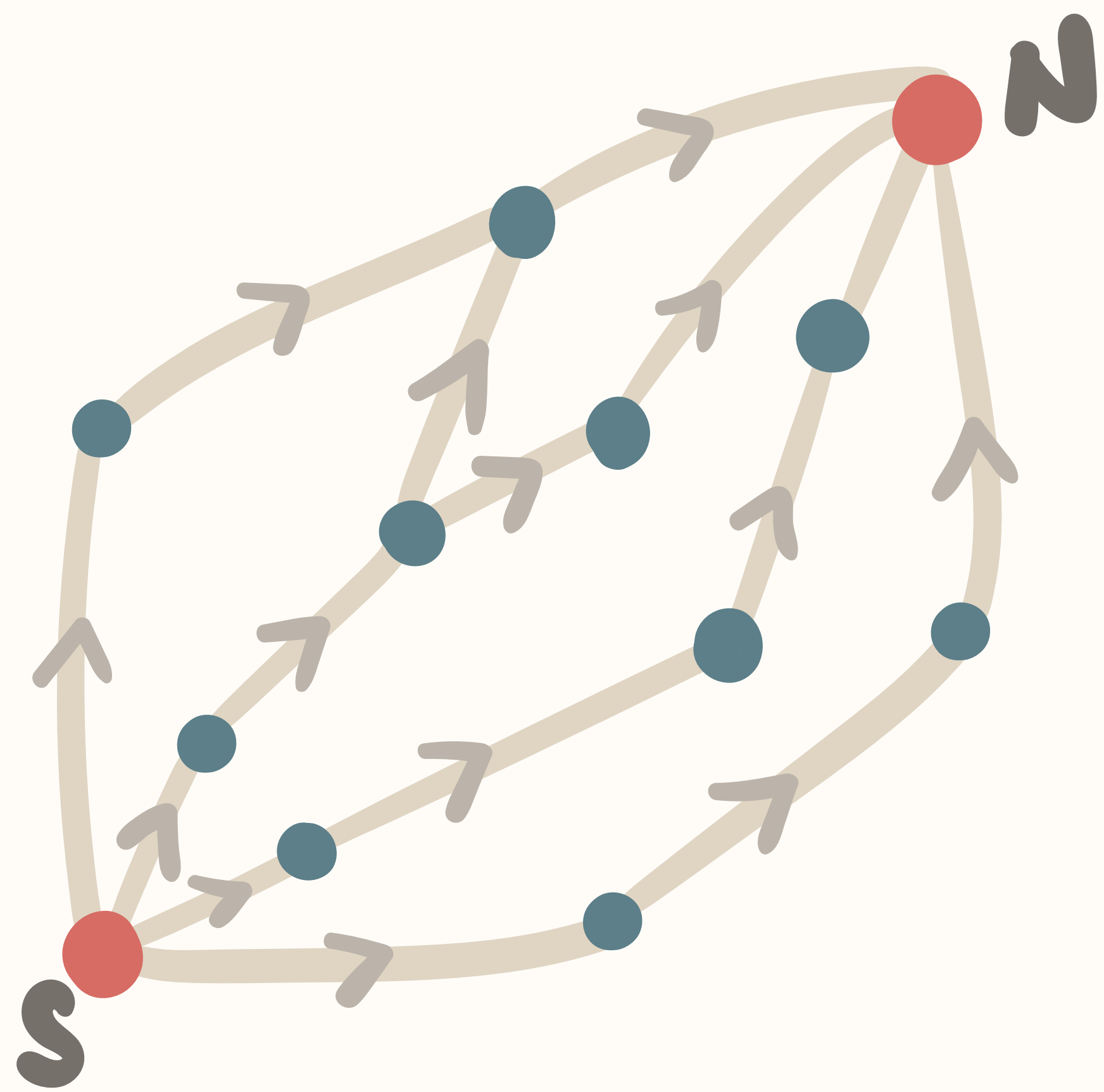
Link with plane permutations

Poset \longrightarrow Plane permutation



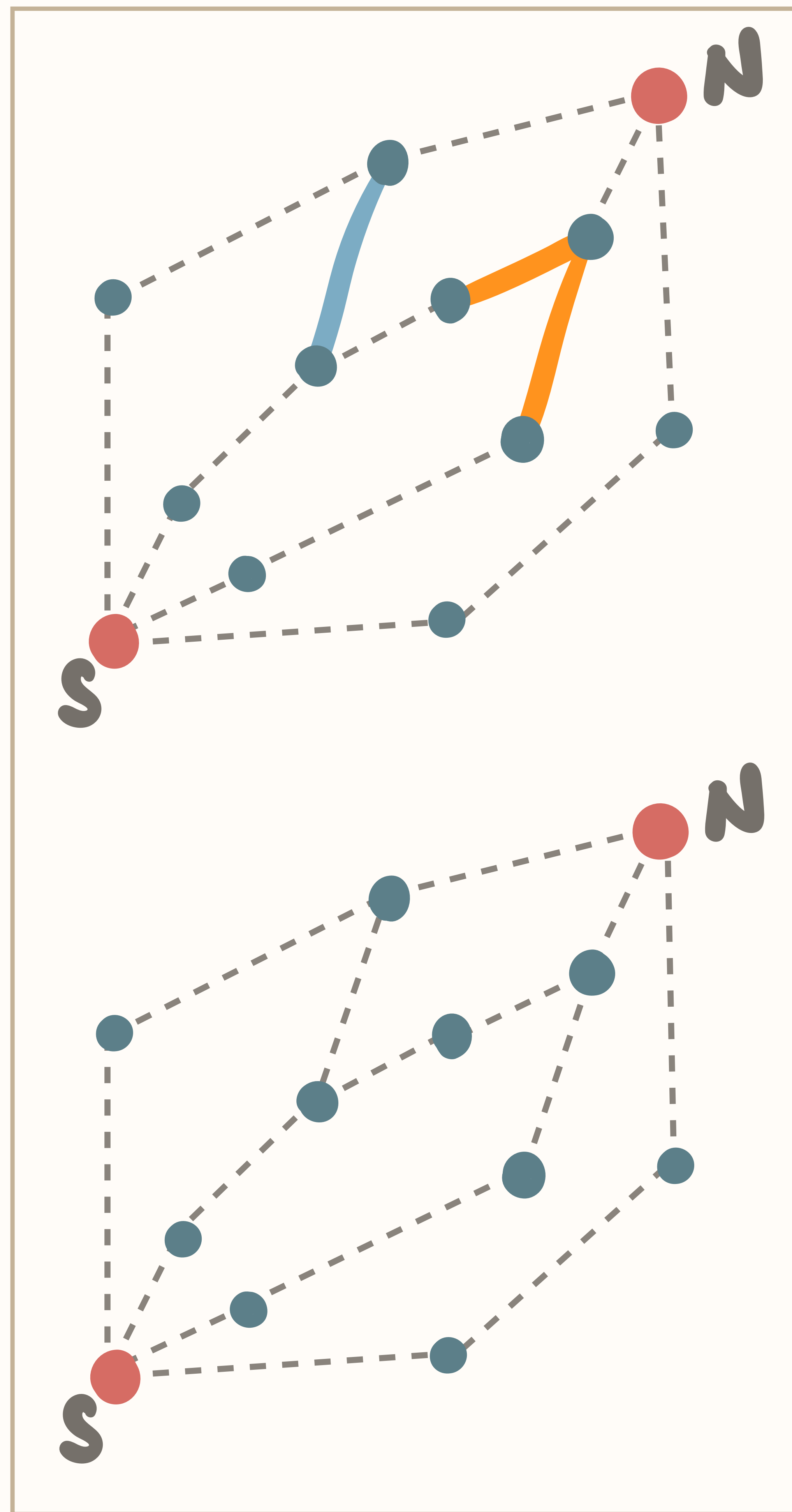
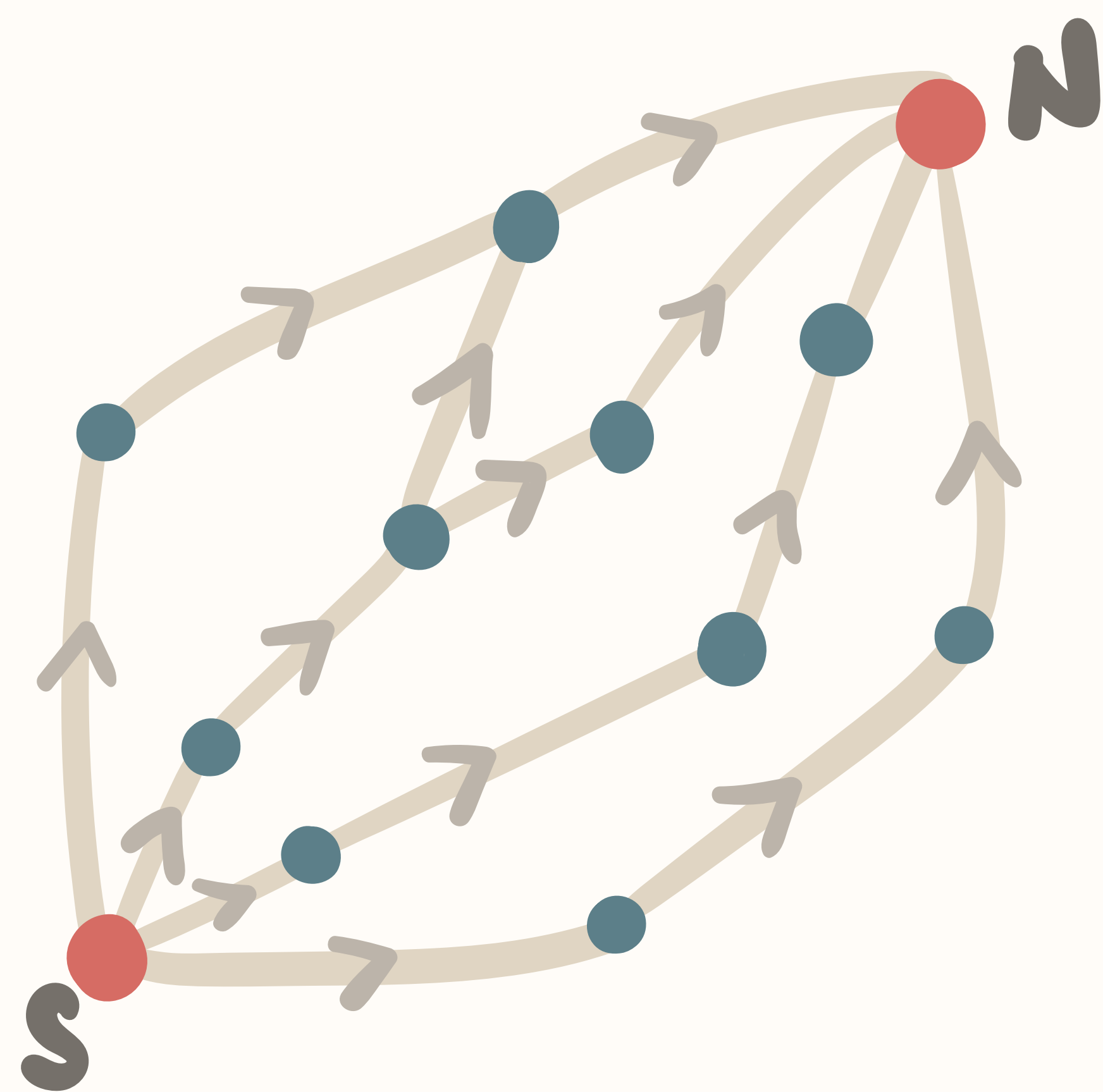
Link with plane permutations

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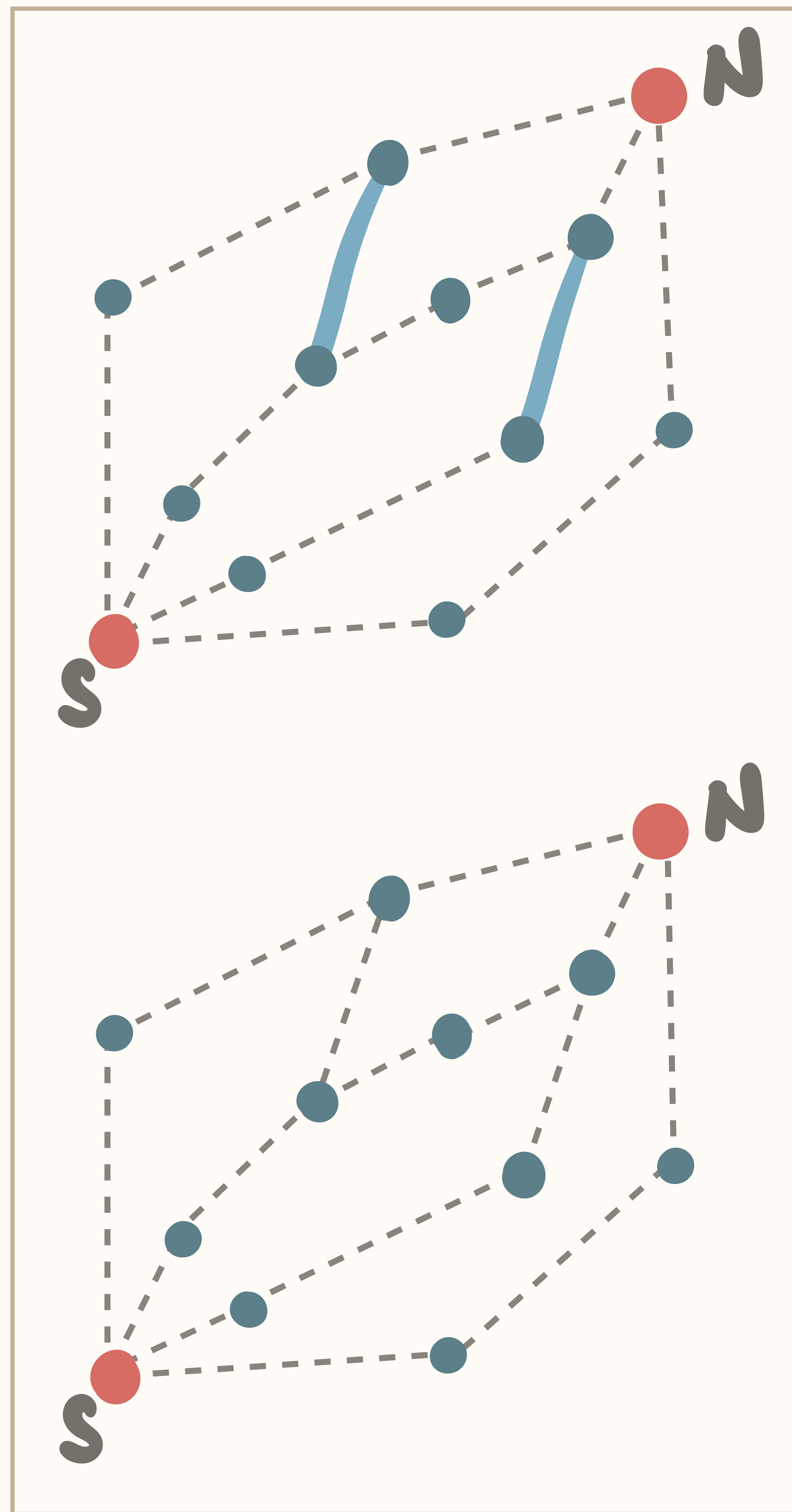
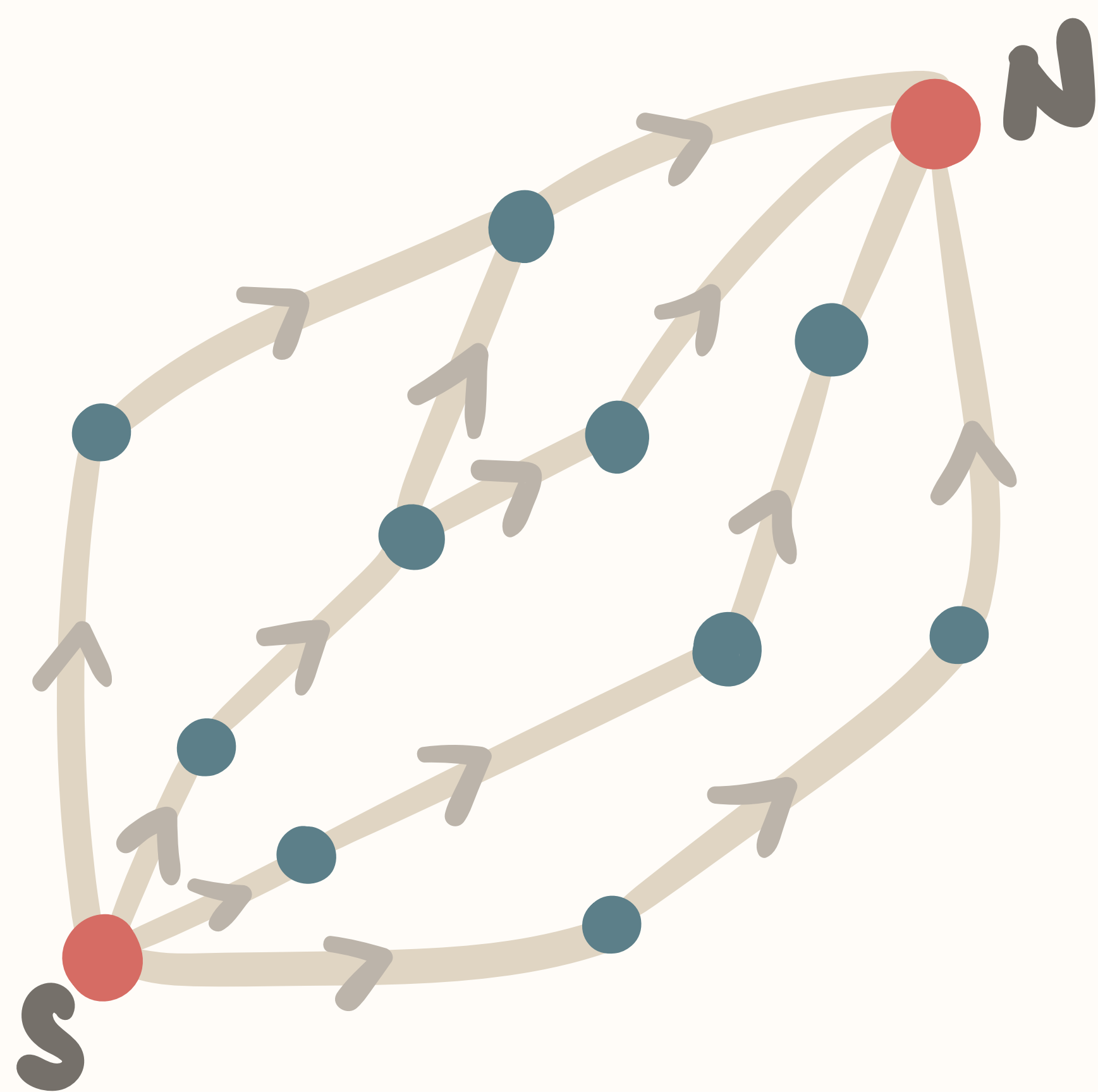
Link with plane permutations

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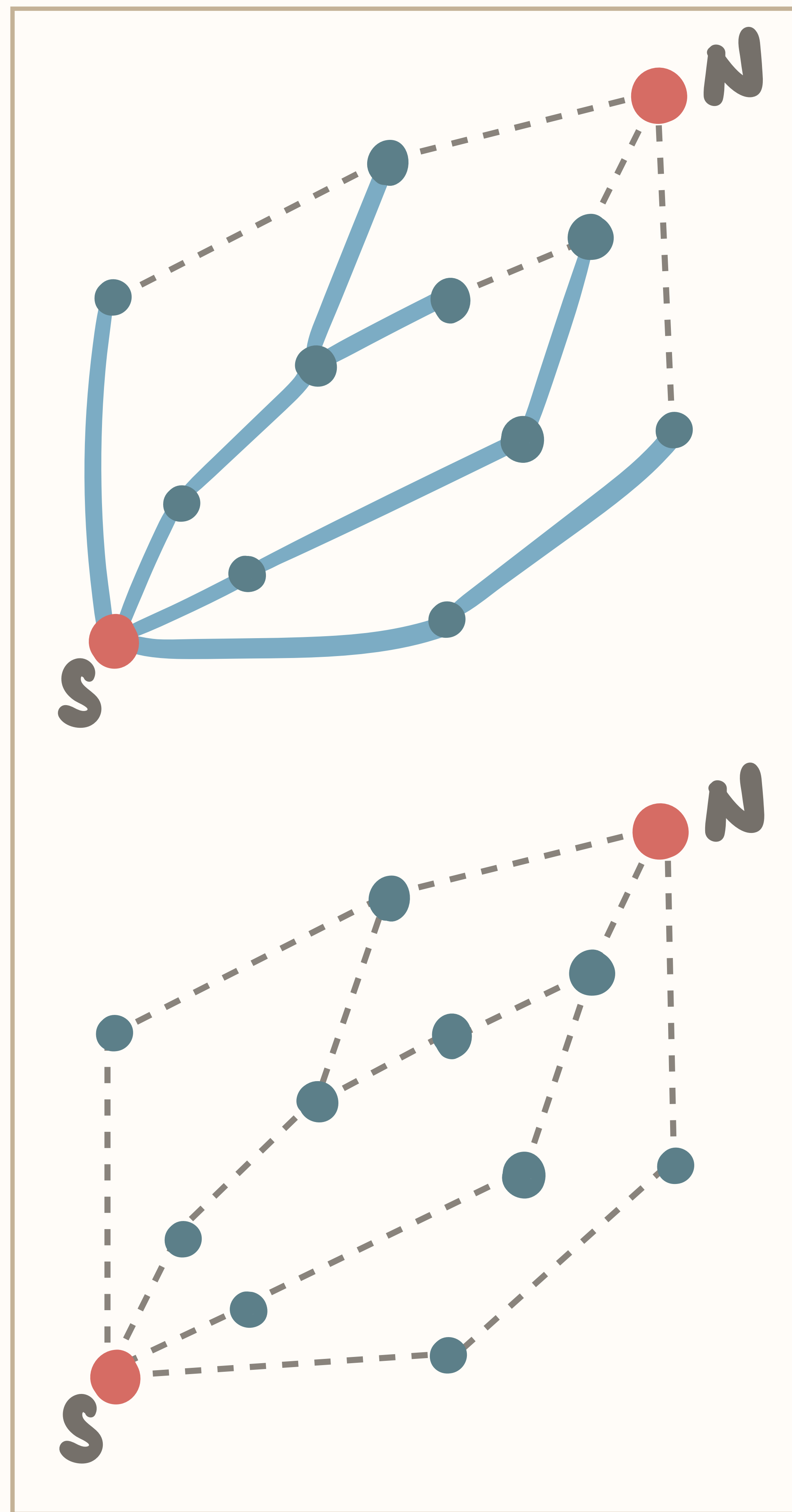
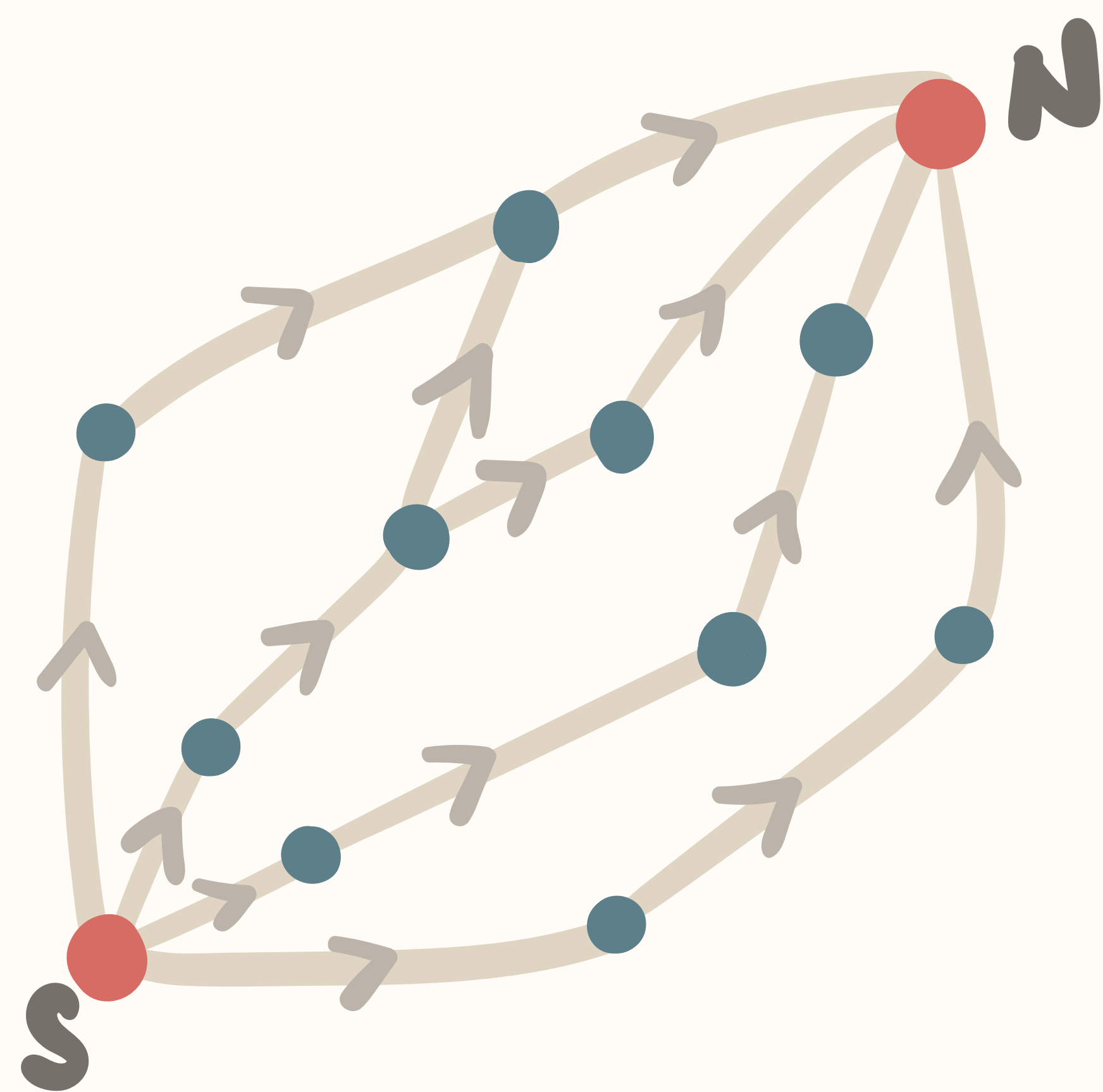
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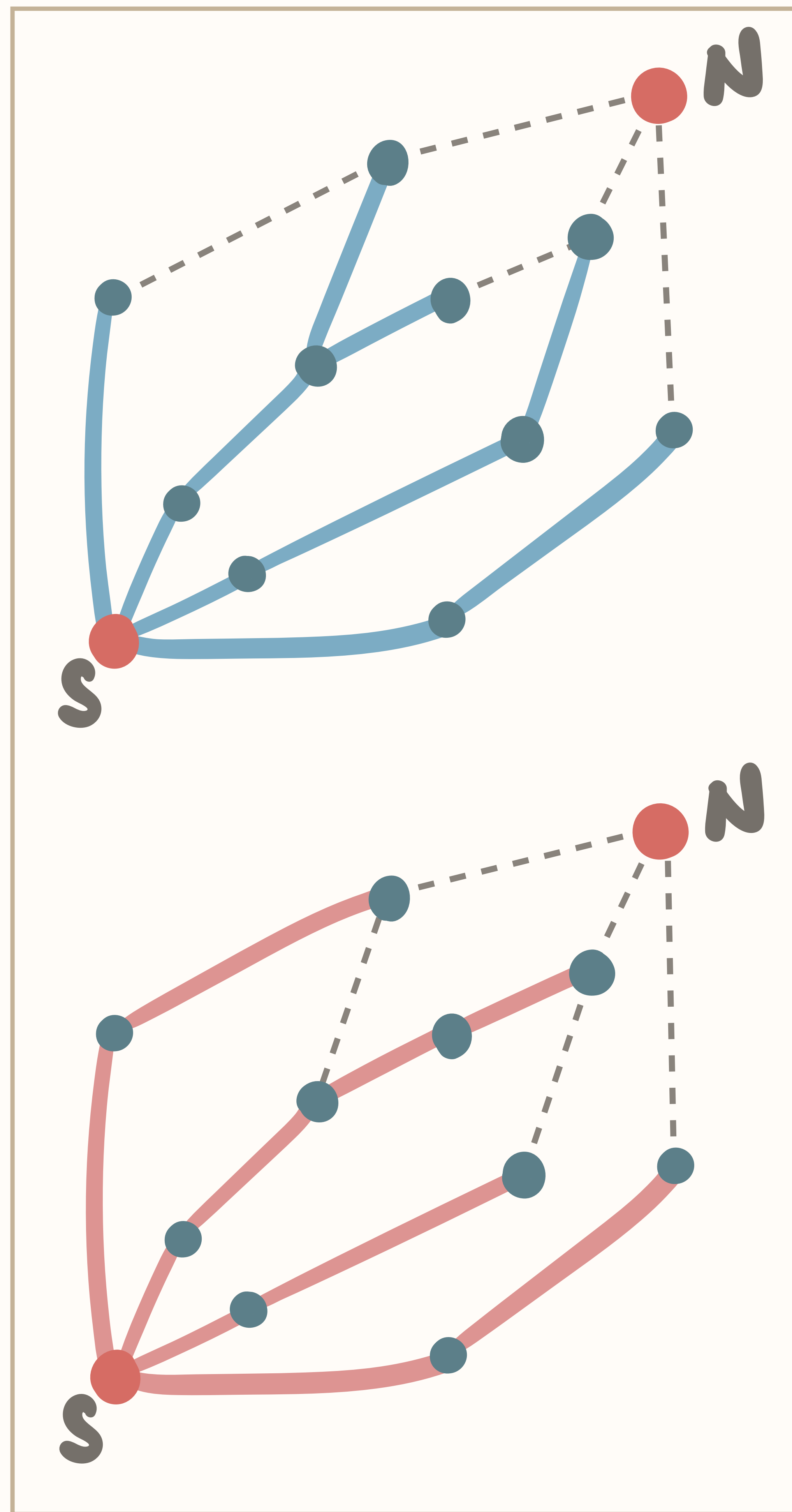
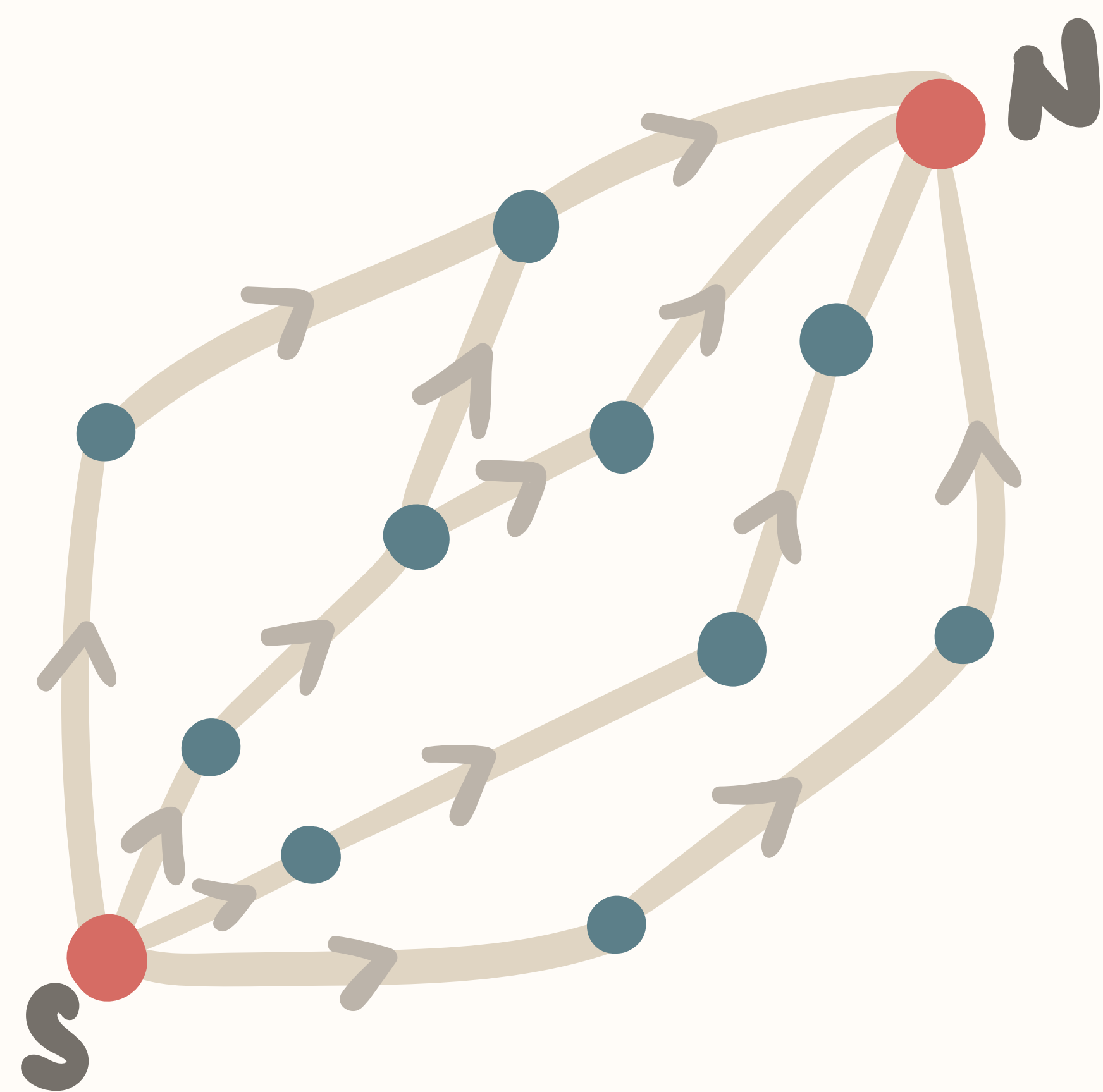
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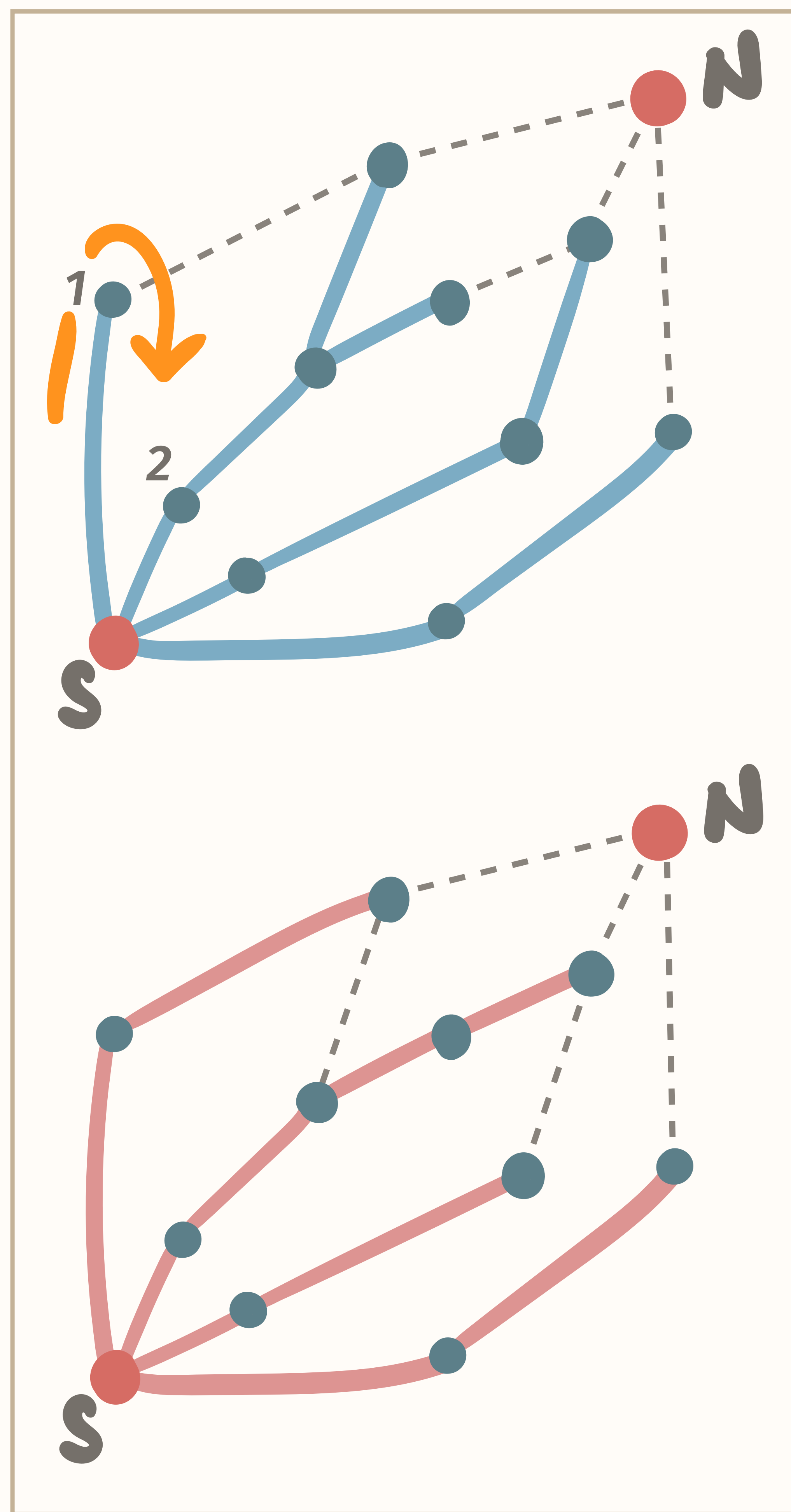
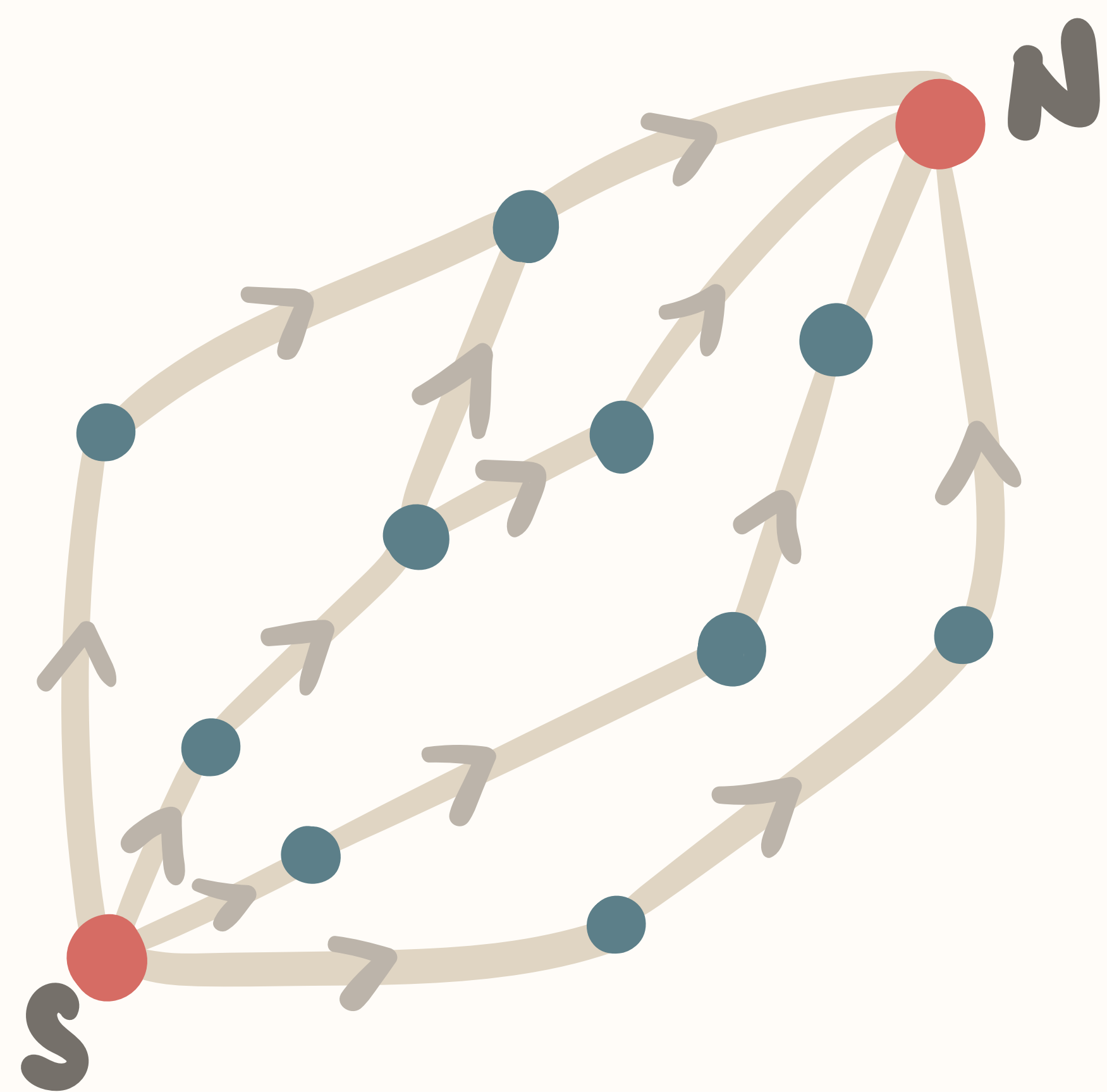
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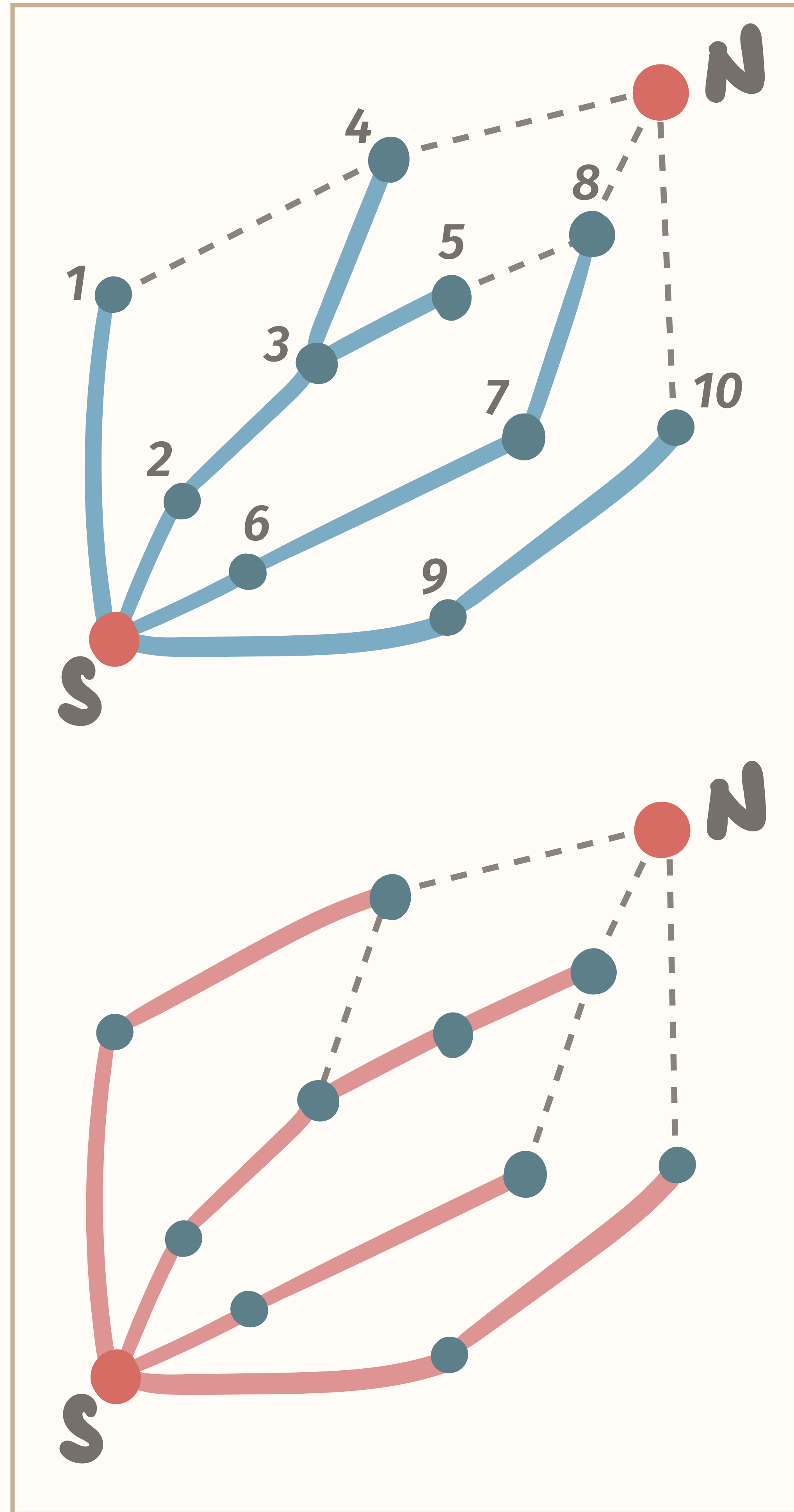
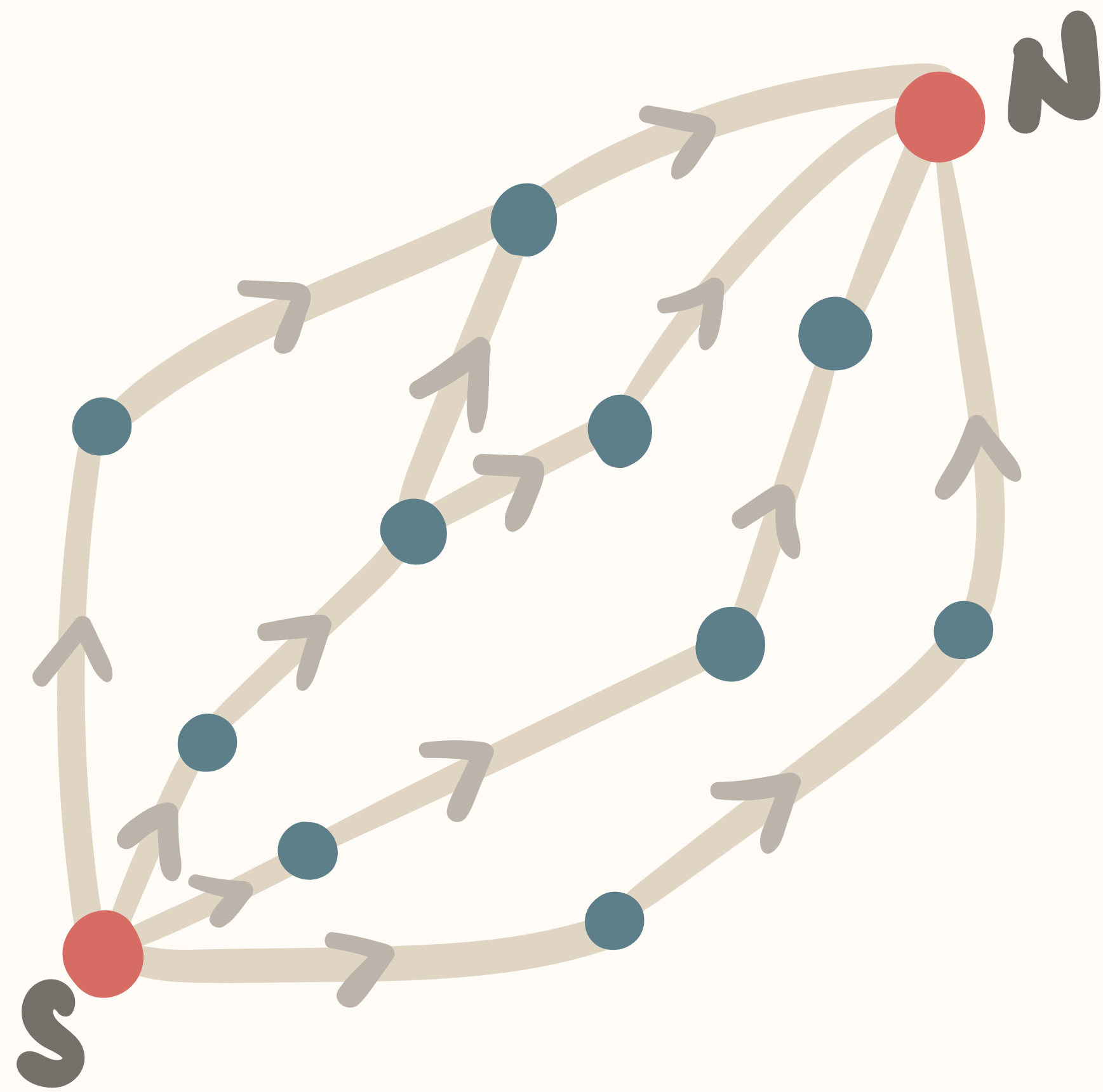
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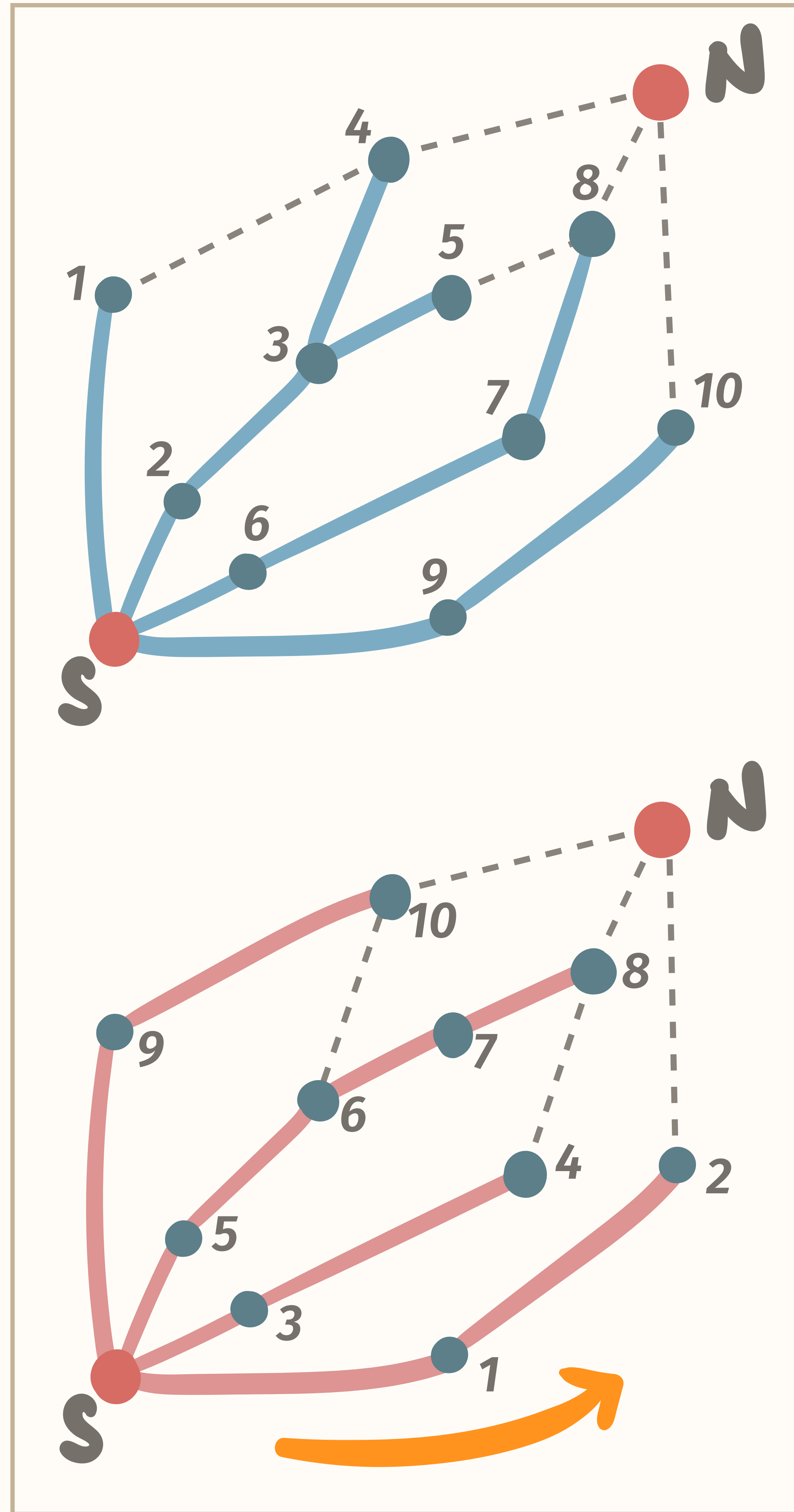
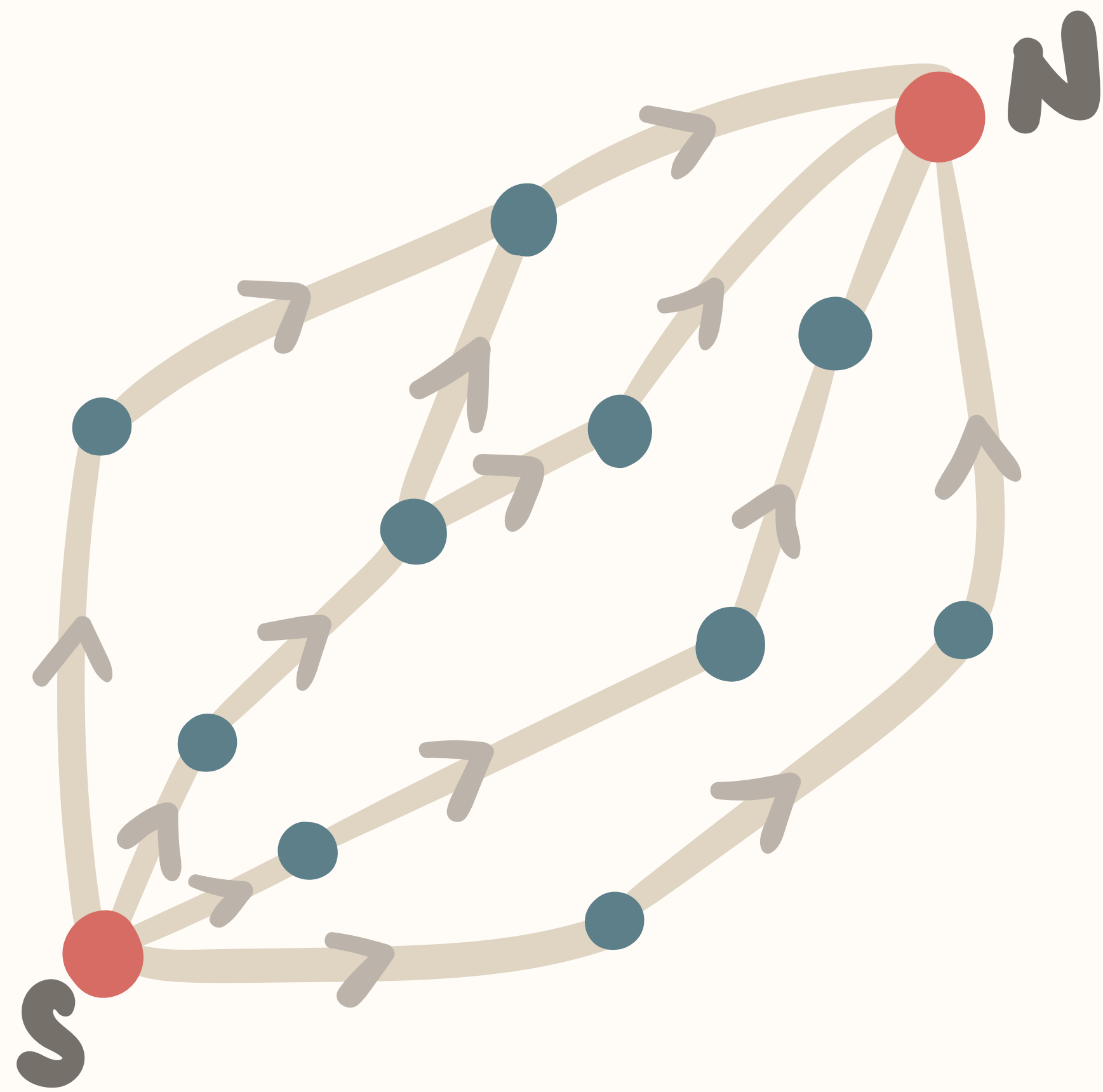
Link with plane permutations

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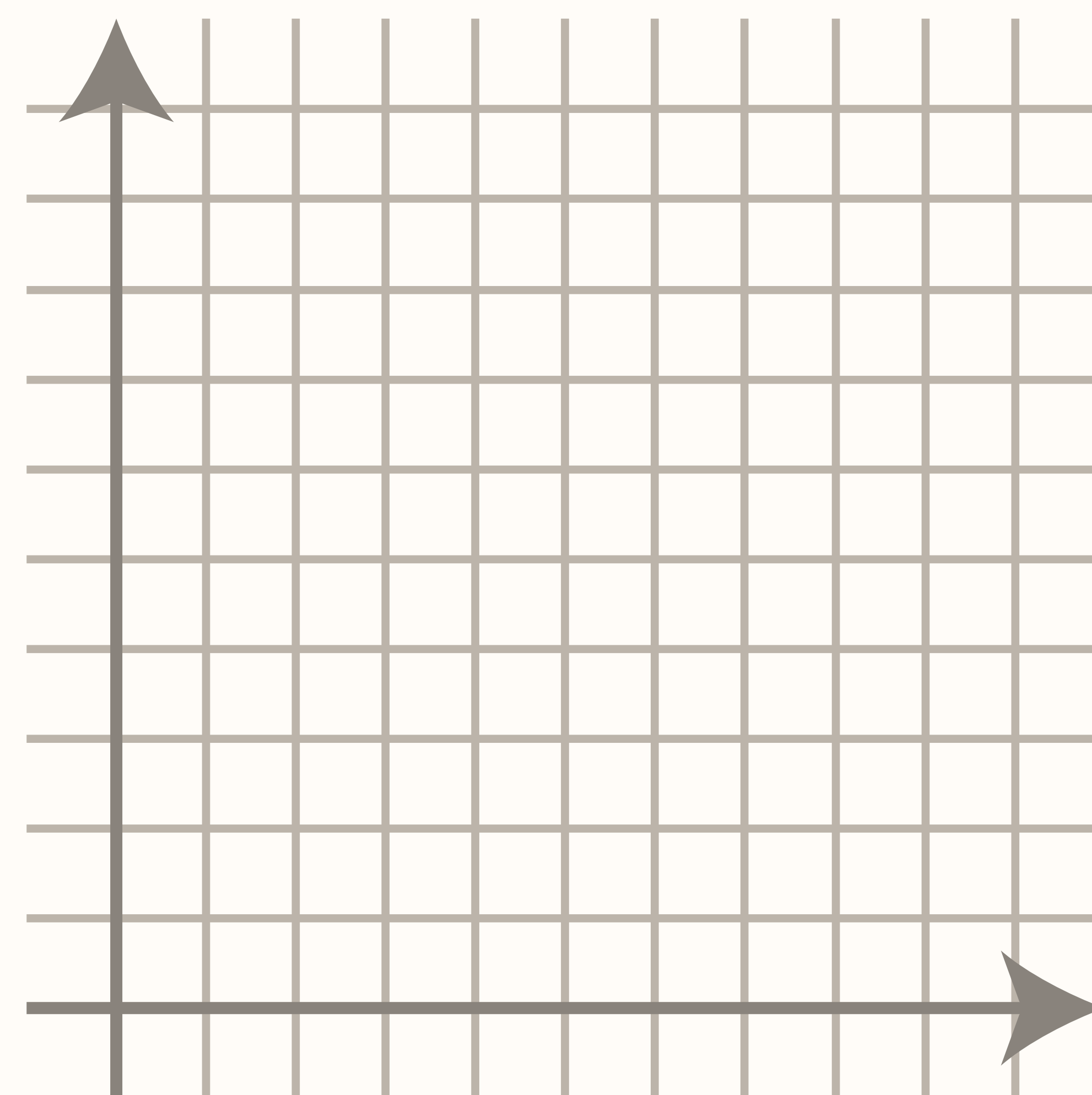
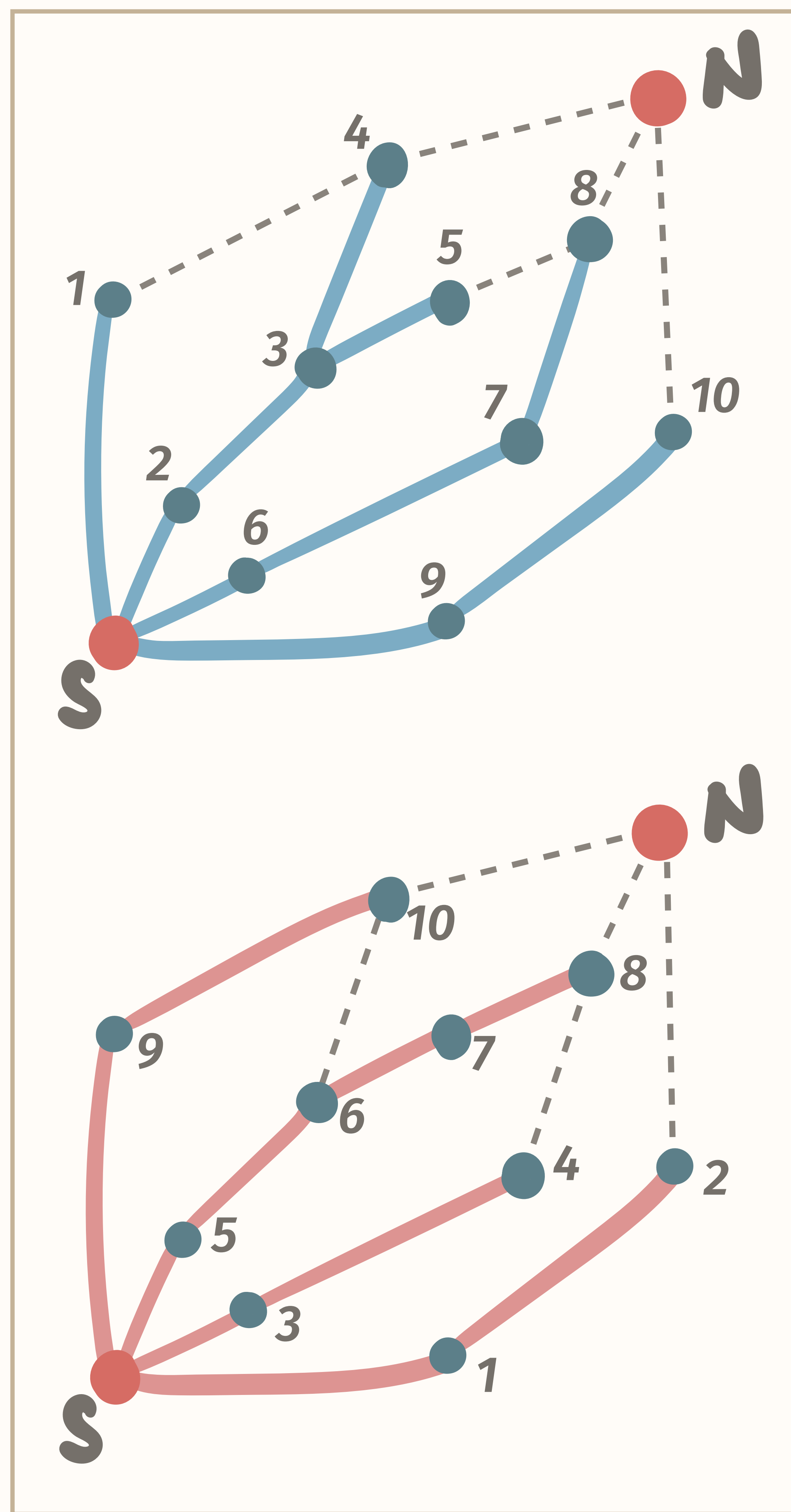
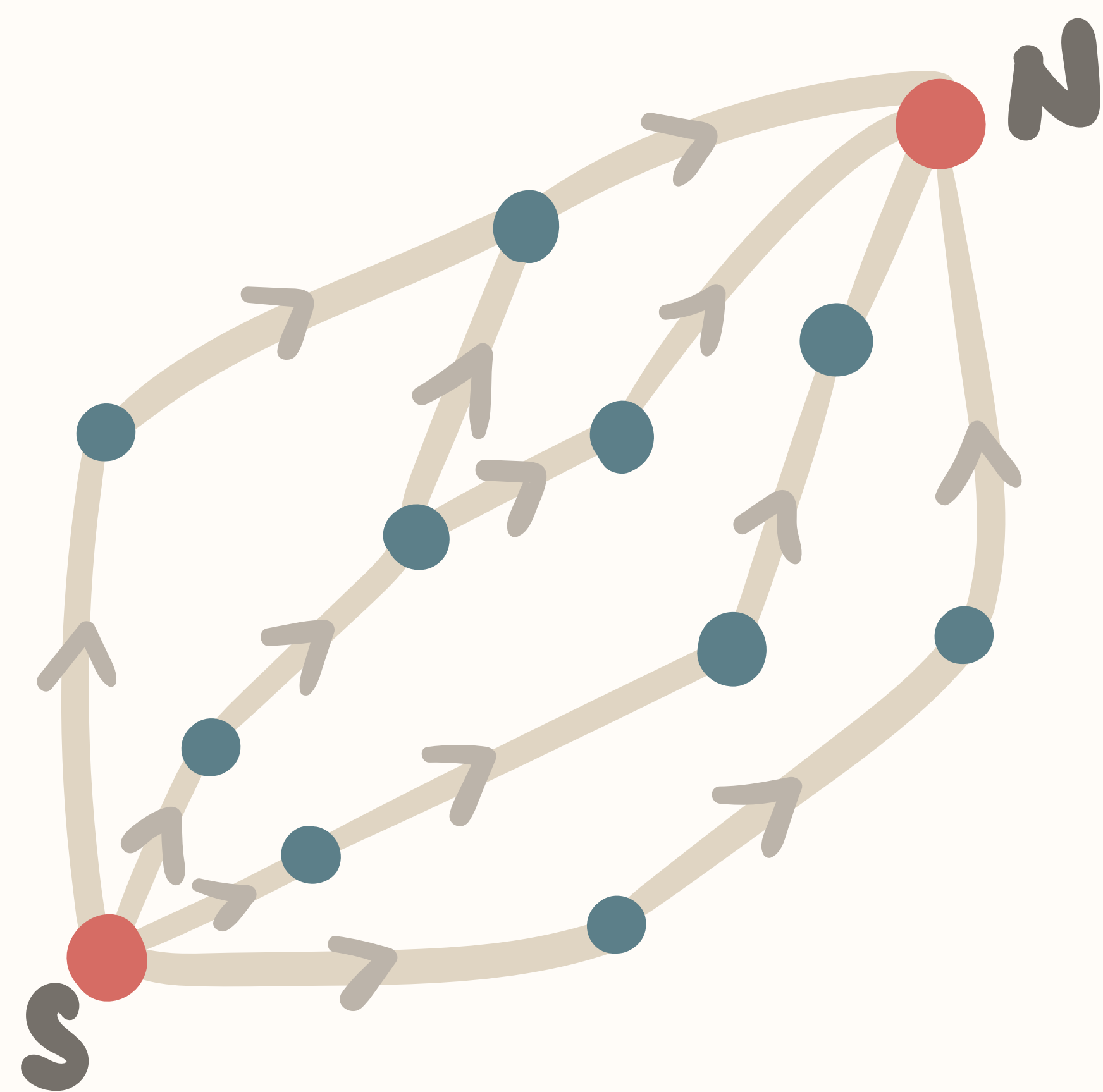
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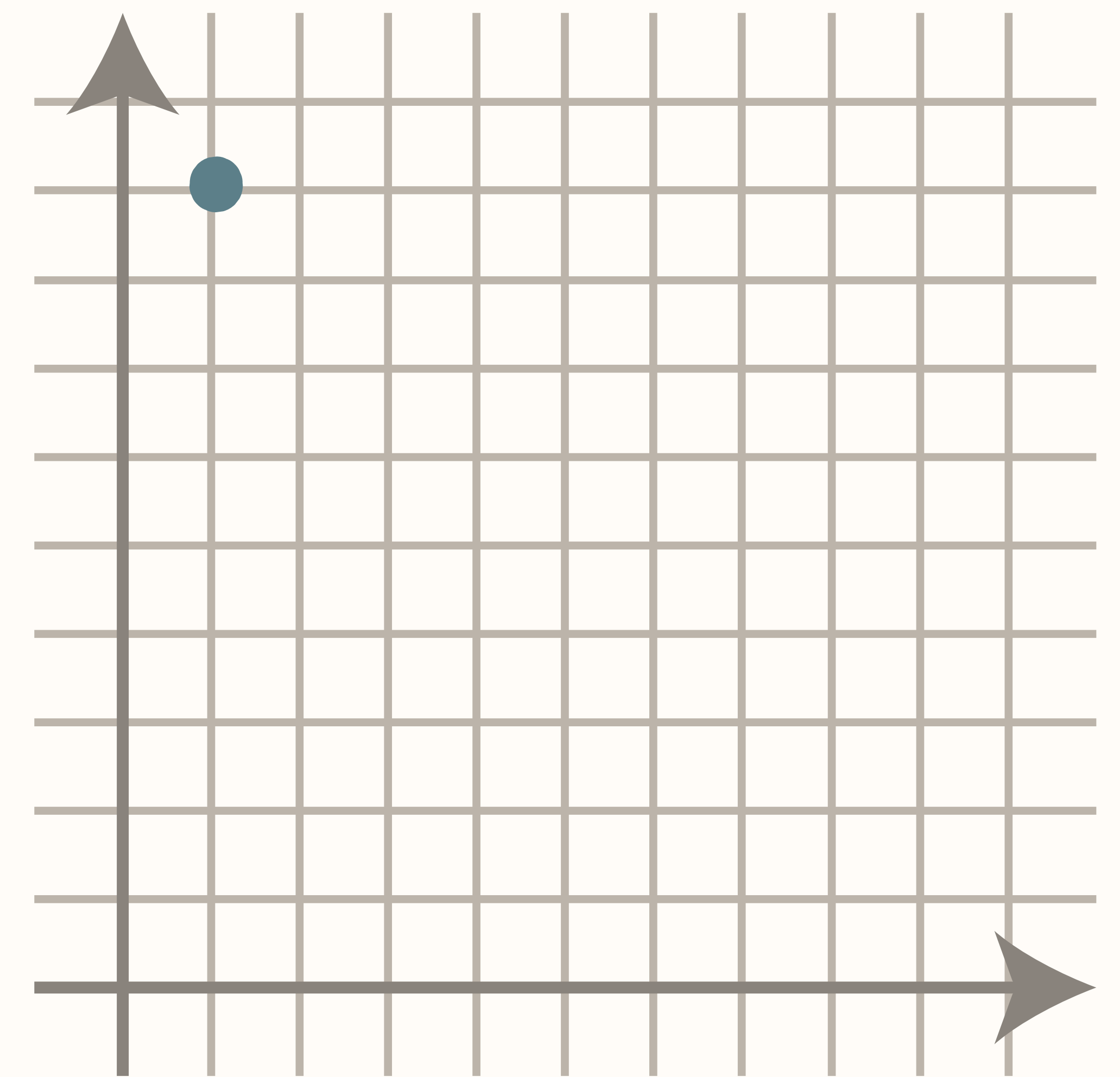
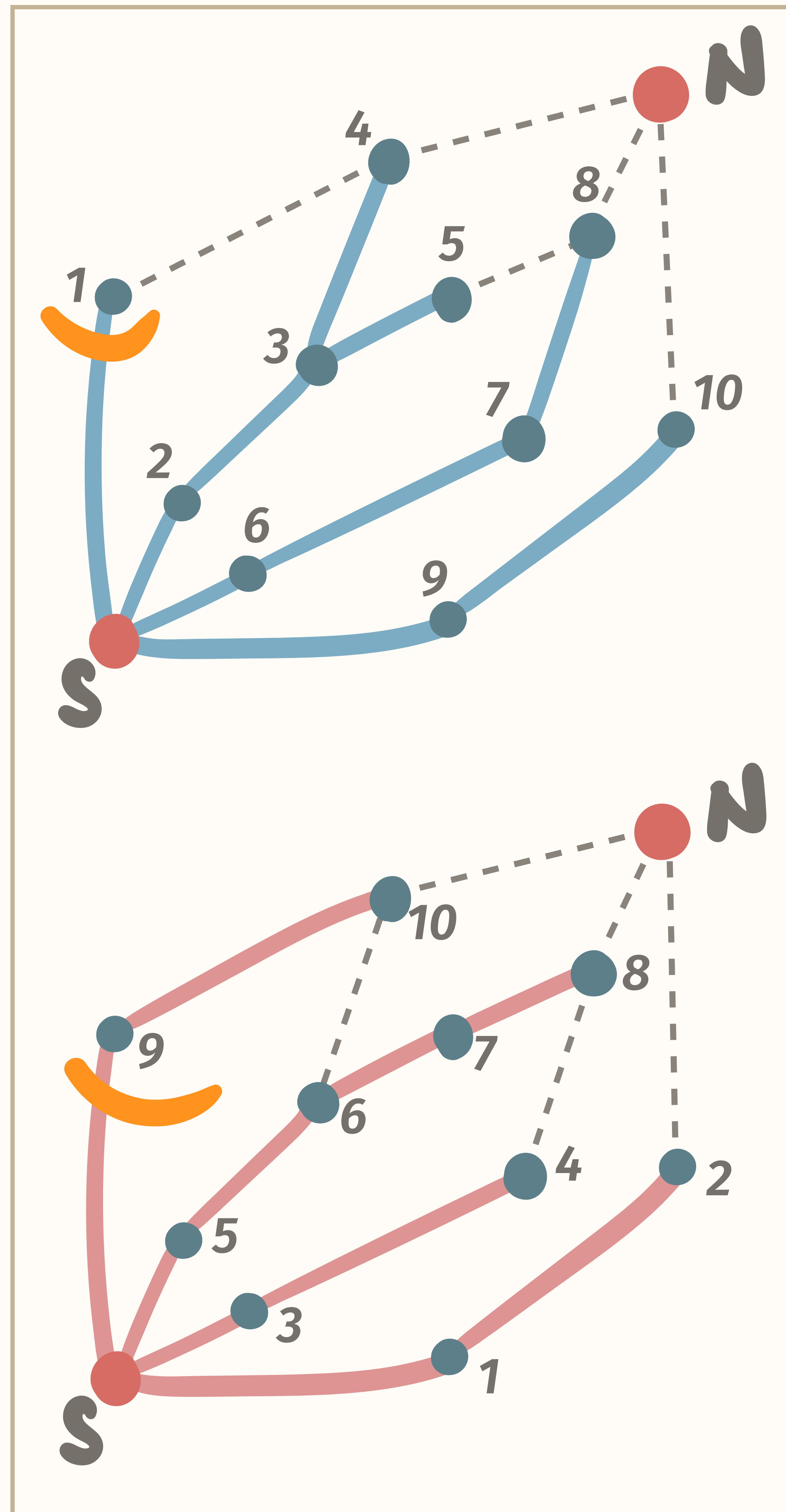
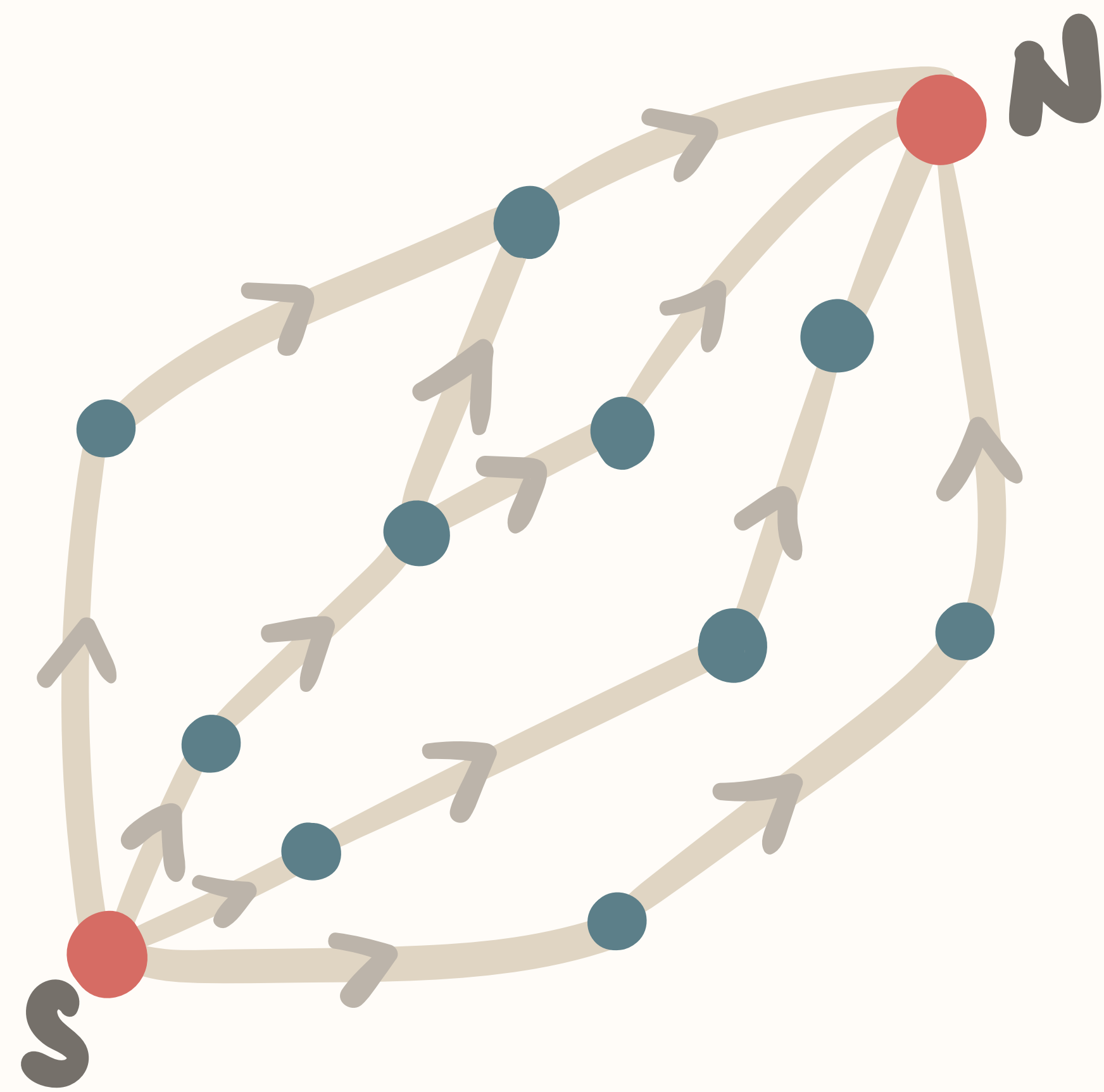
Link with plane permutations

Poset \longrightarrow Plane permutation



Link with plane permutations

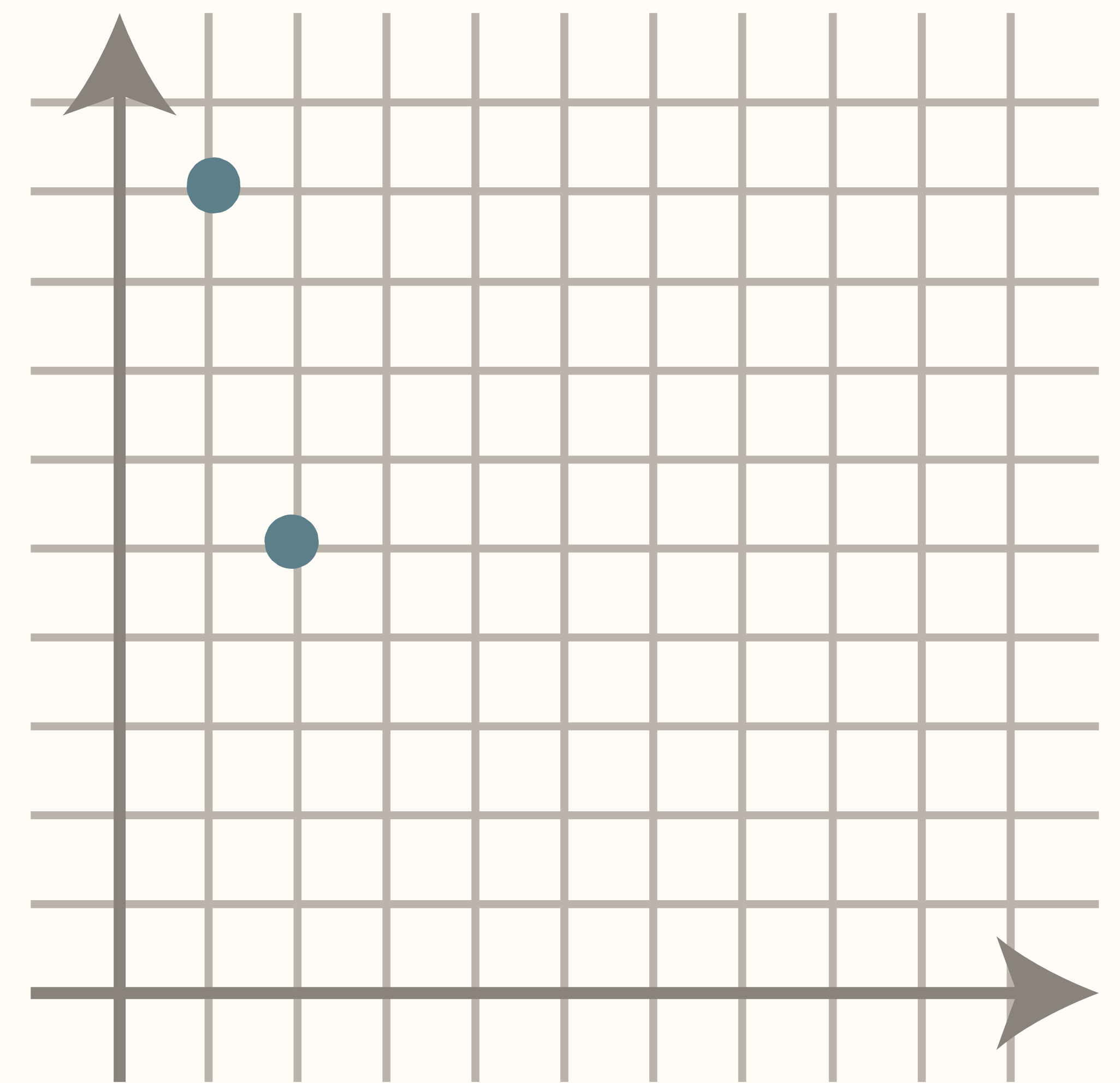
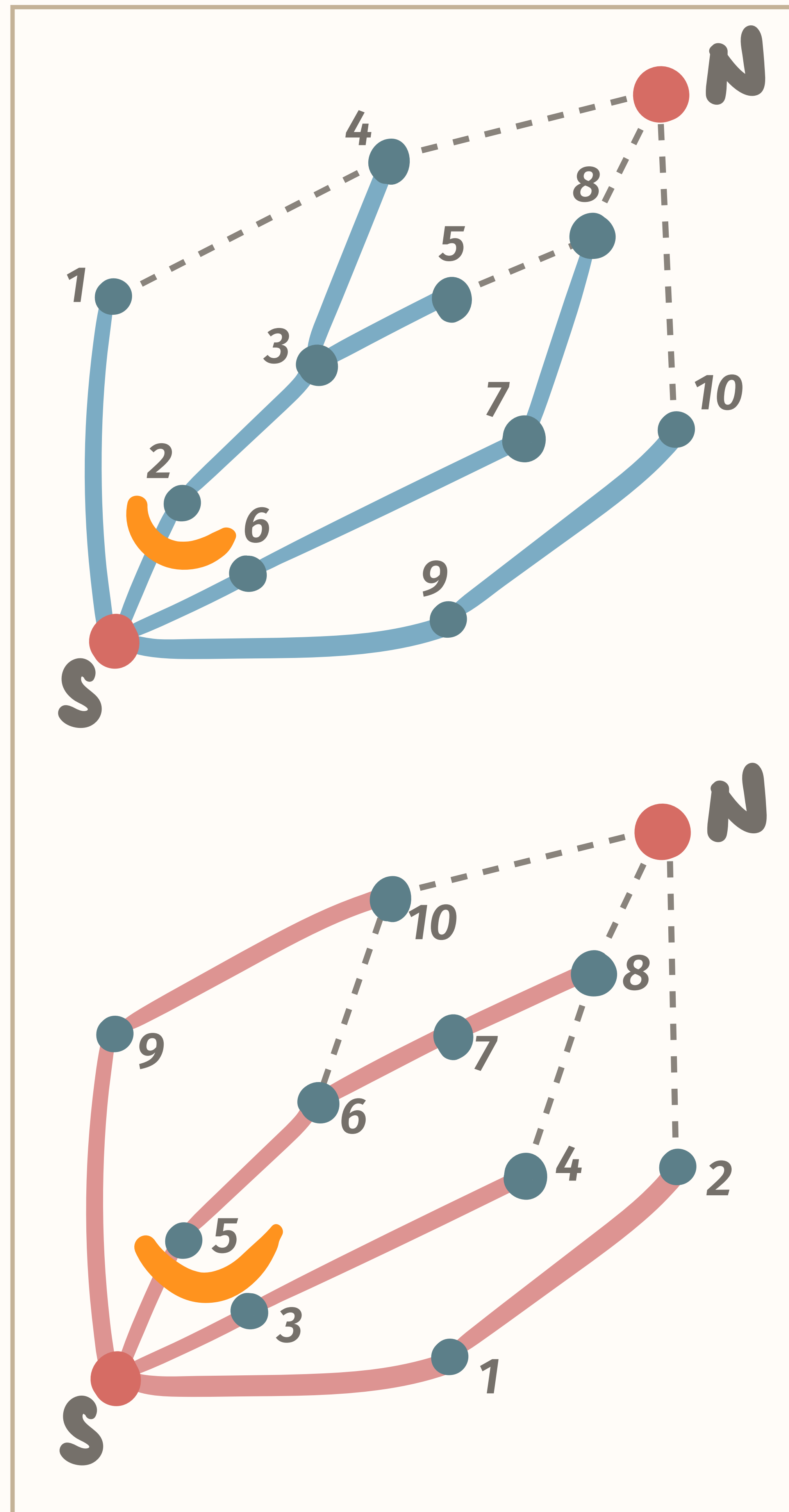
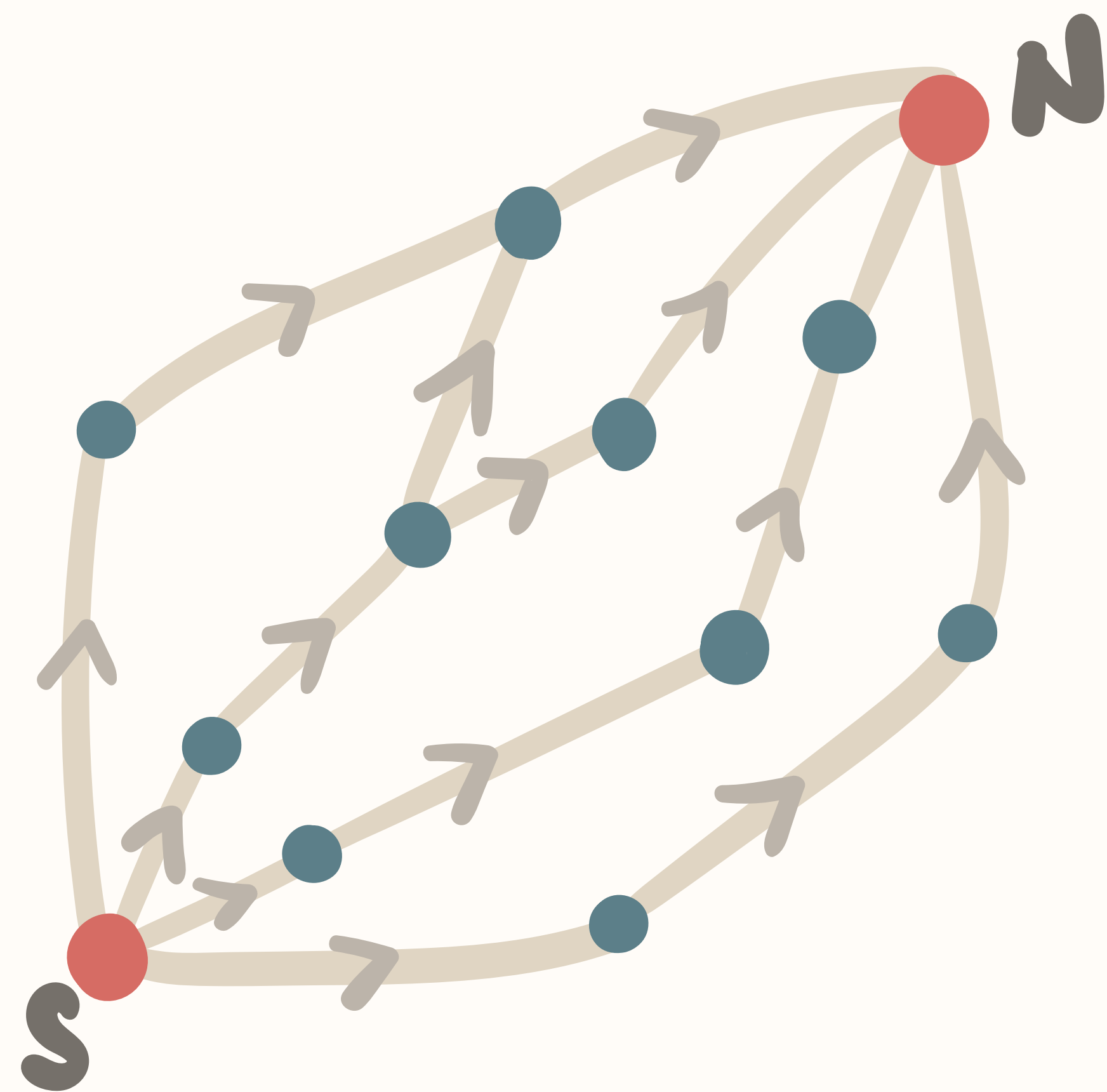
Poset \longrightarrow Plane permutation



$$\pi : \boxed{1 \rightarrow 9}$$

Link with plane permutations

Poset \longrightarrow Plane permutation

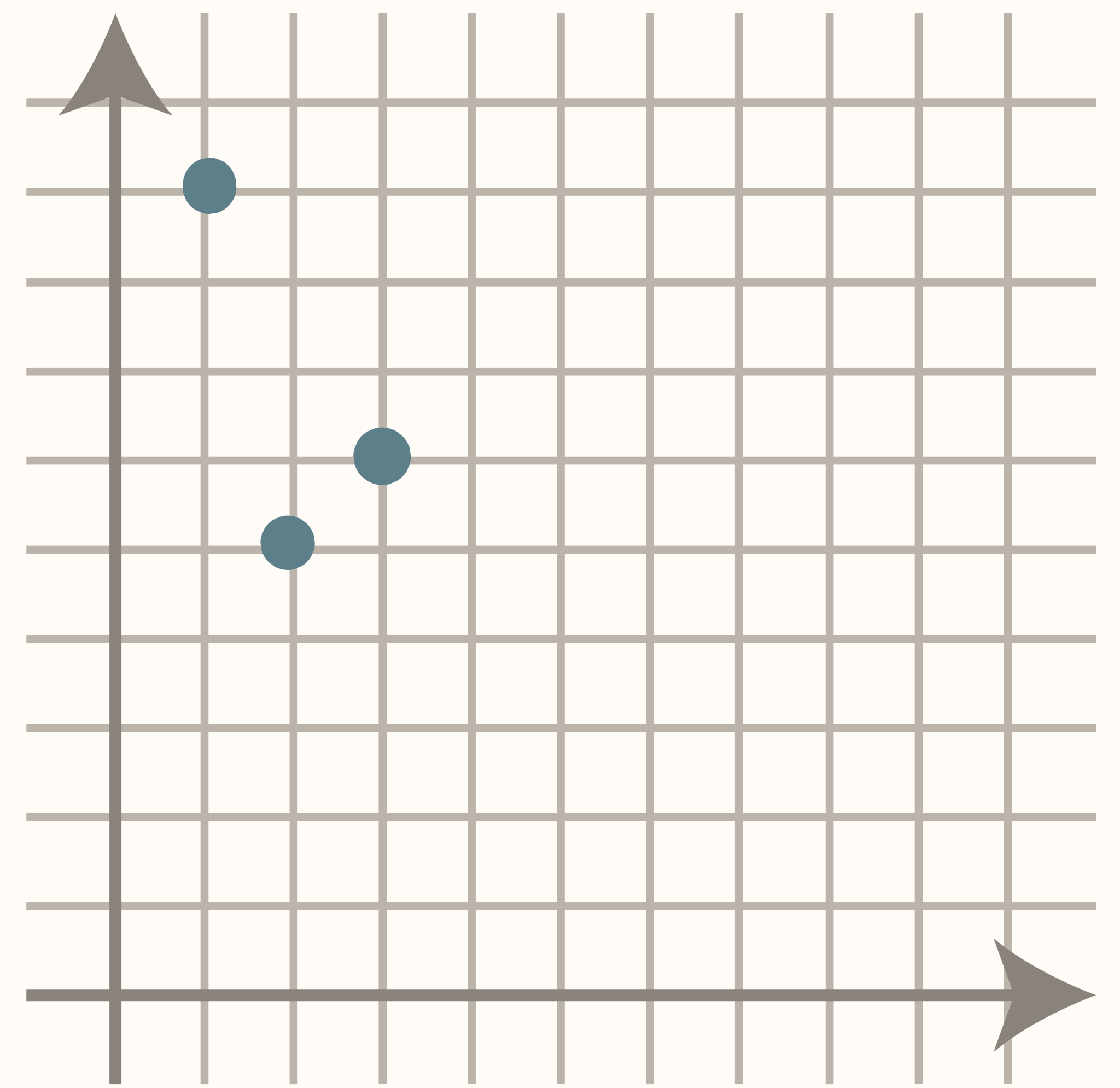
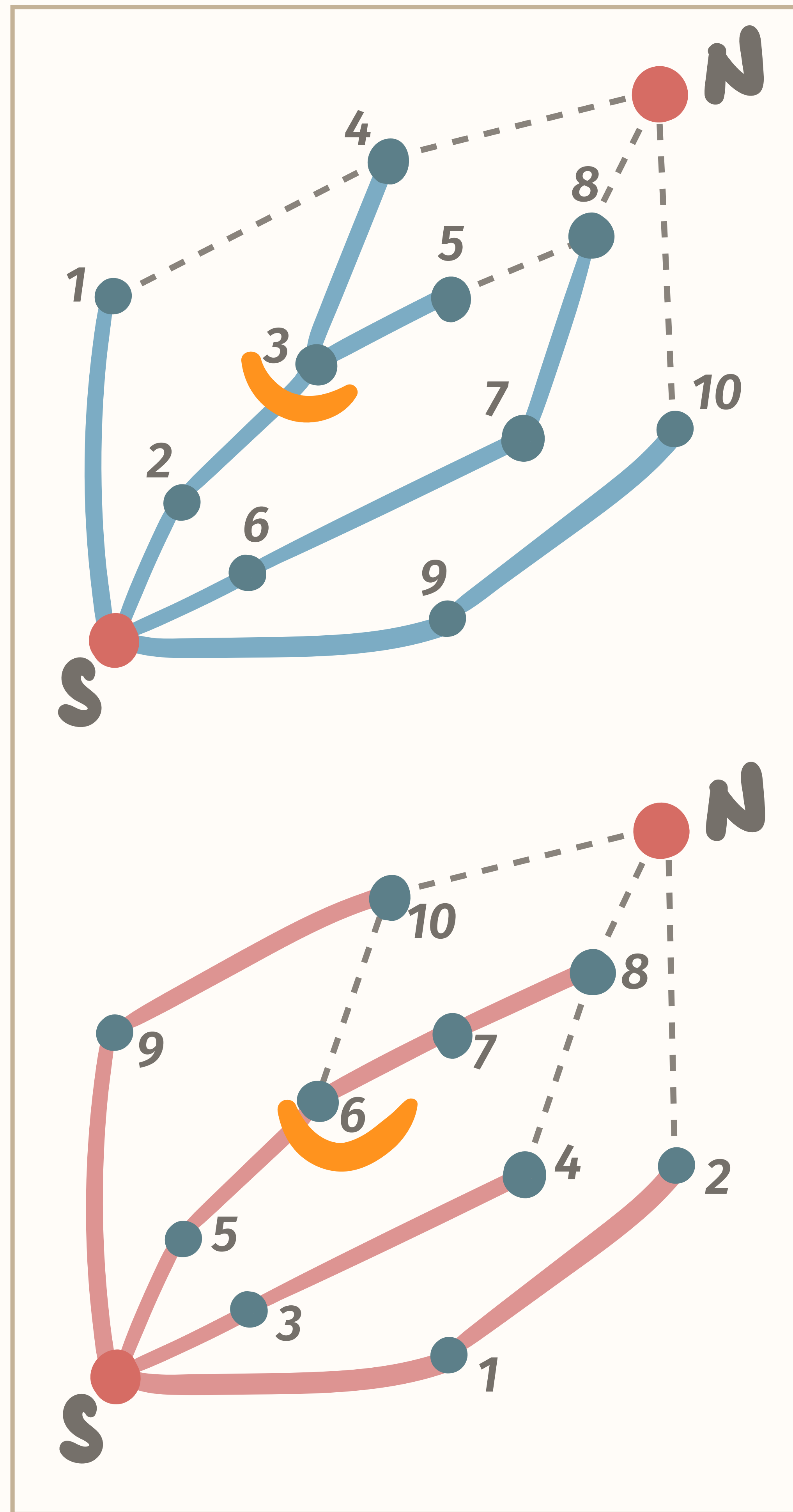
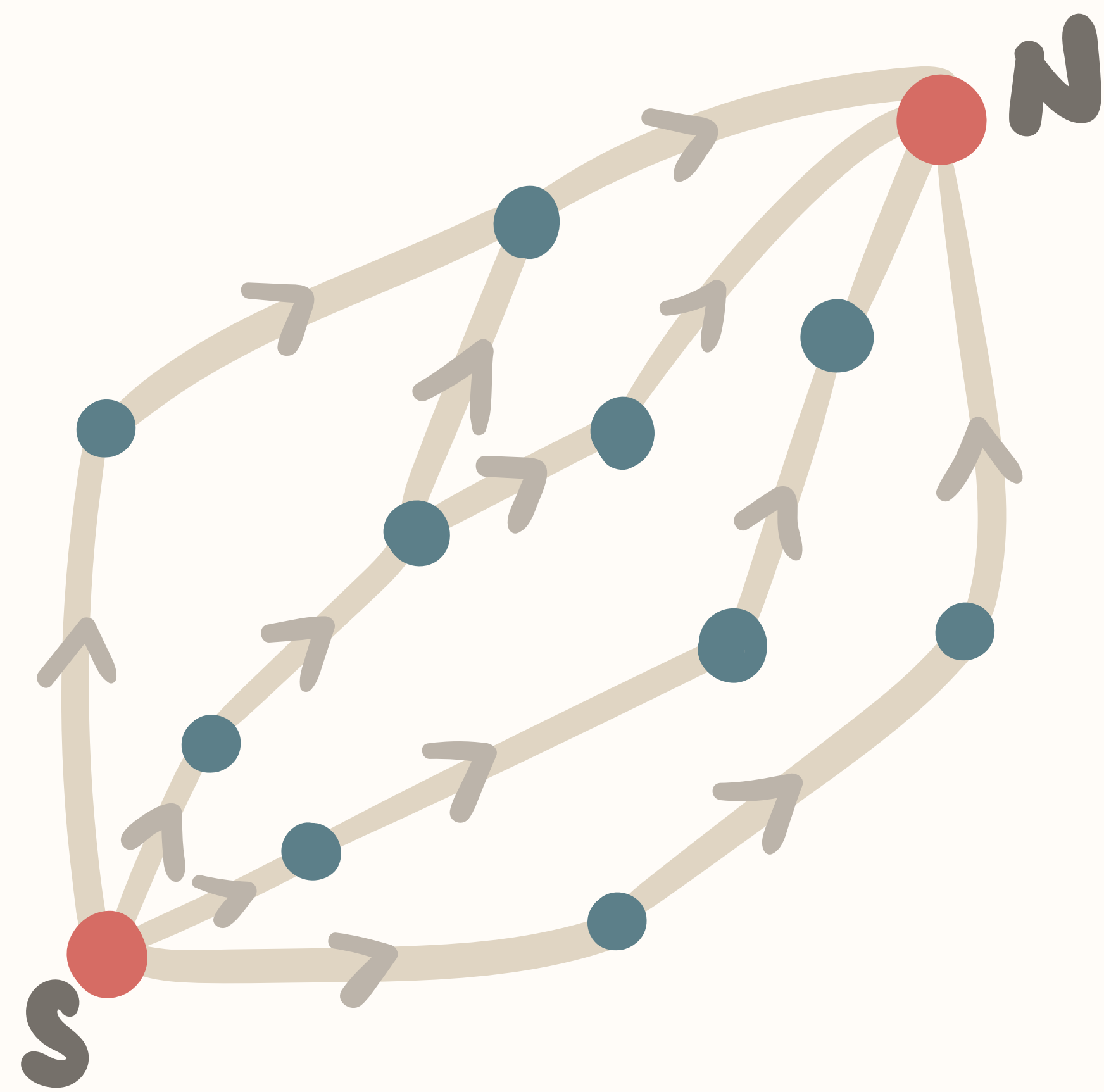


$$\pi: 1 \rightarrow 9$$

$$2 \rightarrow 5$$

Link with plane permutations

Poset \longrightarrow Plane permutation



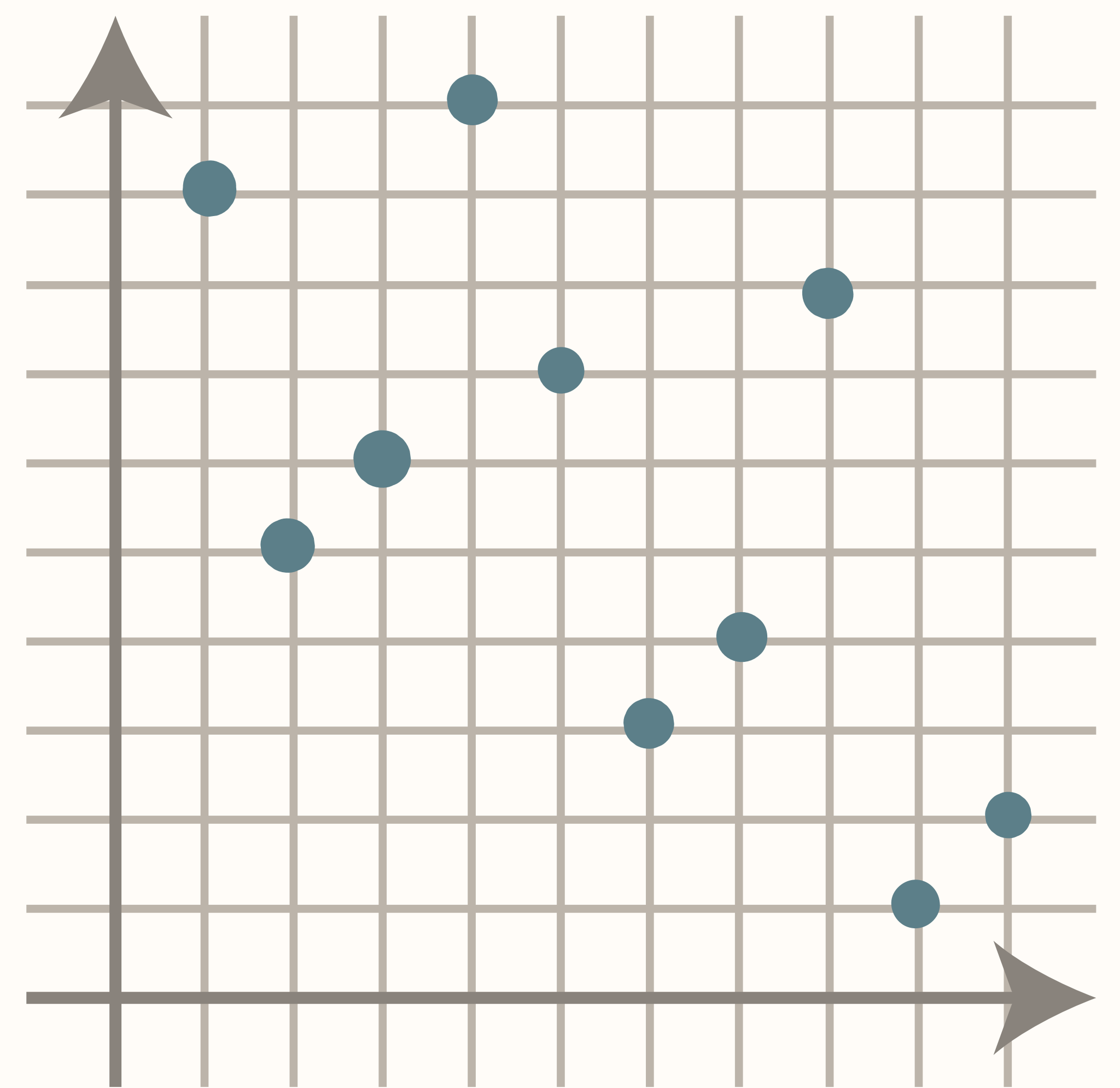
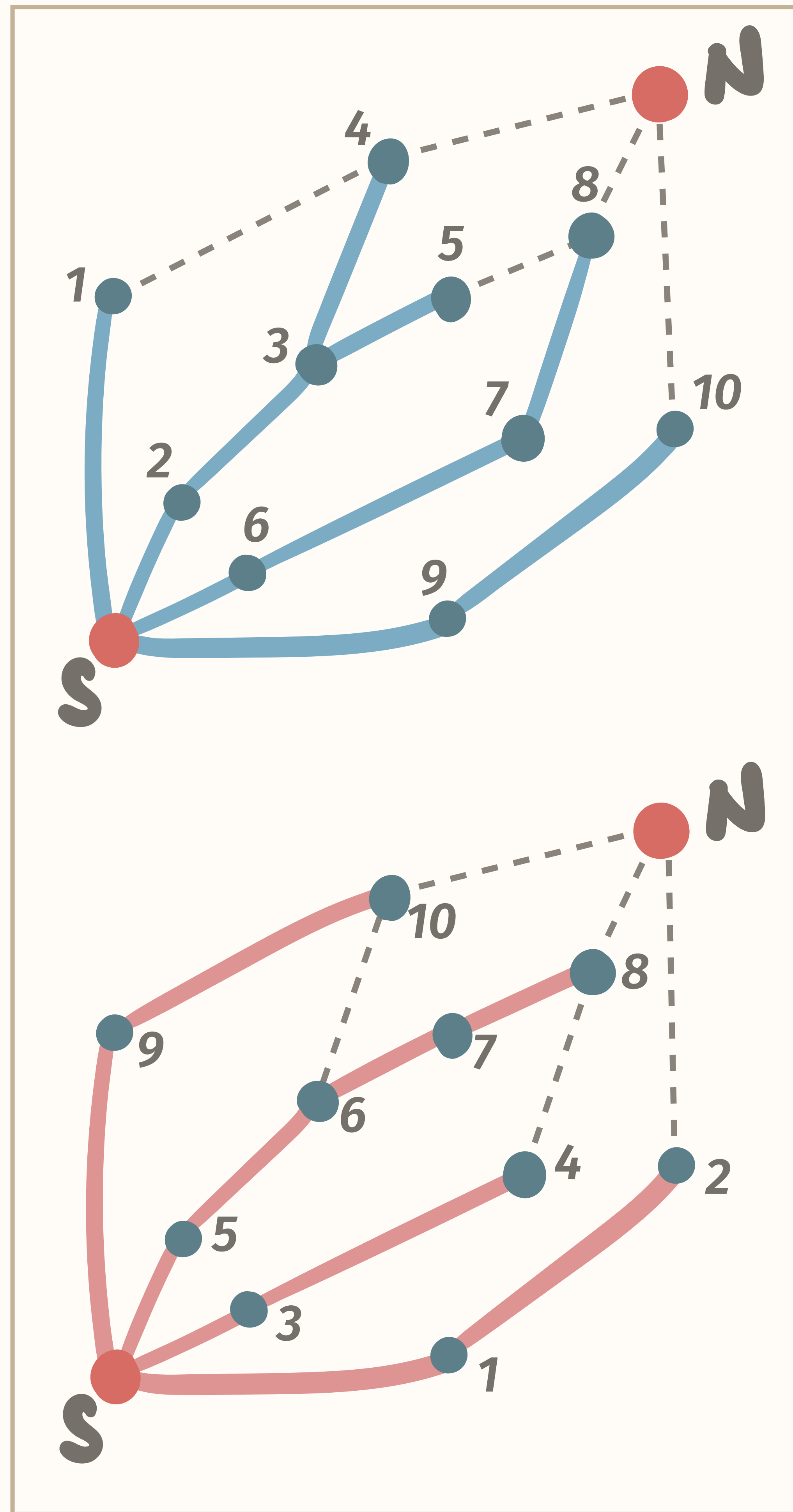
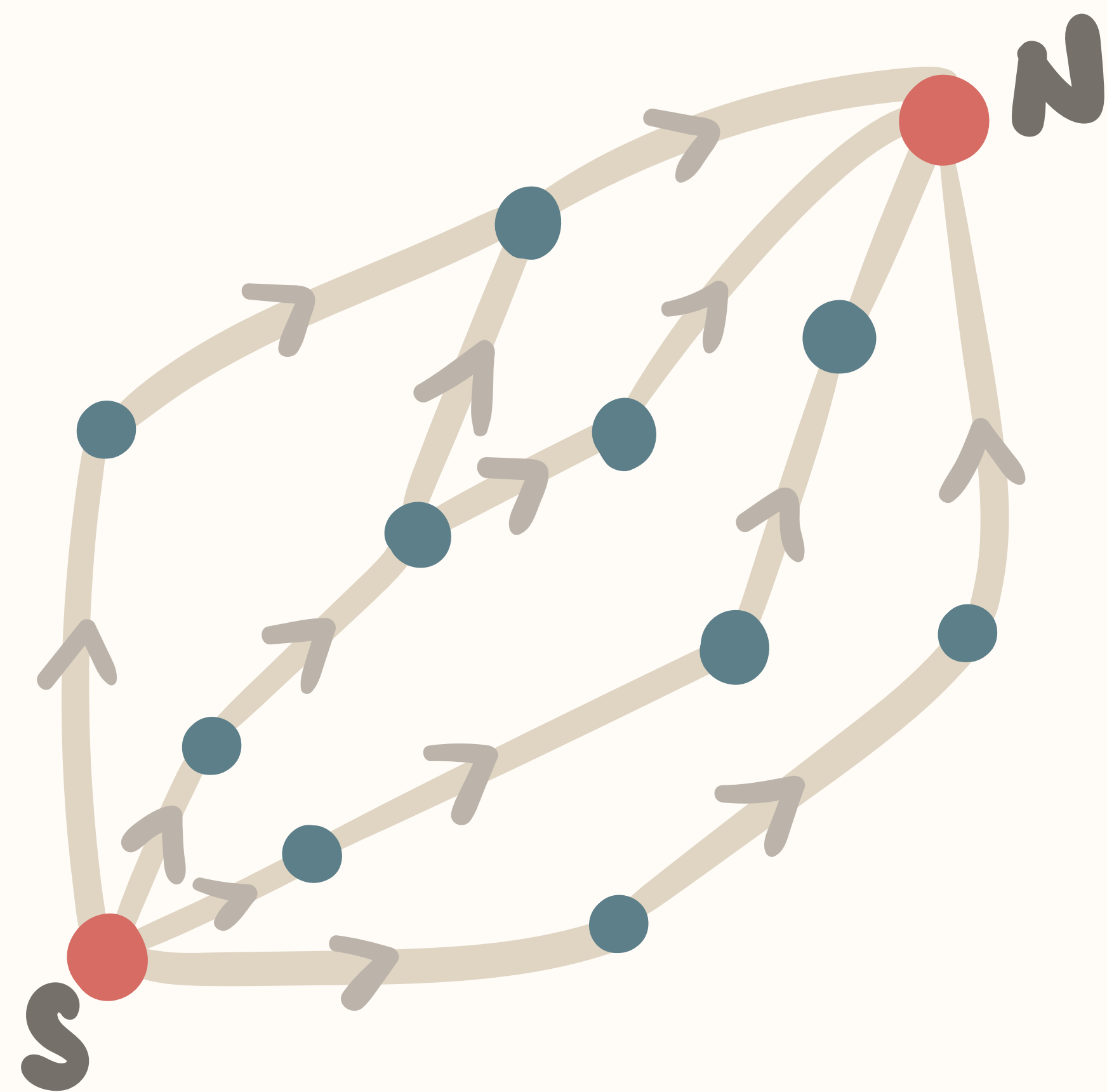
$$\pi: 1 \rightarrow 9$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

Link with plane permutations

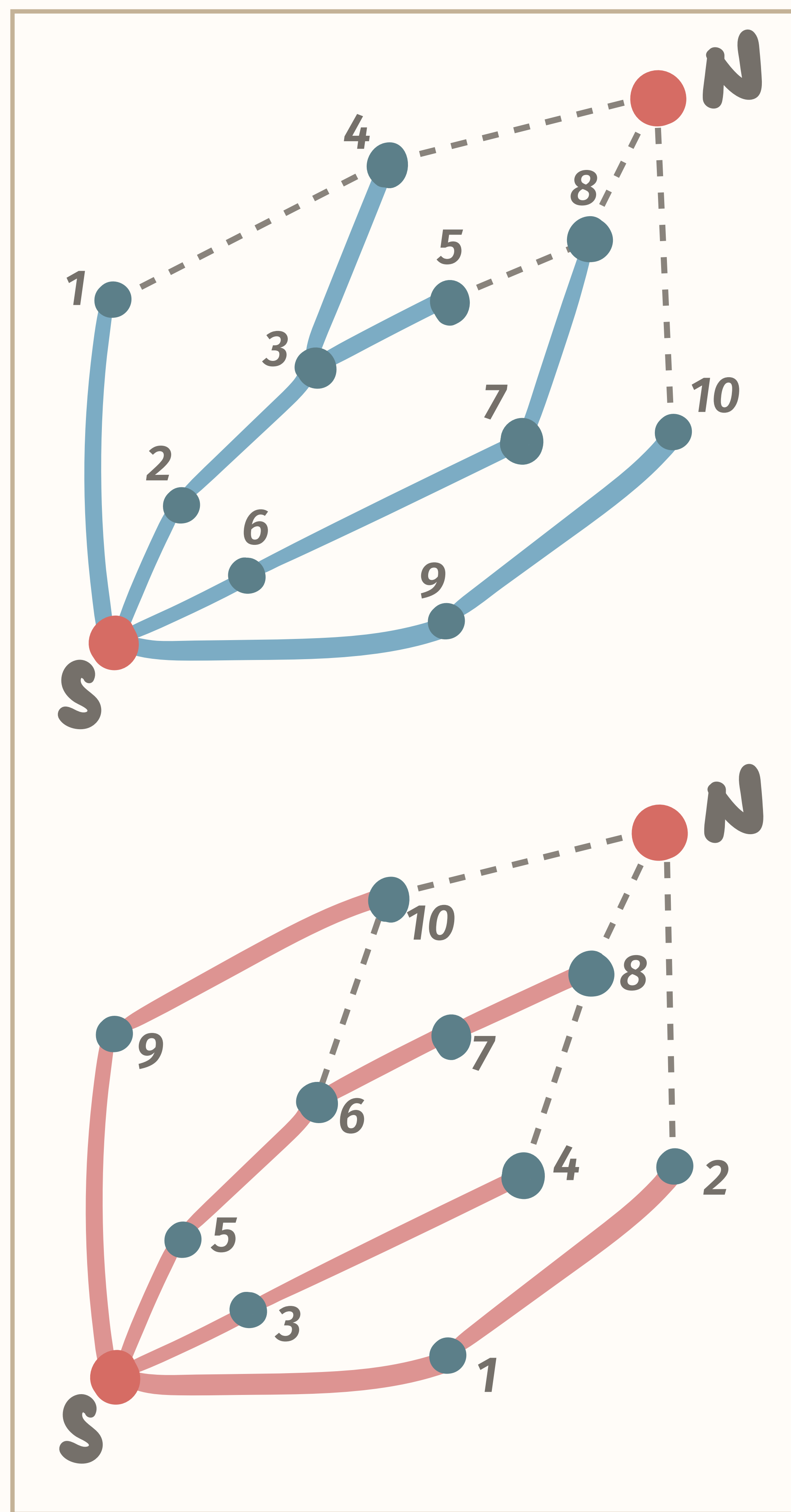
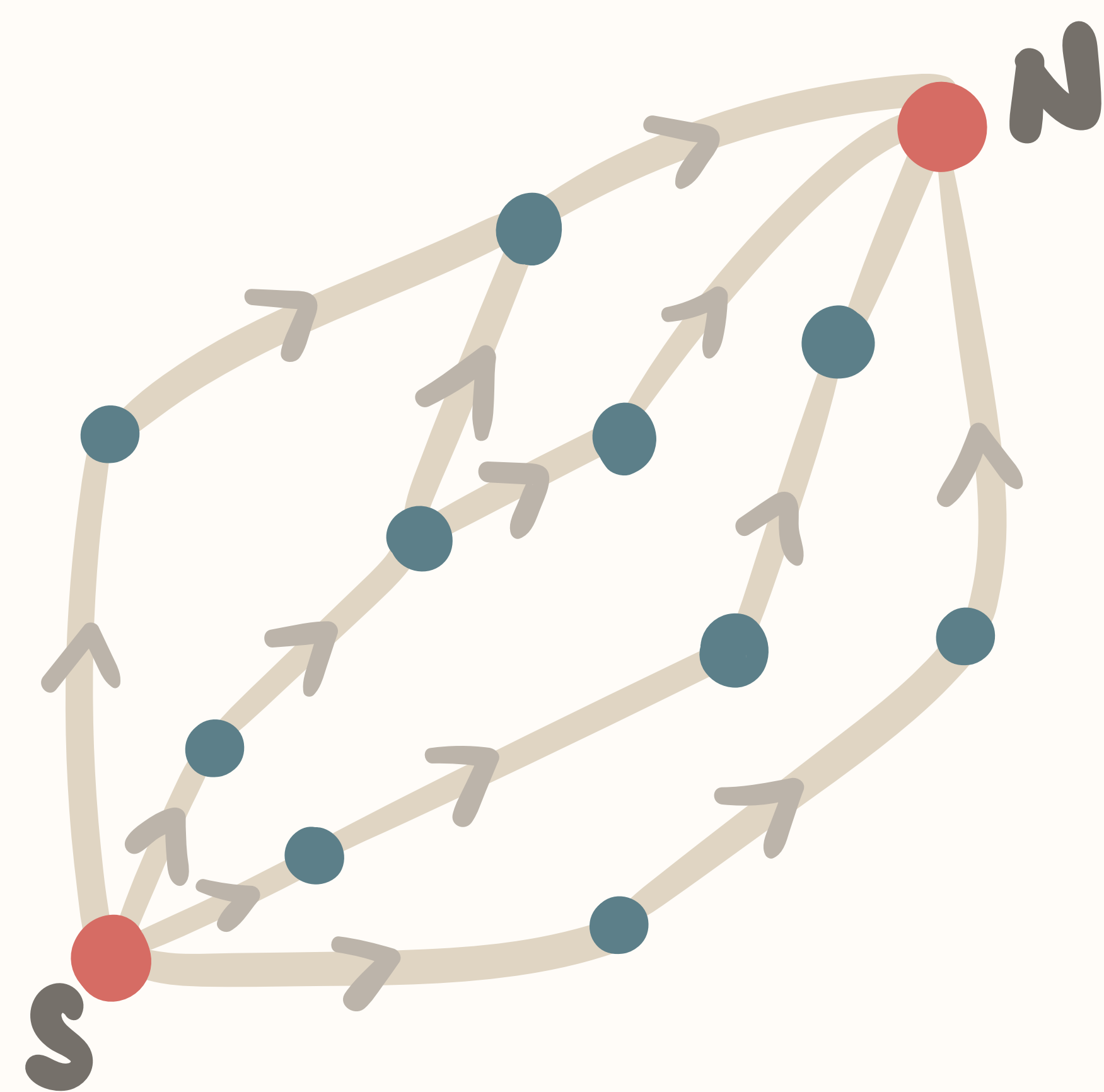
Poset \longrightarrow Plane permutation



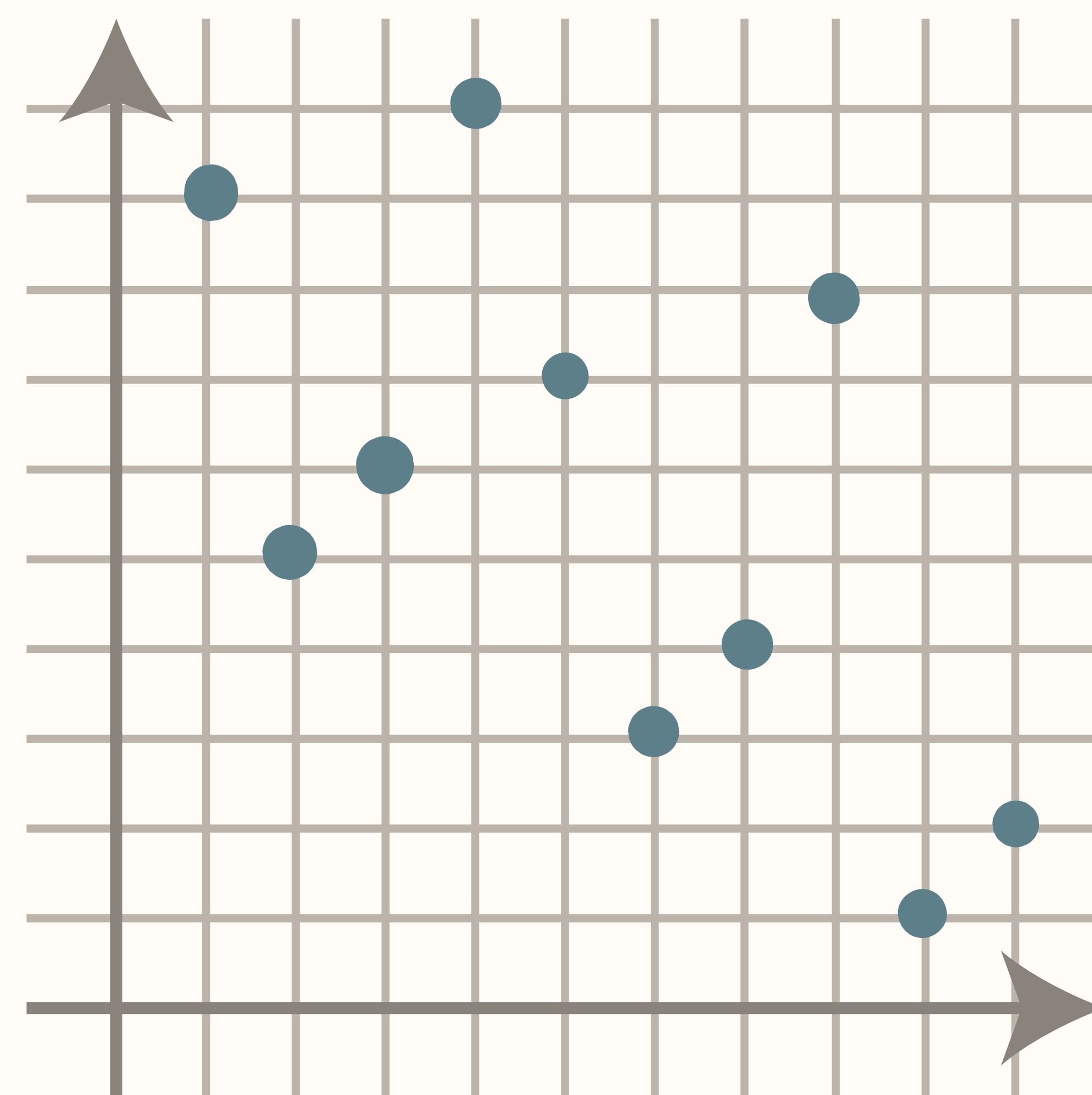
$$\pi: \begin{array}{ll} 1 \rightarrow 9 & 6 \rightarrow 3 \\ 2 \rightarrow 5 & 7 \rightarrow 4 \\ 3 \rightarrow 6 & 8 \rightarrow 8 \\ 4 \rightarrow 10 & 9 \rightarrow 1 \\ 5 \rightarrow 7 & 10 \rightarrow 2 \end{array}$$

Link with plane permutations

Poset \longrightarrow Plane permutation



\Rightarrow Area requirement and symmetry display of planar upward drawings, G. Di Battista, R. Tamassia, and I. G. Tollis (1992)



$$\pi: \begin{array}{ll} 1 \rightarrow 9 & 6 \rightarrow 3 \\ 2 \rightarrow 5 & 7 \rightarrow 4 \\ 3 \rightarrow 6 & 8 \rightarrow 8 \\ 4 \rightarrow 10 & 9 \rightarrow 1 \\ 5 \rightarrow 7 & 10 \rightarrow 2 \end{array}$$

Sommaire

The KMSW Bijection

1. Application to three maps model

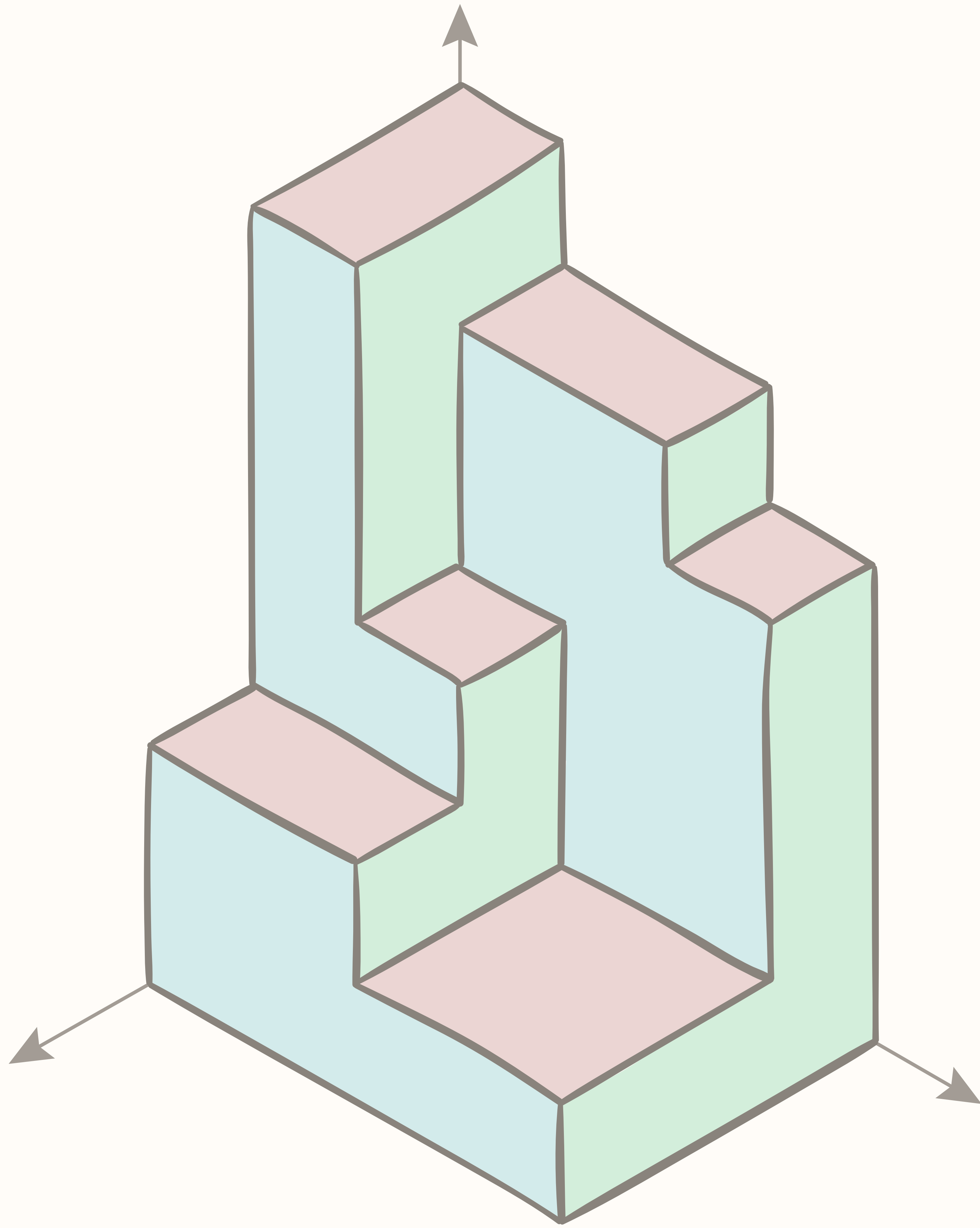
- a. Plane bipolar posets*
- b. Plane bipolar posets by vertices*
- c. Transversal structures*
- d. Asymtotoxic*

2. Interlude : plane permutations

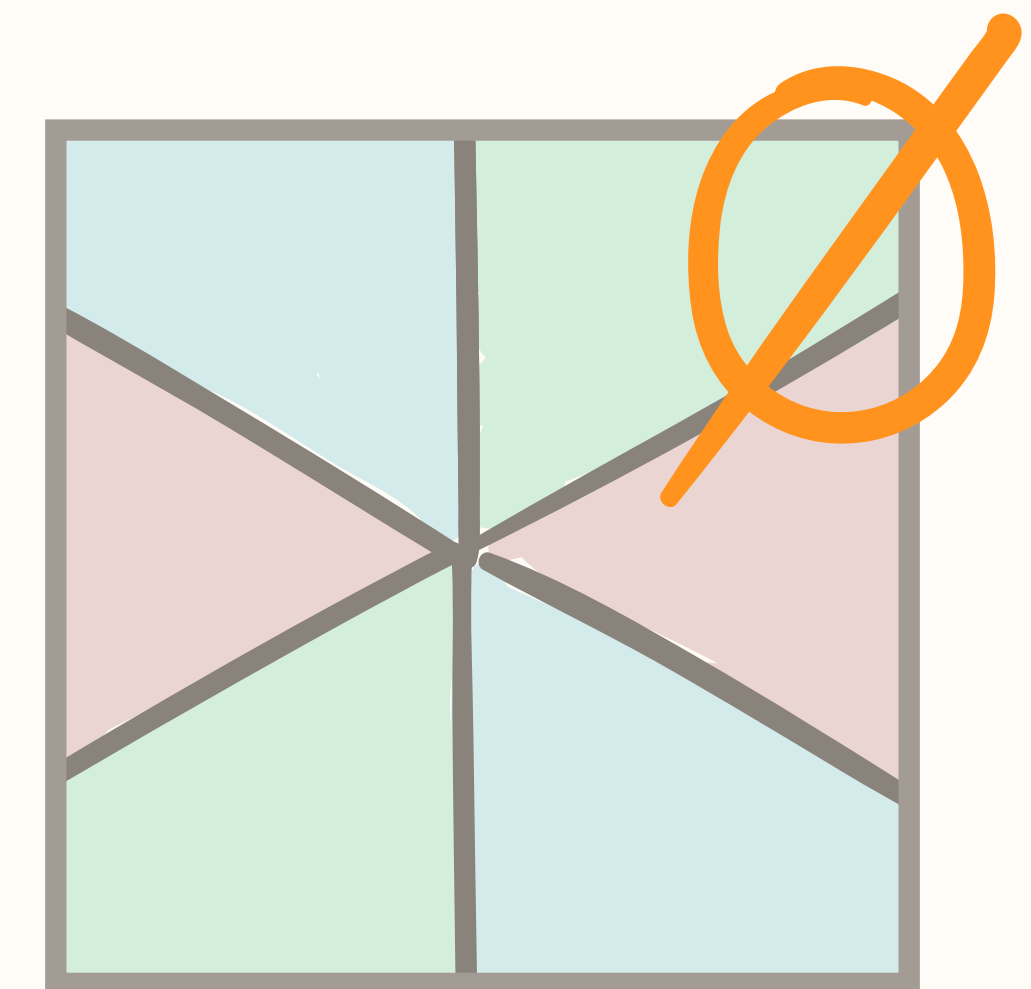
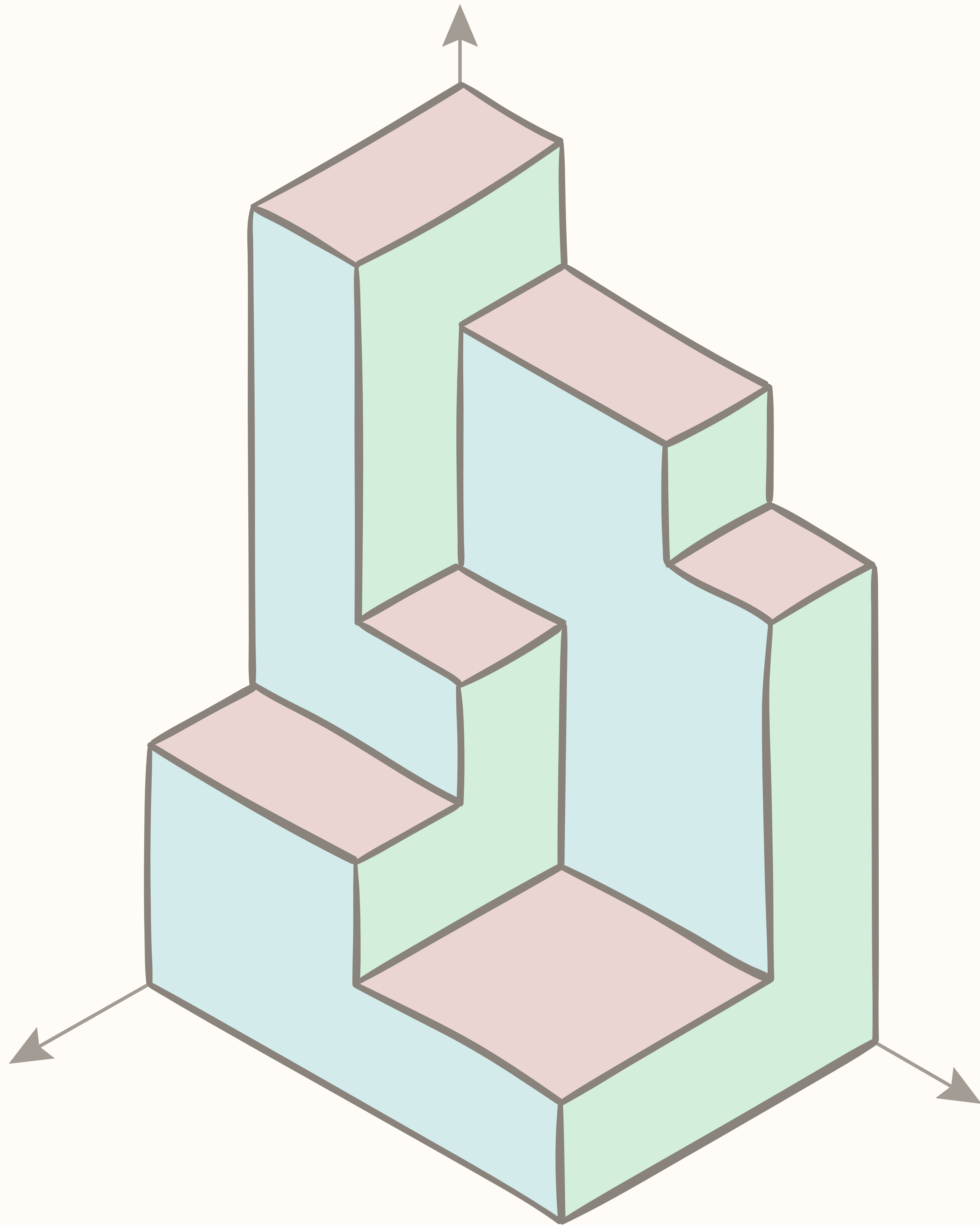
3. Application to corner polyhedra

- a. Via polyhedral orientations*
- b. Via Schnyder colorings*
- c. Asymtotoxic*

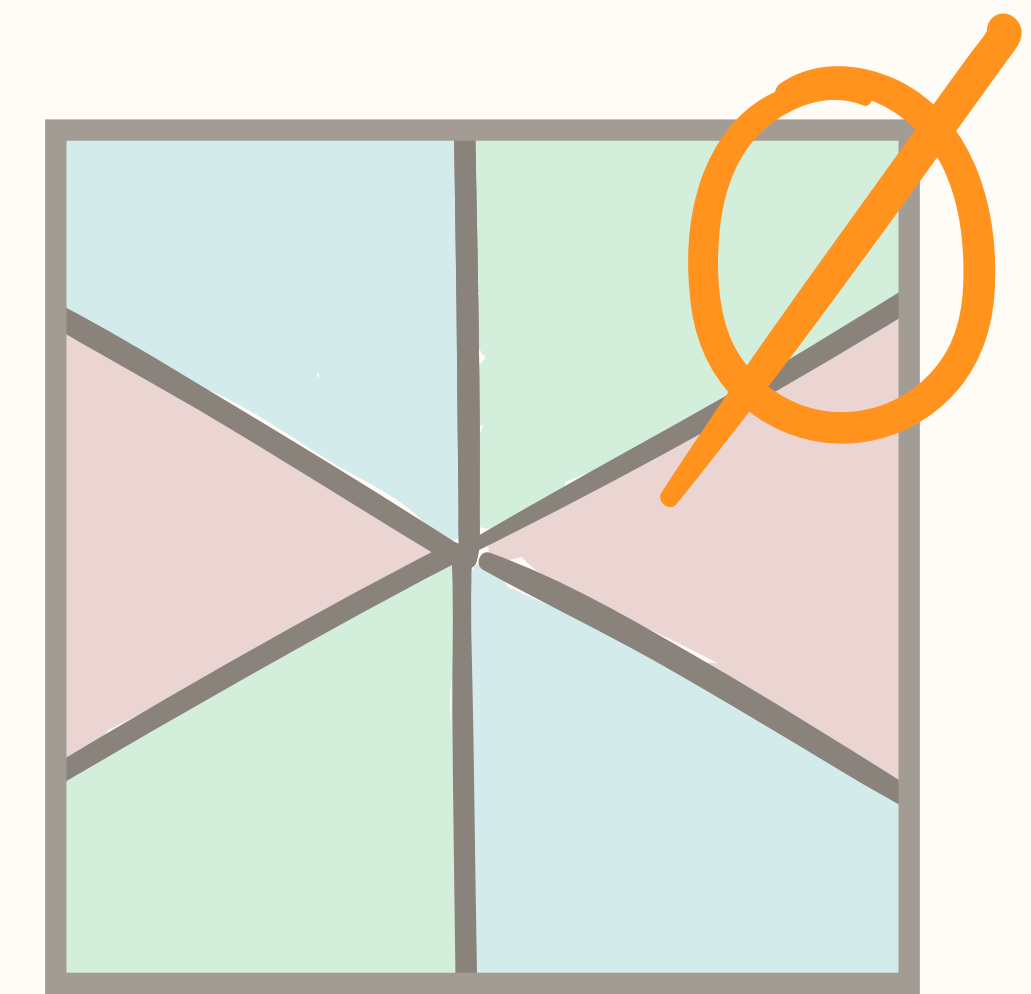
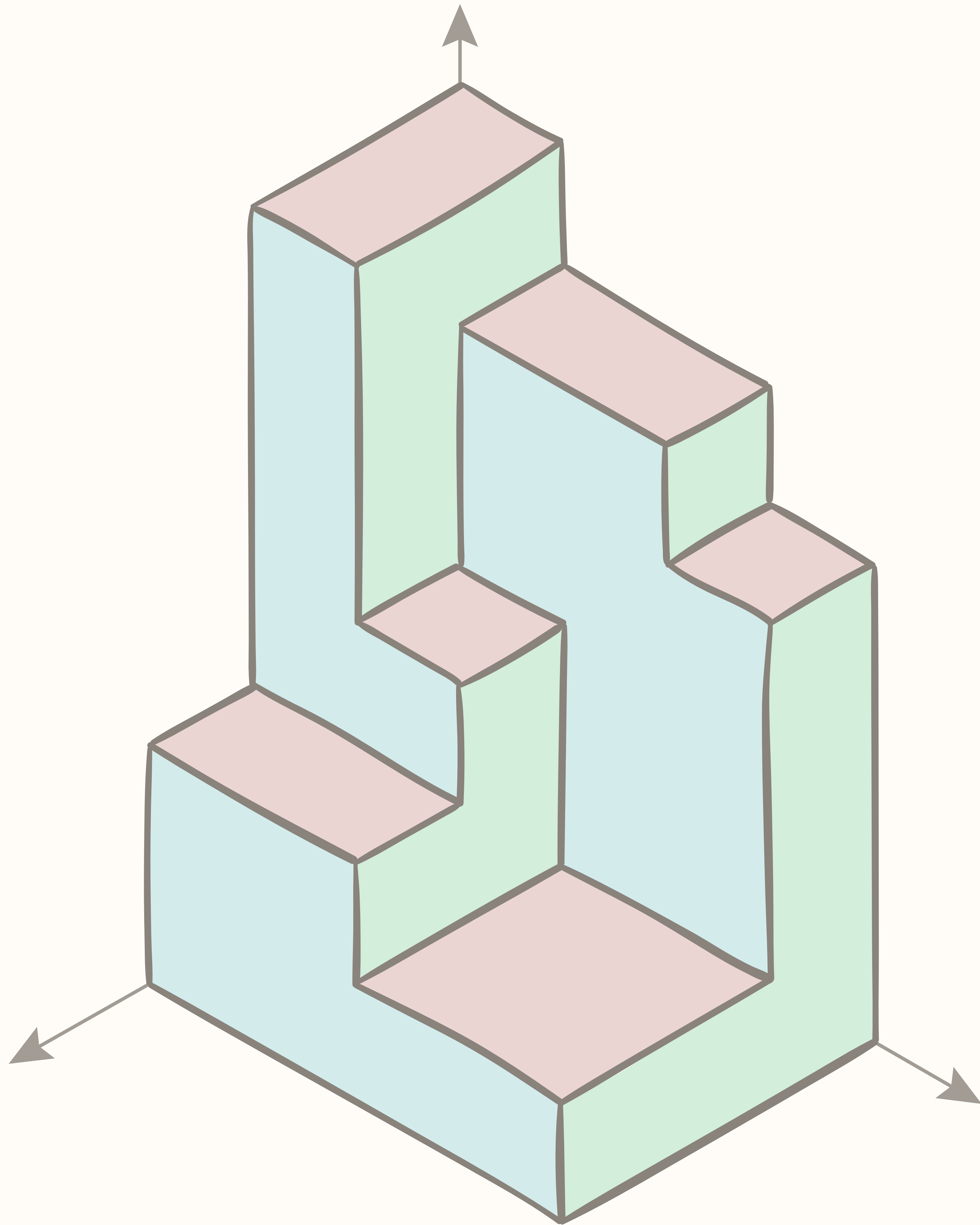
Corner polyhedra



Corner polyhedra



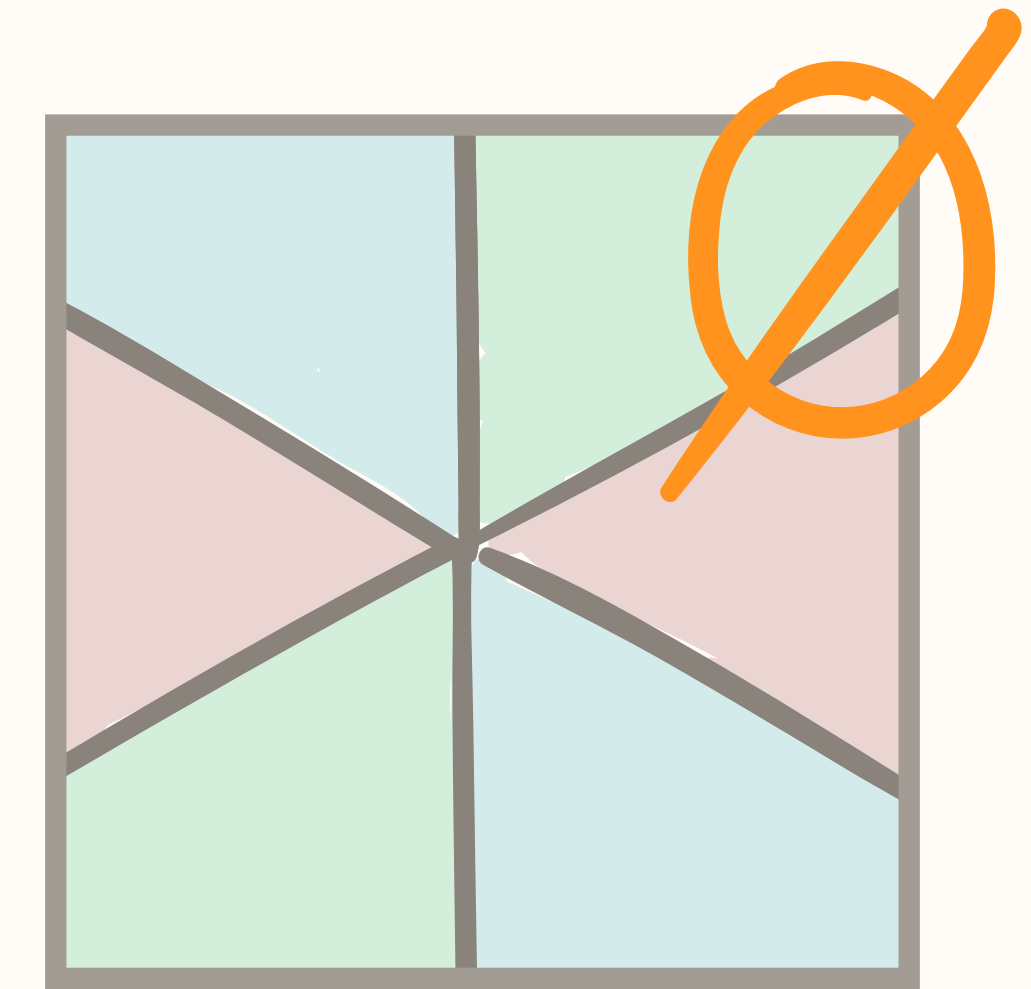
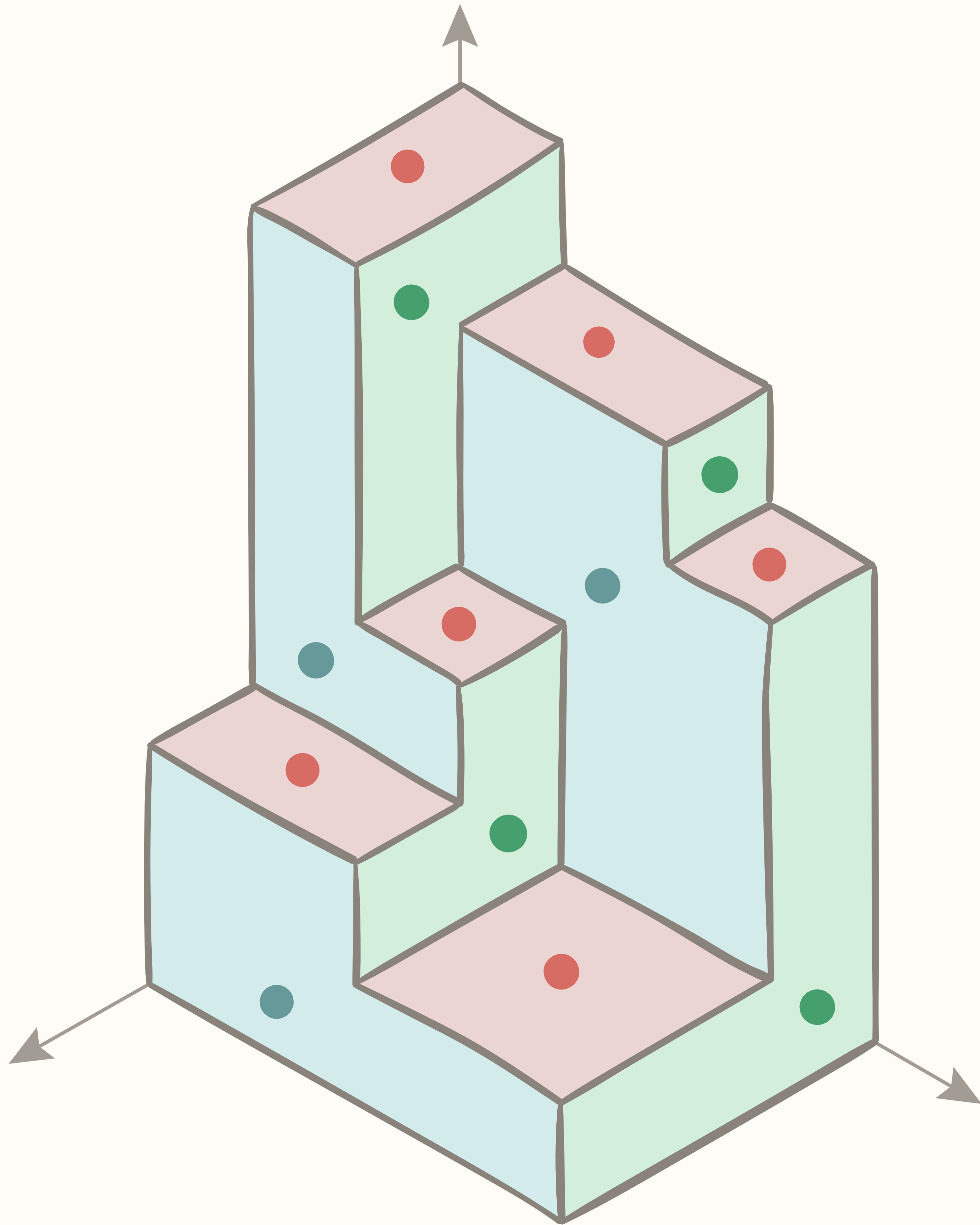
Corner polyhedra



corner polyhedra \leftrightarrow **polyhedral orientations**

\Rightarrow Steinitz theorems for orthogonal polyhedra,
D. Eppstein & E. Mumford (2010)

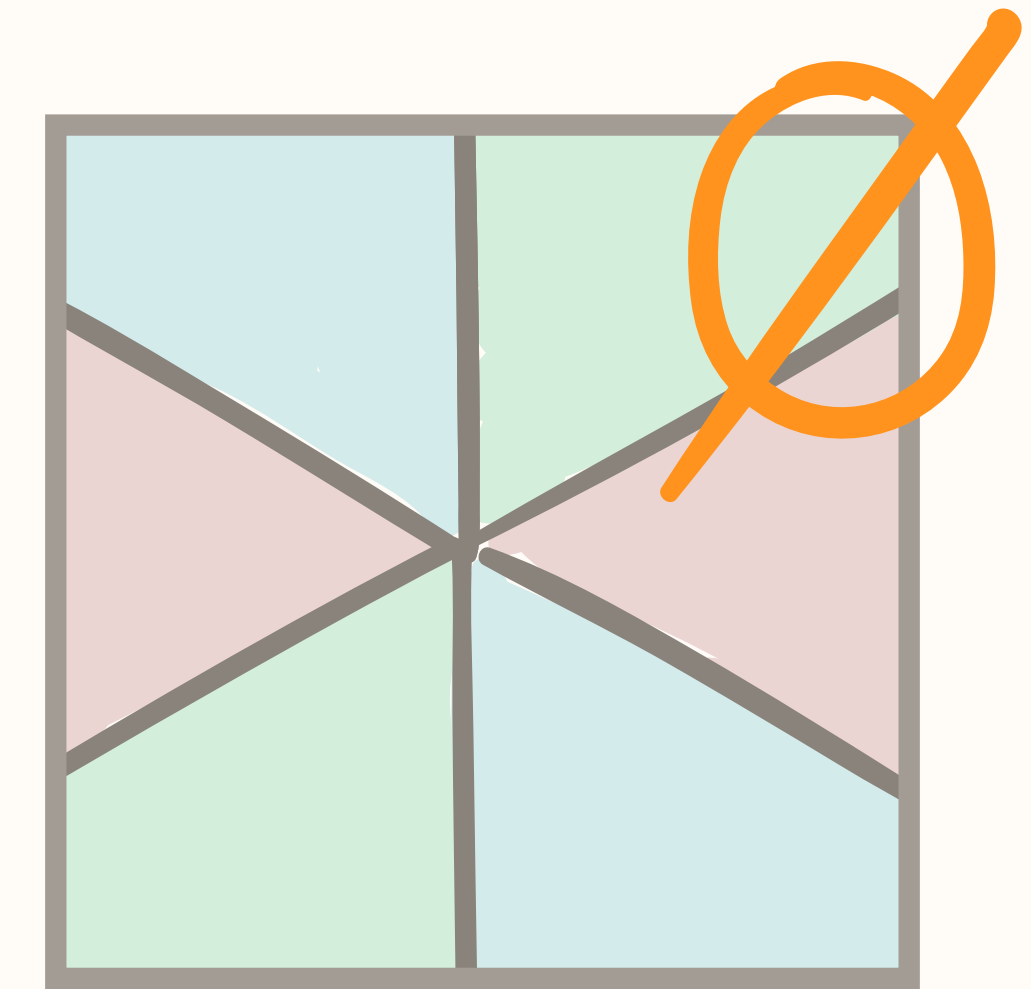
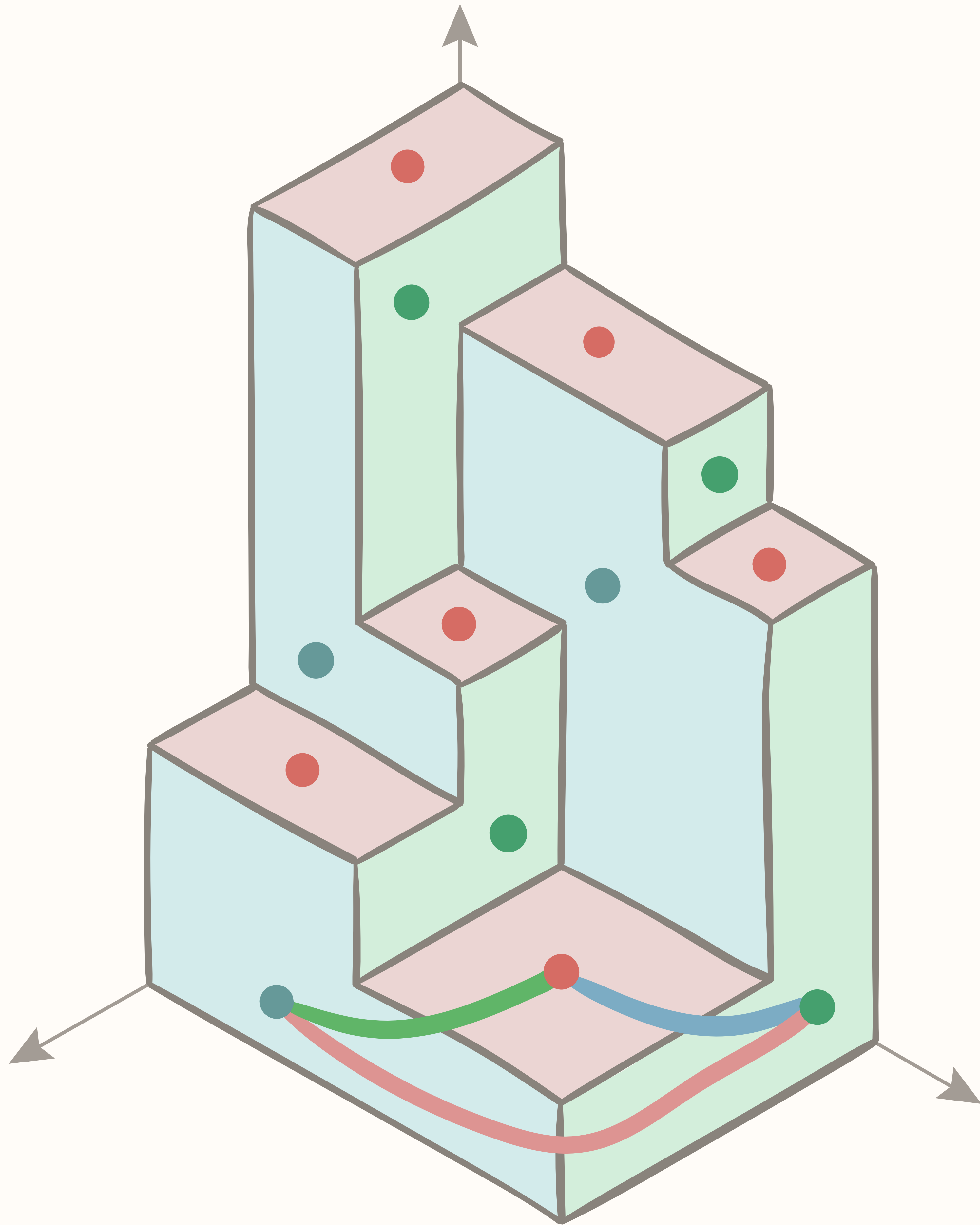
Corner polyhedra



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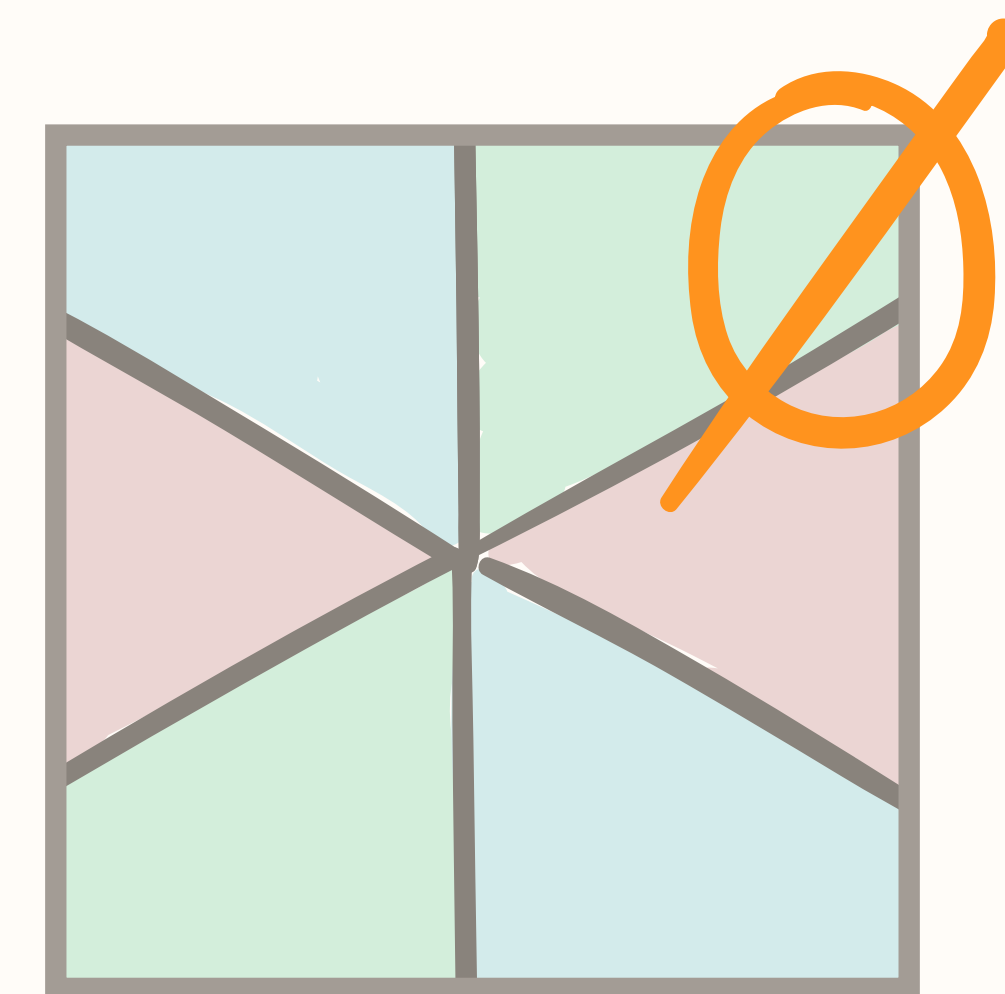
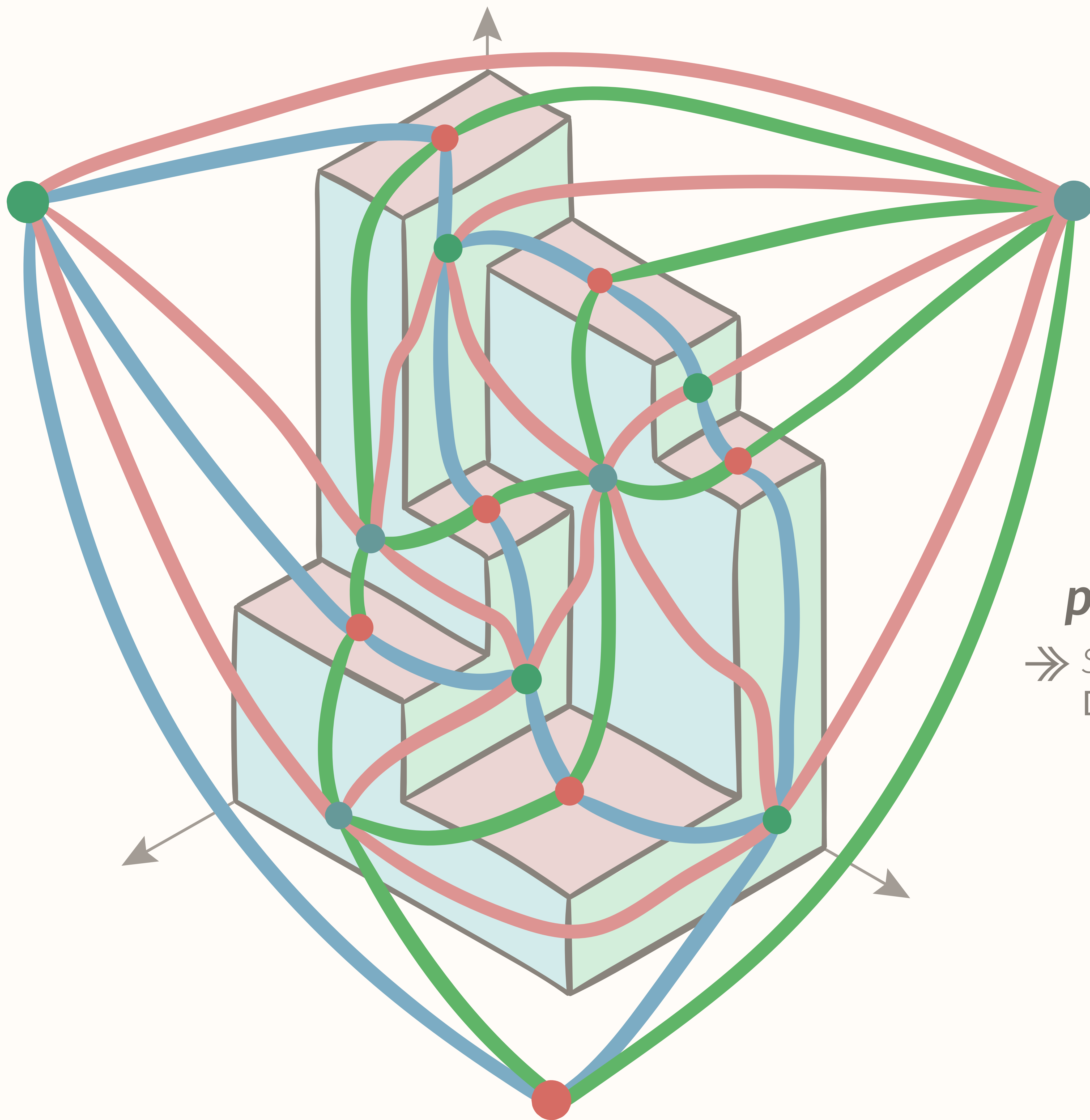
Corner polyhedra



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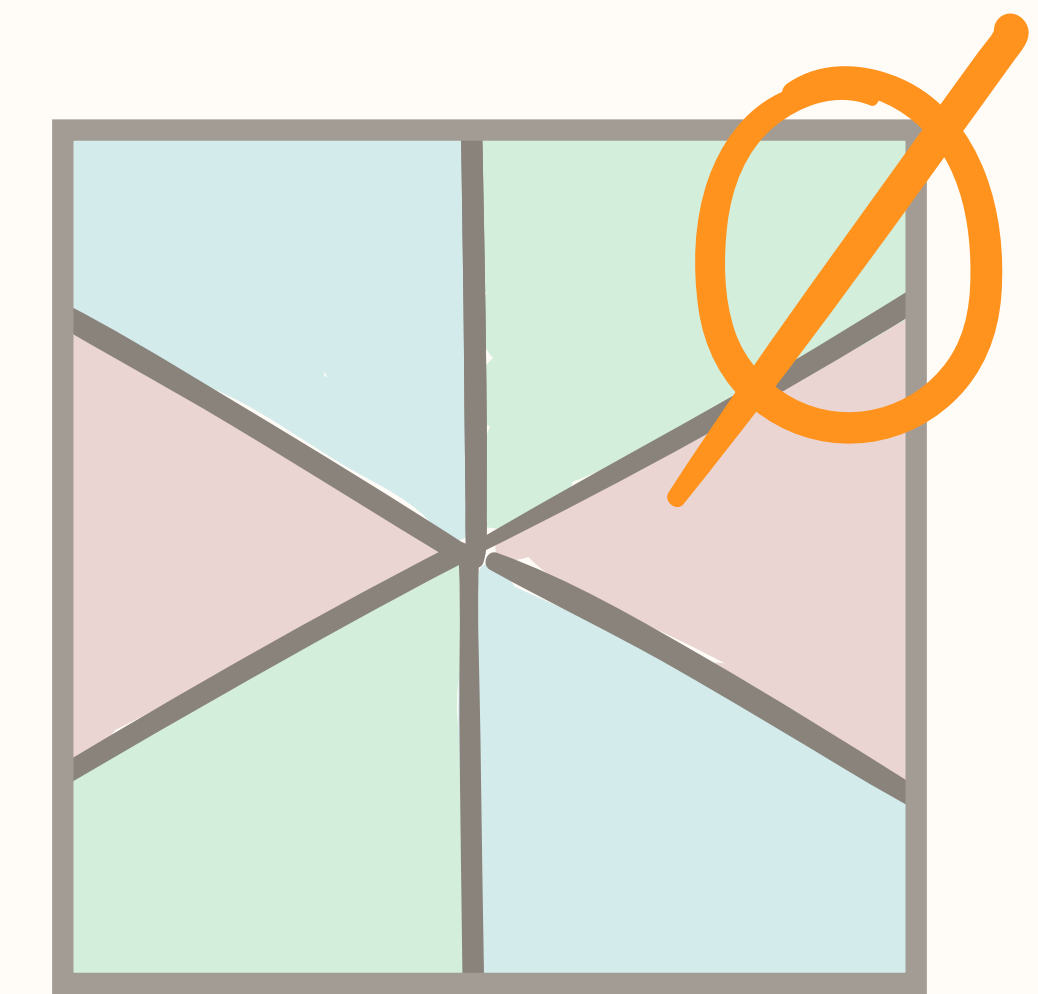
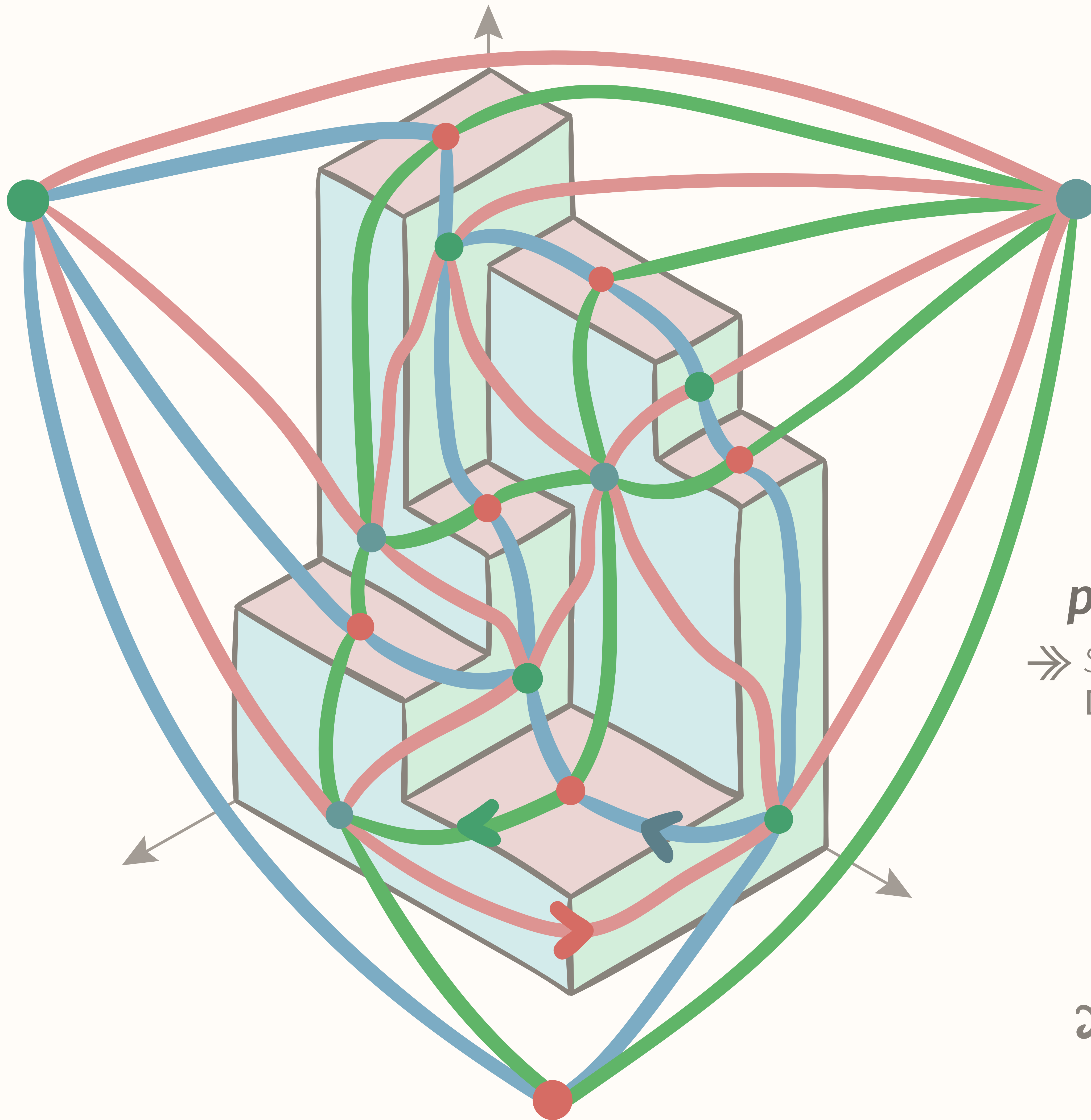
Corner polyhedra



corner polyhedra \leftrightarrow *polyhedral orientations*

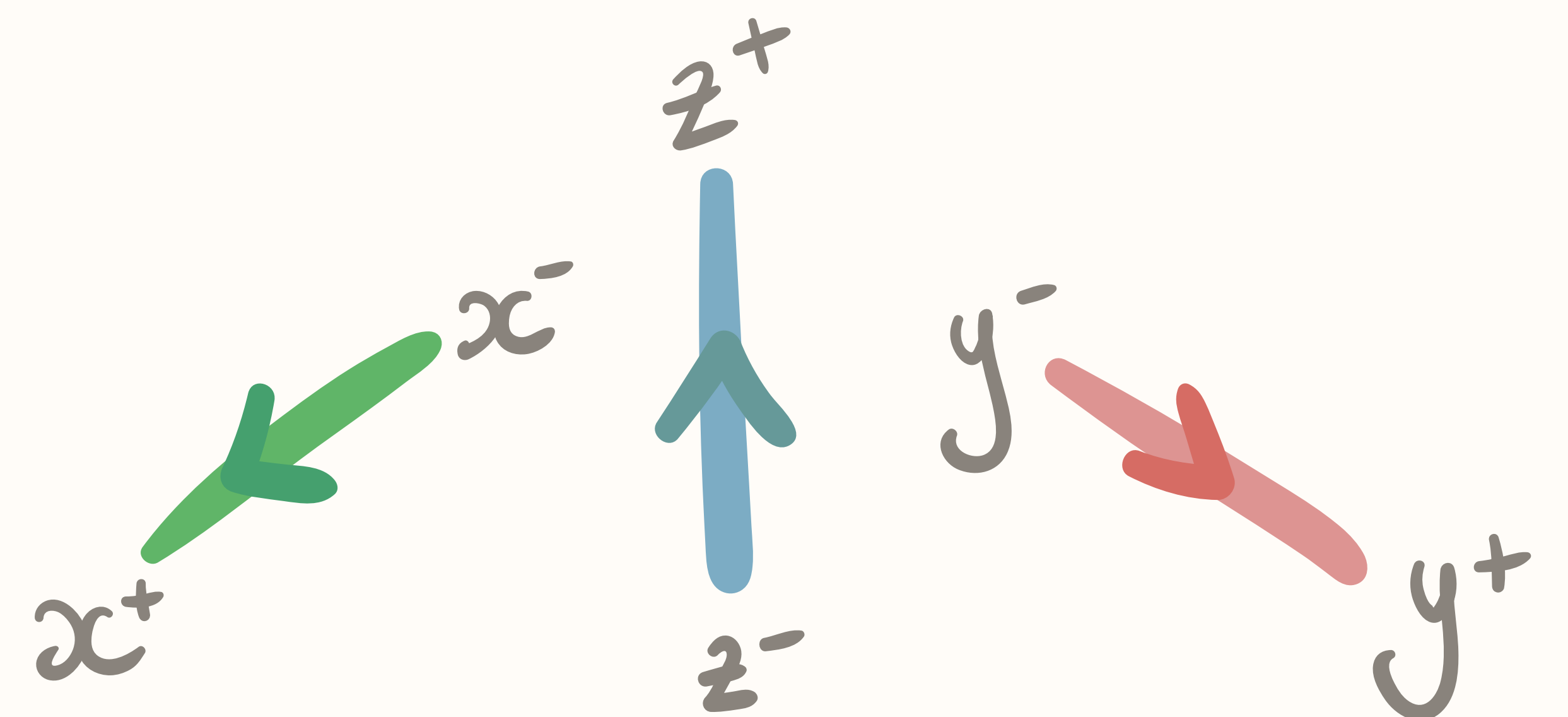
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Corner polyhedra

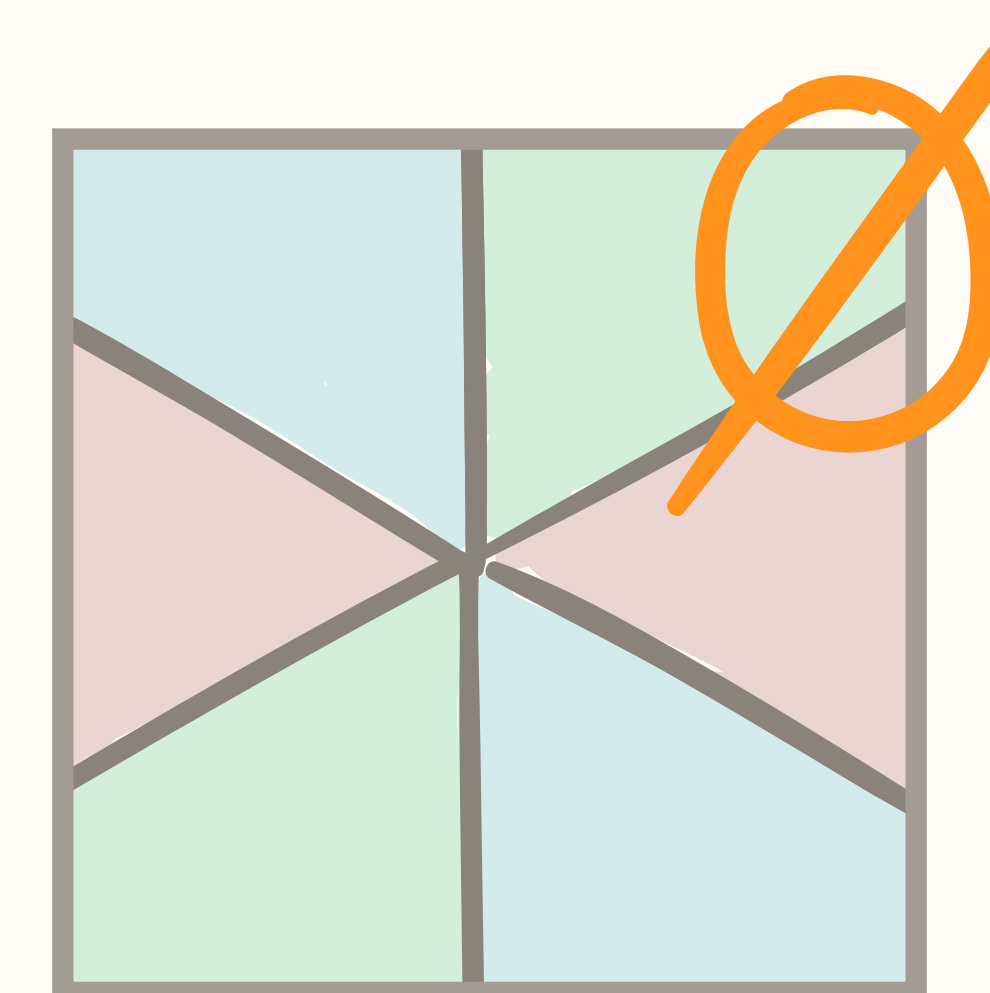
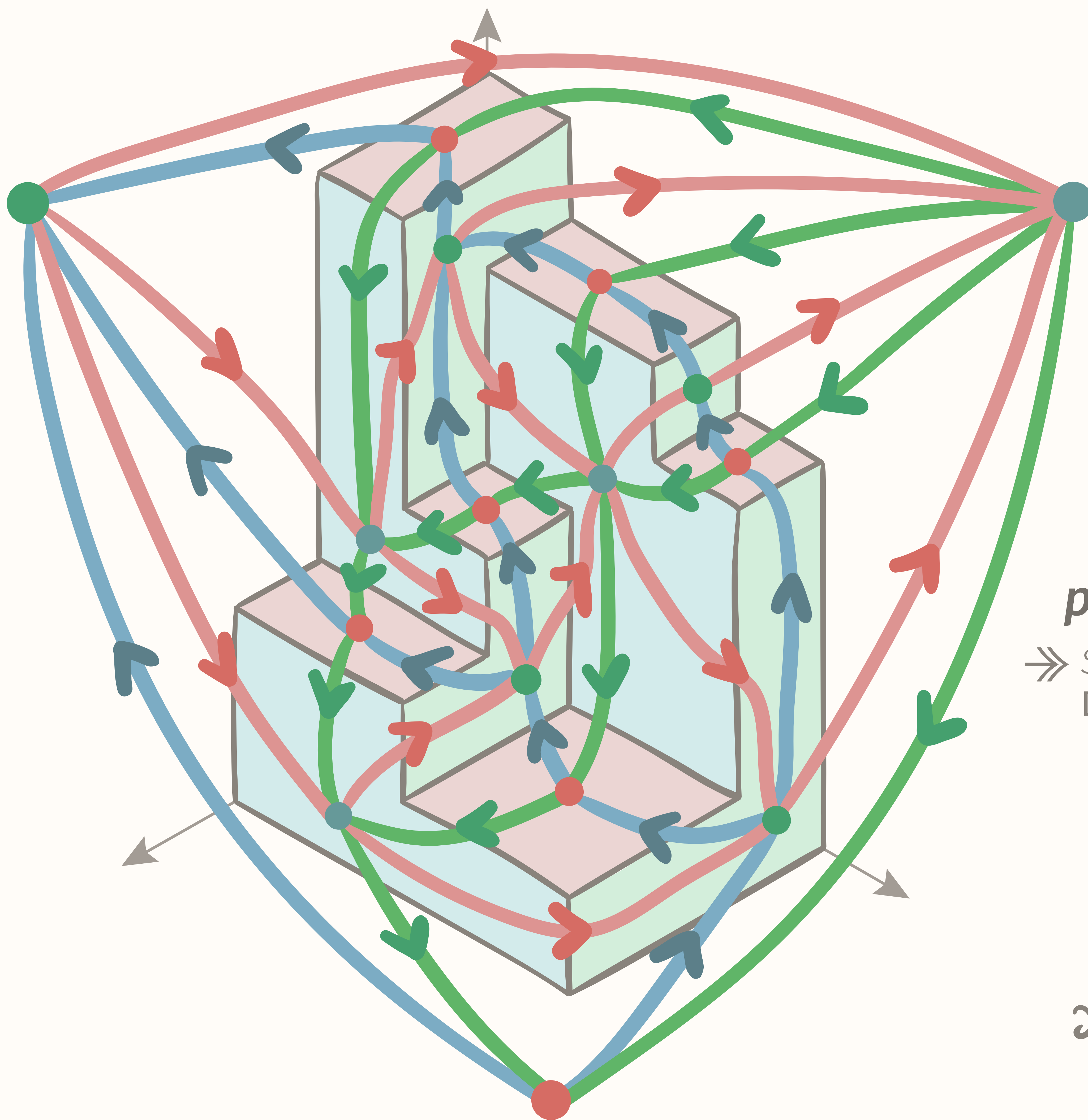


corner polyhedra \leftrightarrow **polyhedral orientations**

\Rightarrow Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)

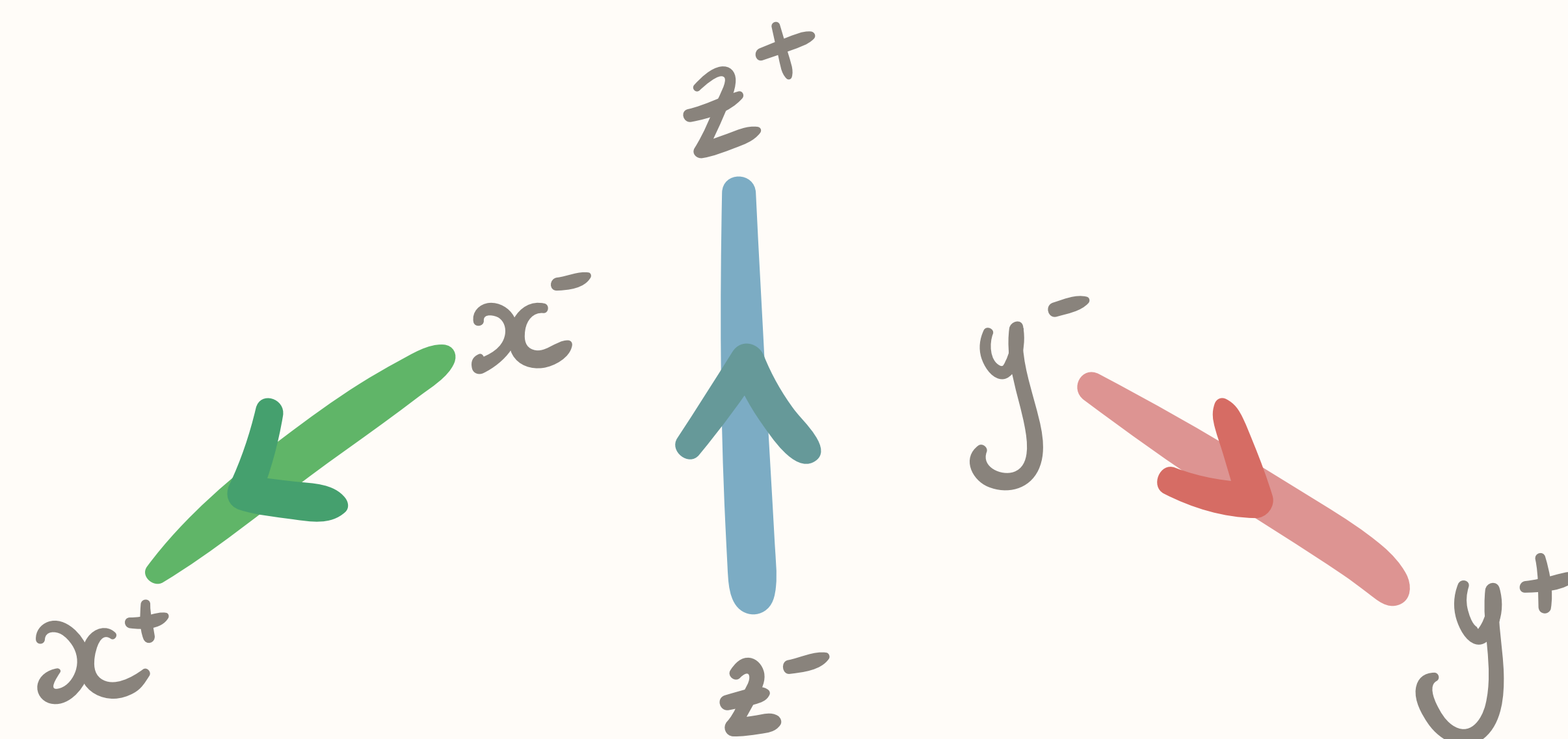


Corner polyhedra

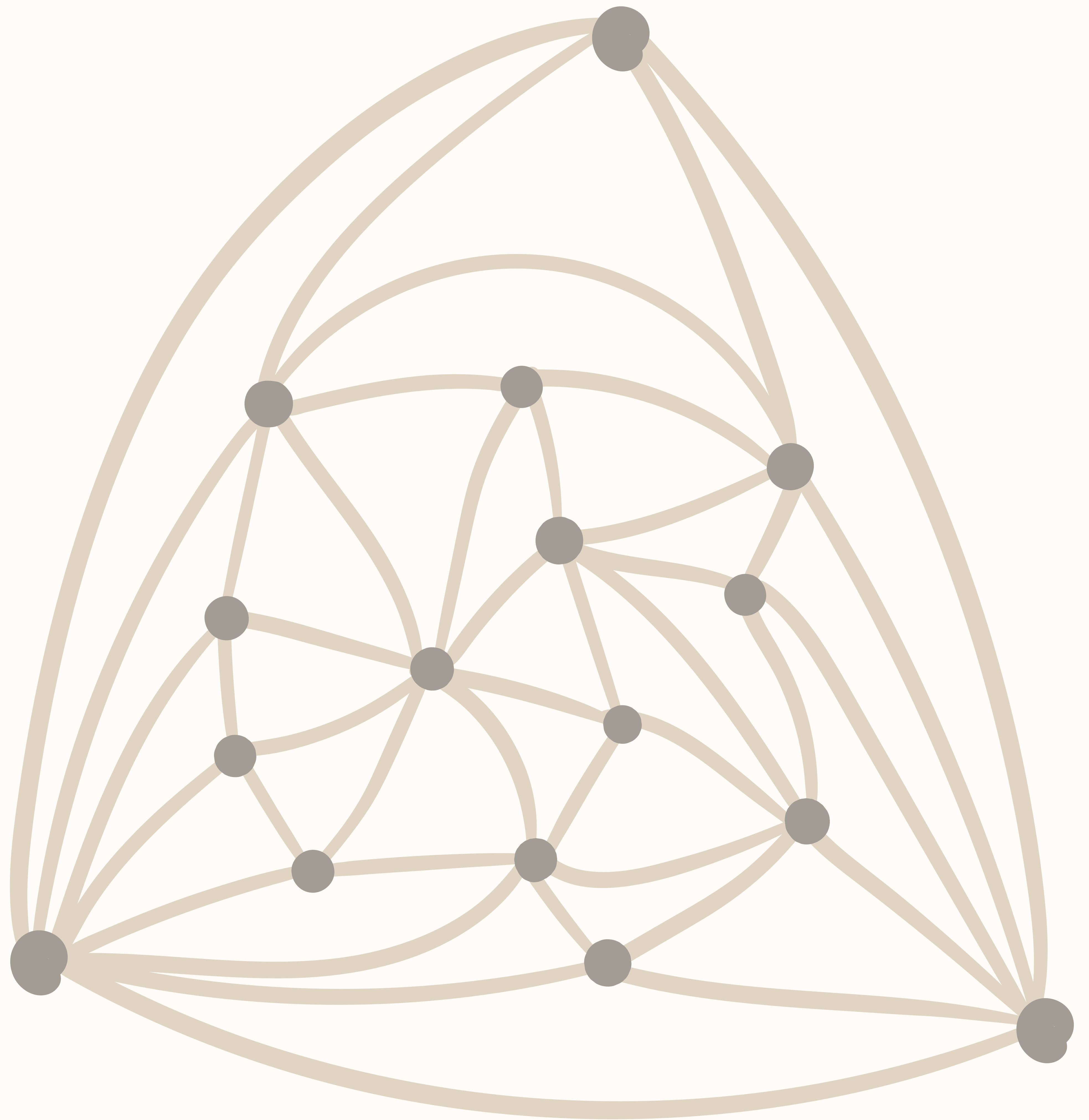


corner polyhedra \leftrightarrow **polyhedral orientations**

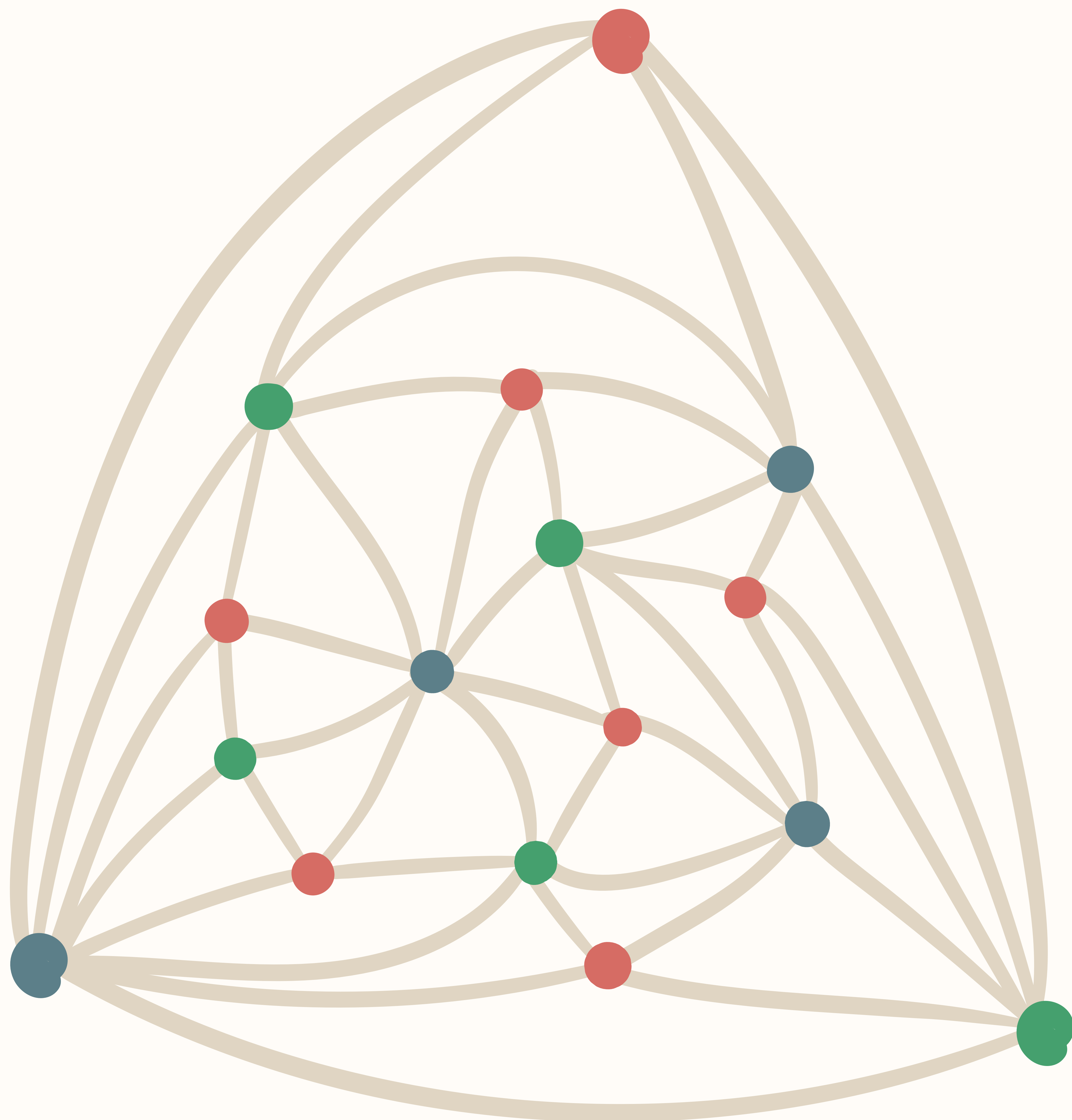
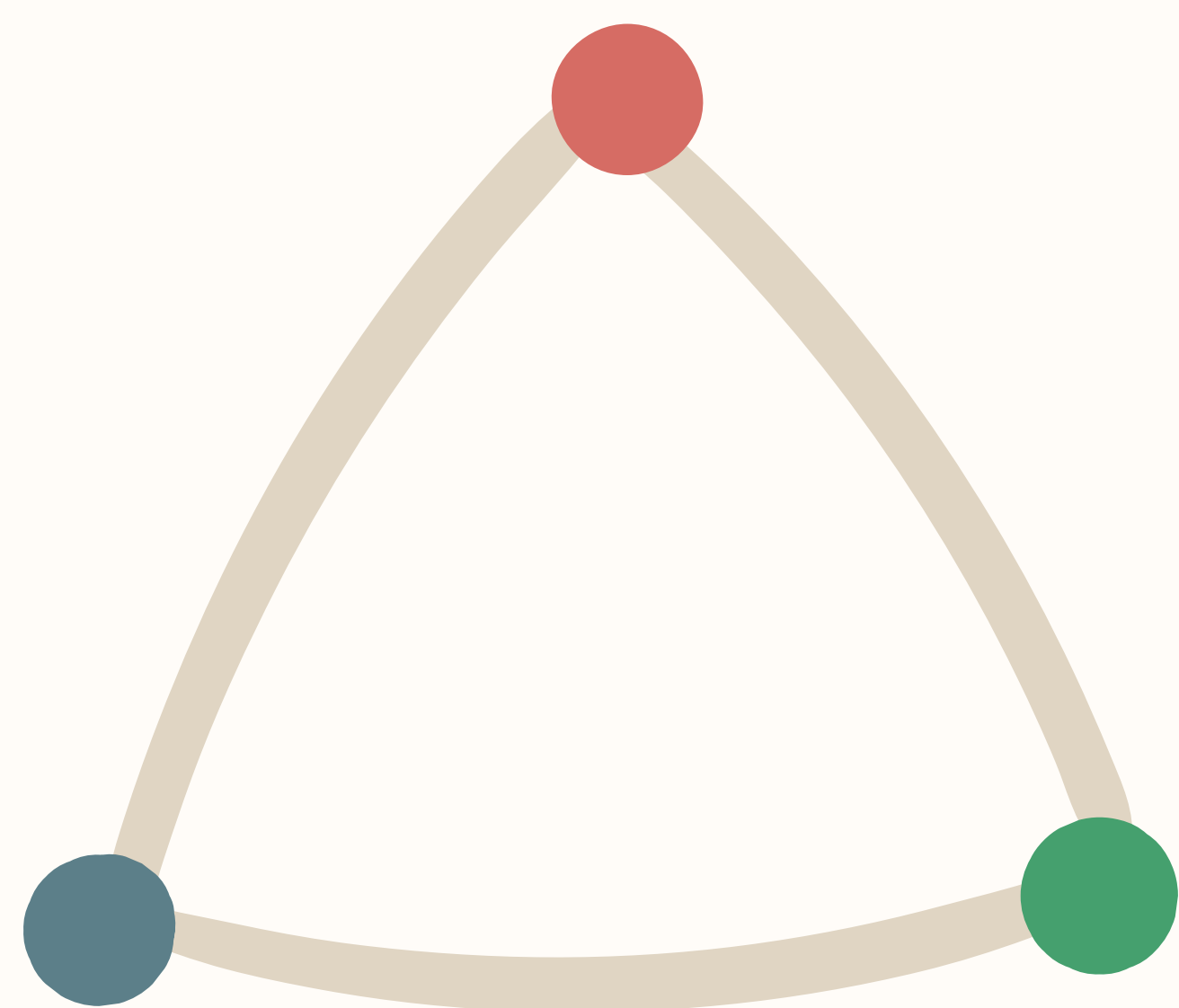
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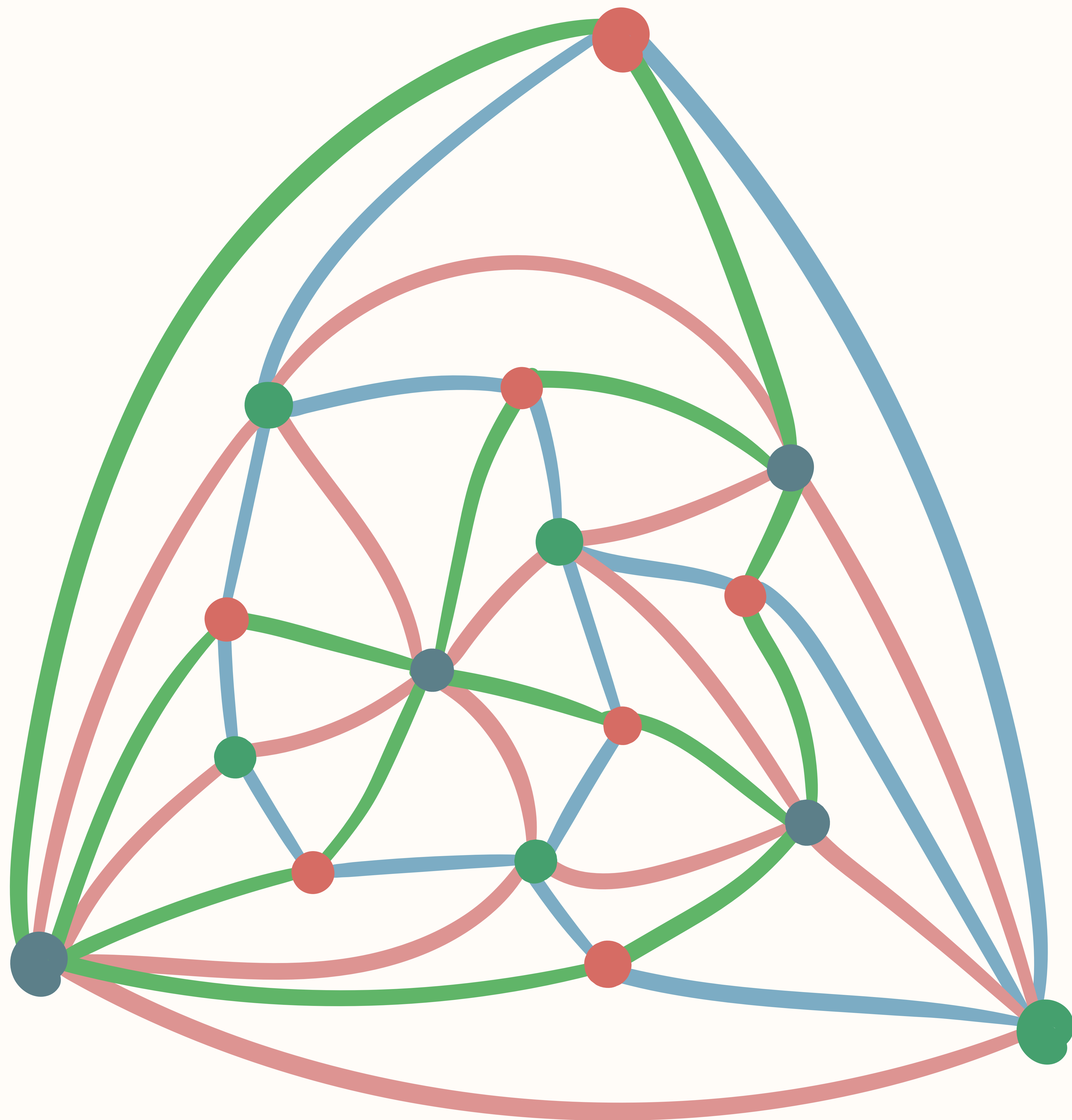
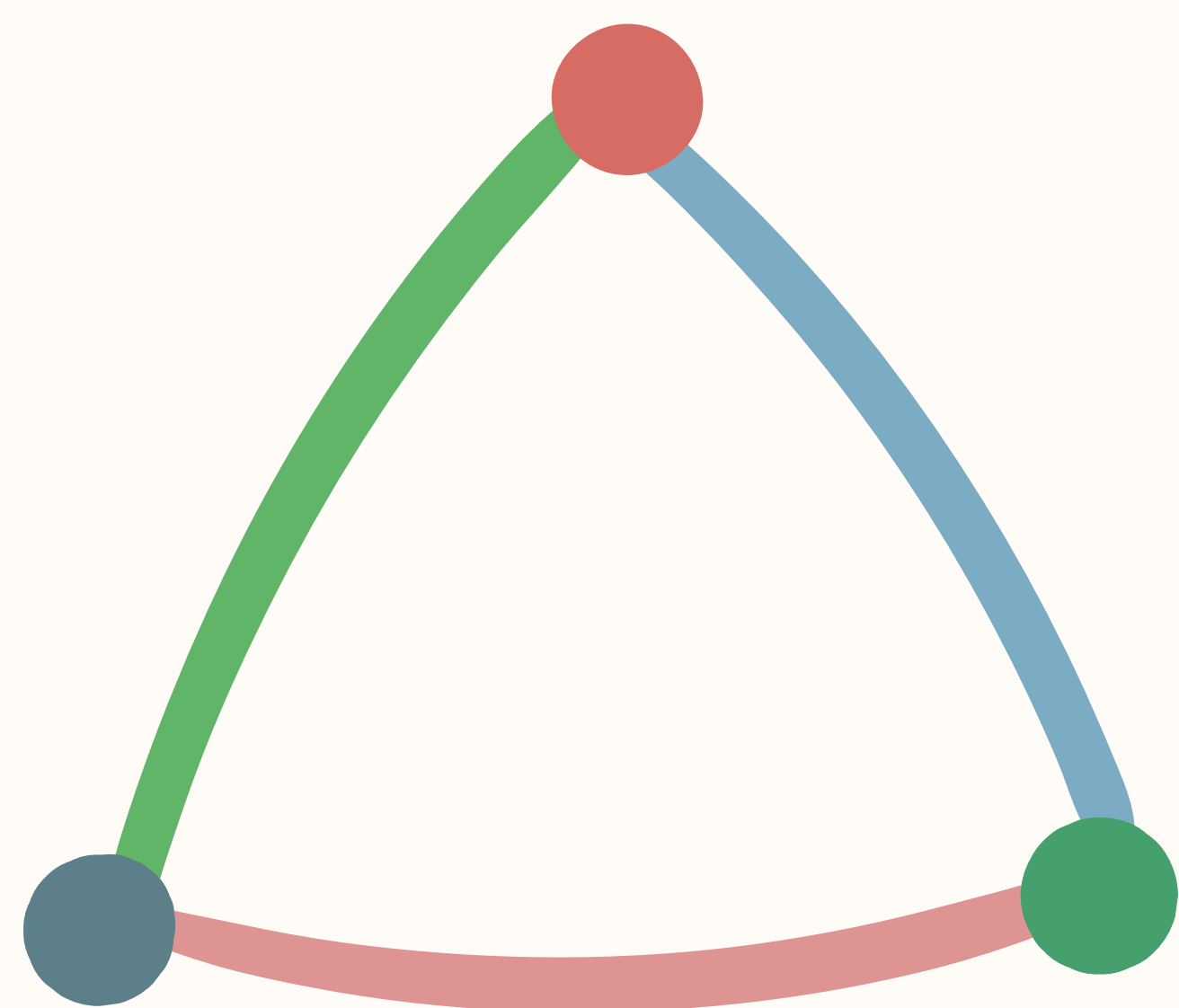
Polyhedral orientations



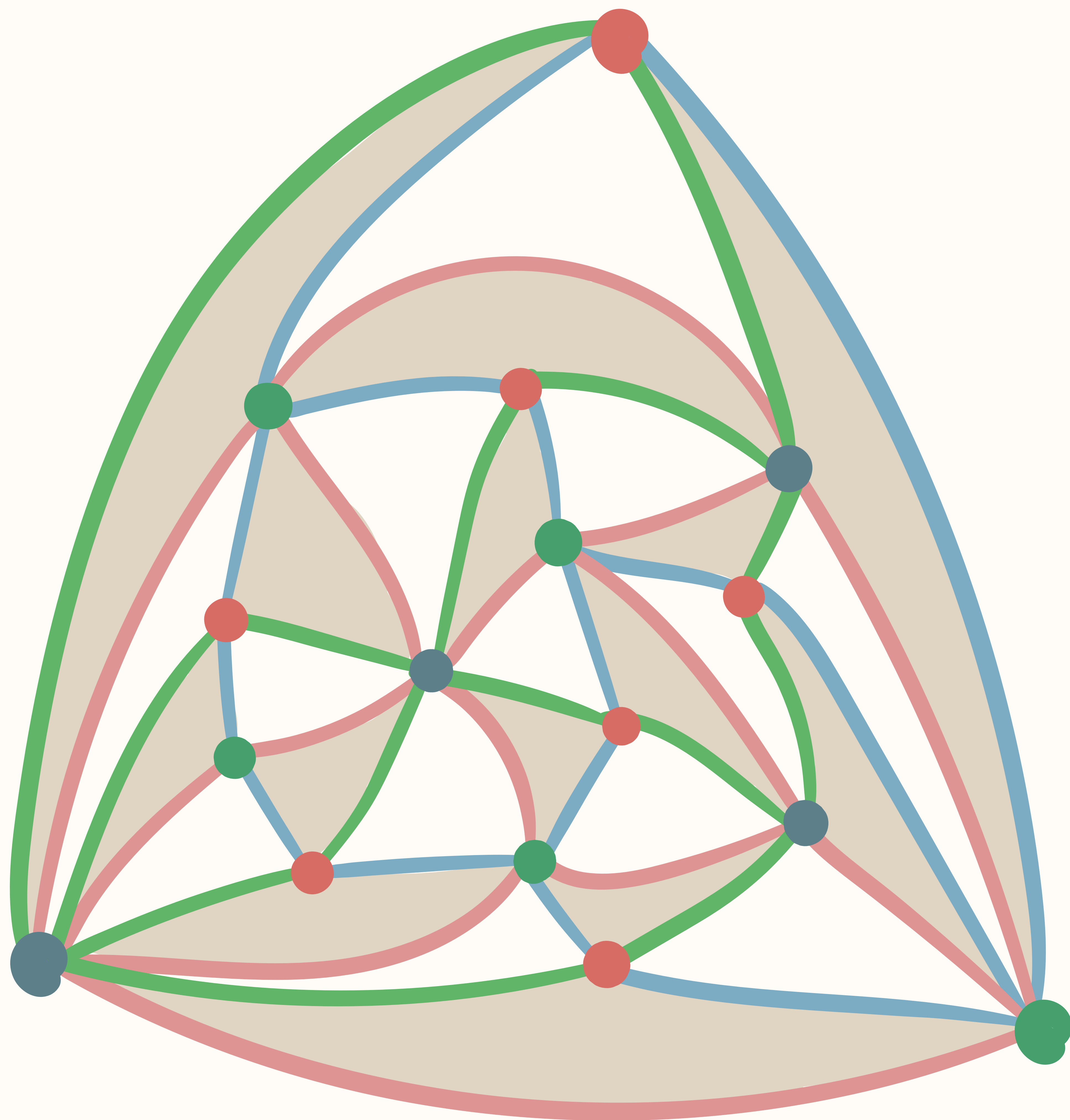
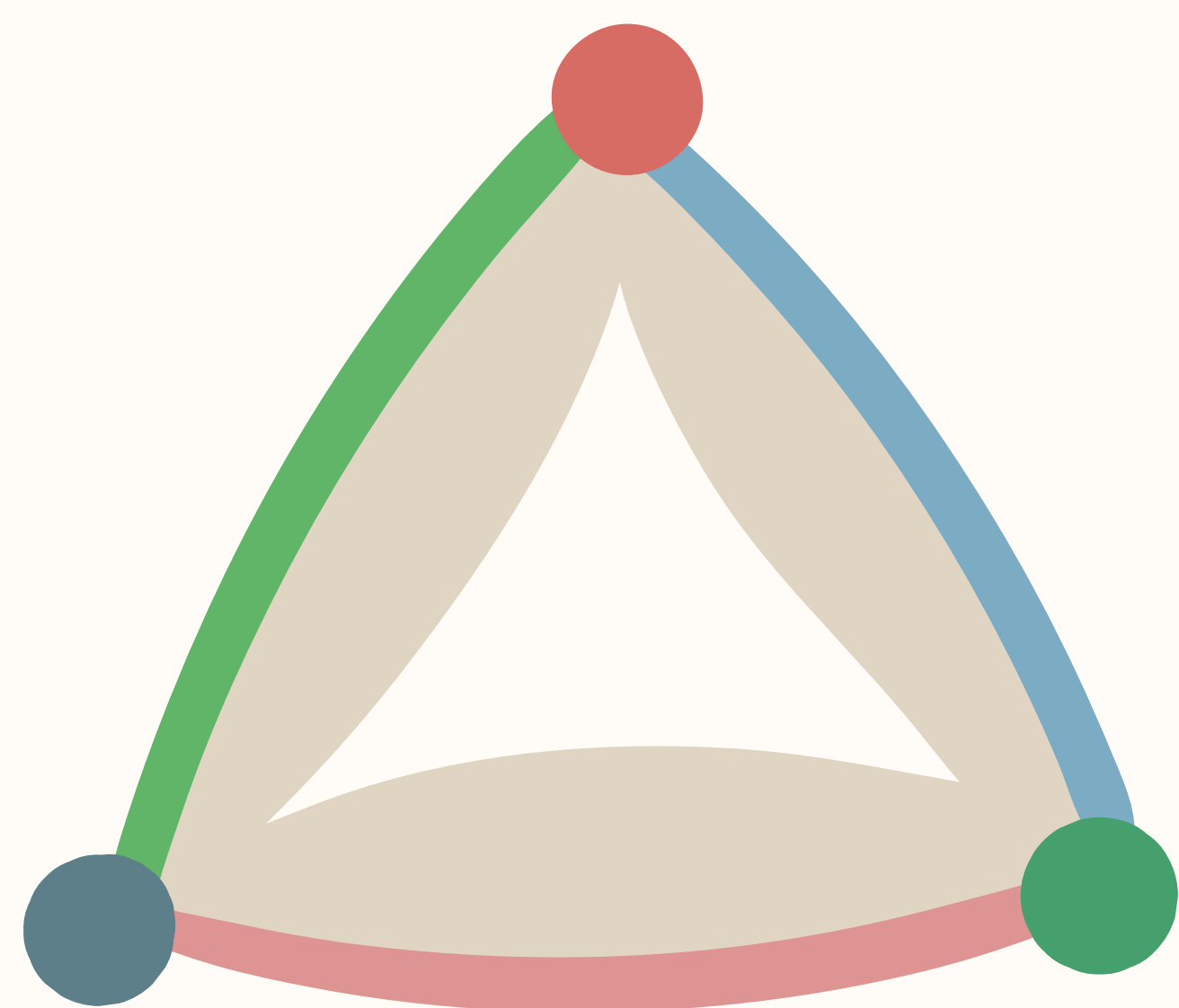
Polyhedral orientations



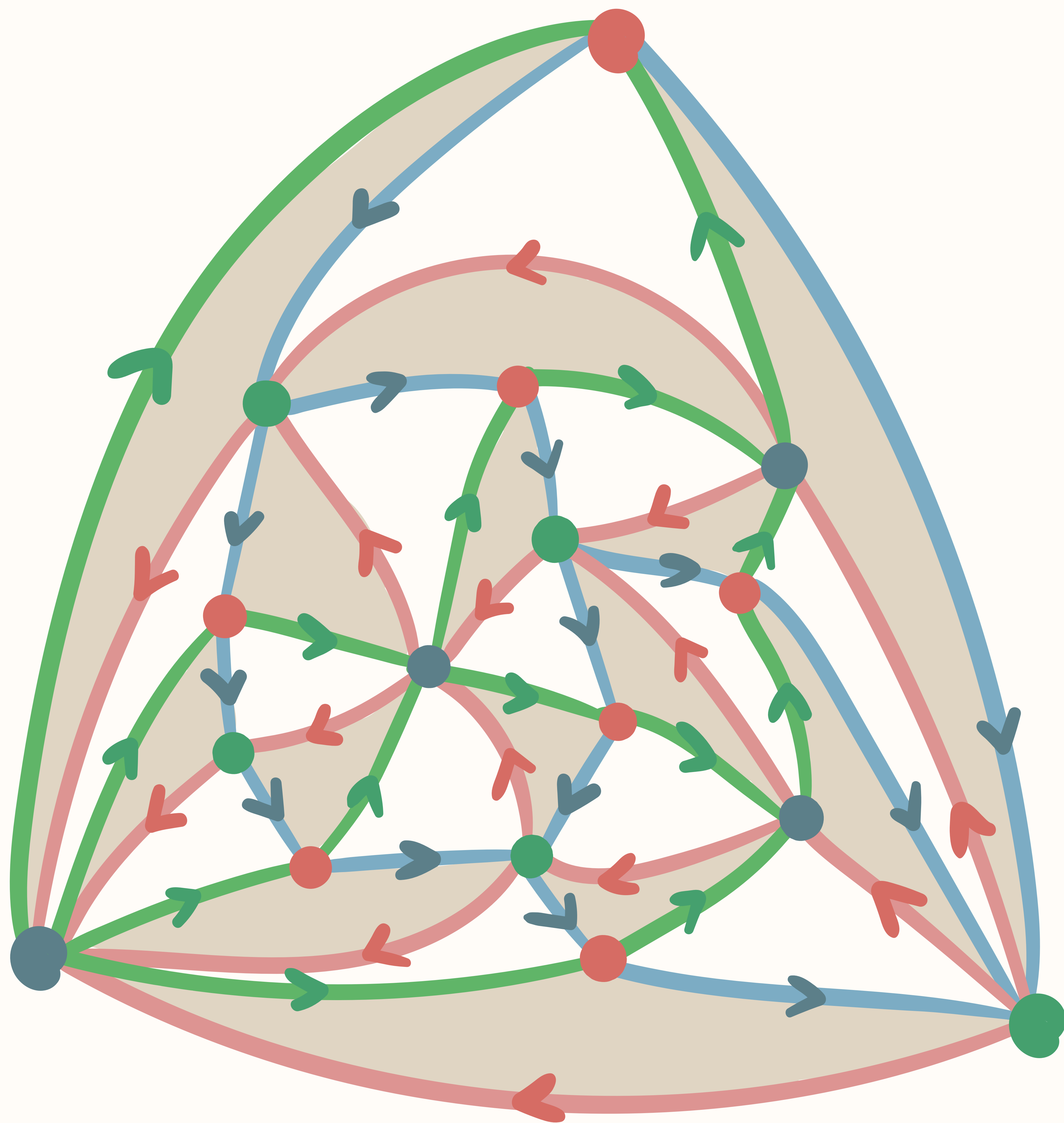
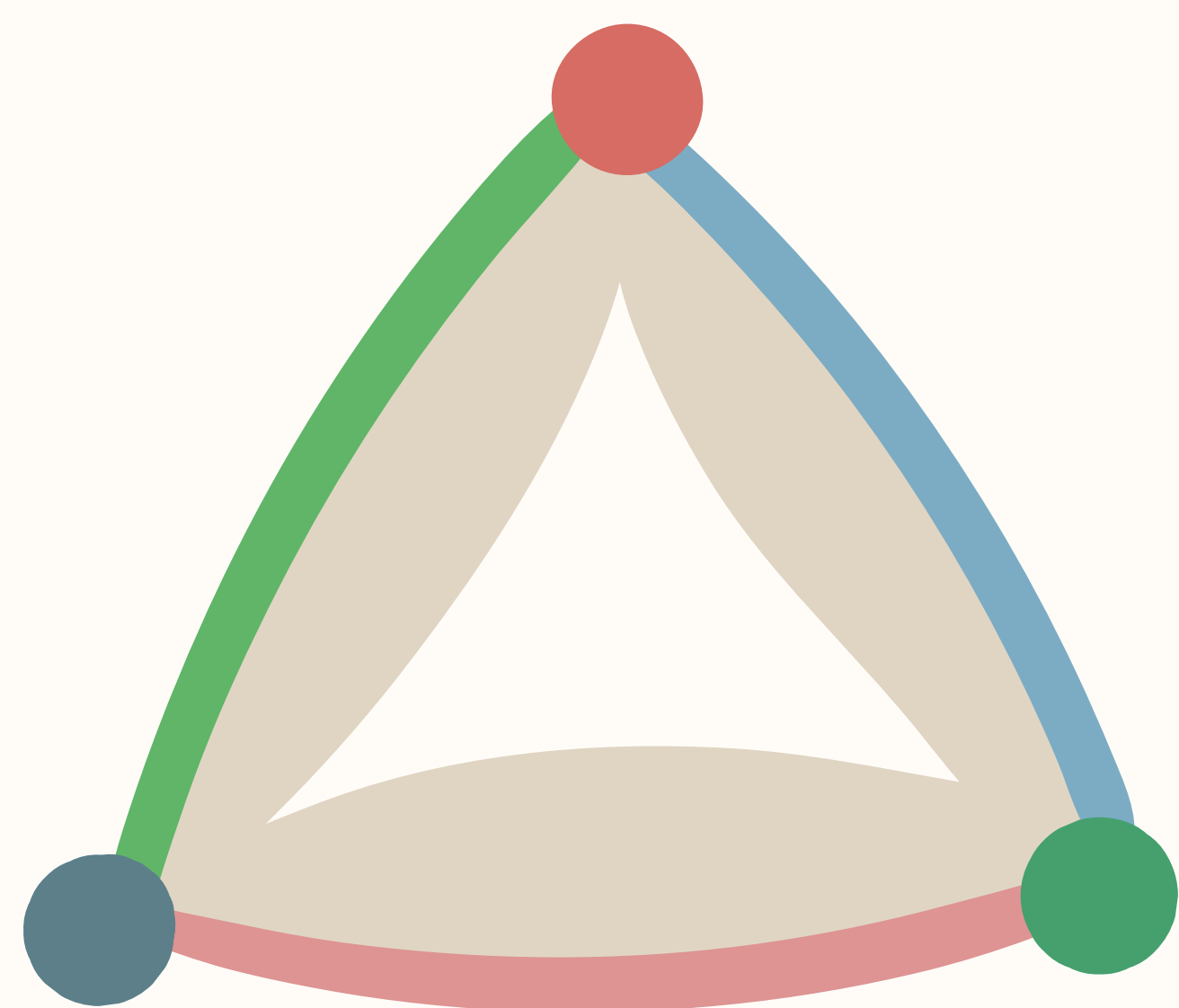
Polyhedral orientations



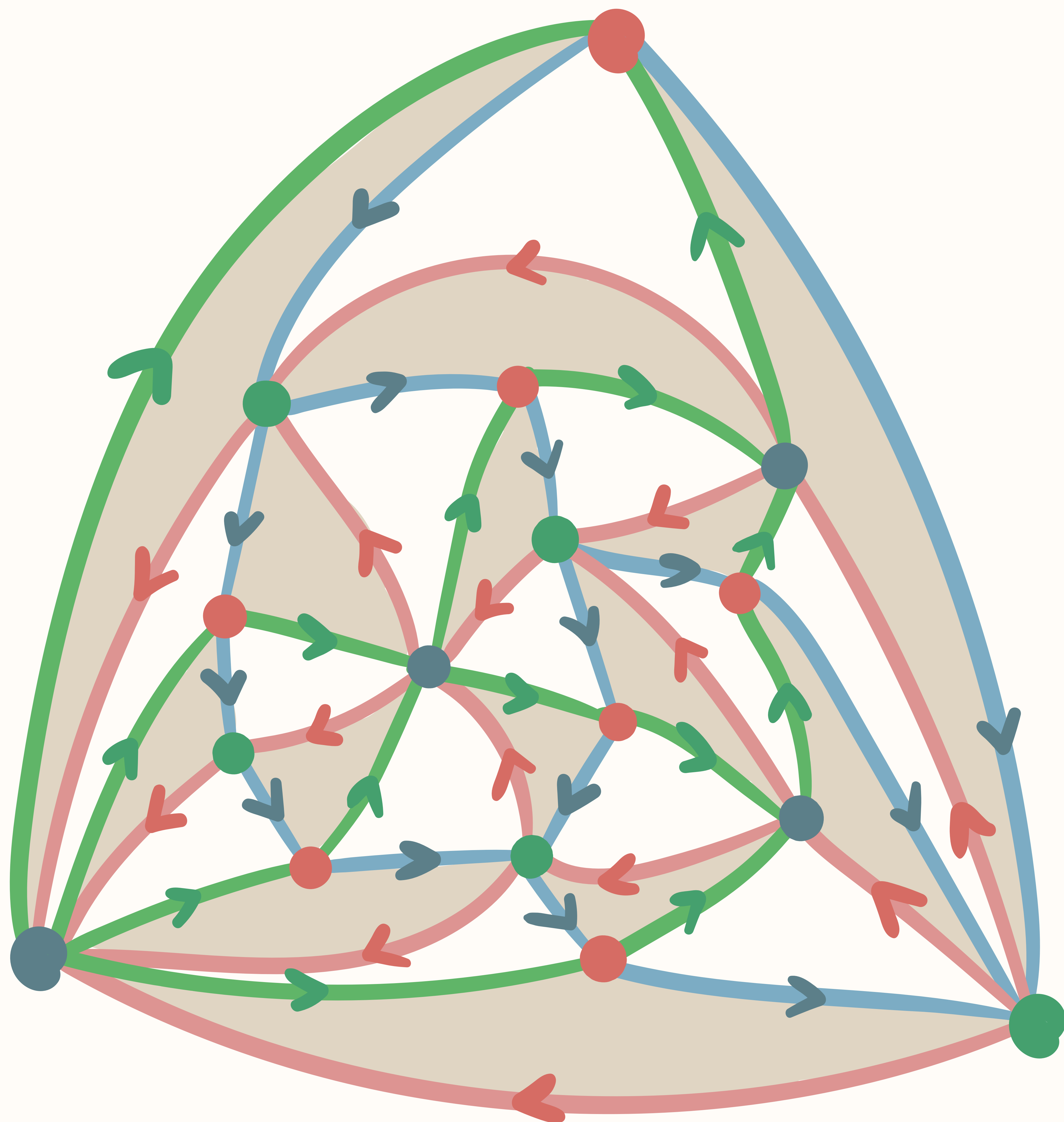
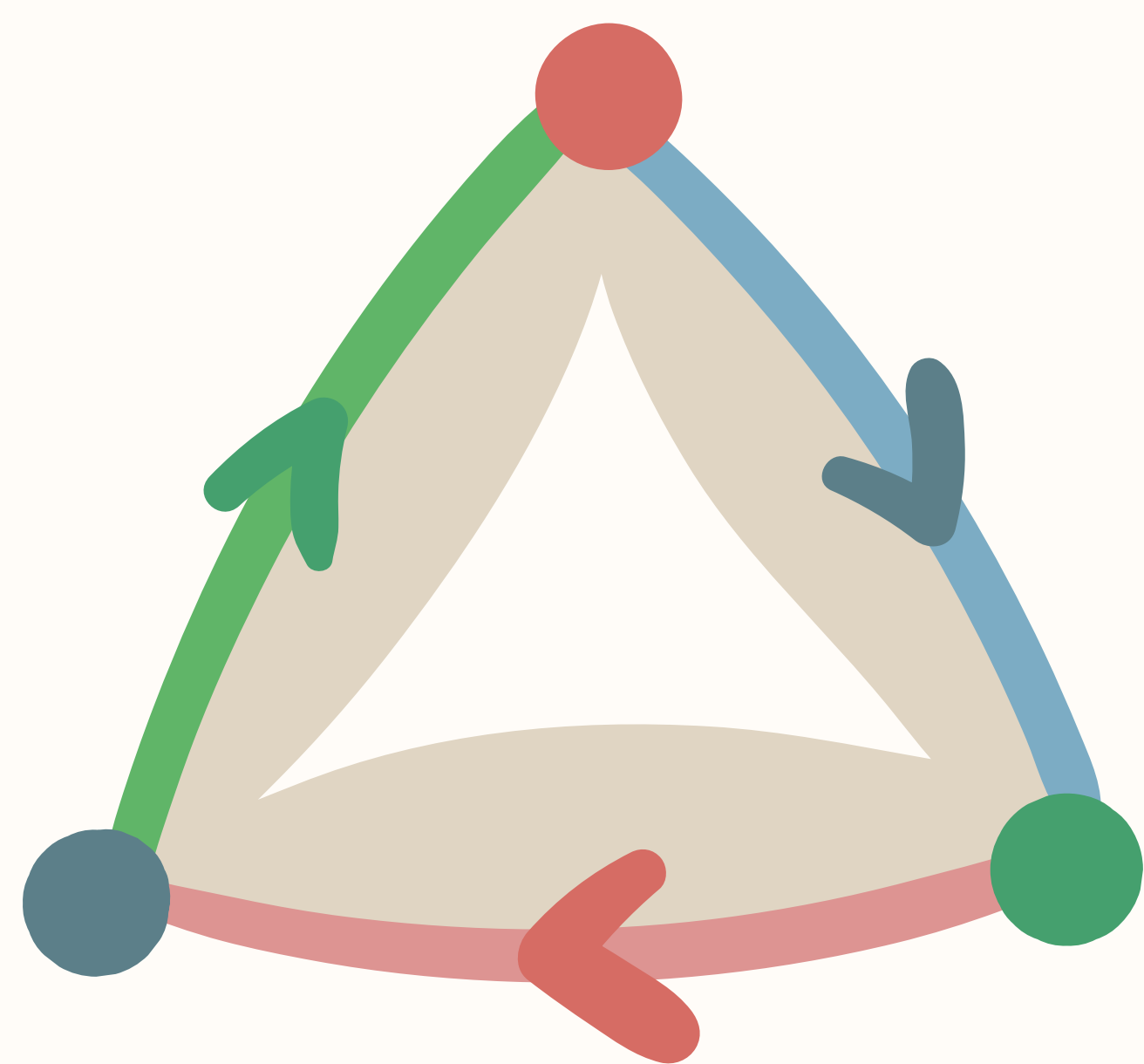
Polyhedral orientations



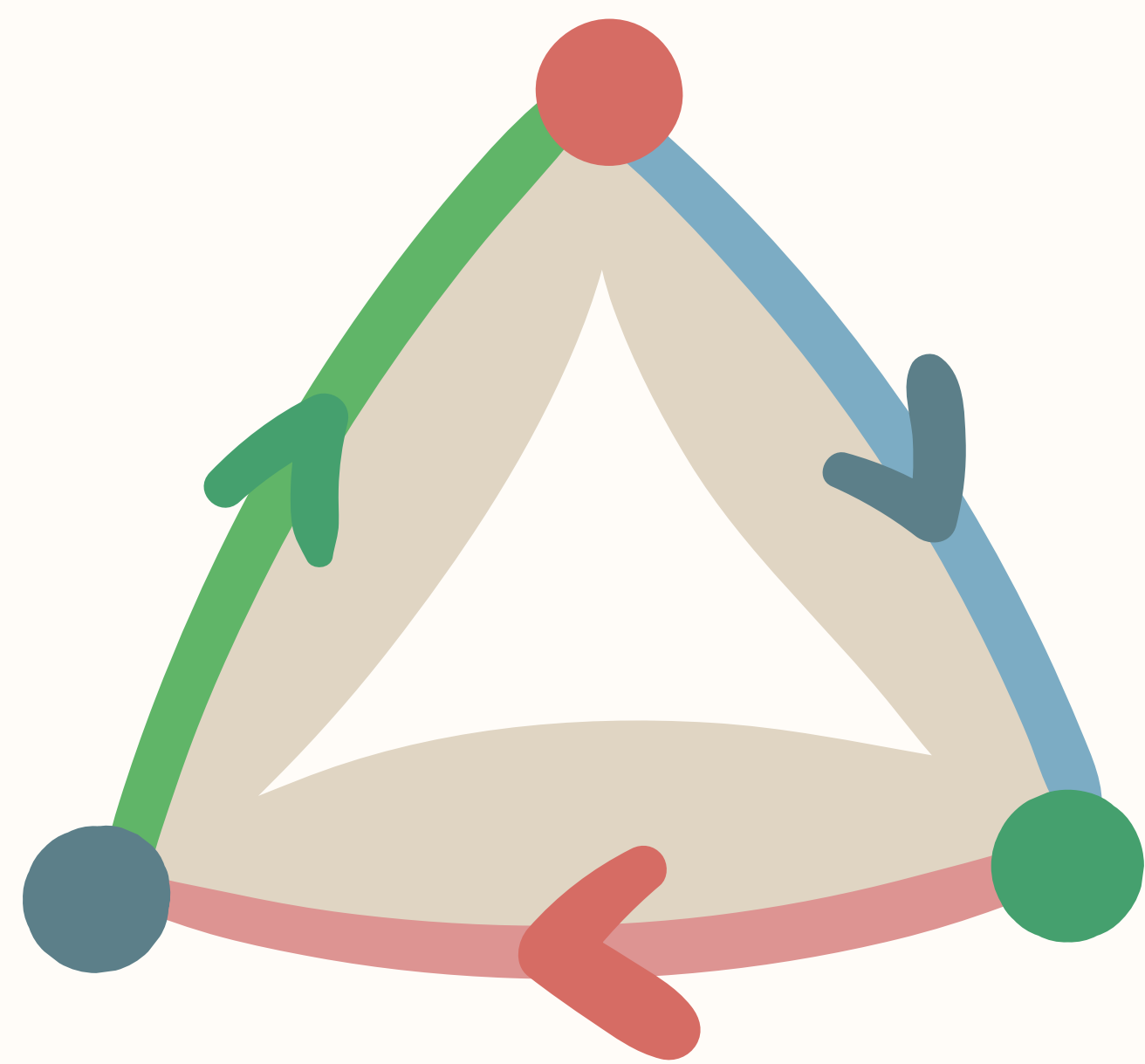
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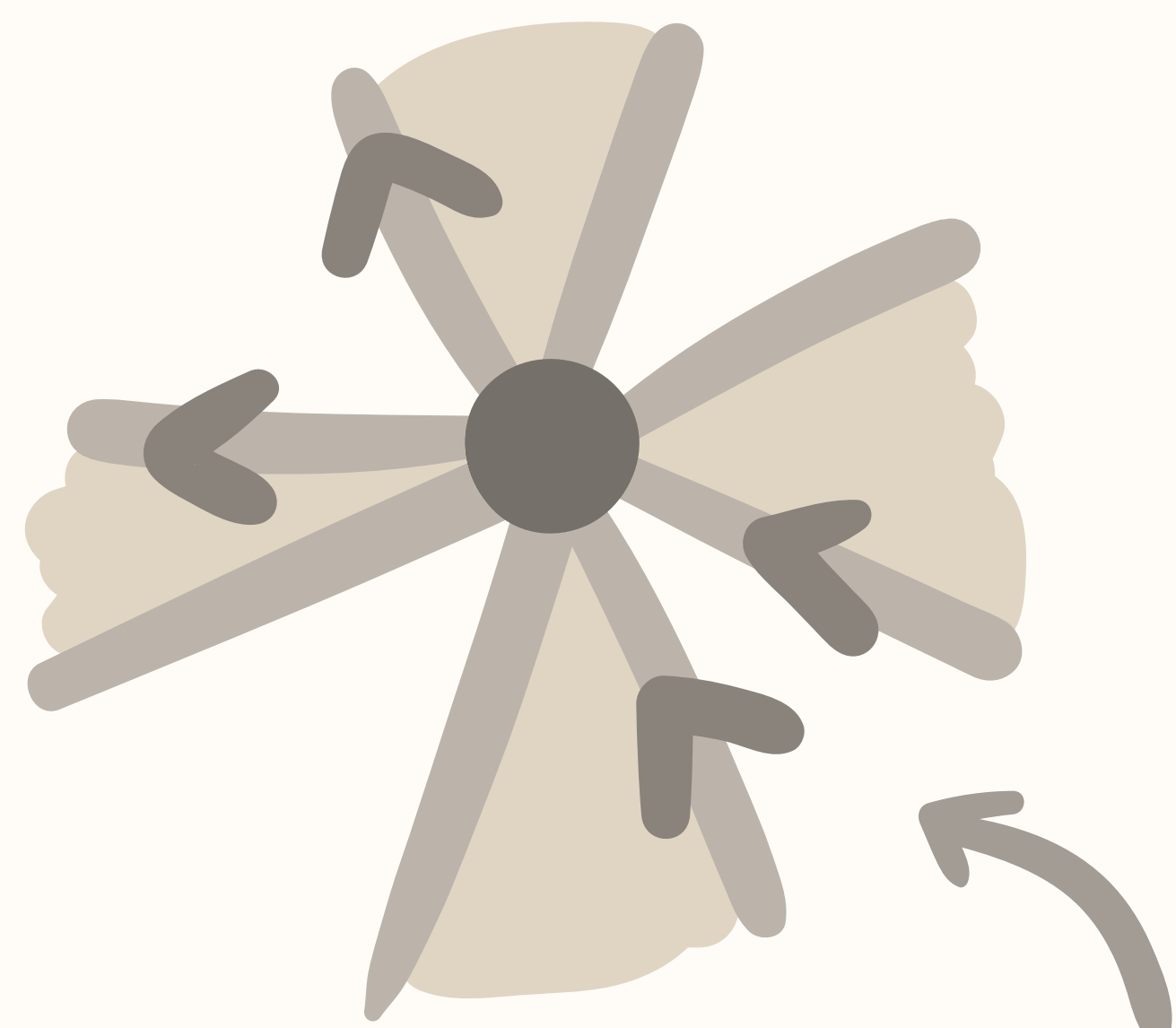
Polyhedral orientations



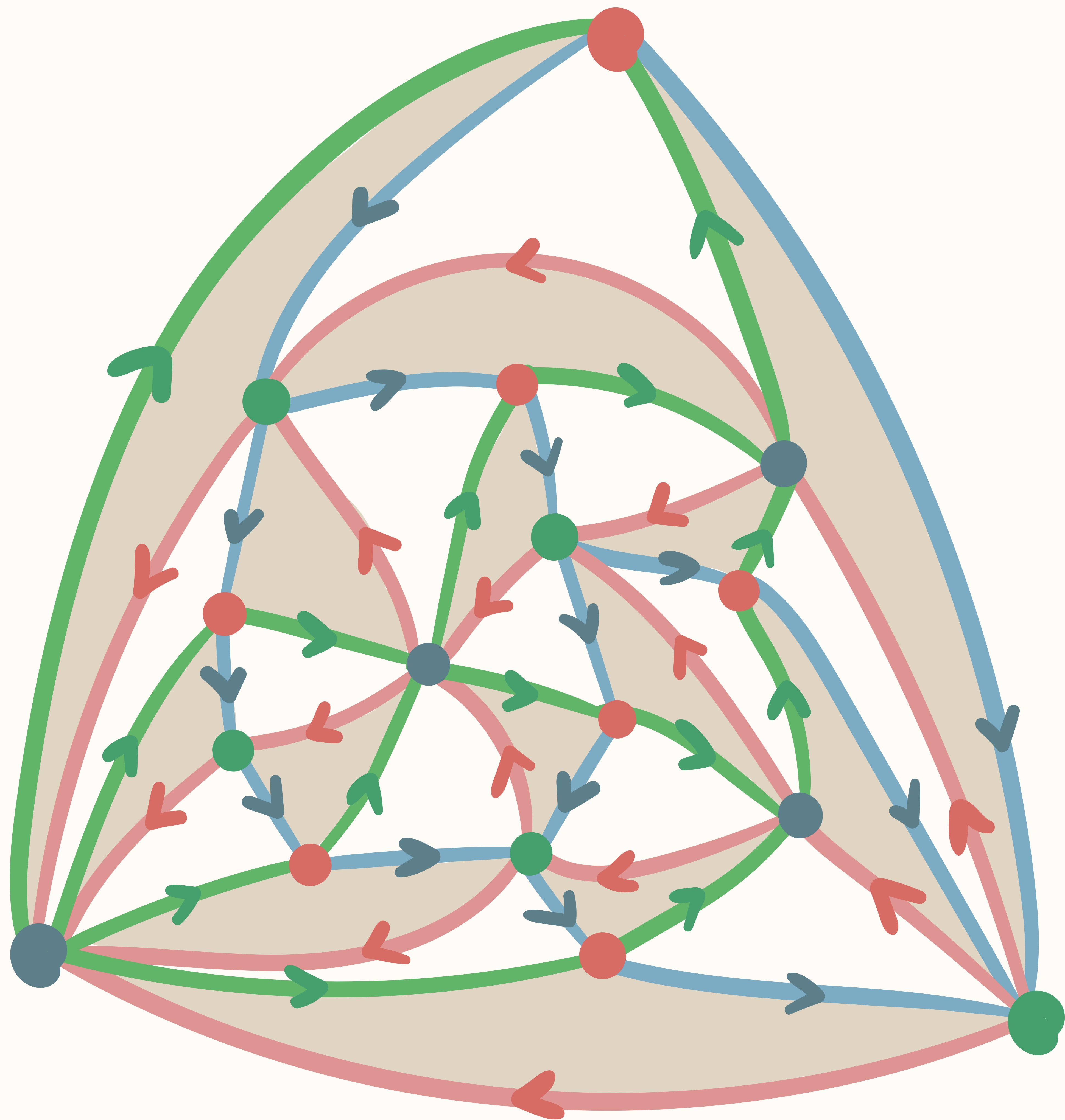
Polyhedral orientations



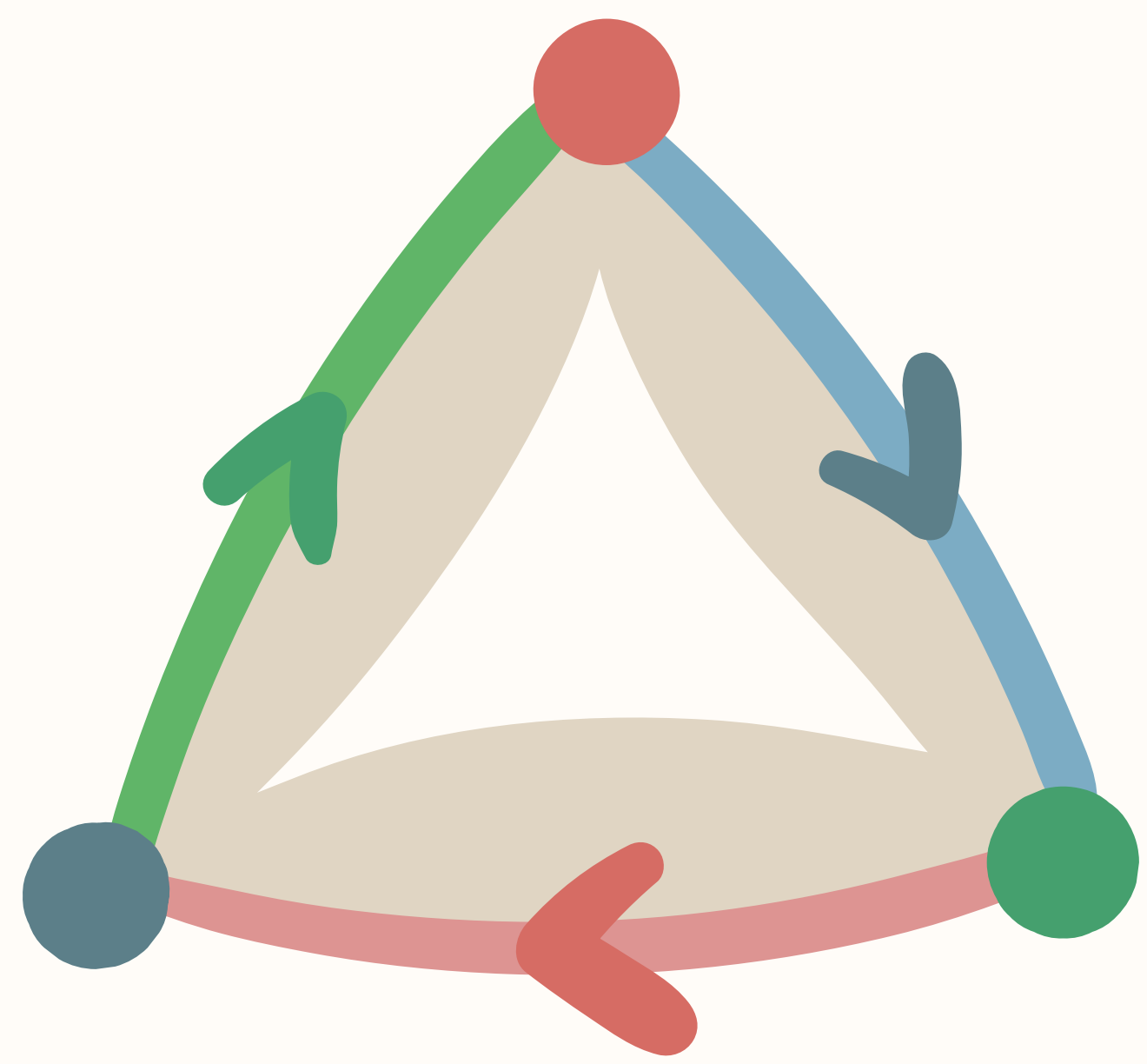
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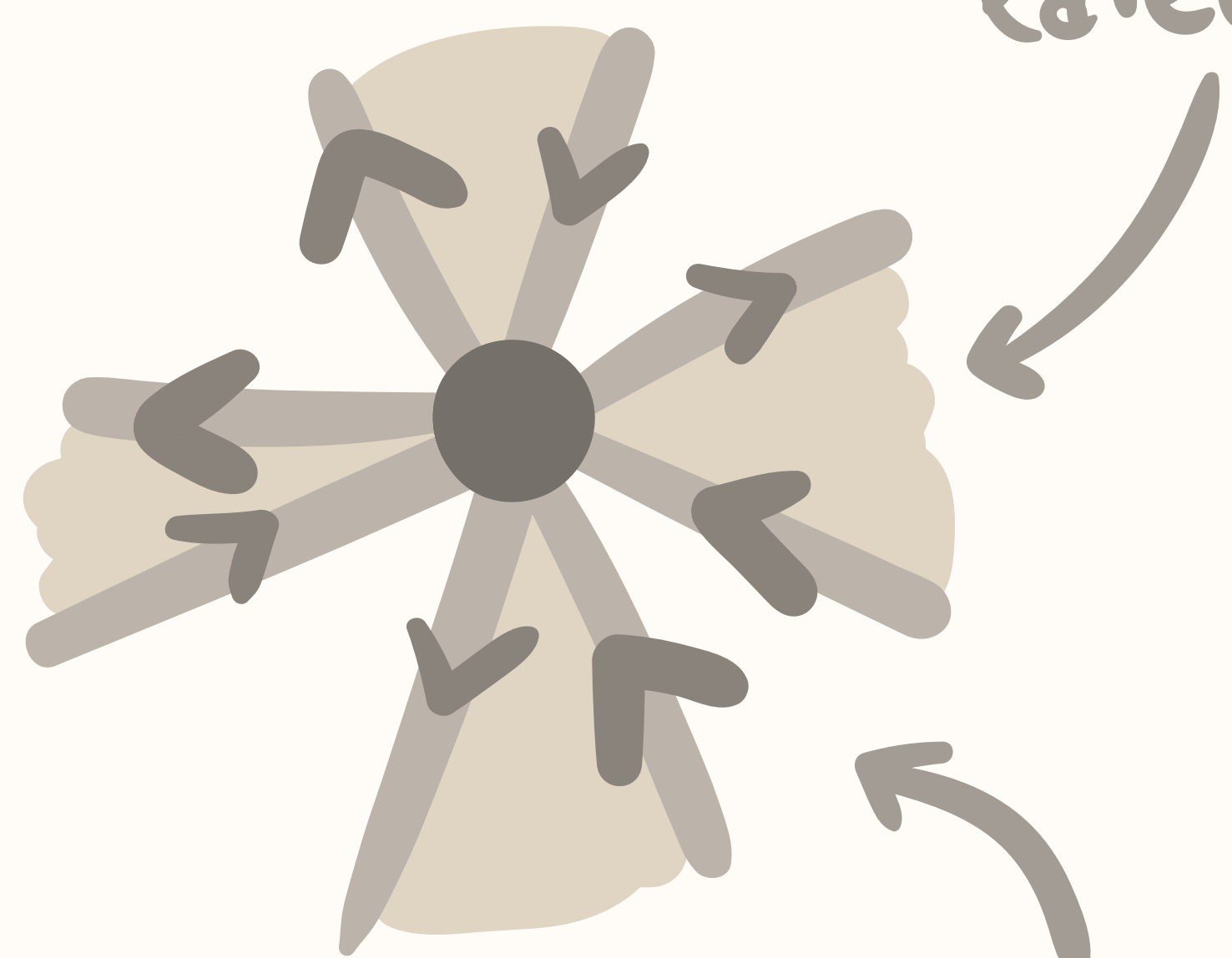
extremal x 2



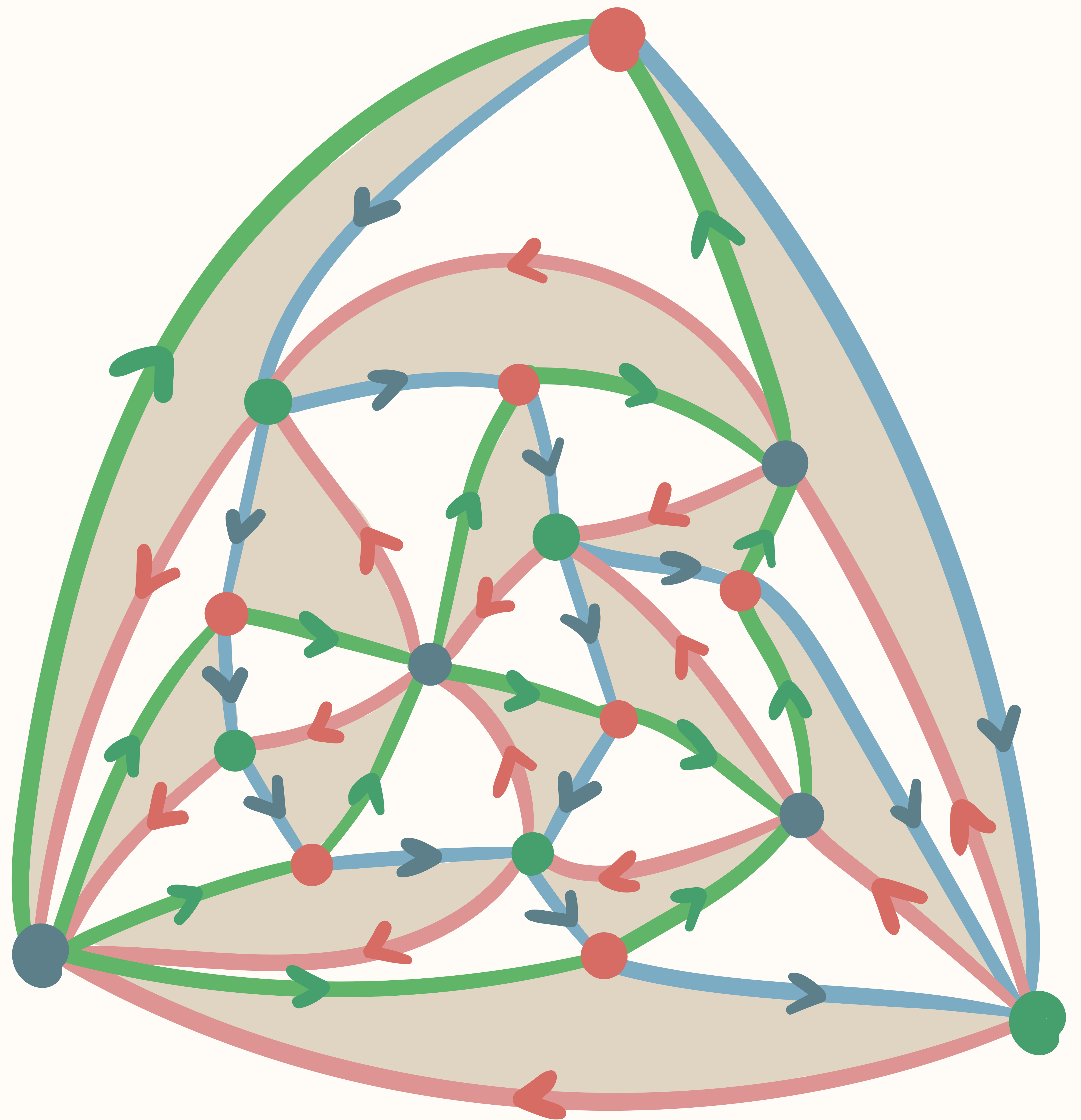
Polyhedral orientations



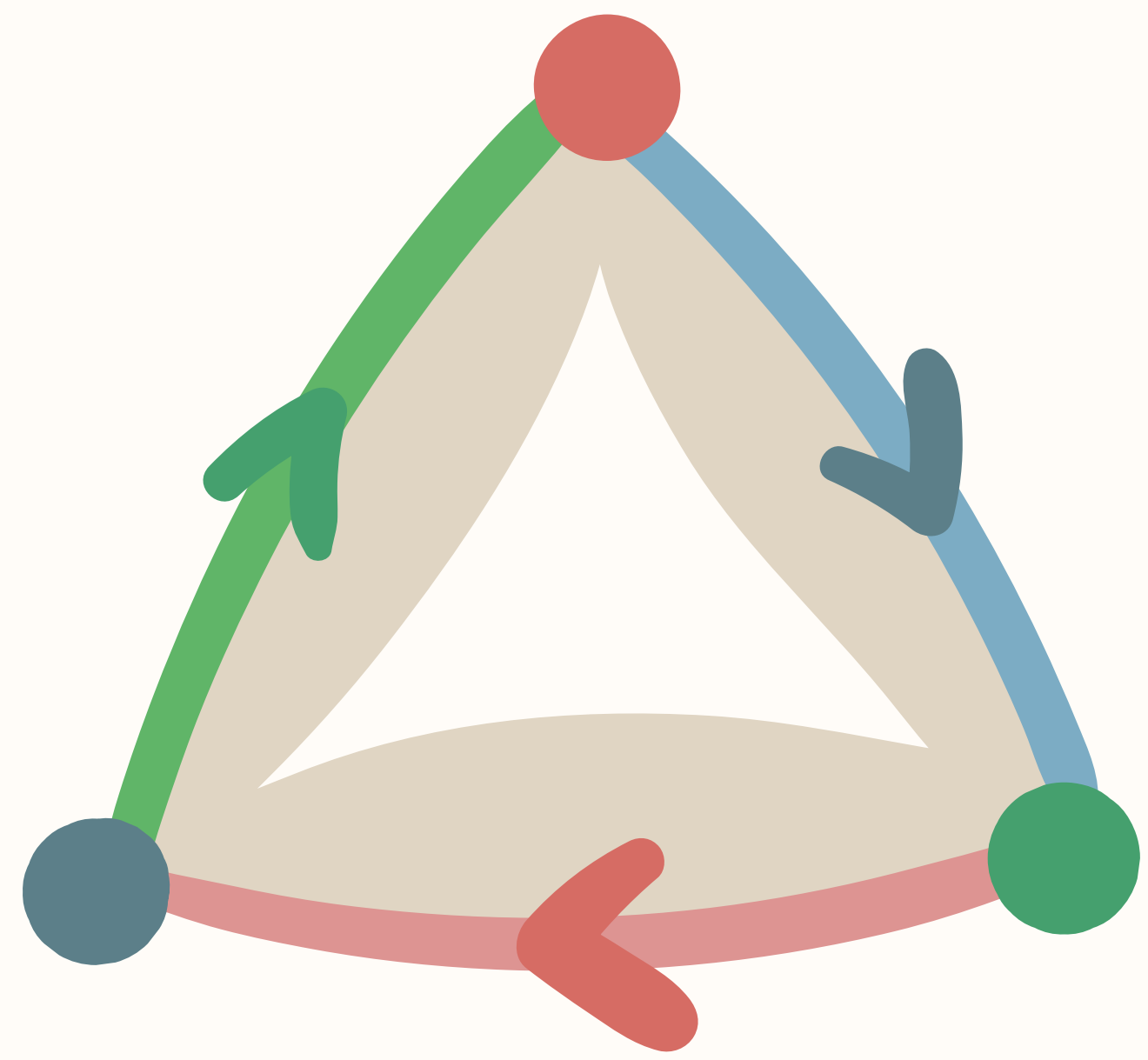
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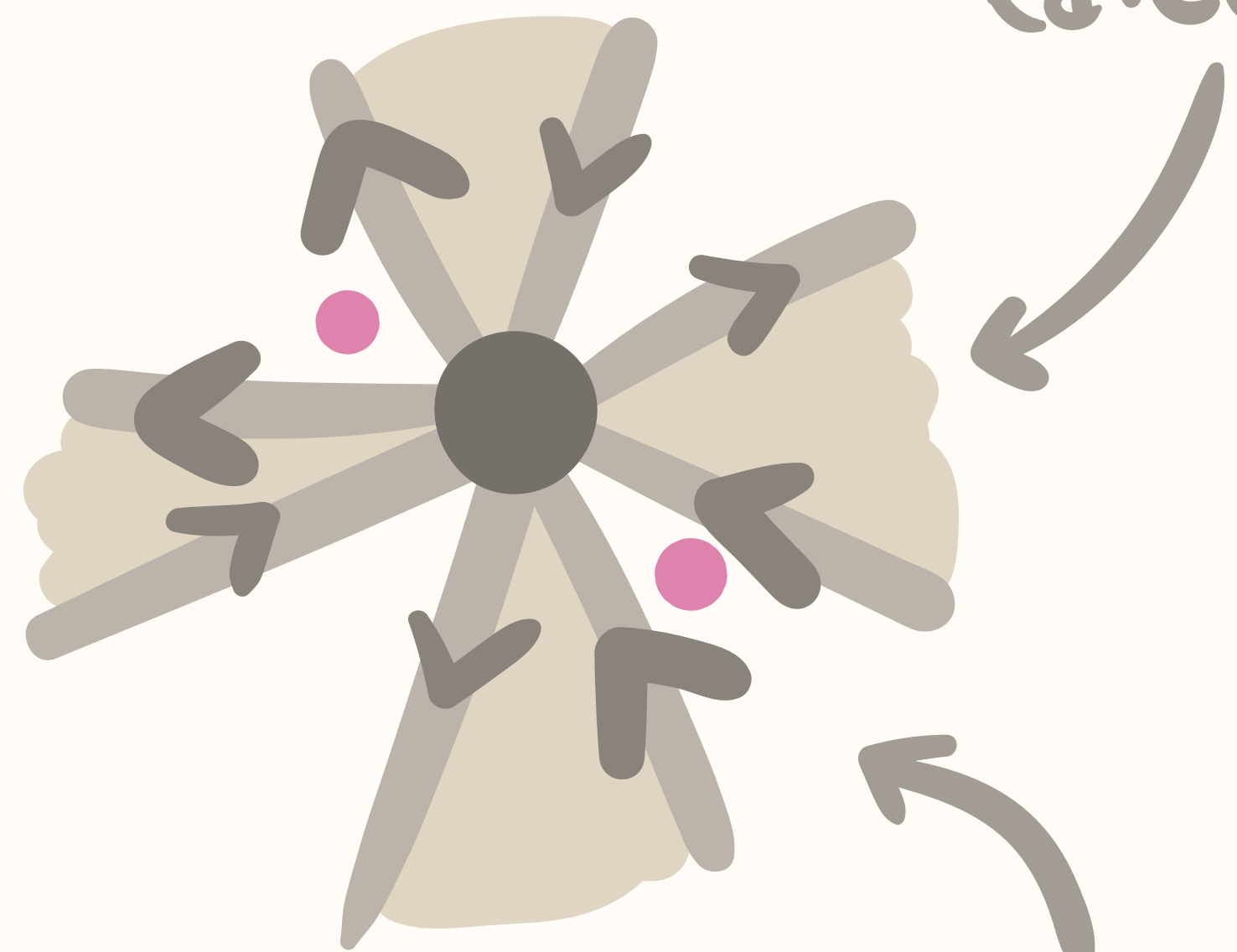
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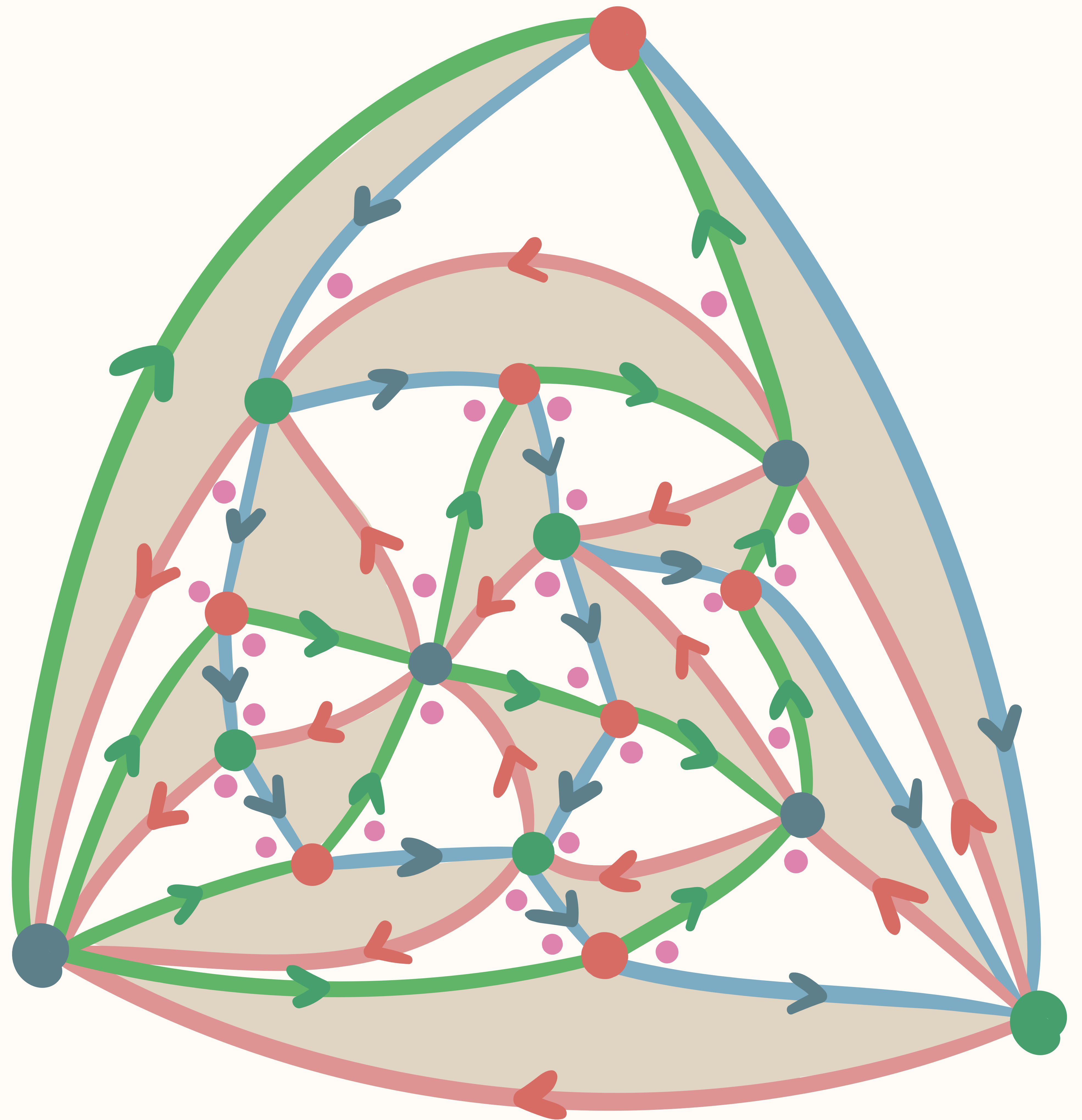
Polyhedral orientations



inner:

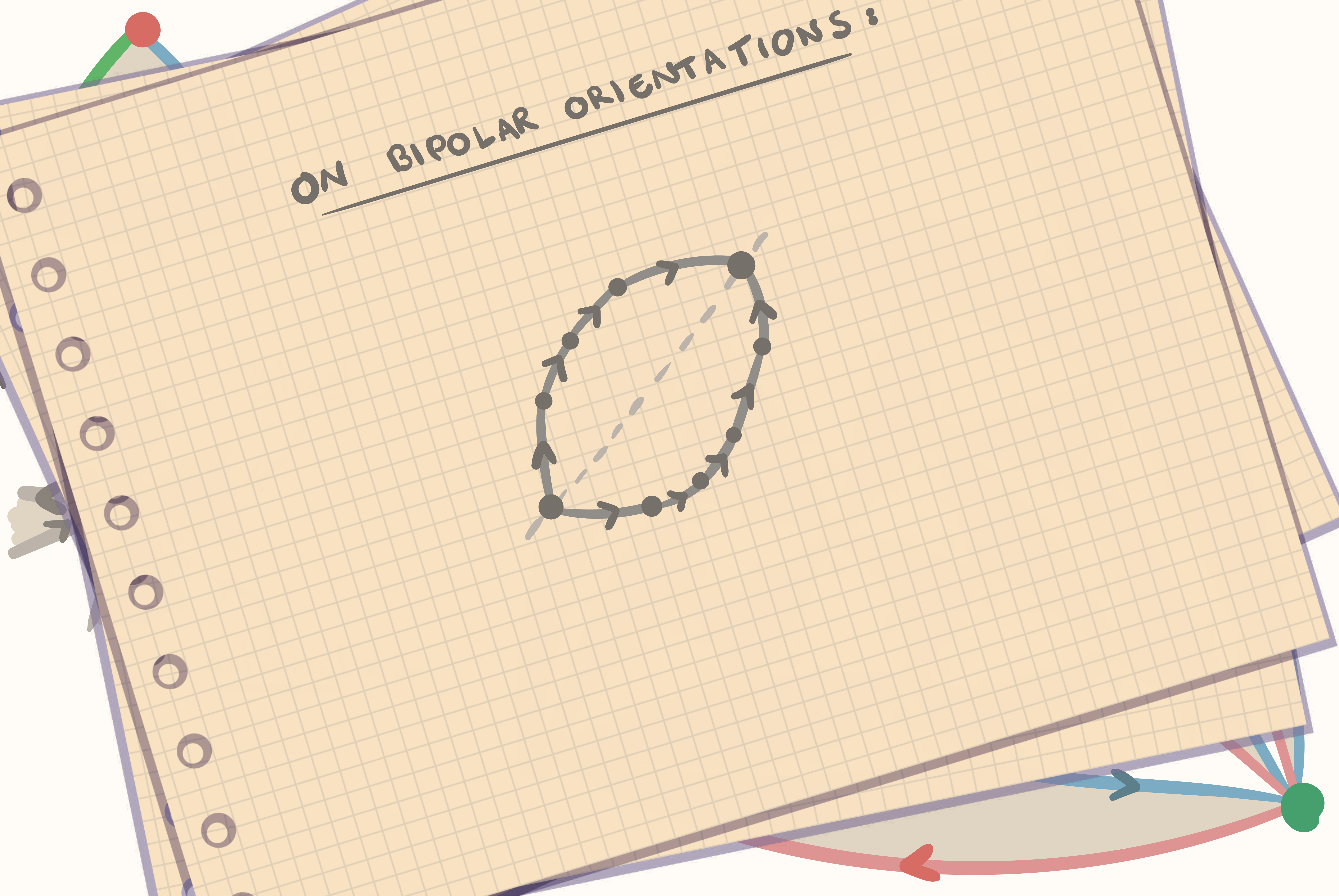


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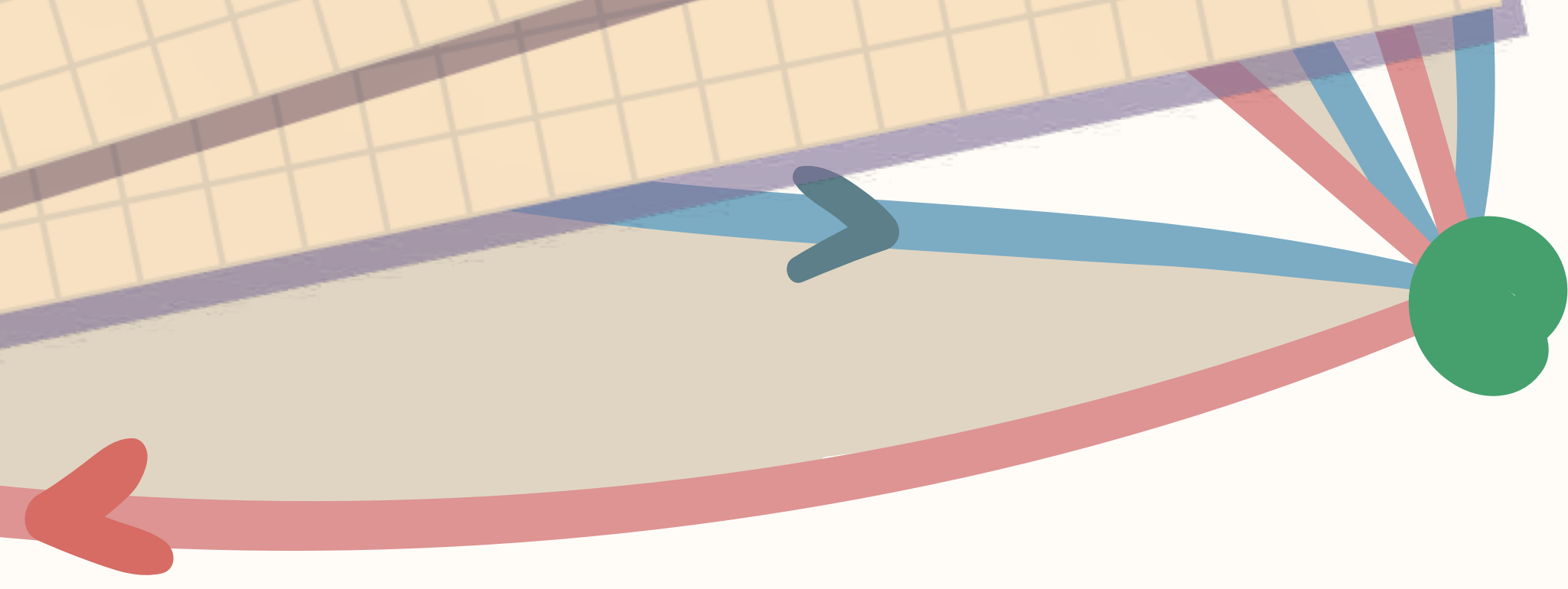
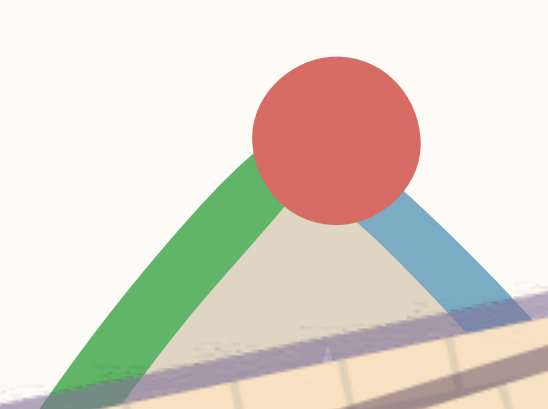
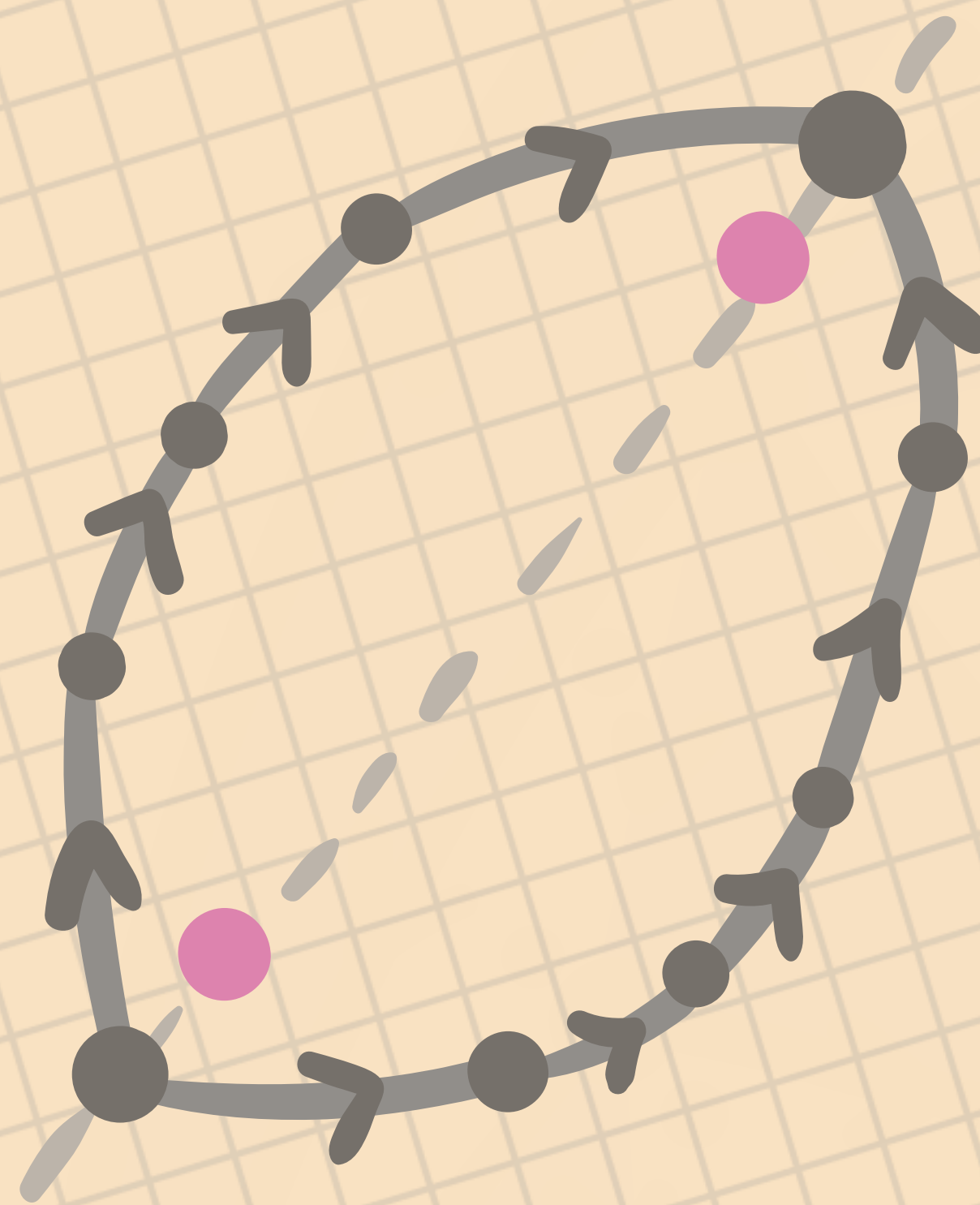
Polyhedral orientati

ON BIPOLAR ORIENTATIONS :



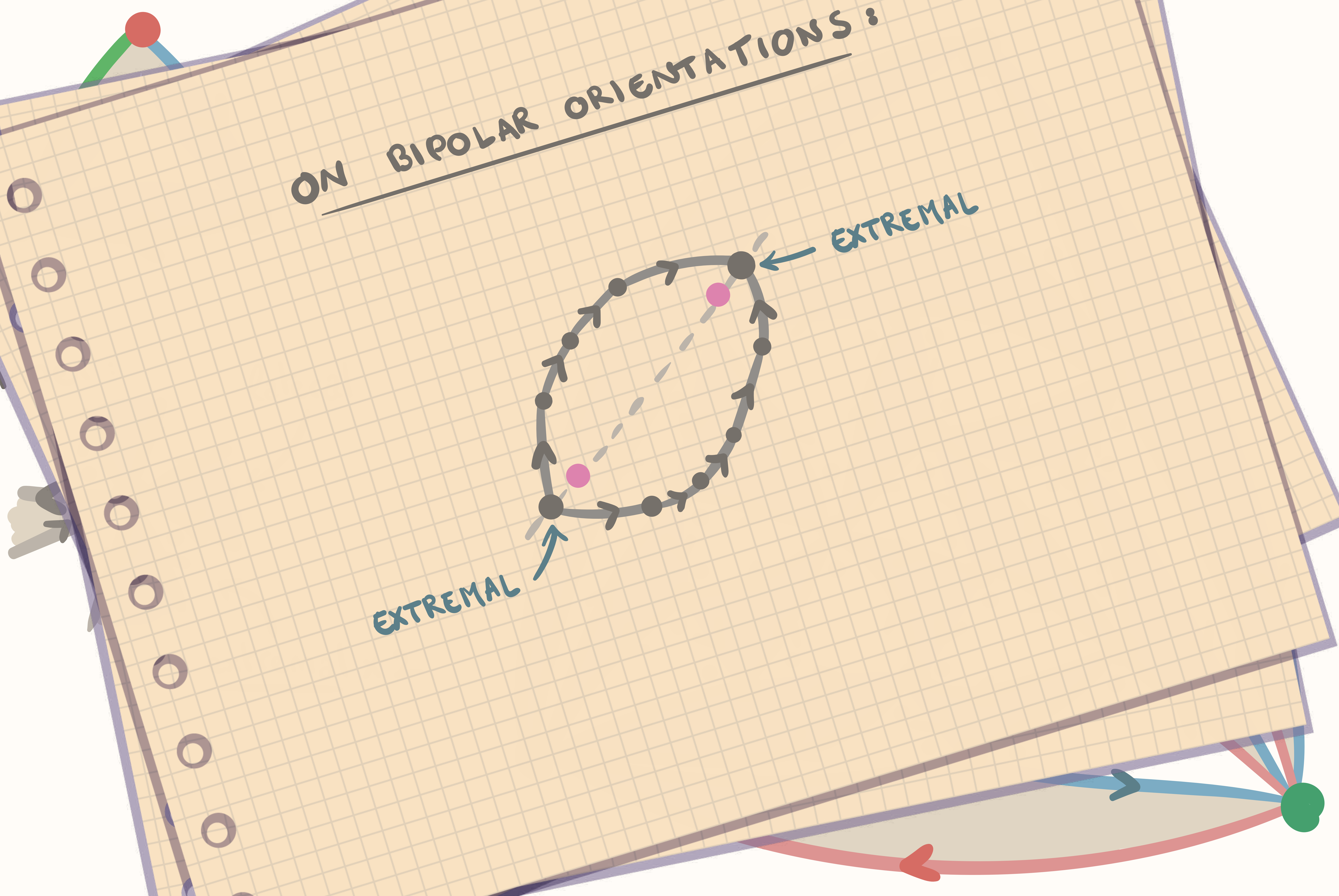
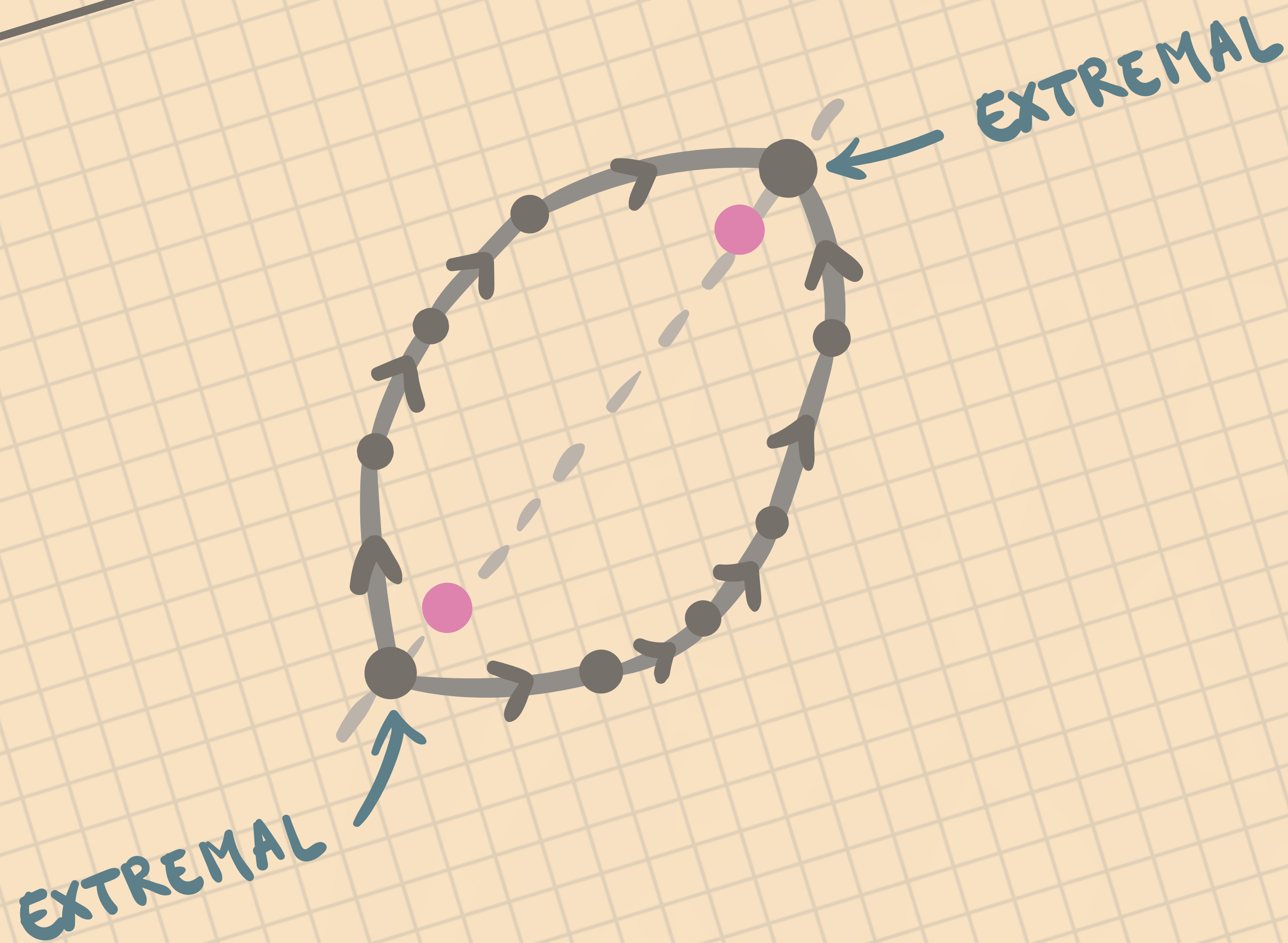
Polyhedral orientati

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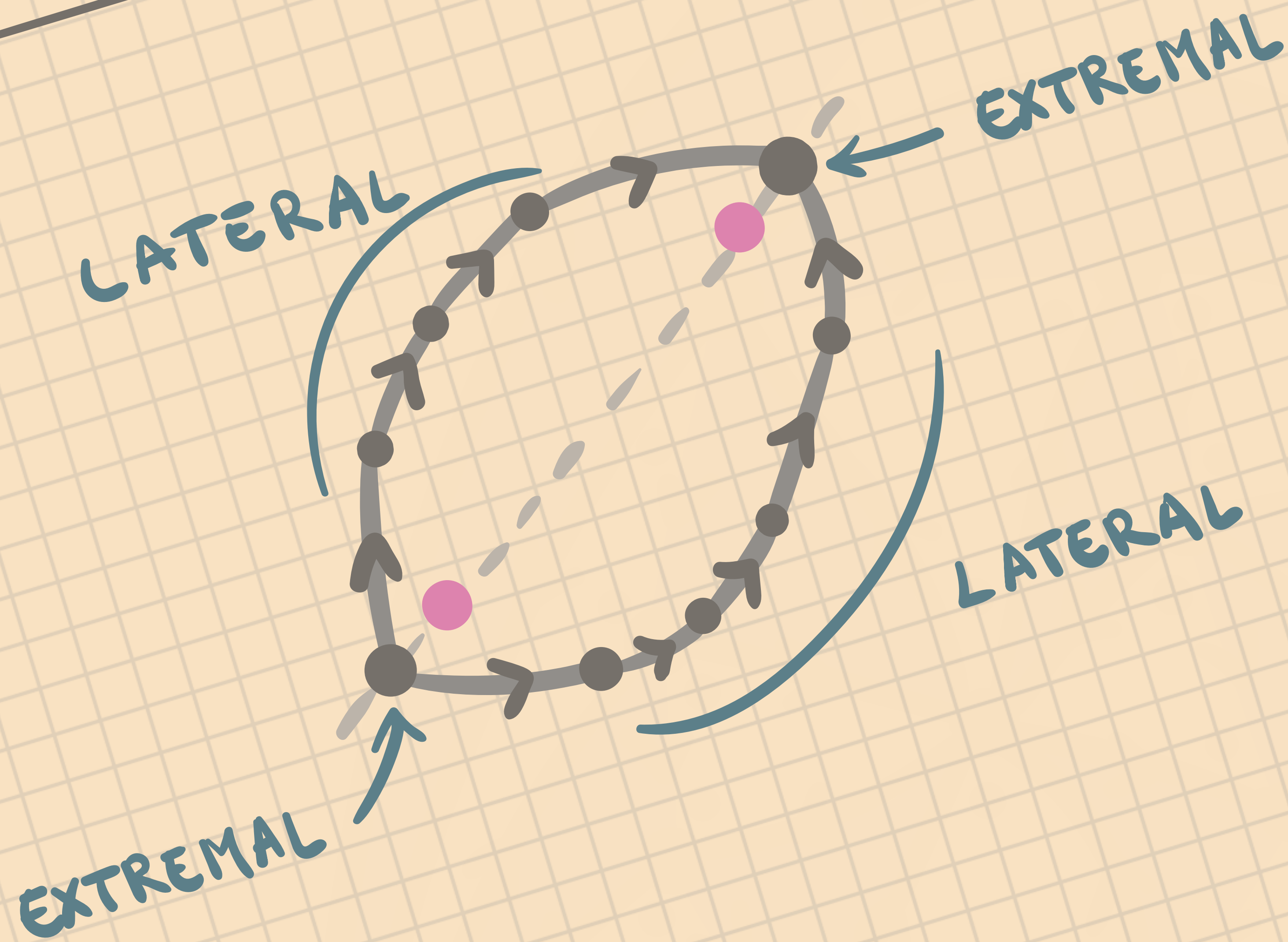
Polyhedral orientation

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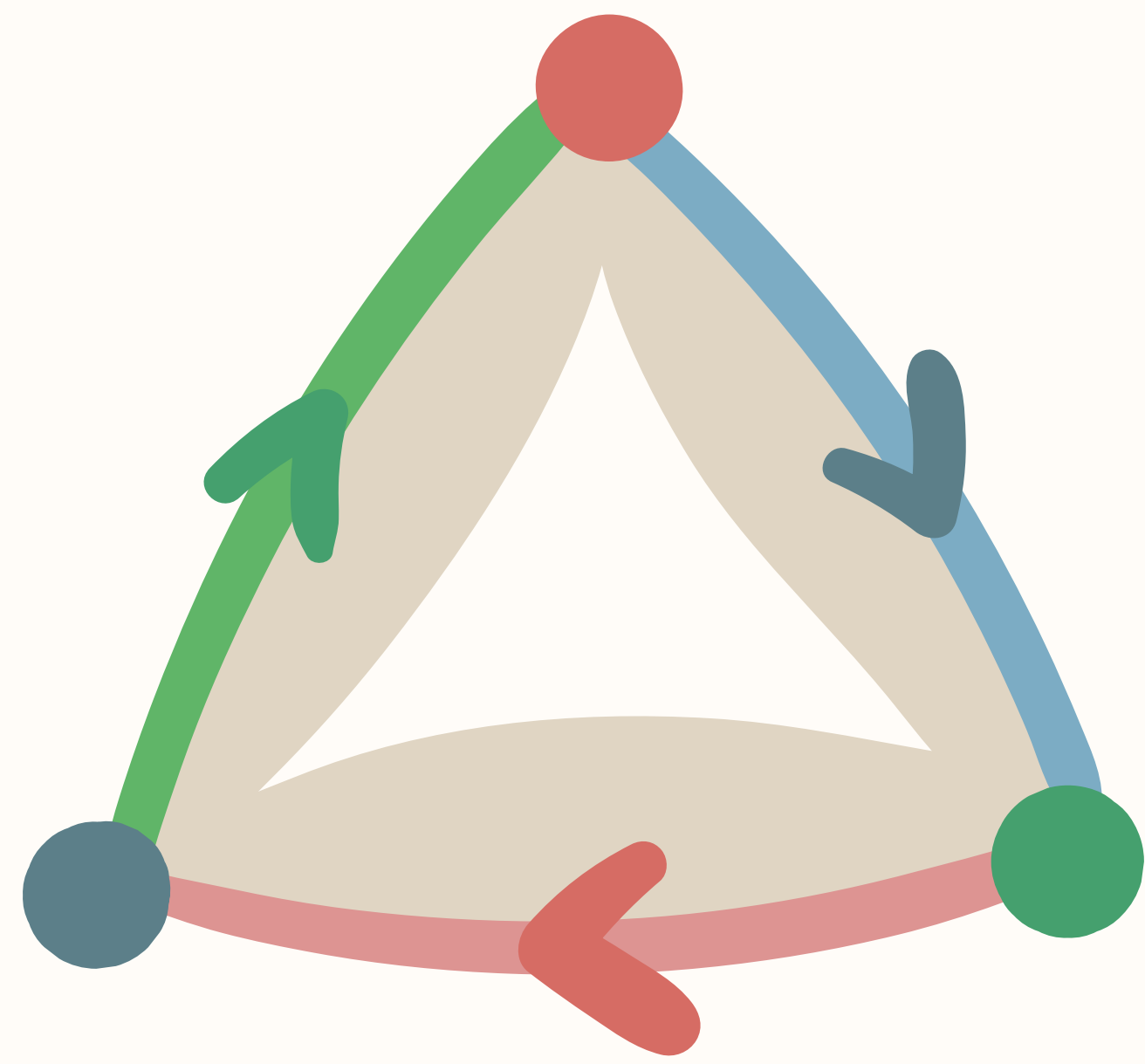


Polyhedral orientation

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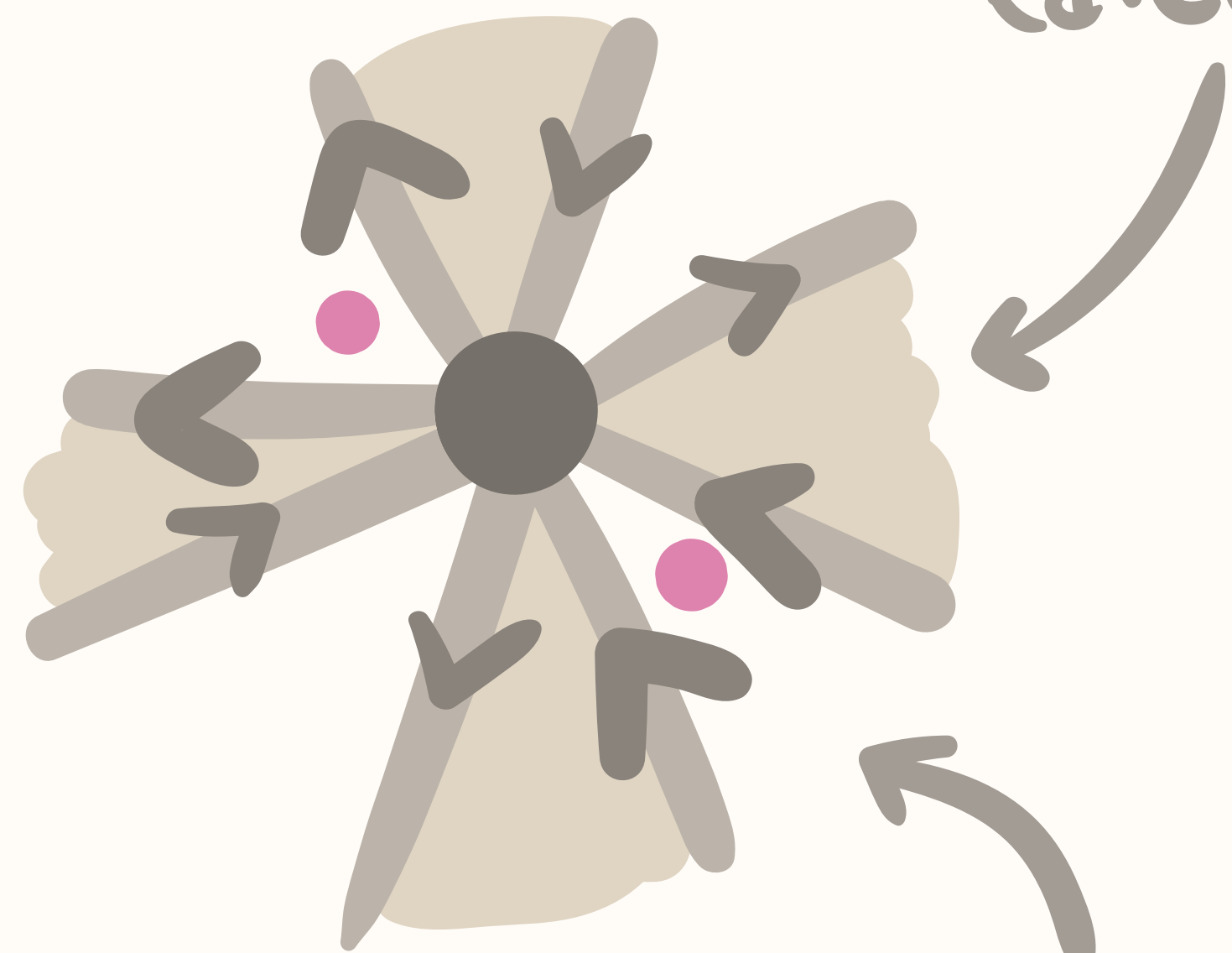


Polyhedral orientations

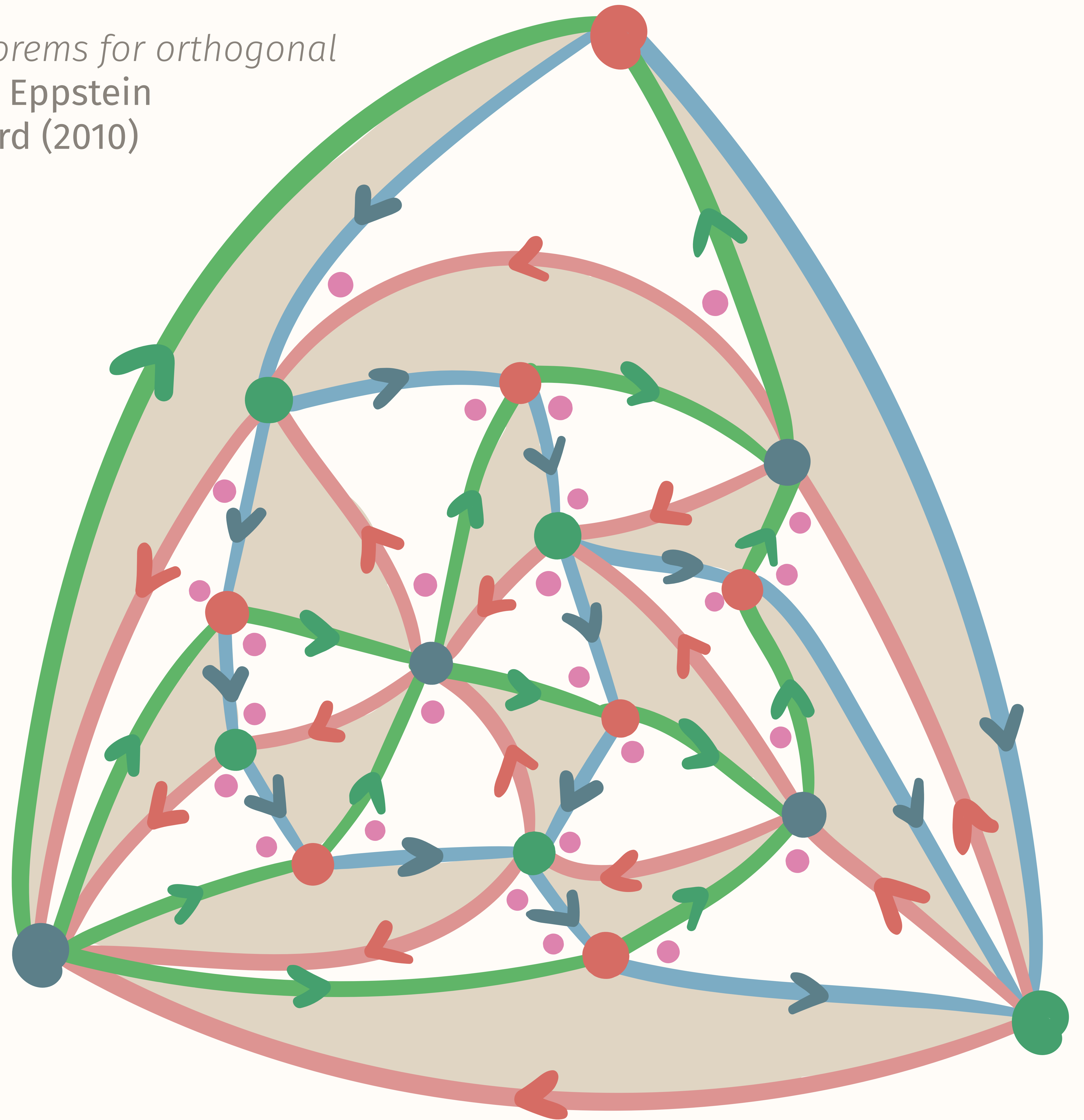


⇒ Steinitz theorems for orthogonal polyhedra, D. Eppstein & E. Mumford (2010)

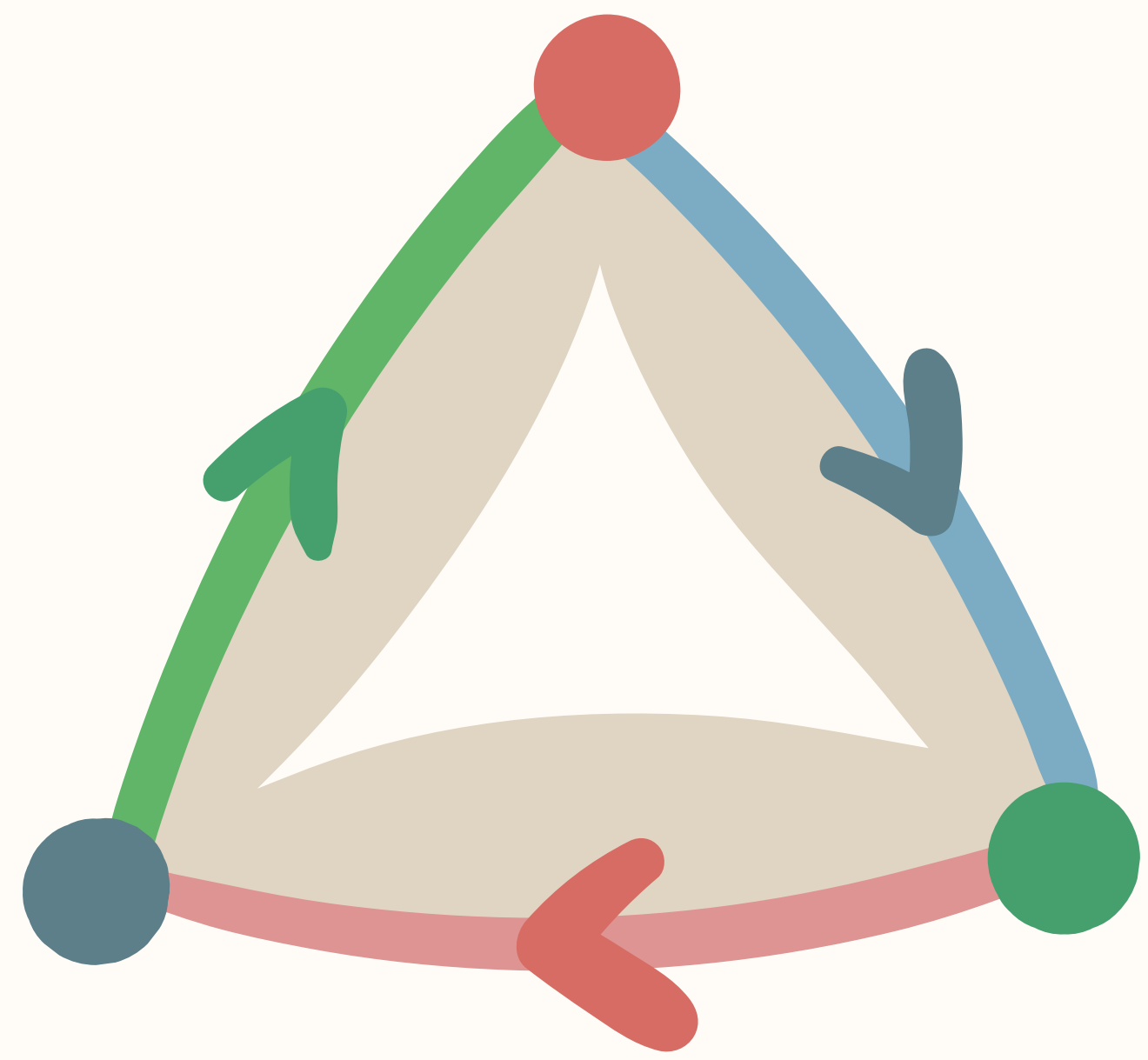
inner:



extremal x 2

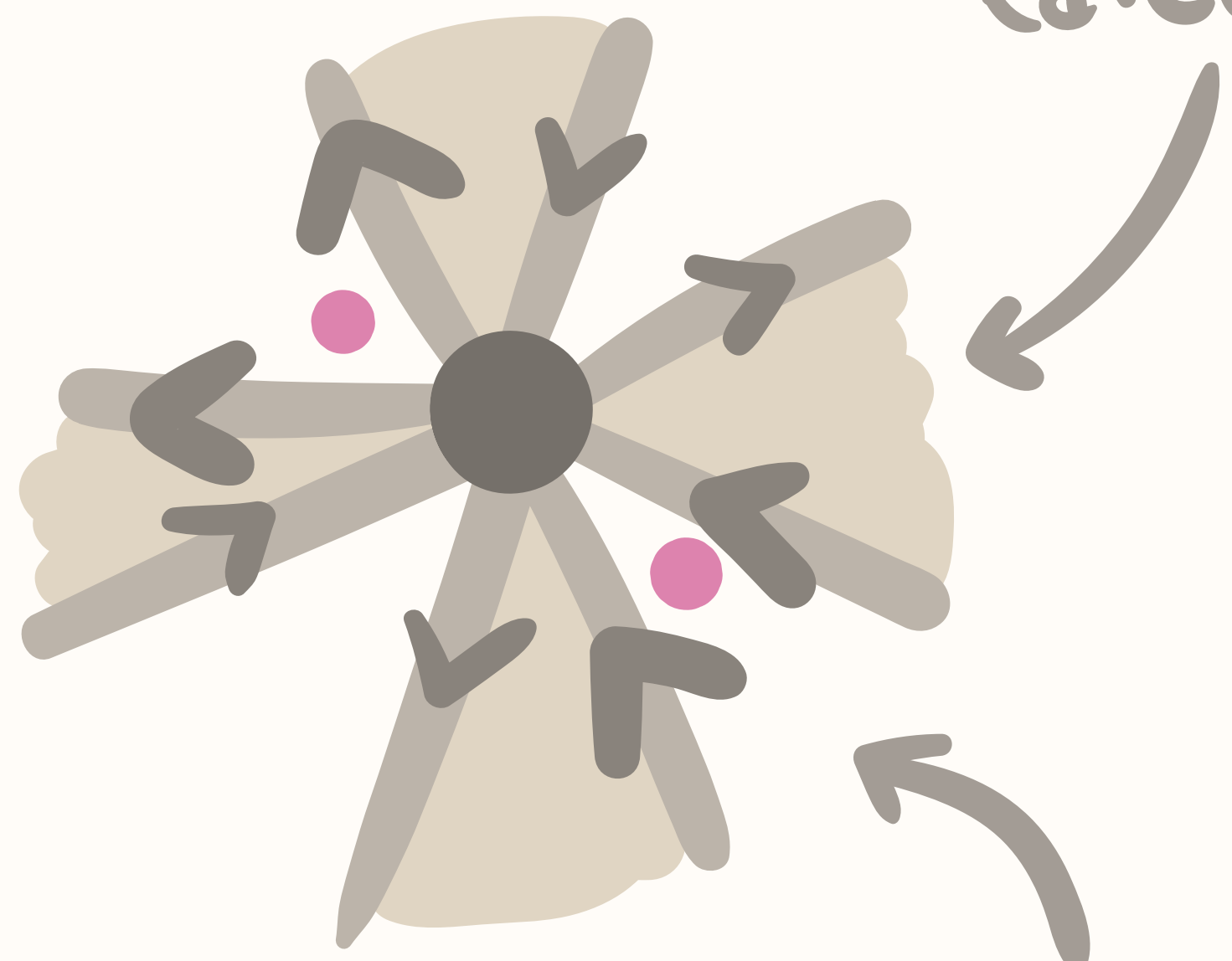


Polyhedral orientations

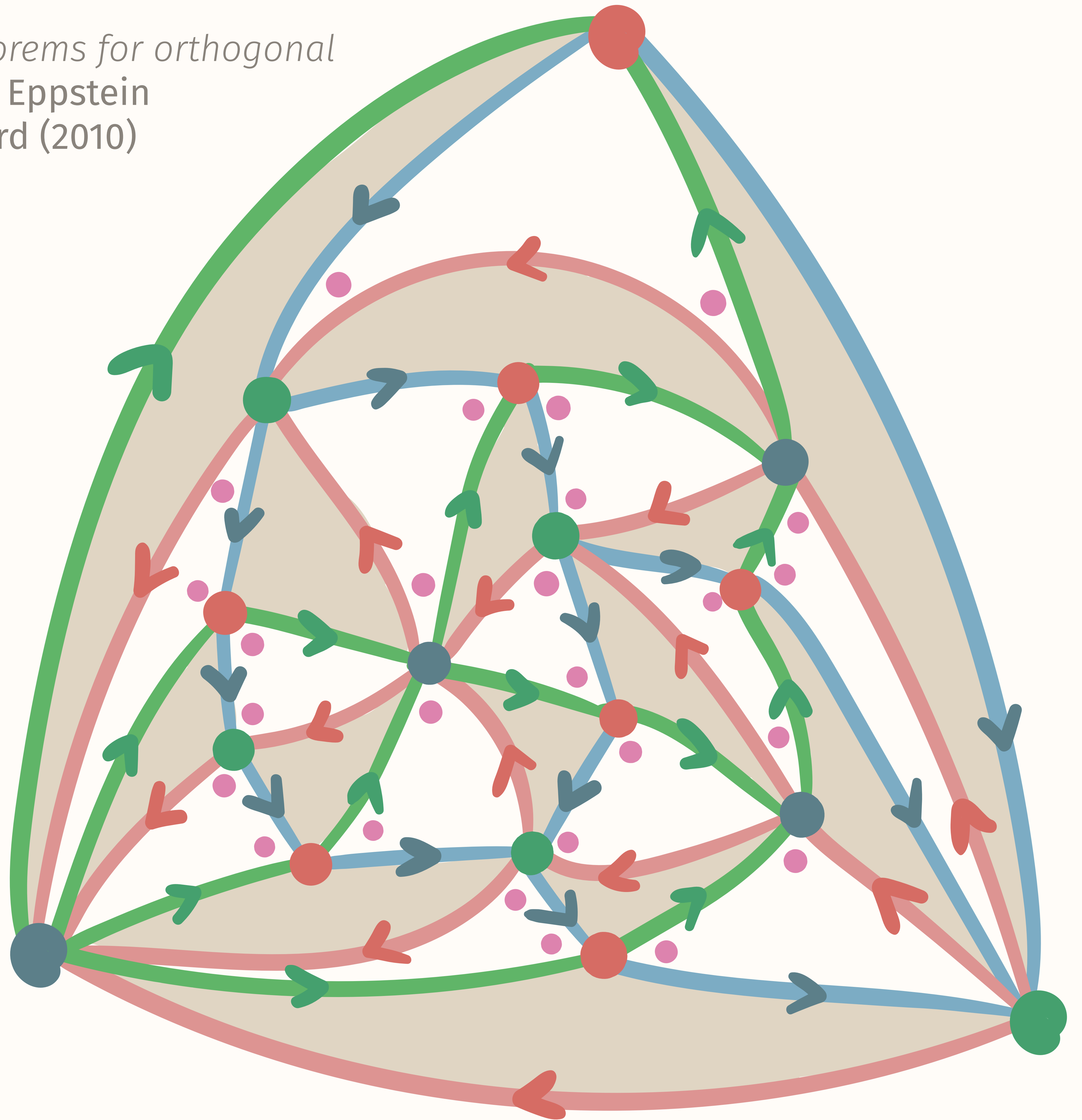
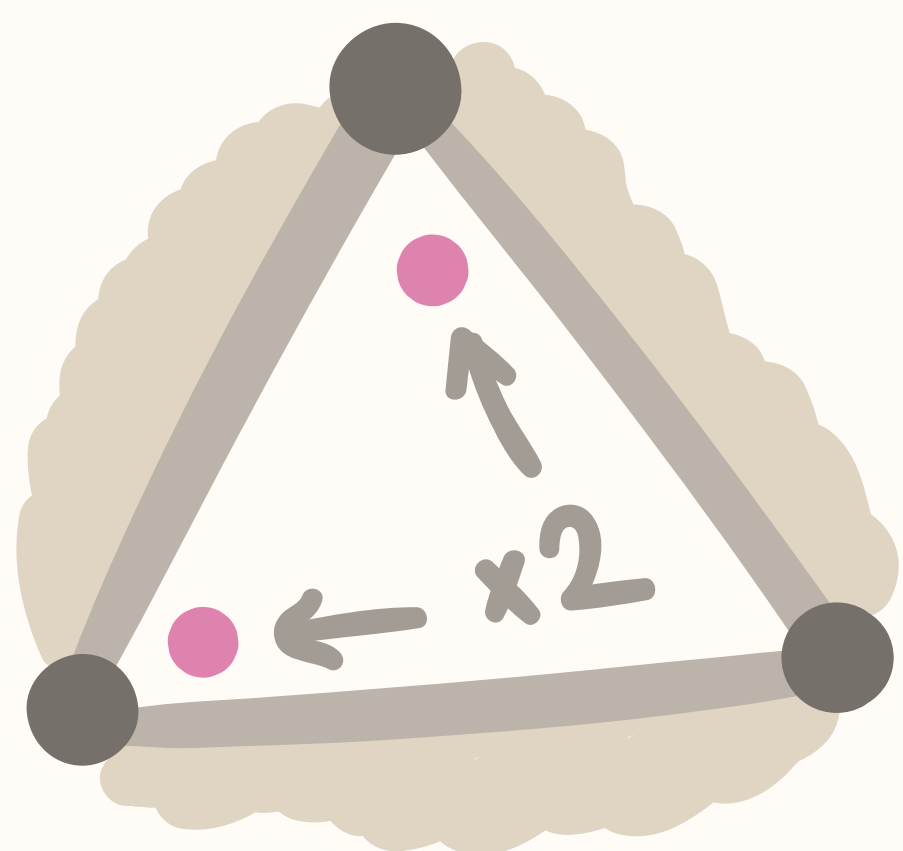


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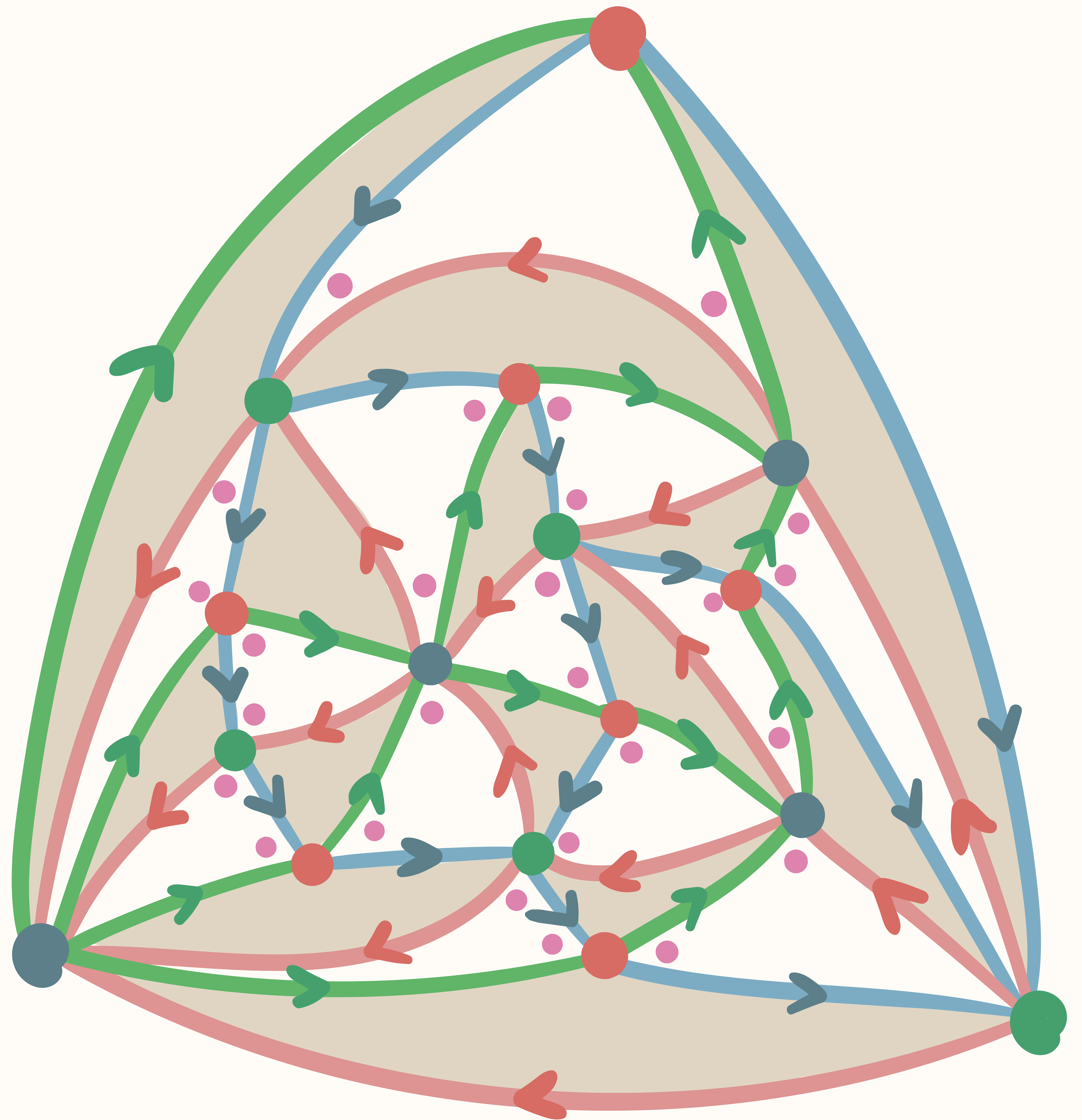
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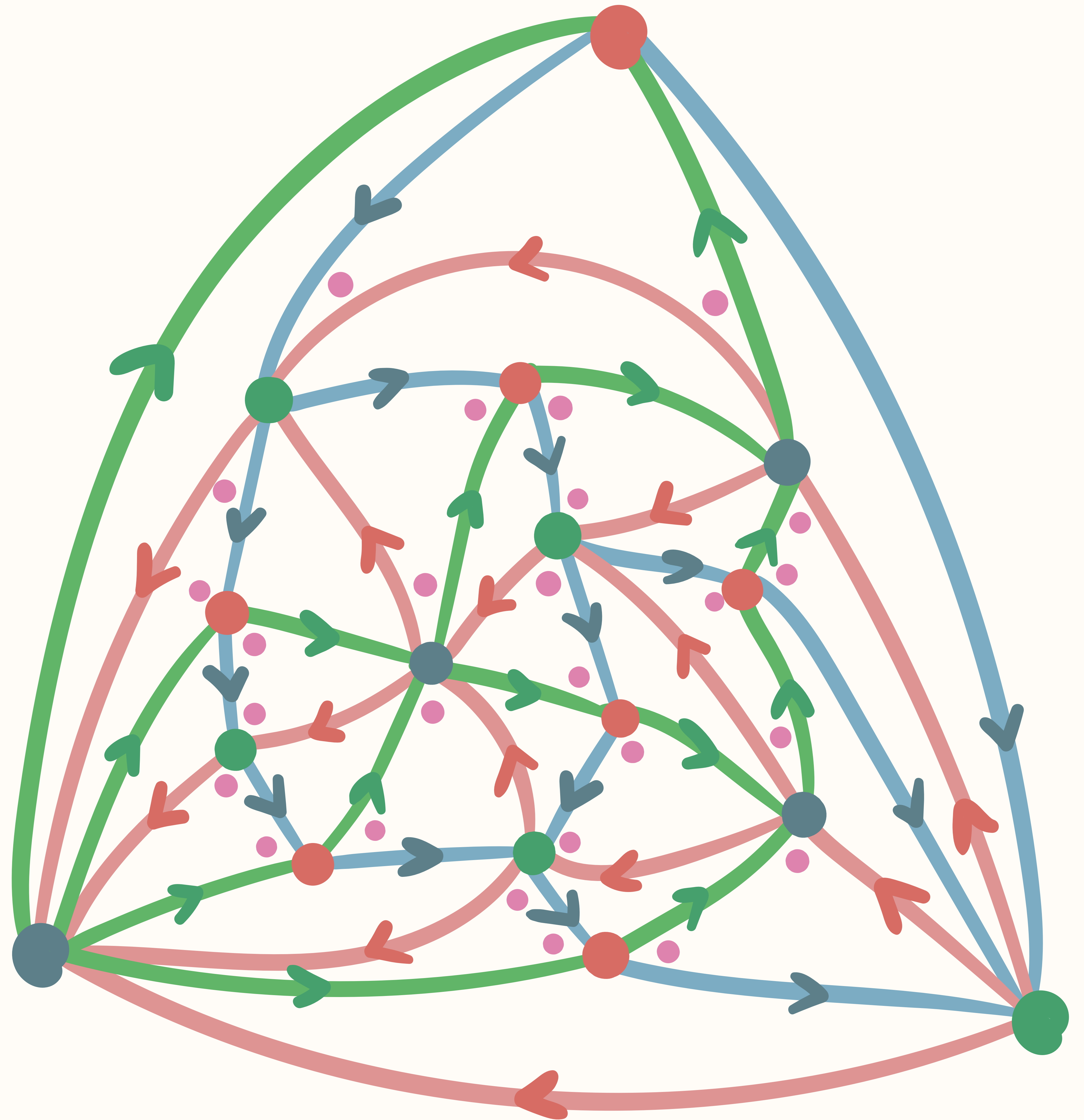
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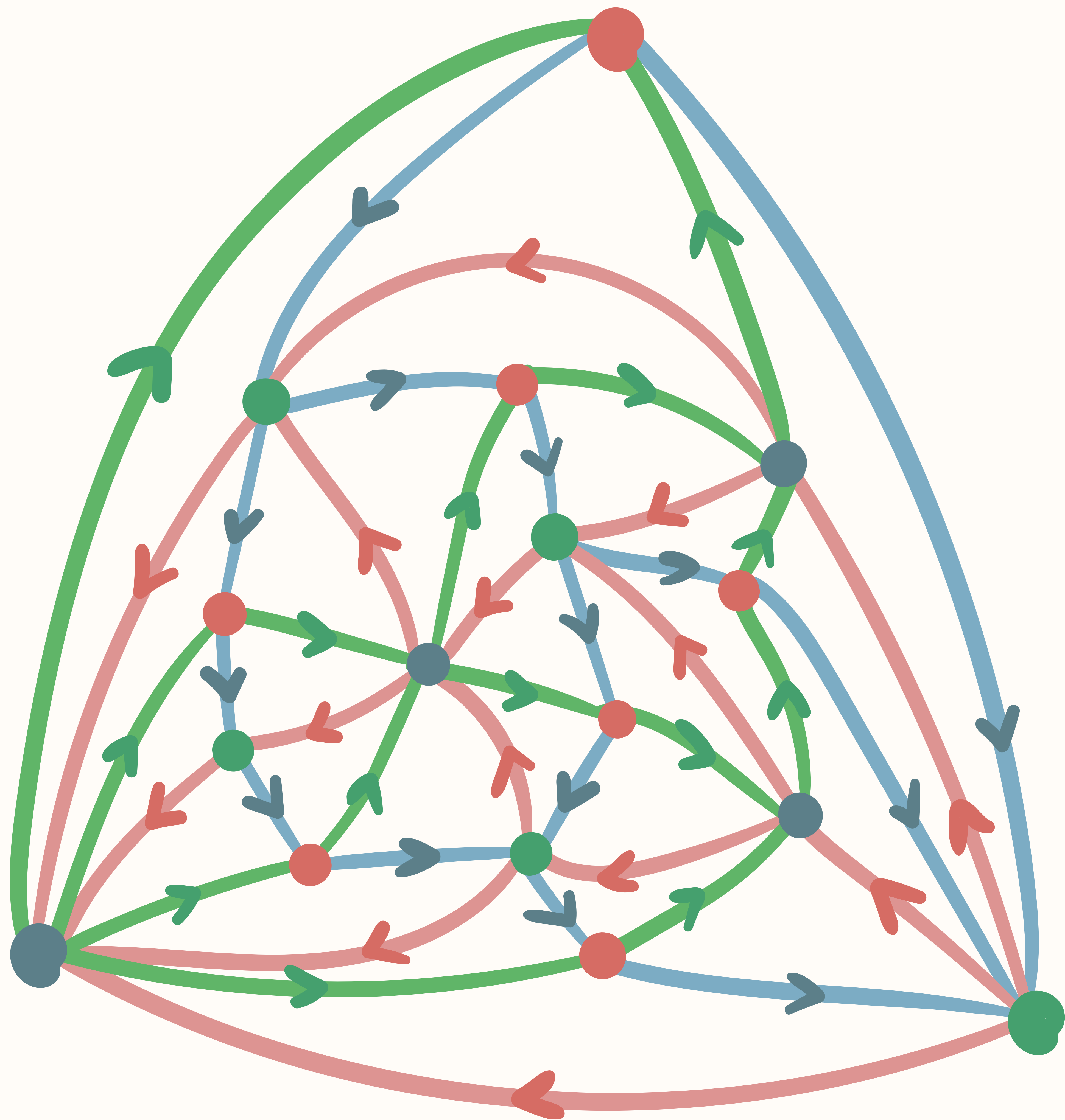
Polyhedral orientations



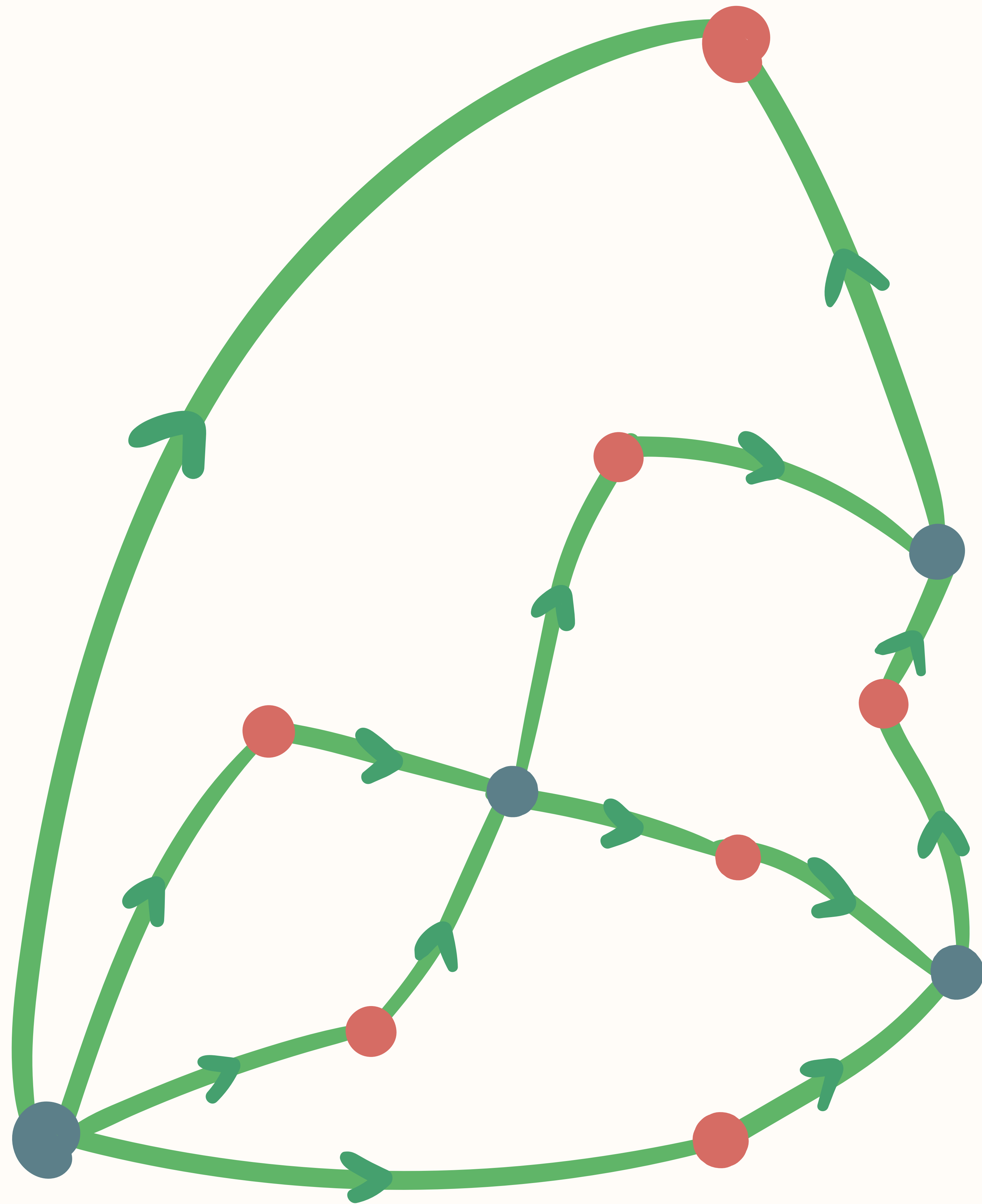
Polyhedral orientations



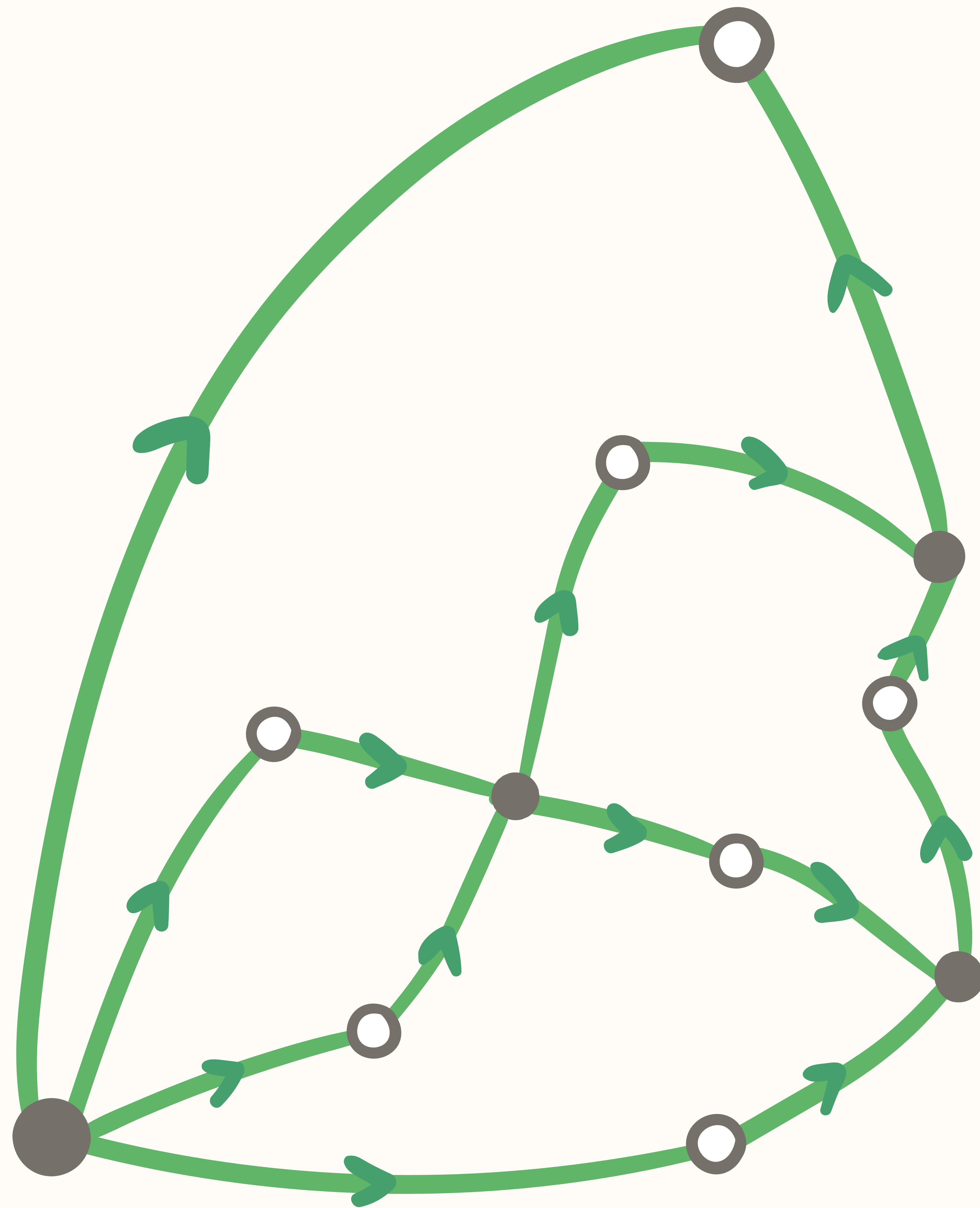
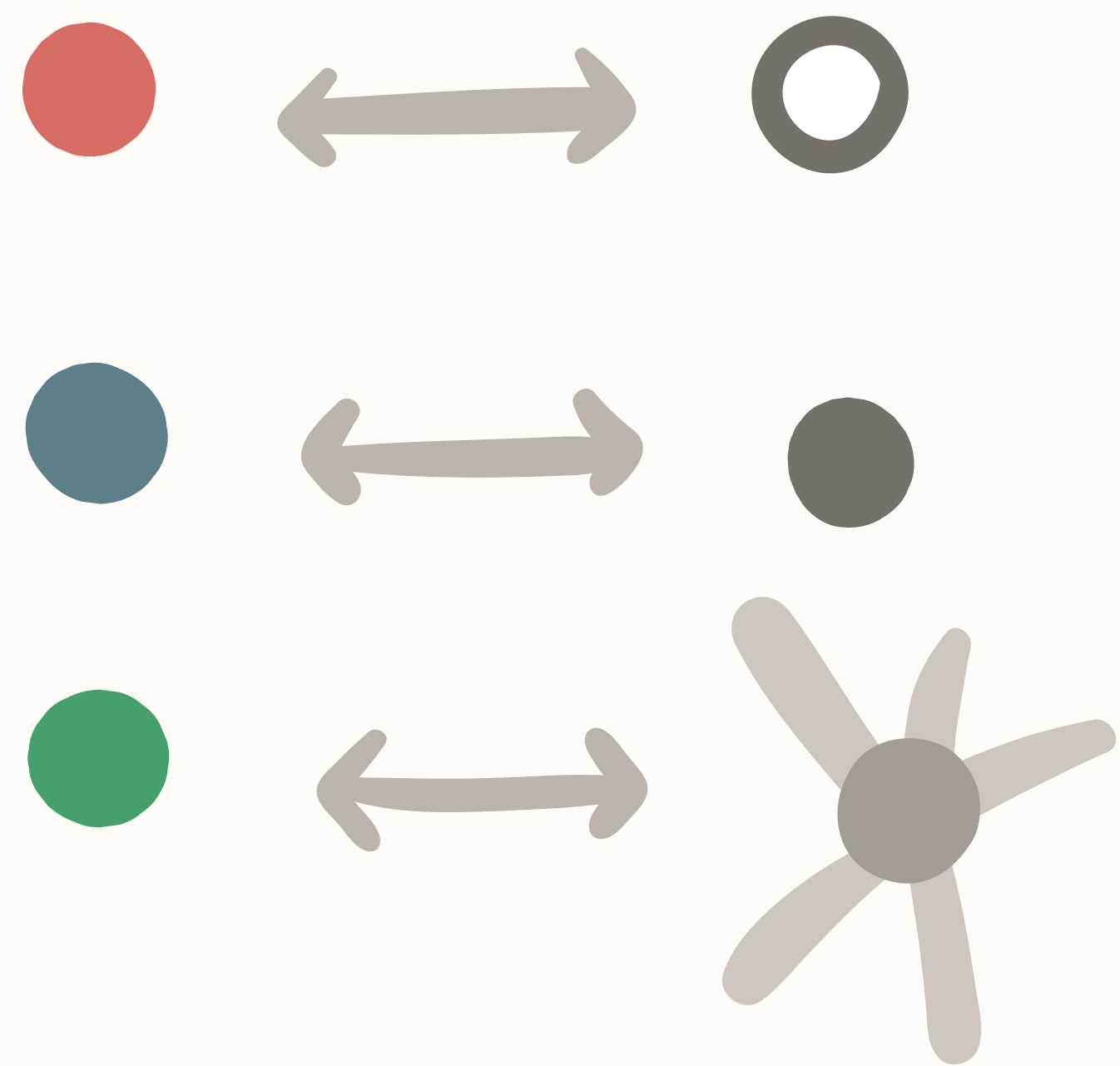
Polyhedral orientations



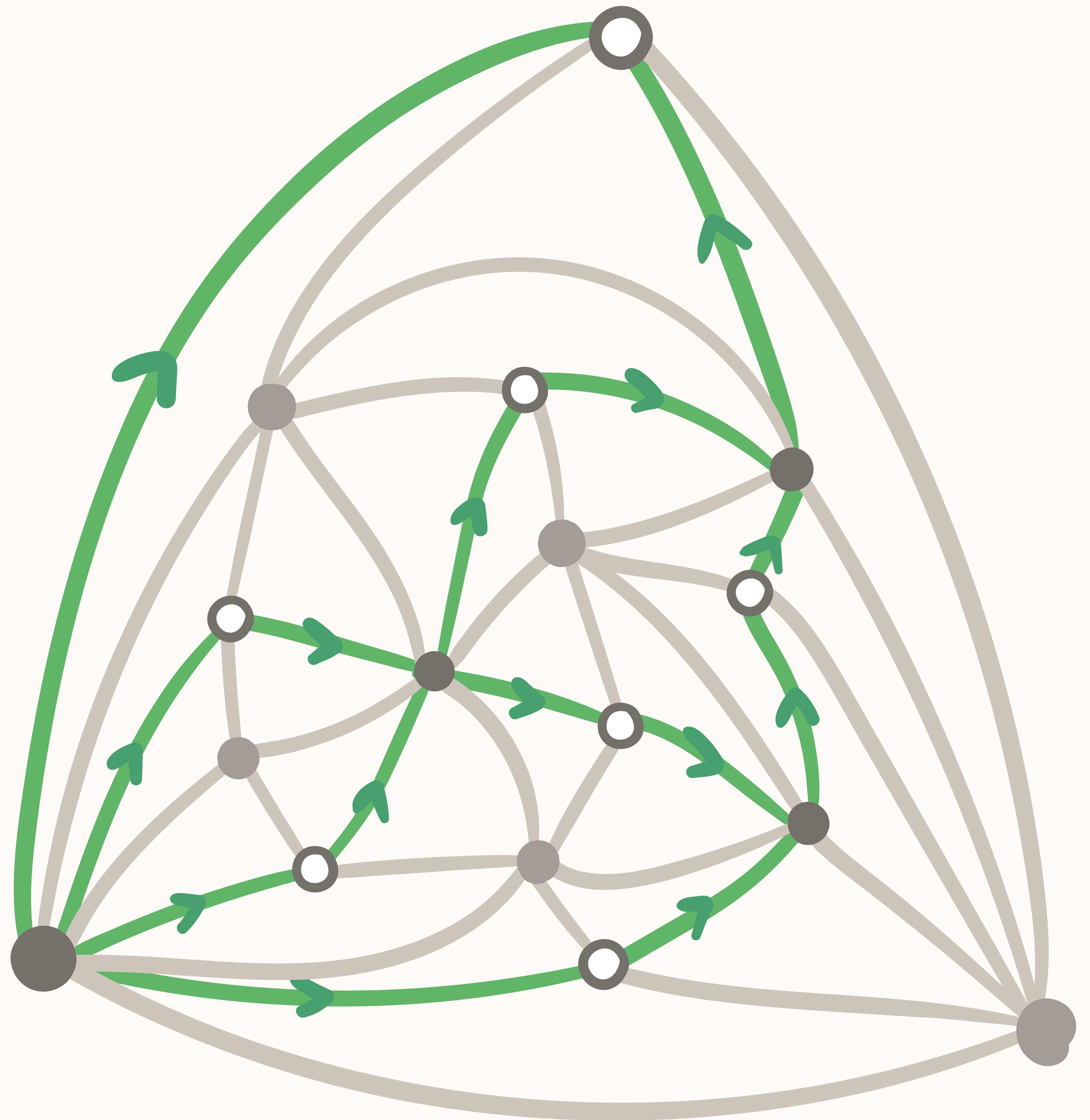
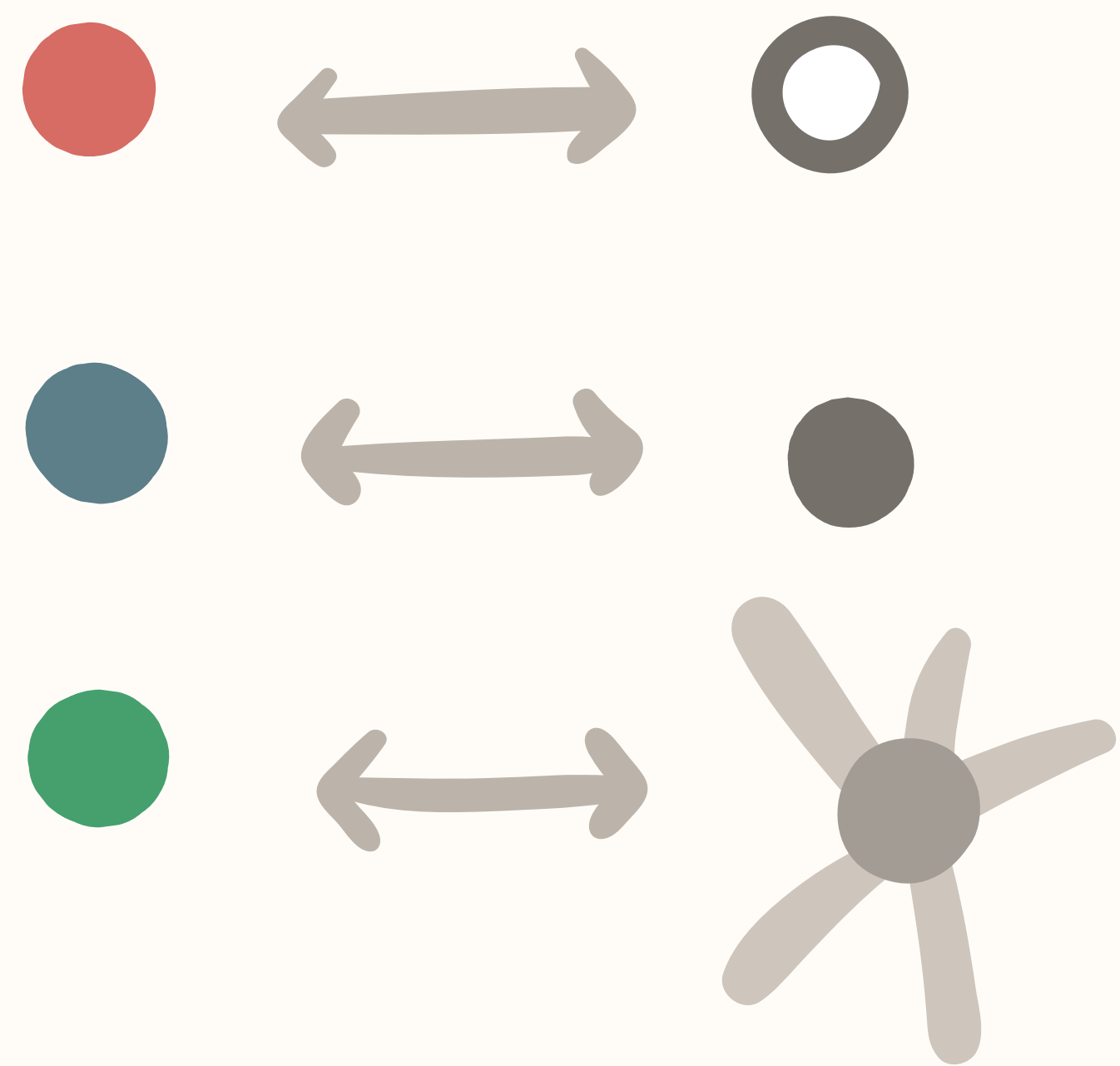
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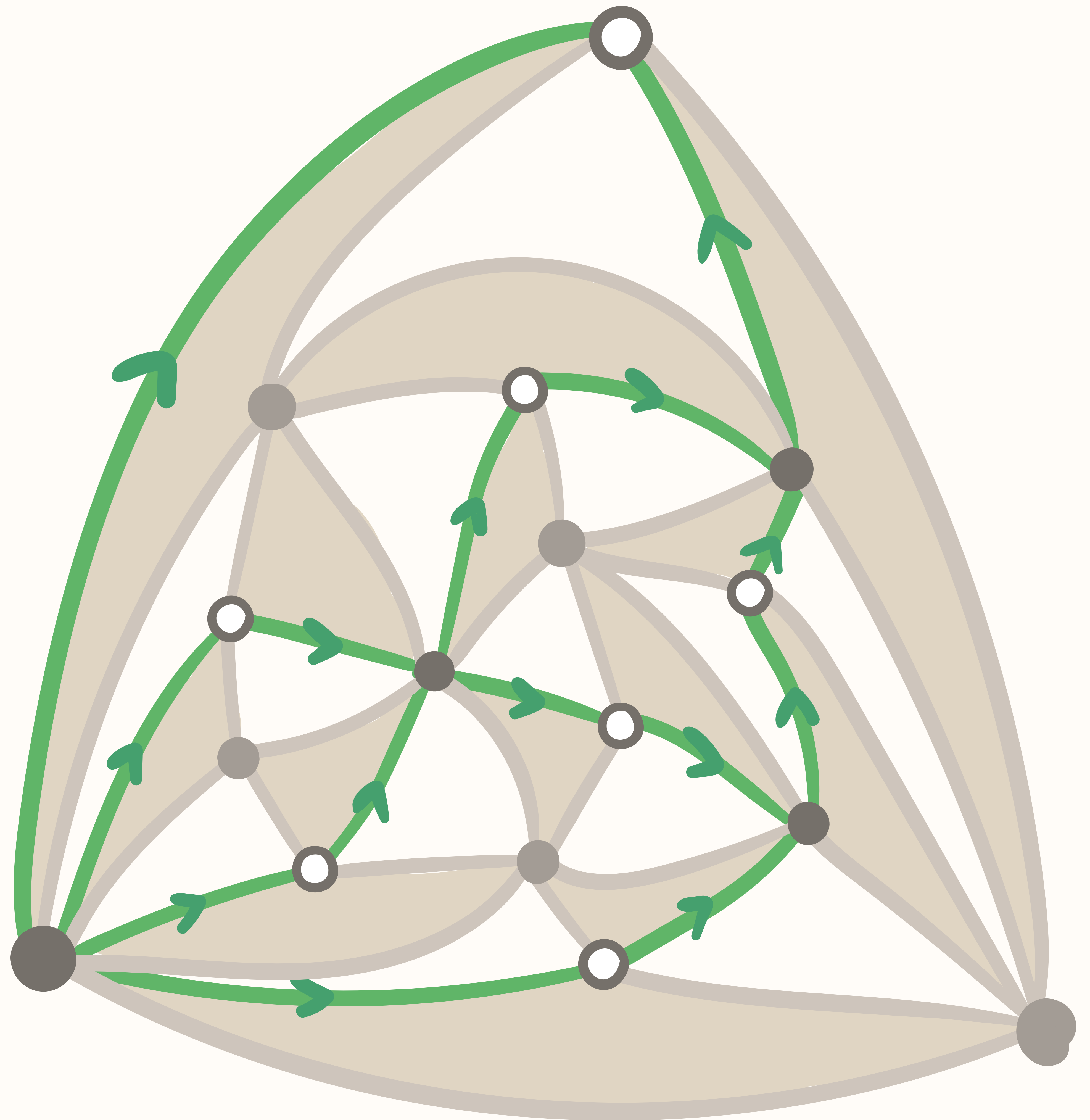
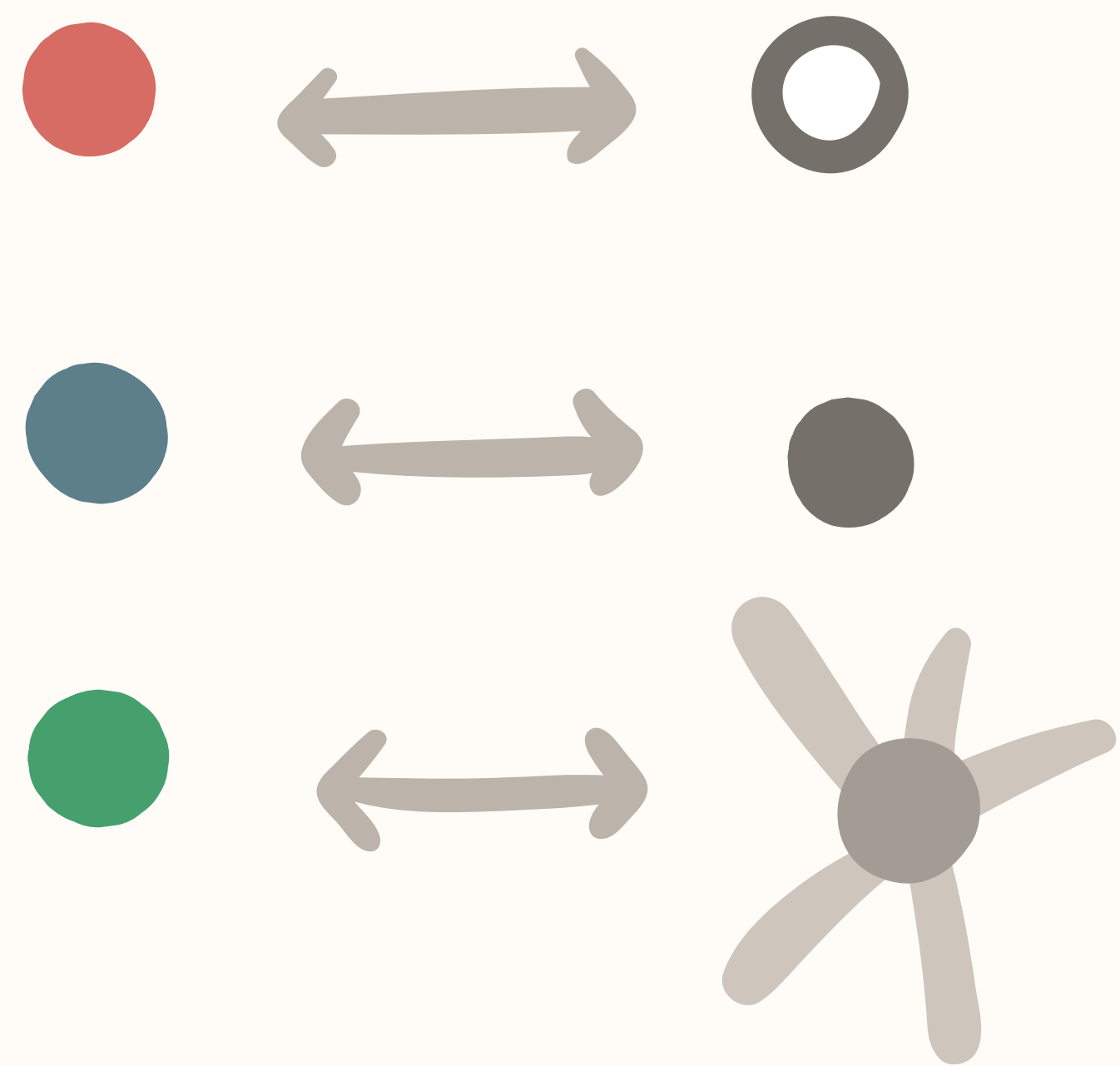
Polyhedral orientations



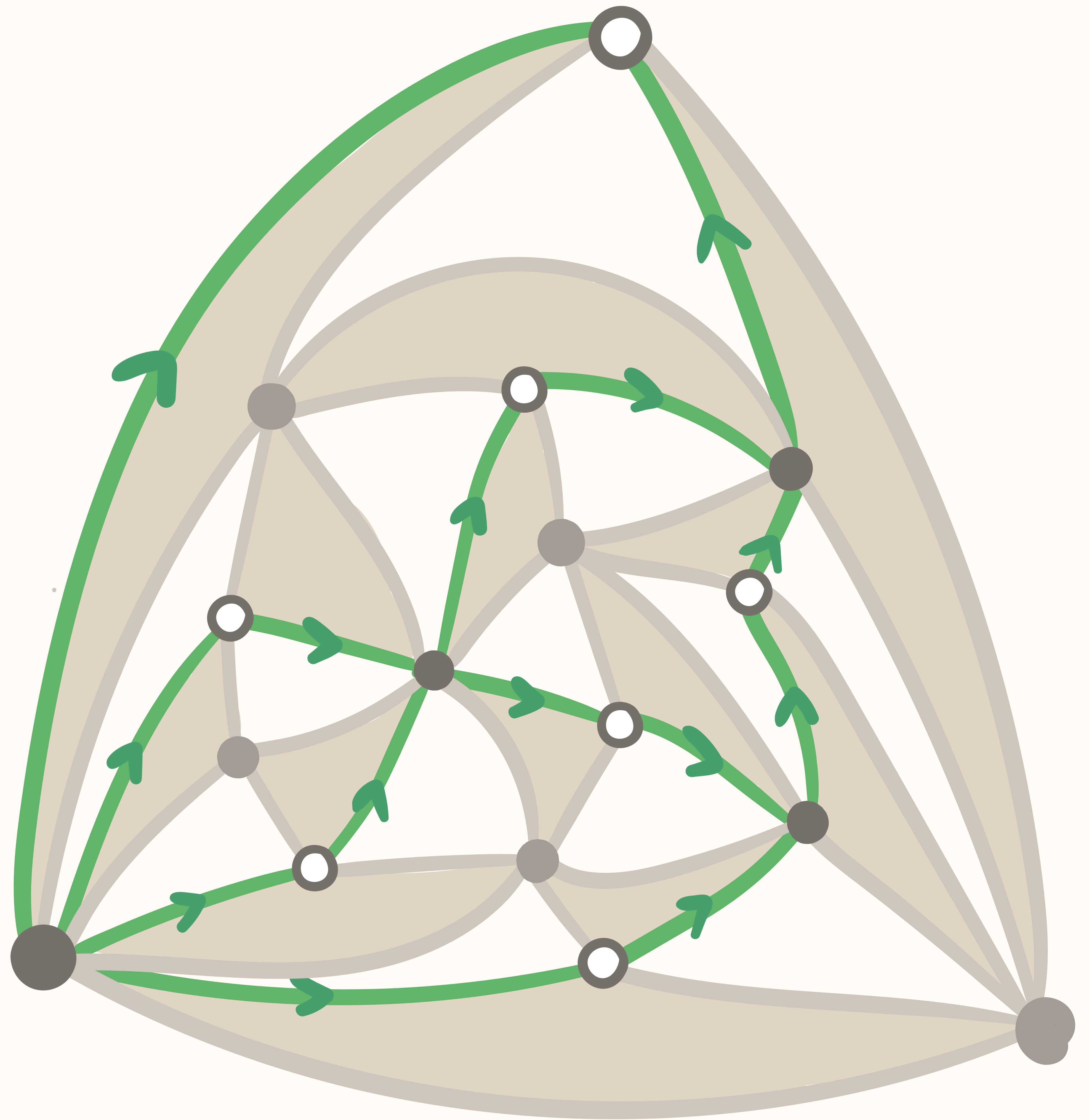
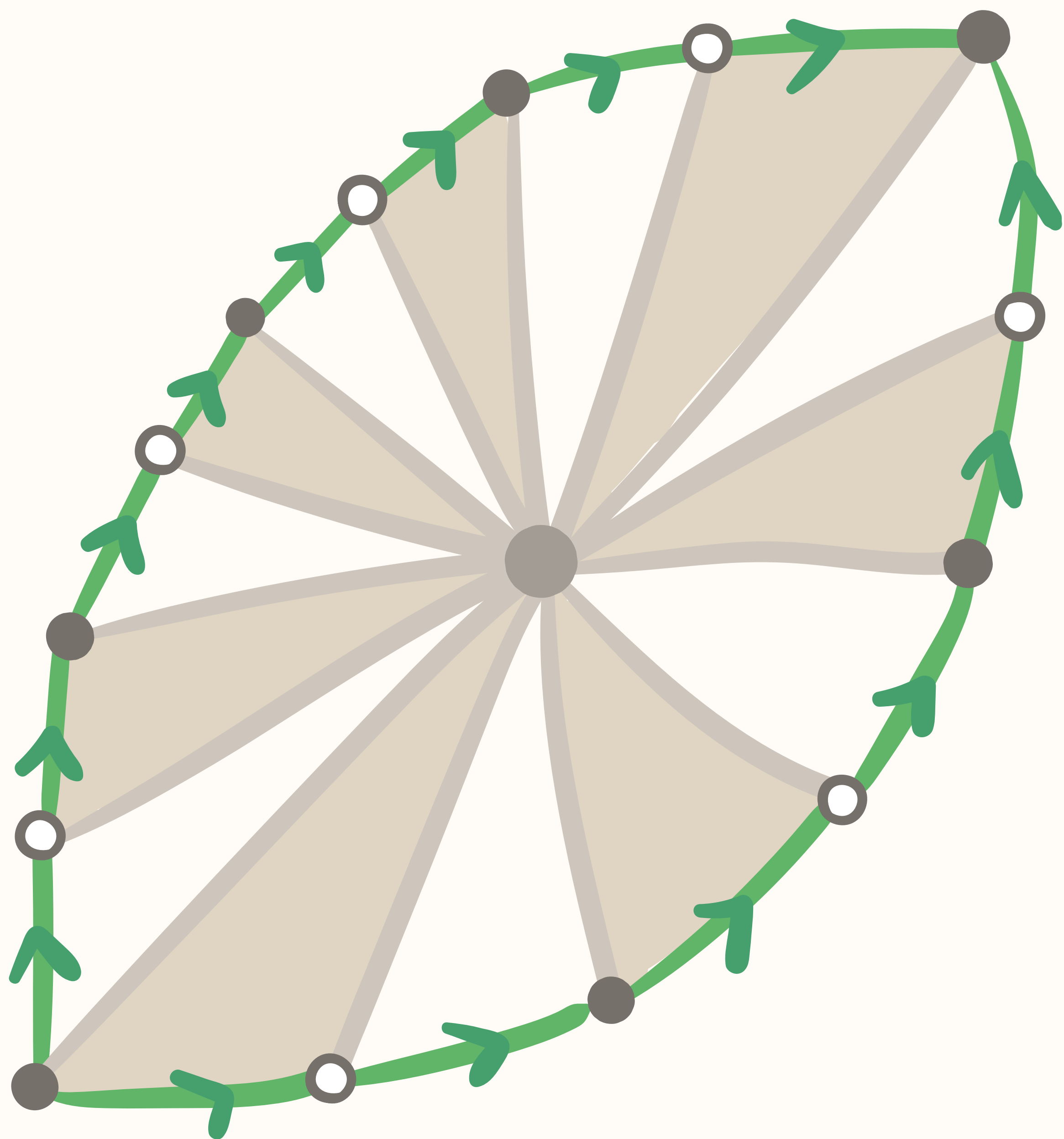
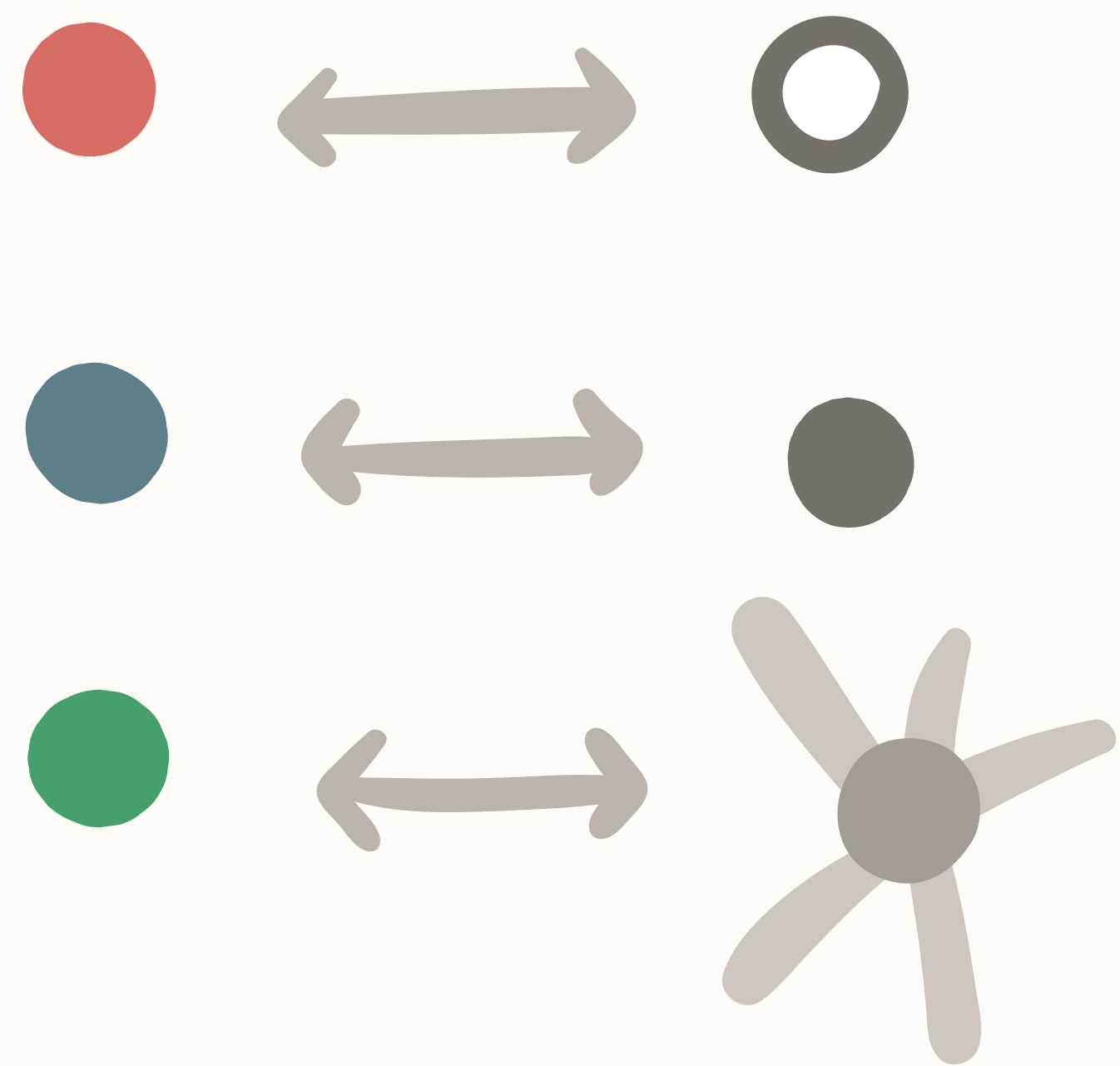
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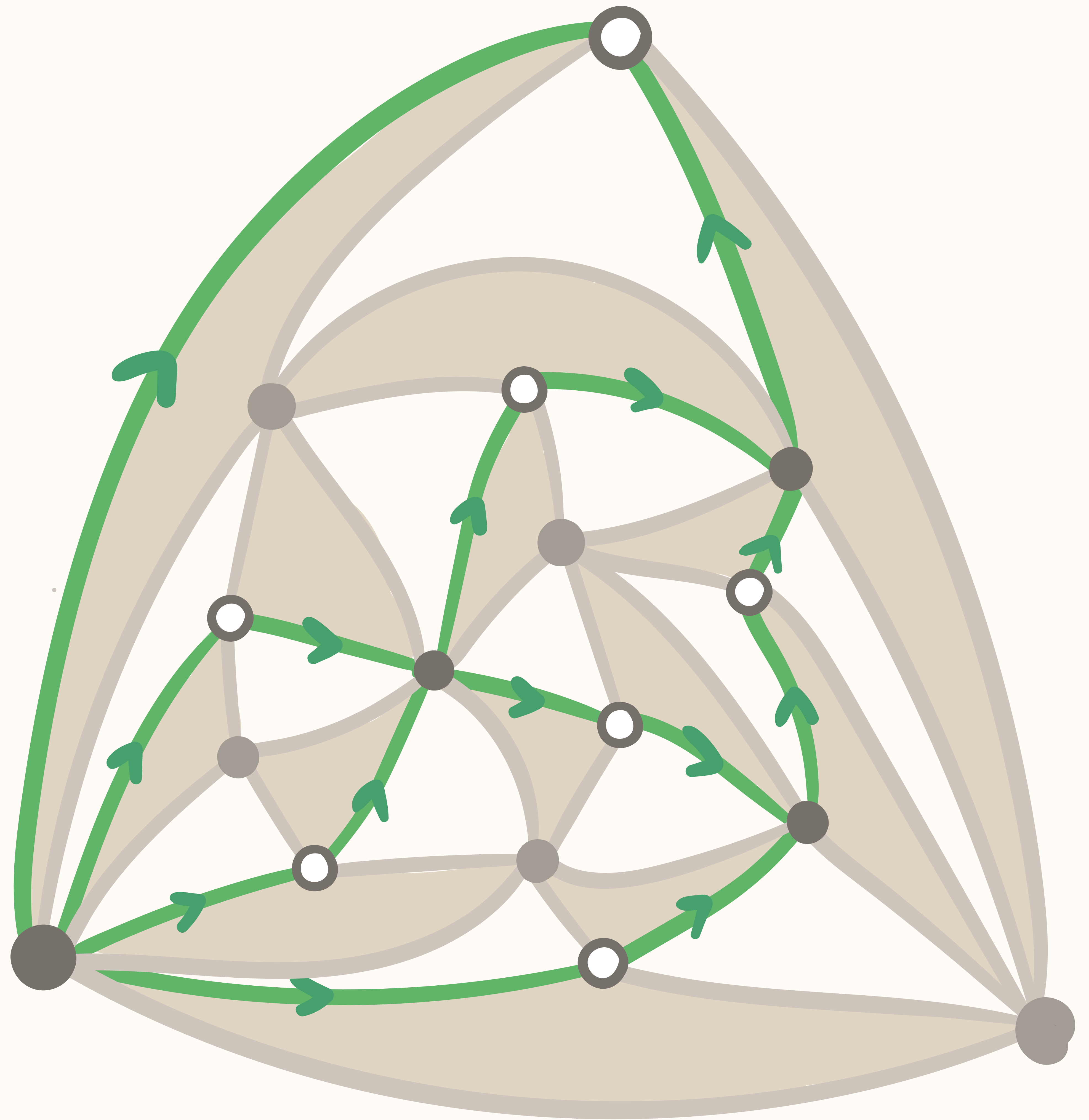
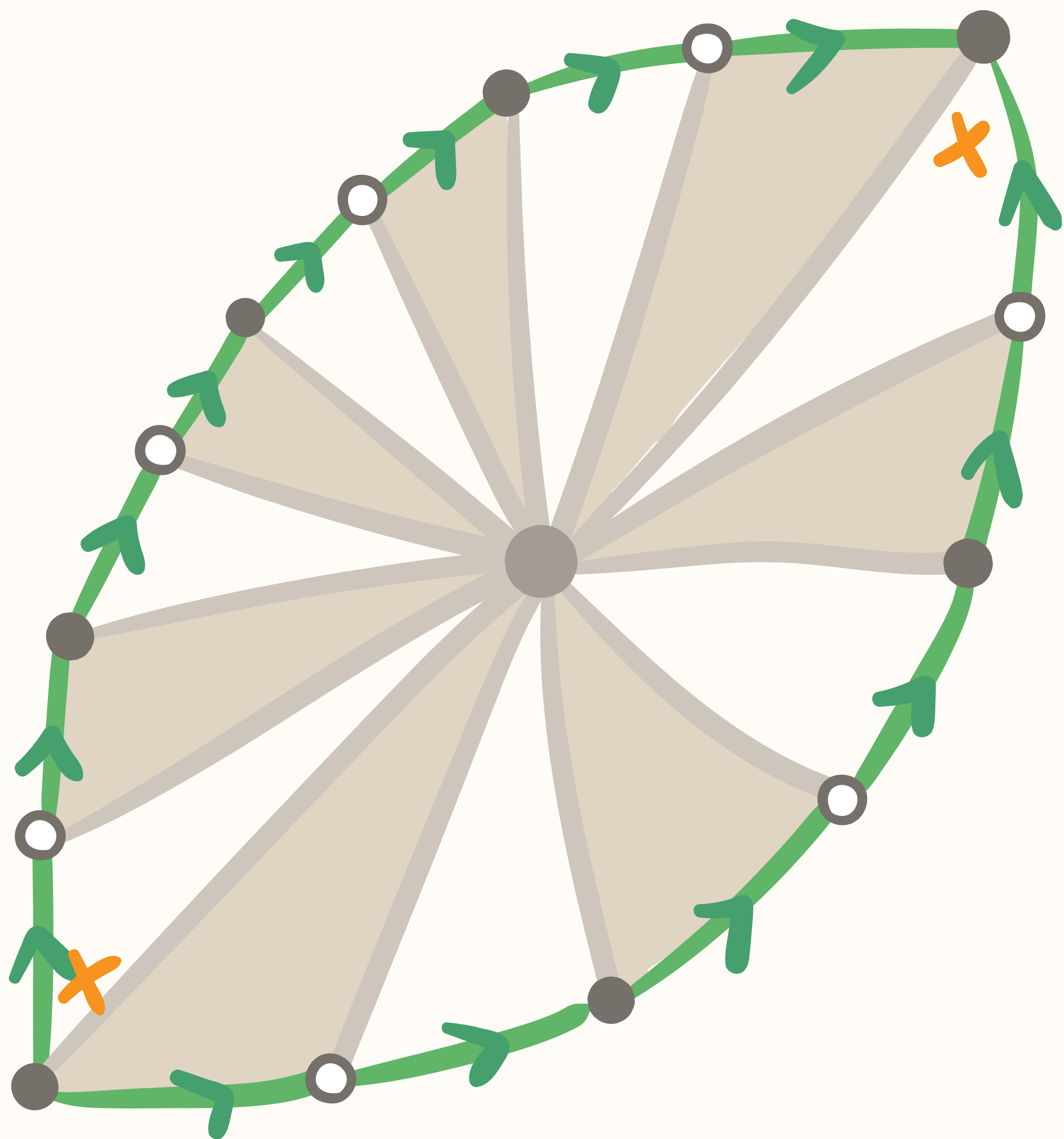
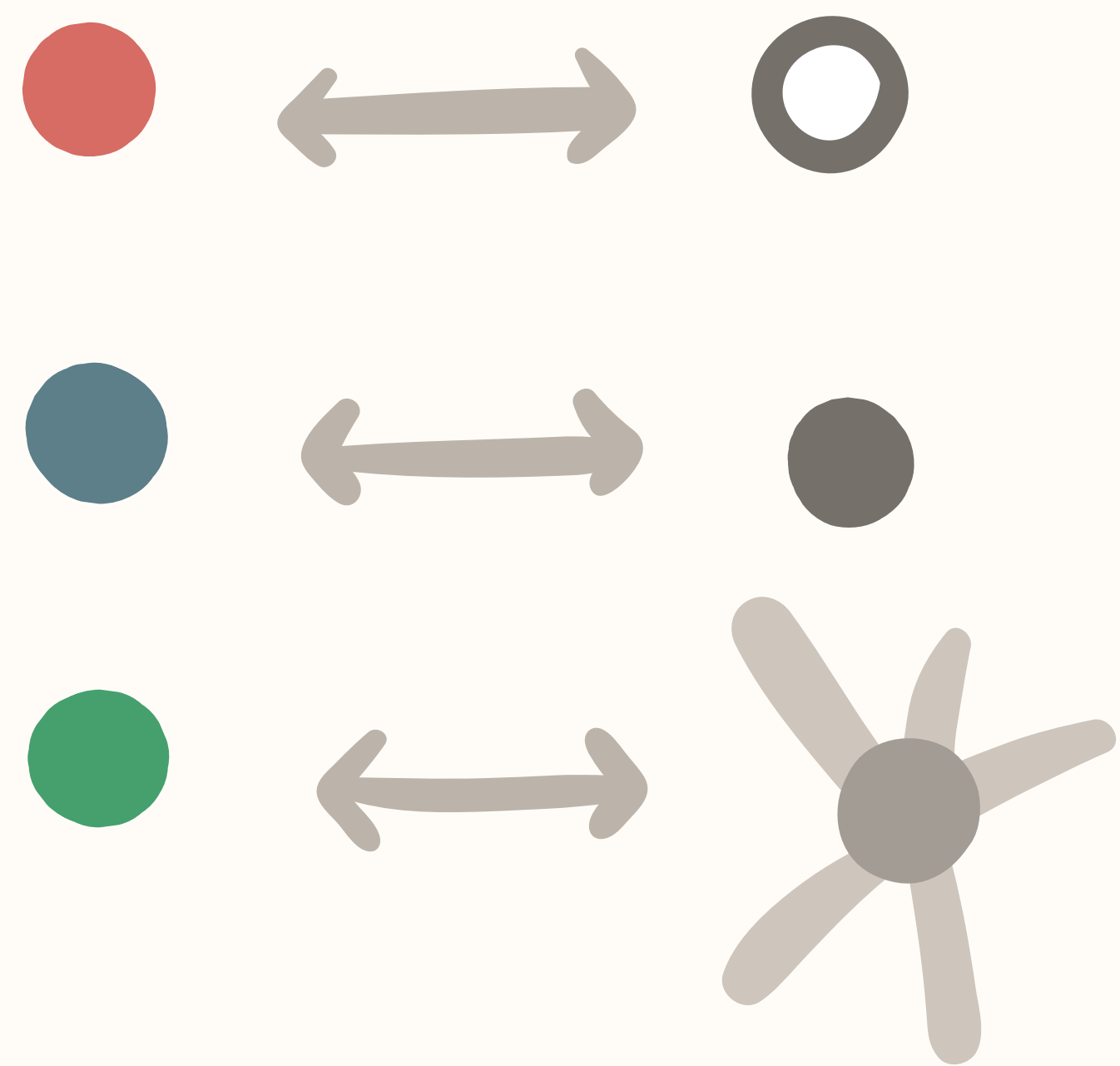
Polyhedral orientations



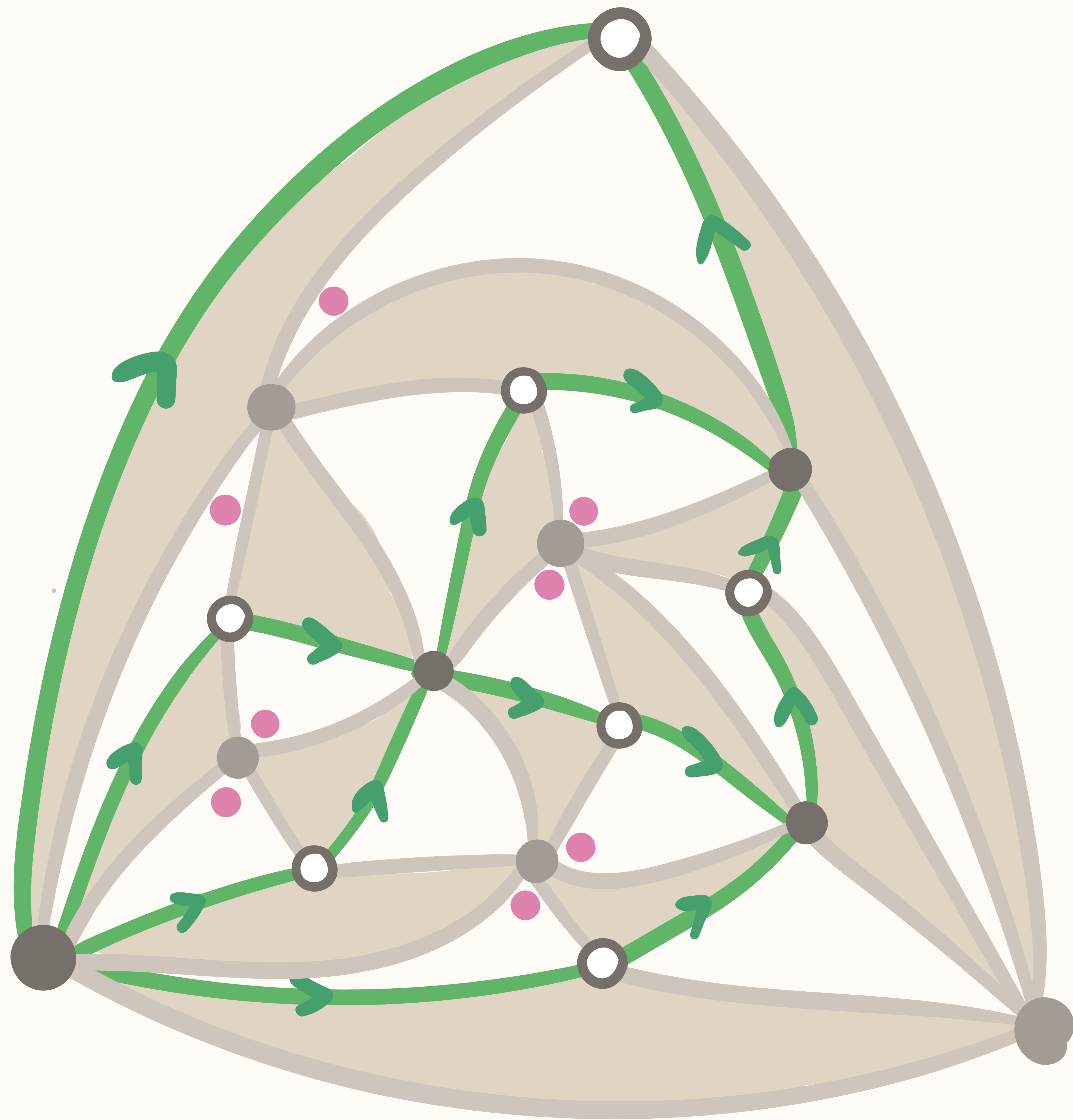
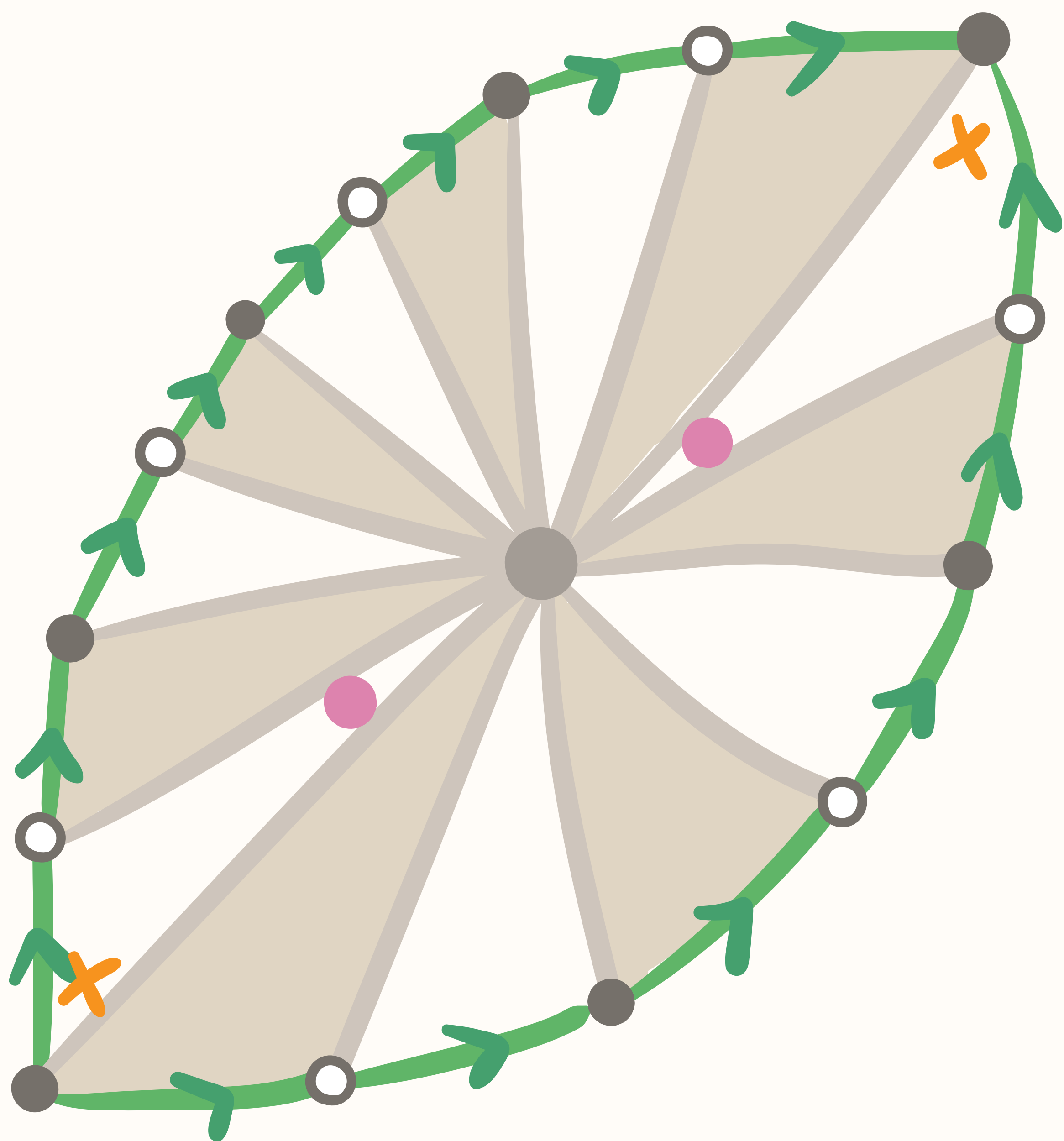
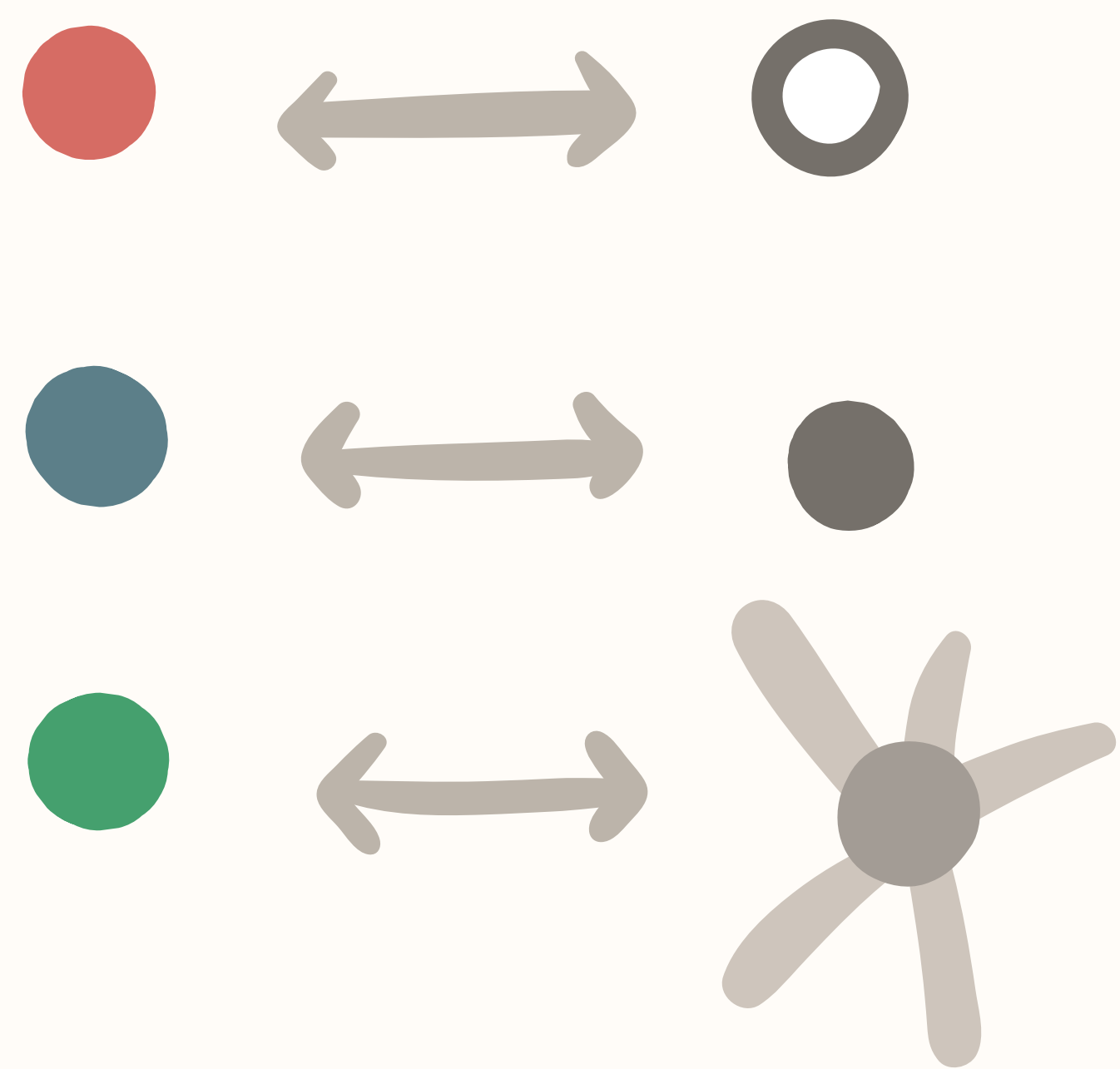
Polyhedral orientations



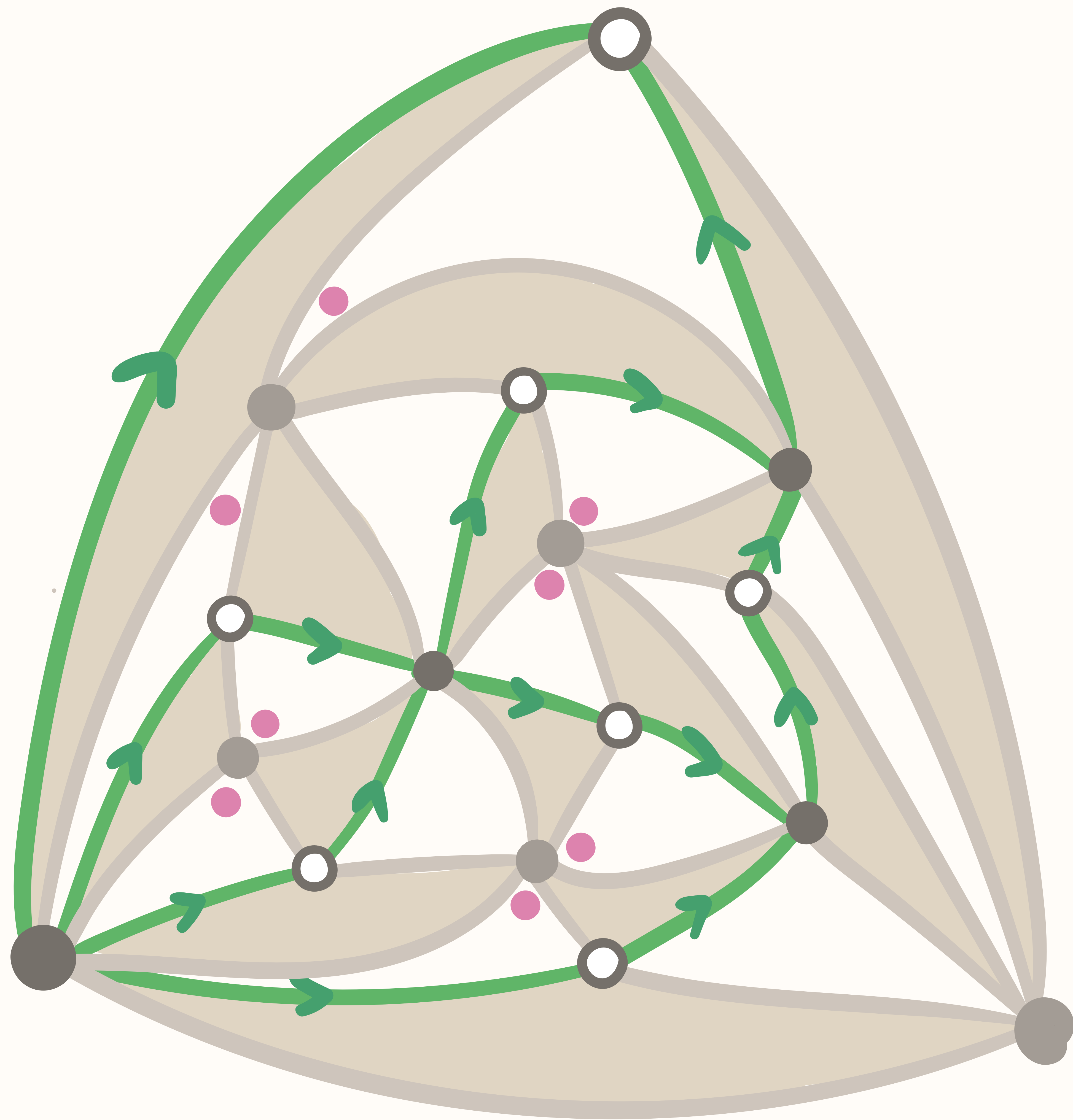
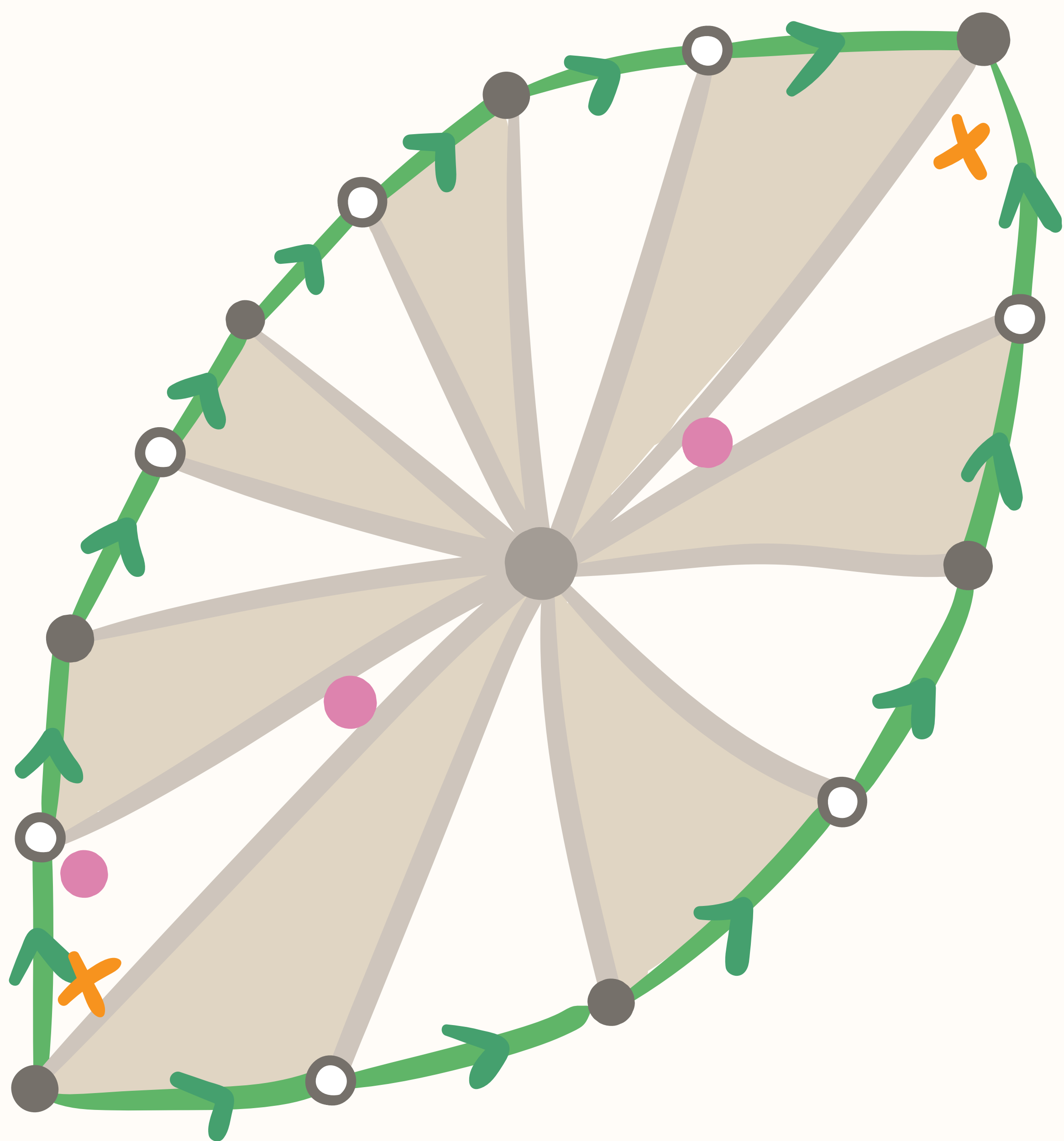
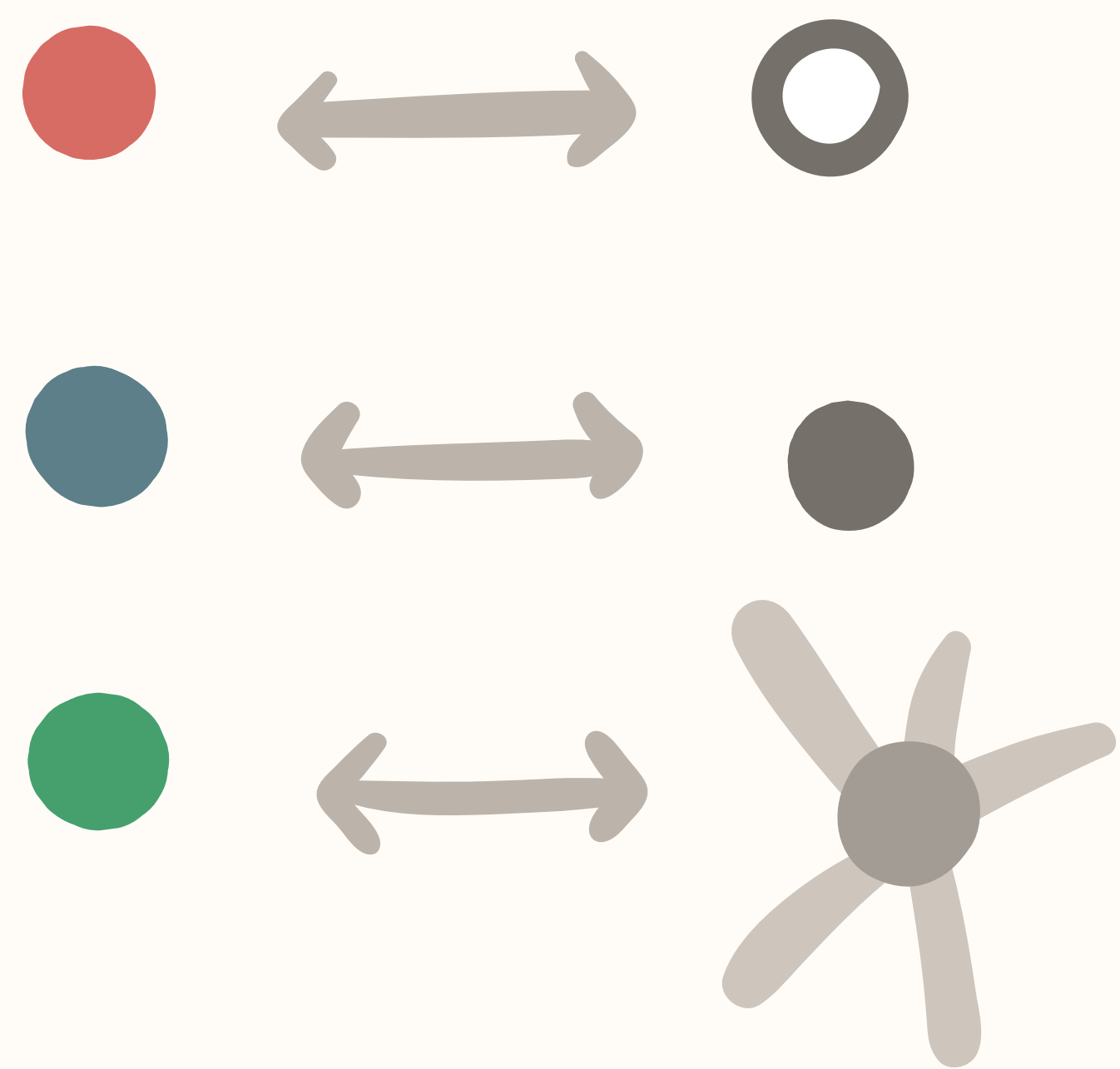
Polyhedral orientations



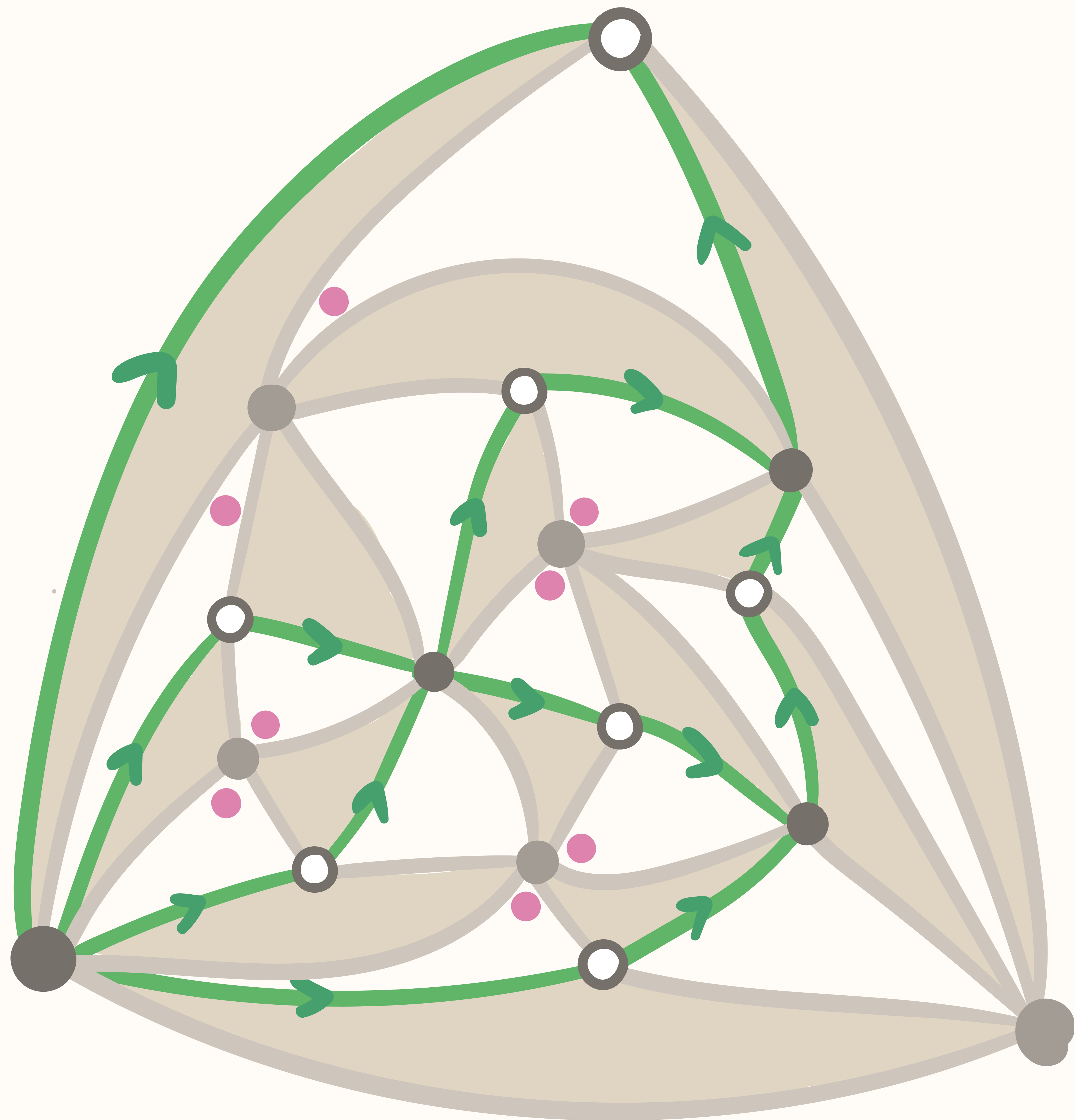
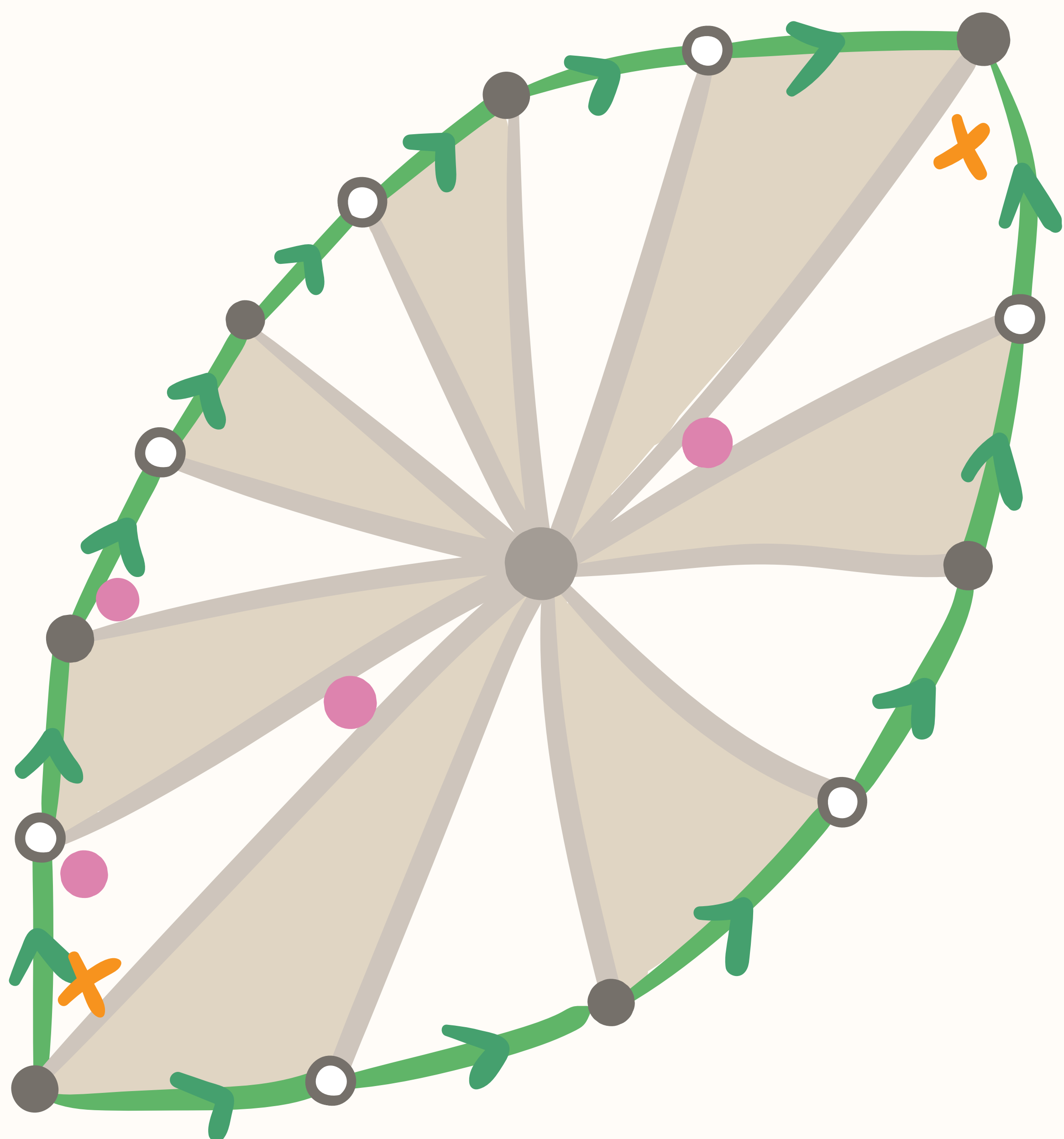
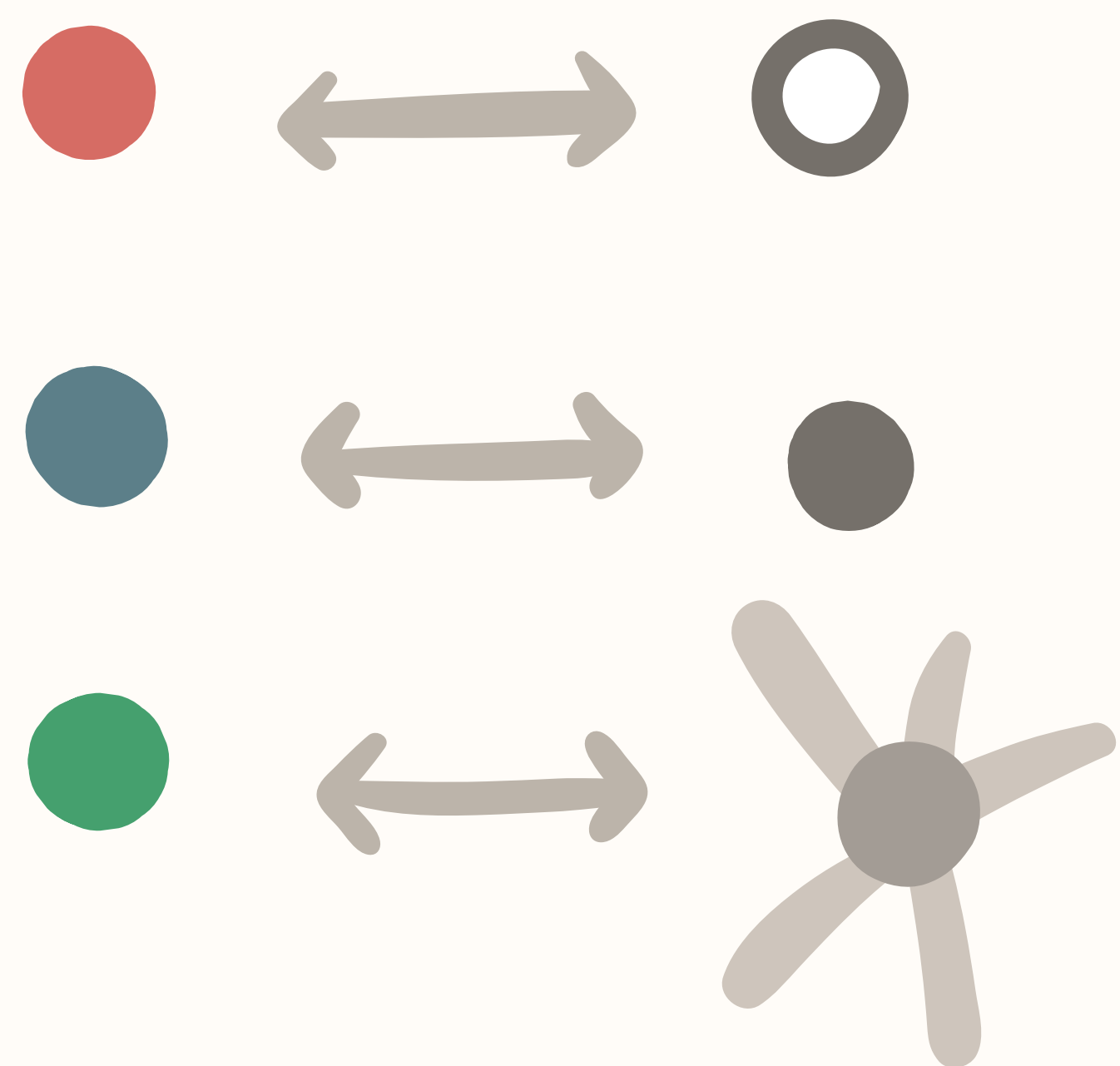
Polyhedral orientations



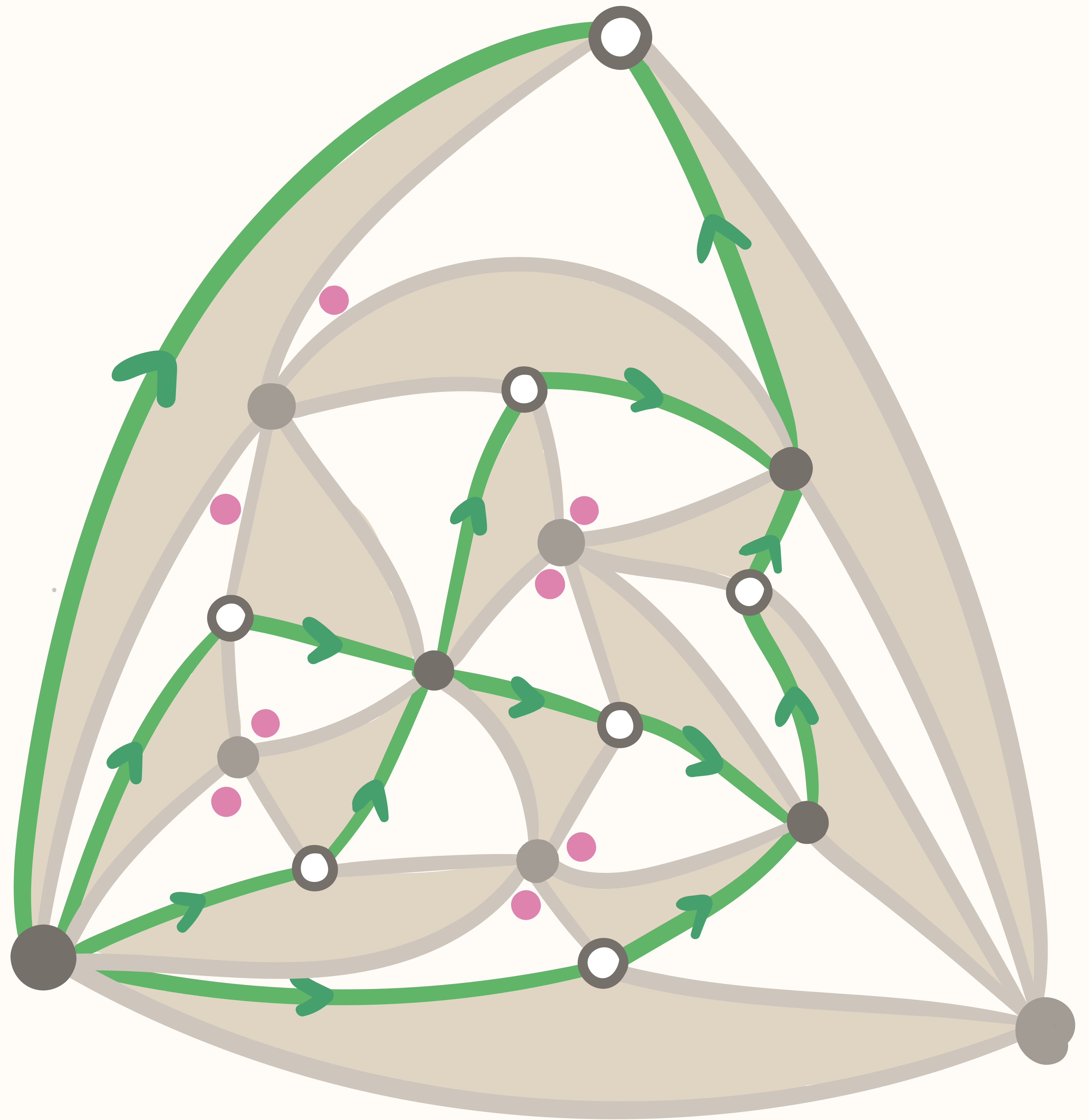
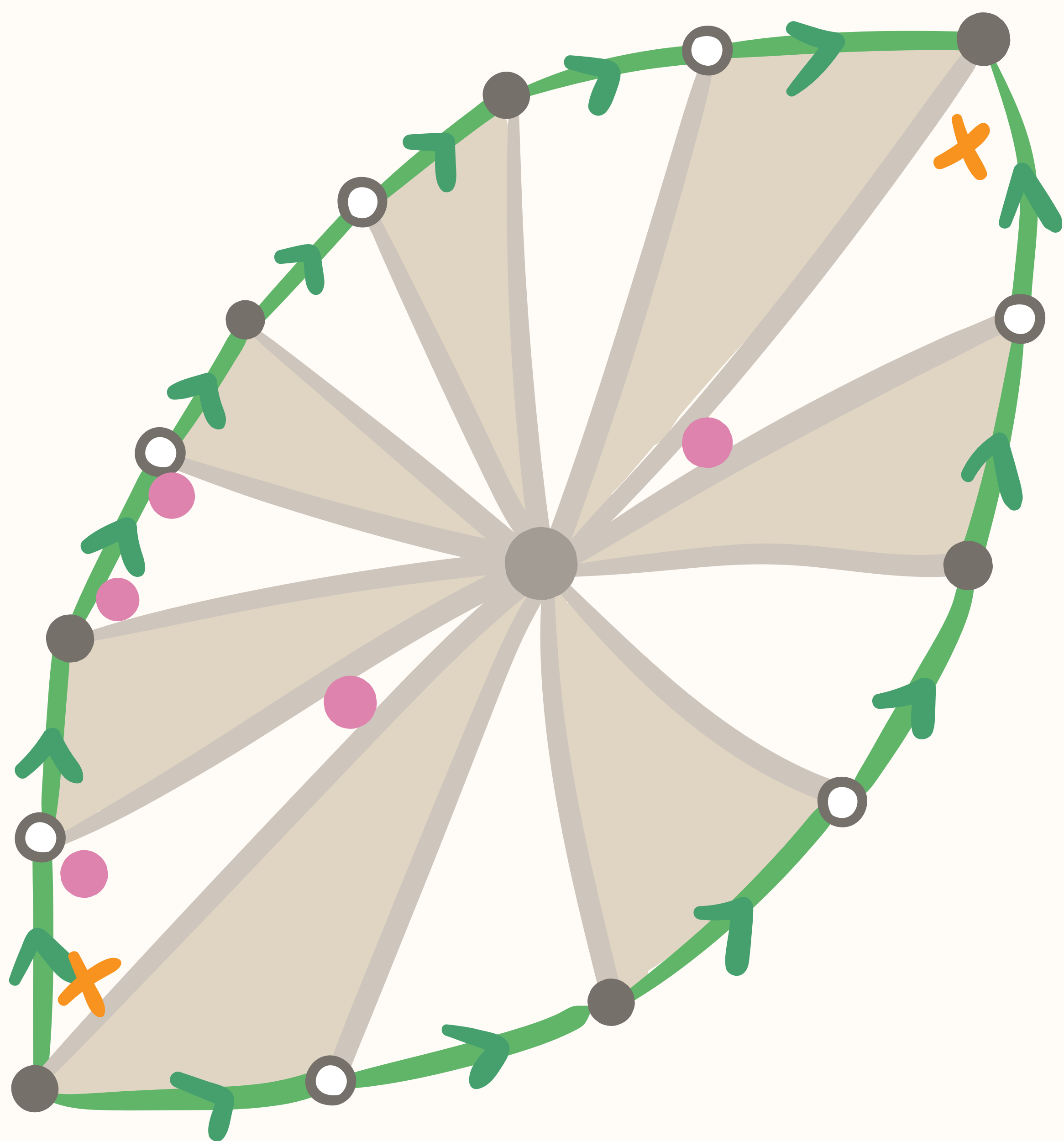
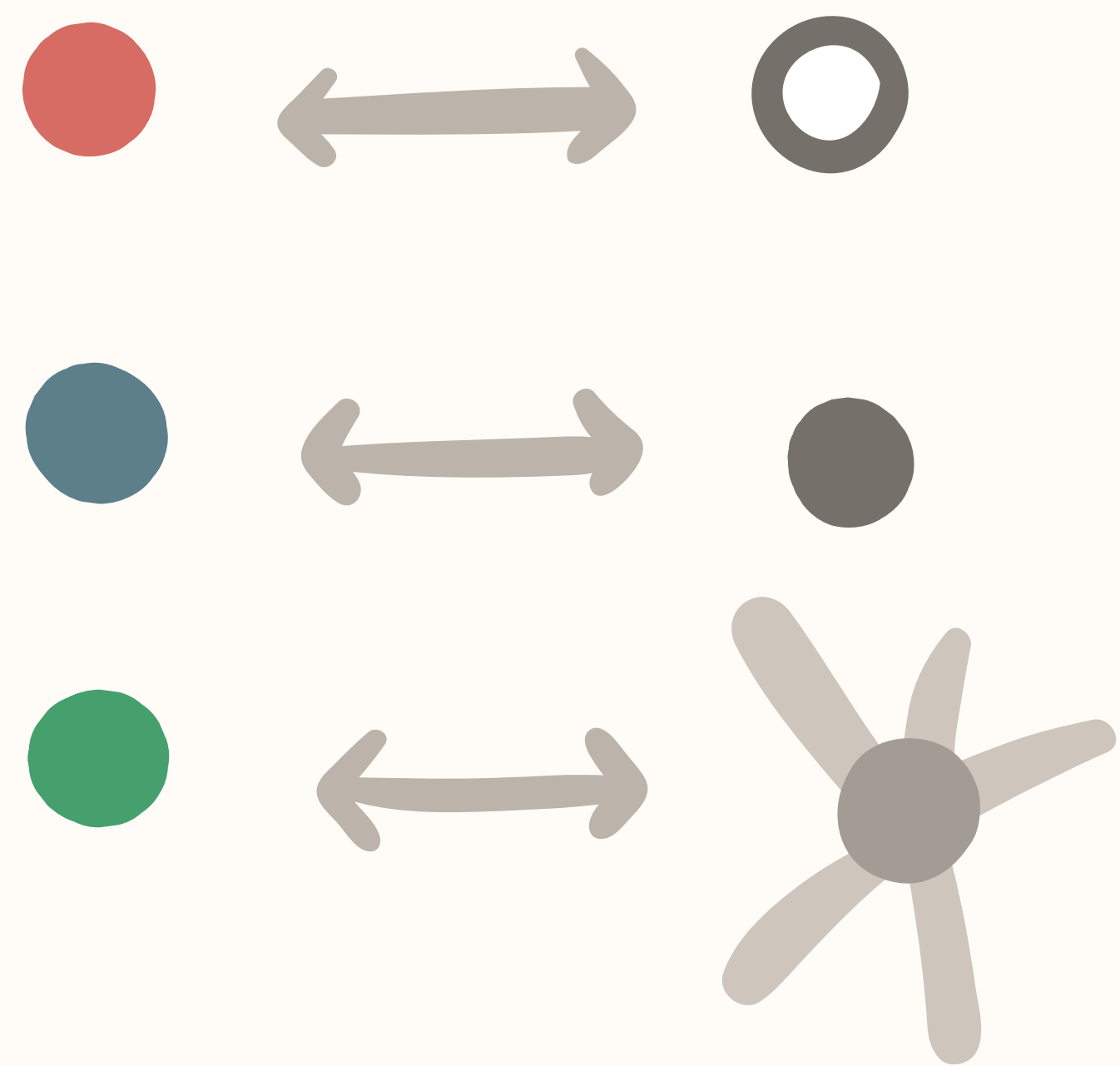
Polyhedral orientations



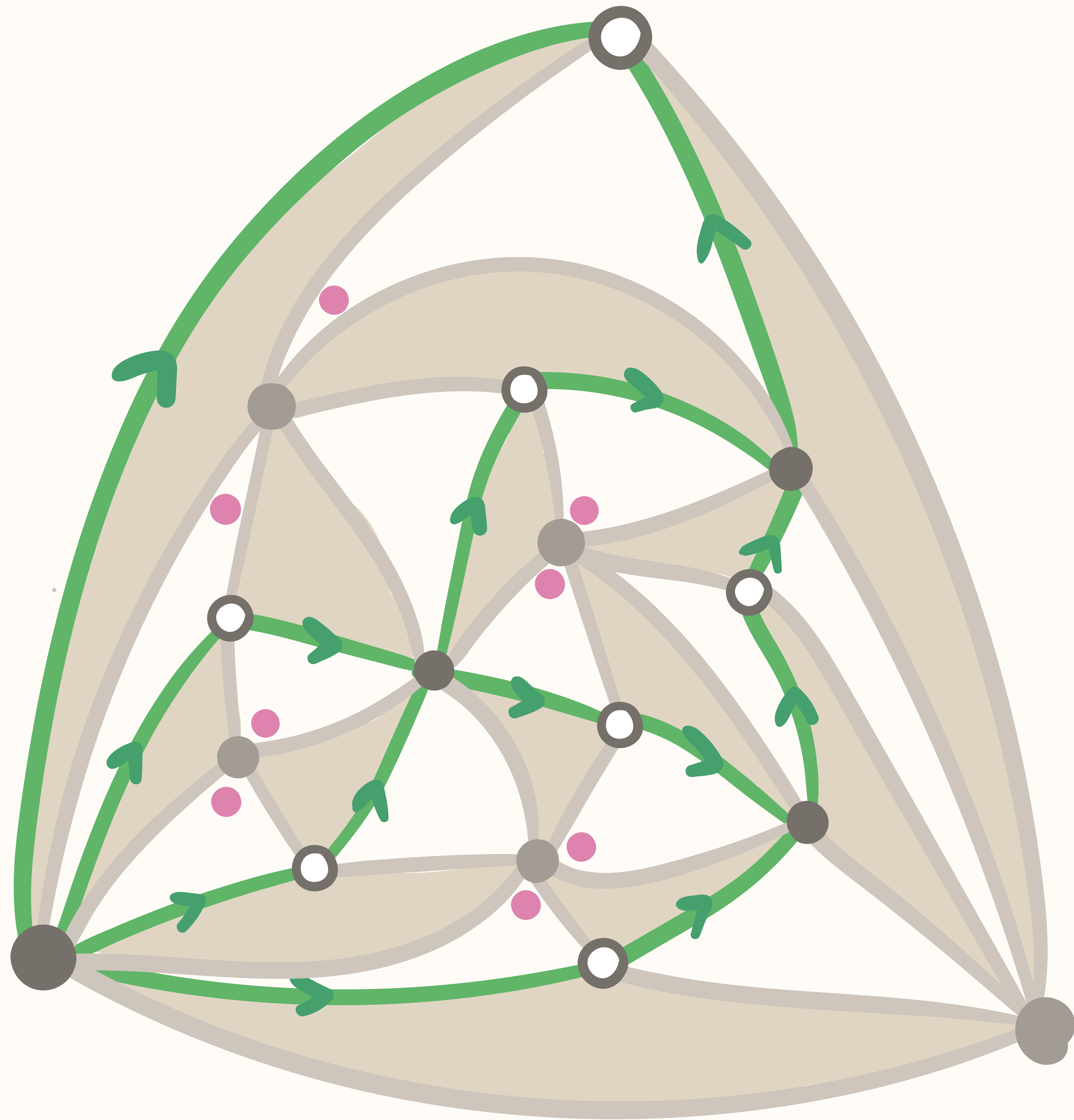
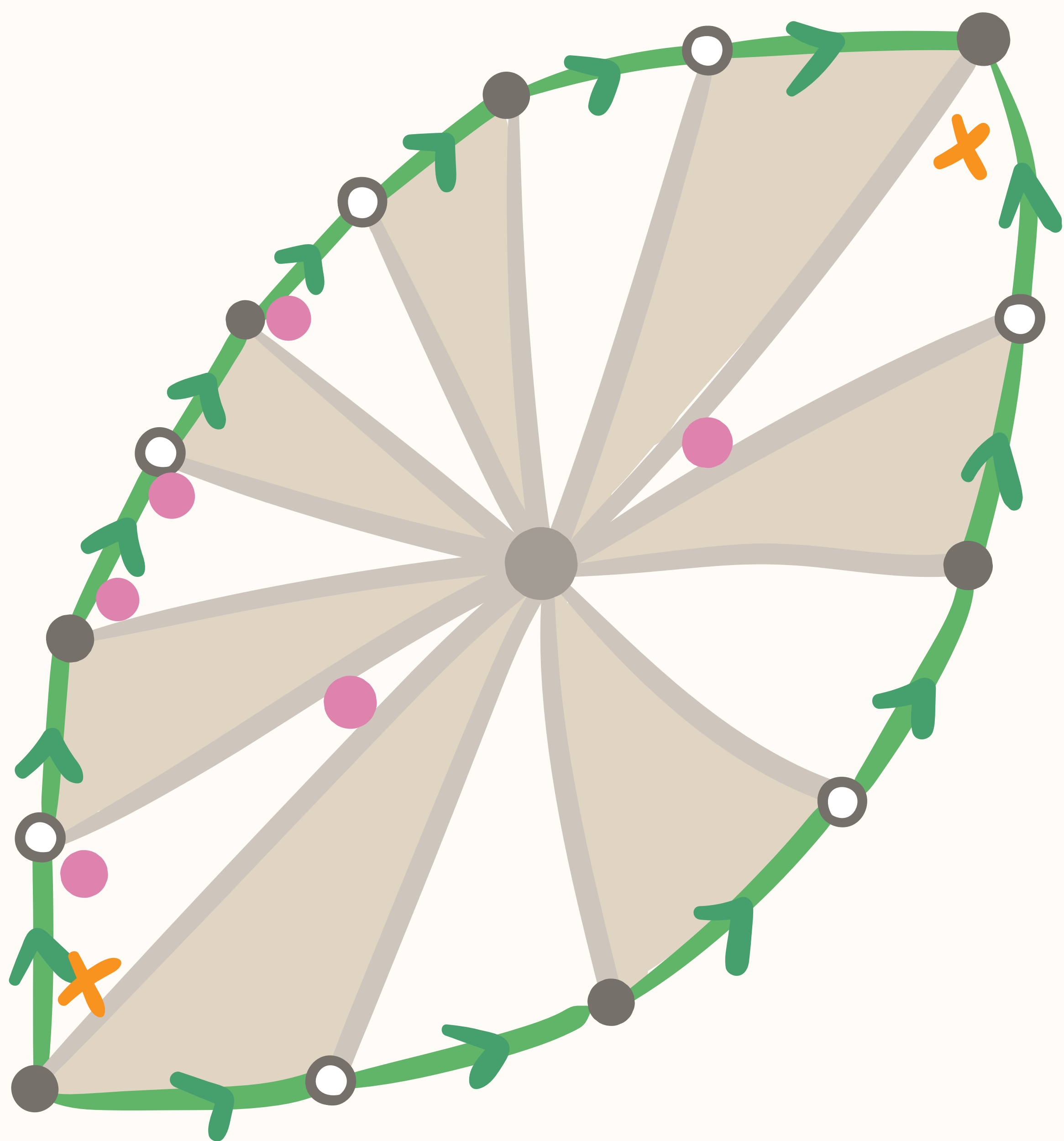
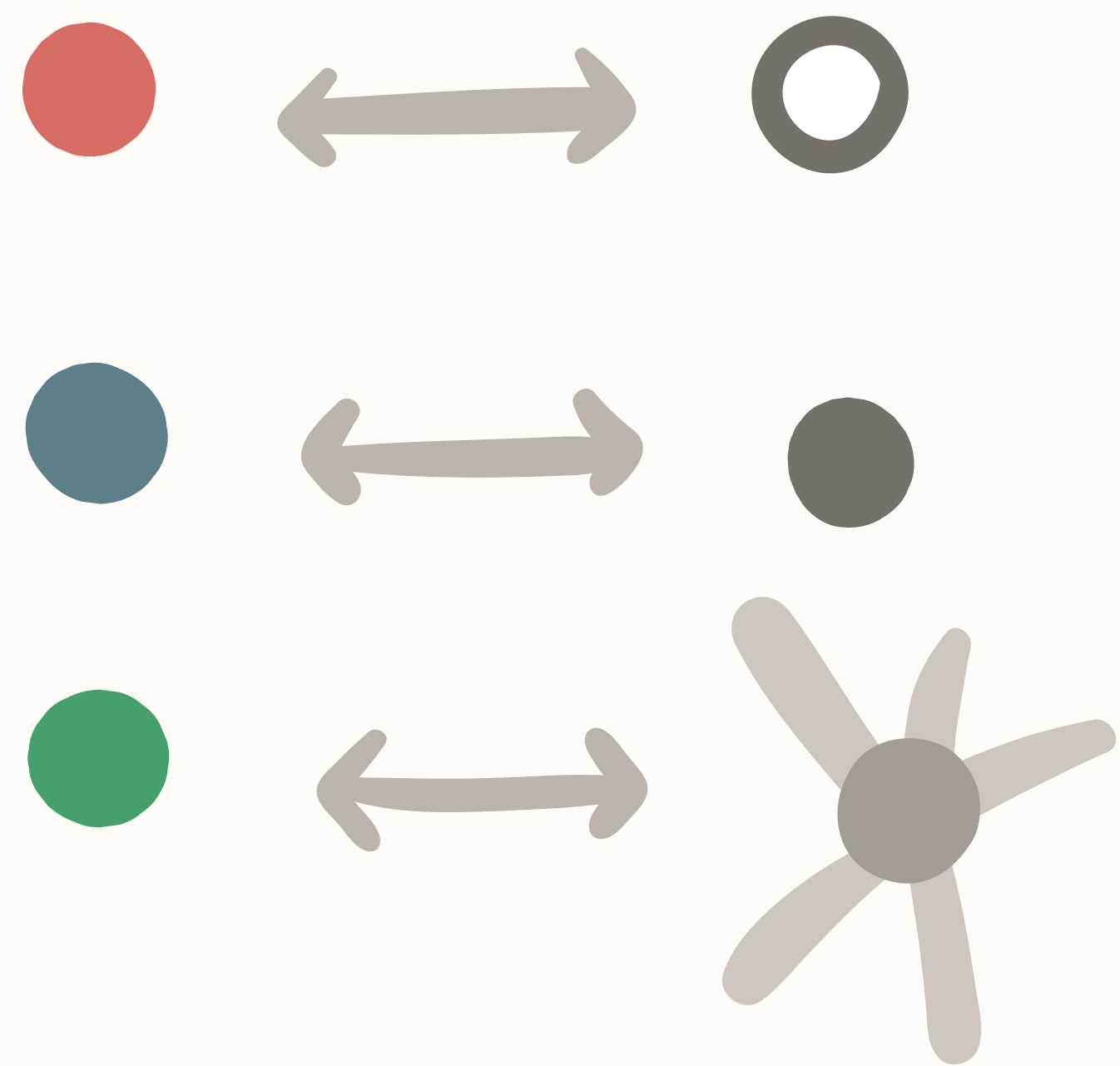
Polyhedral orientations



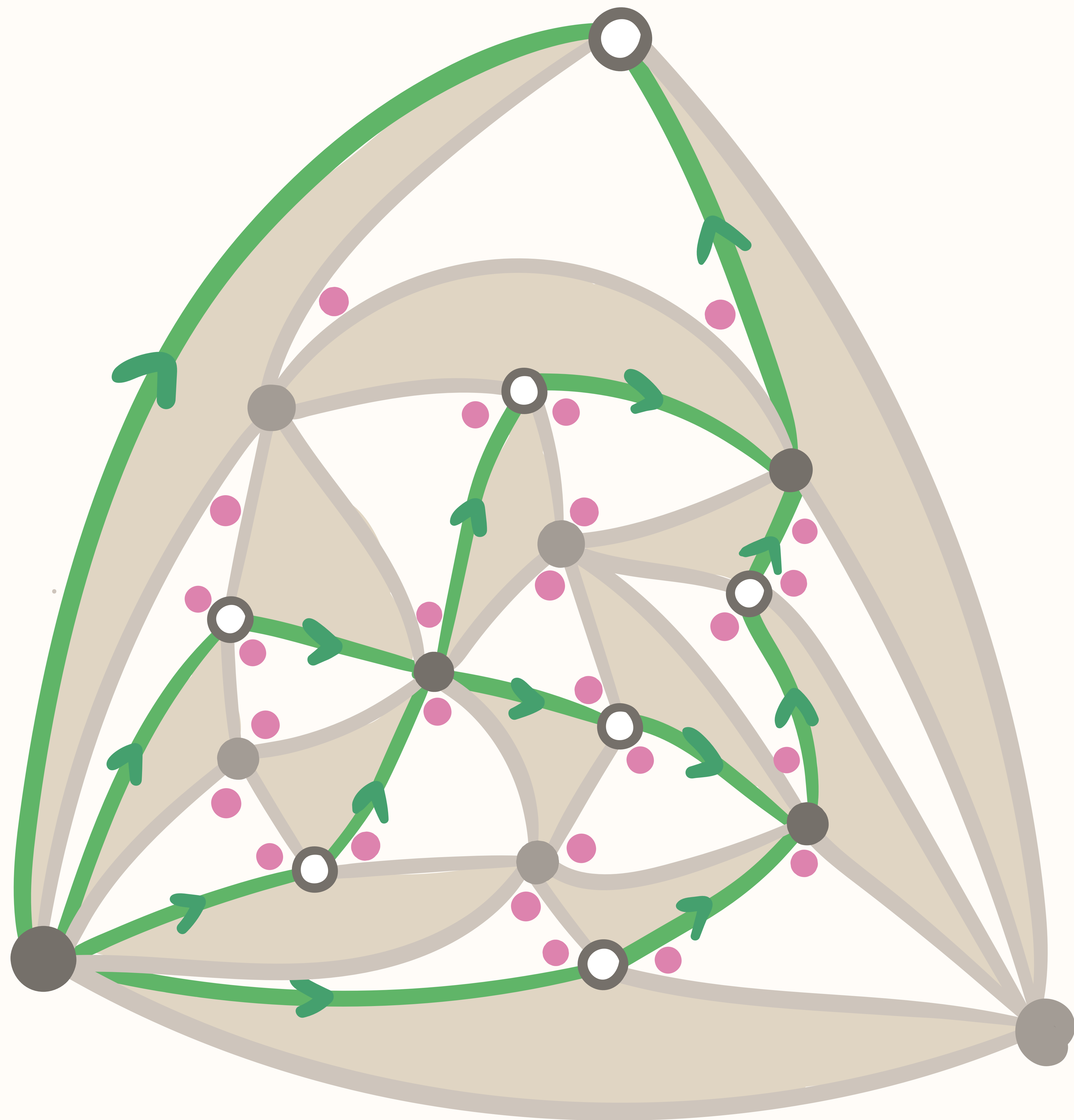
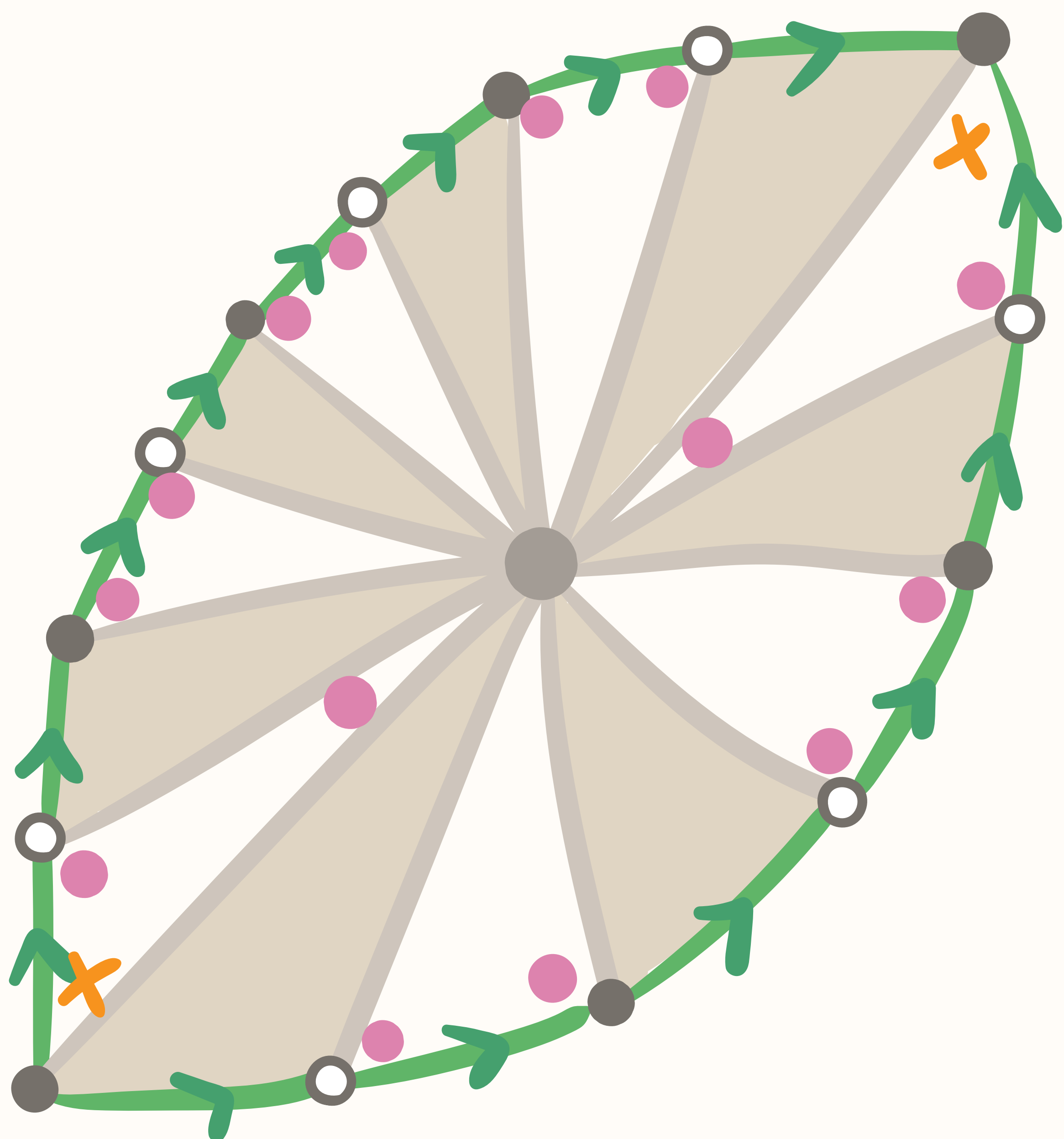
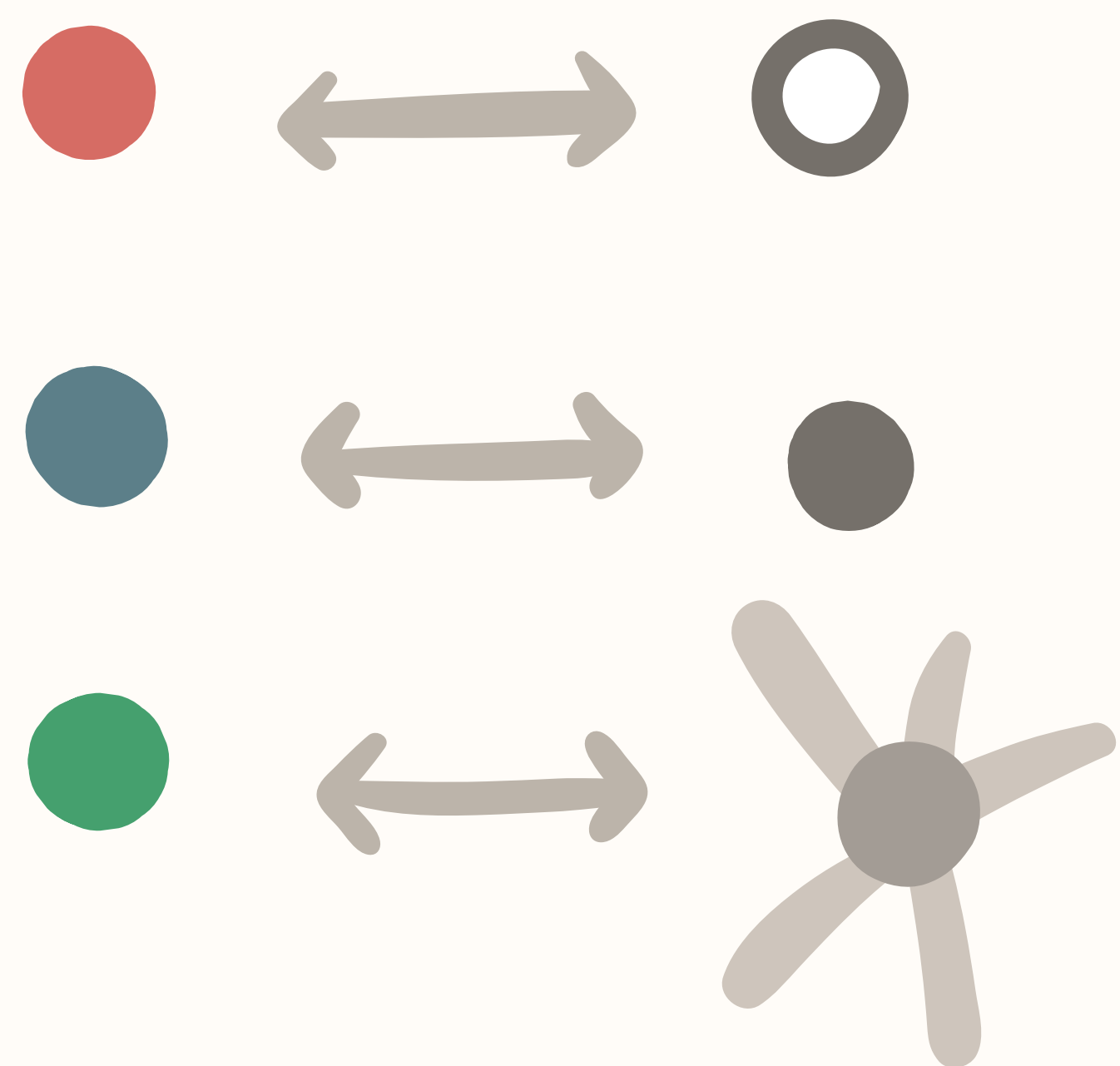
Polyhedral orientations



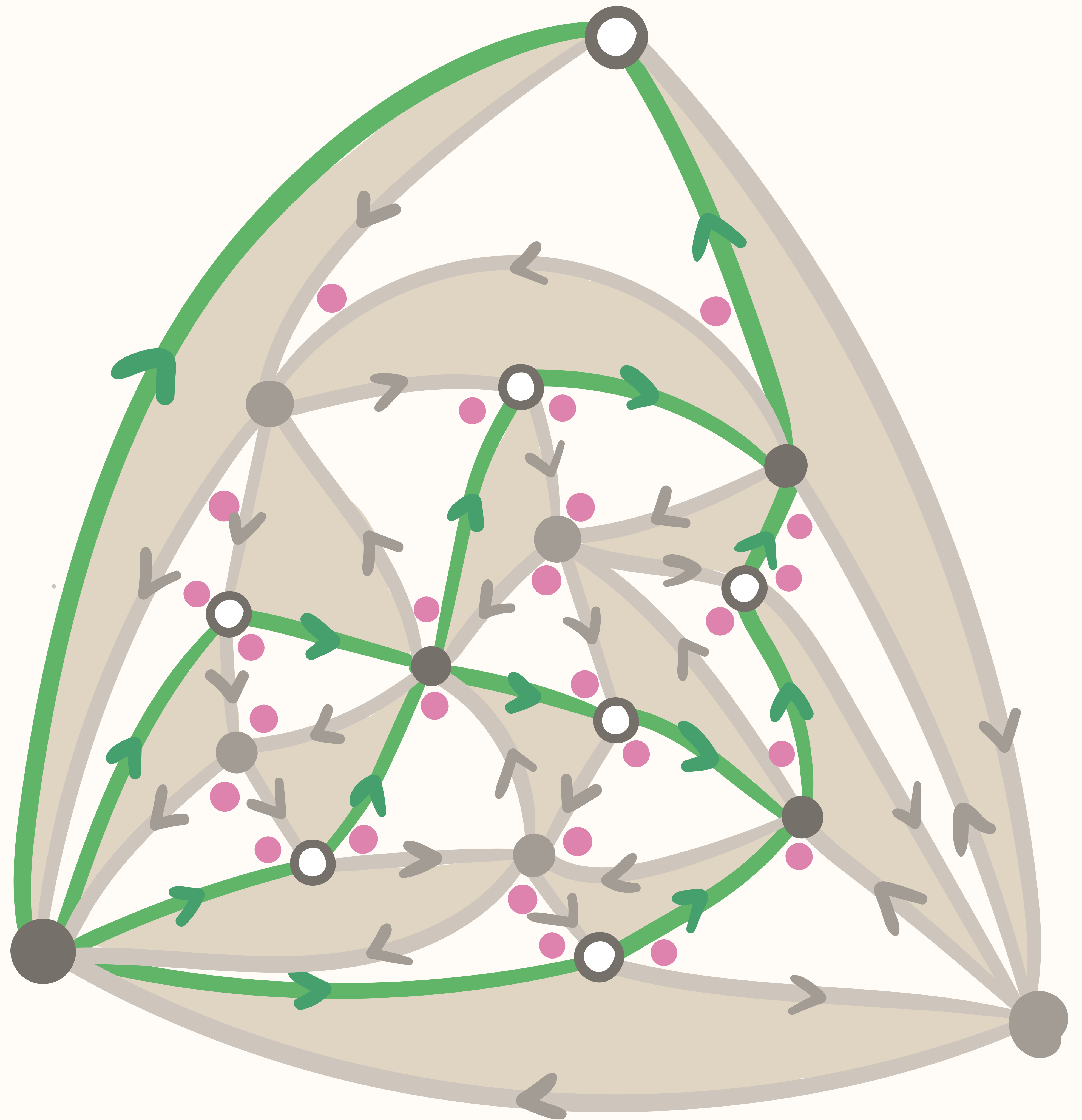
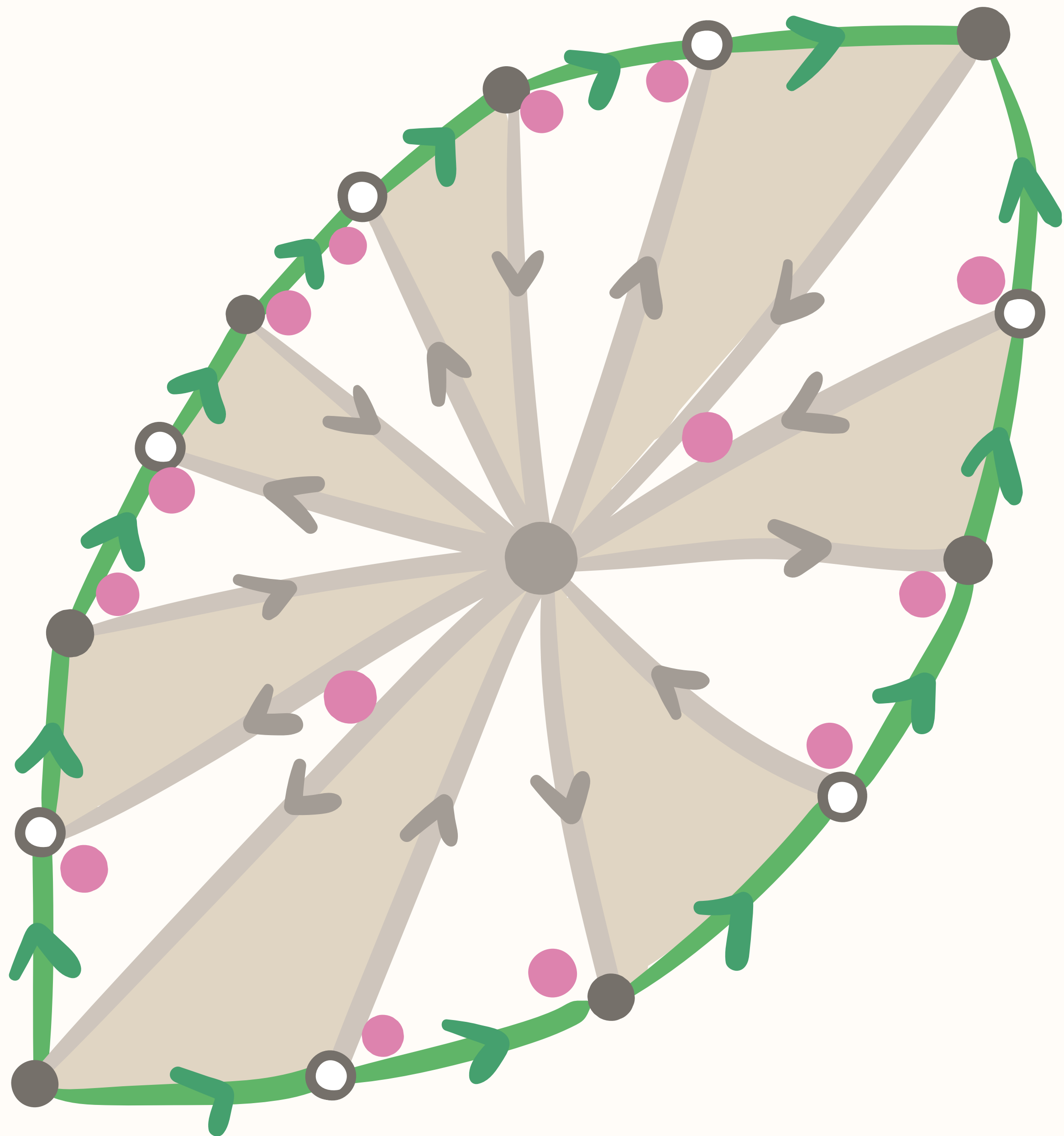
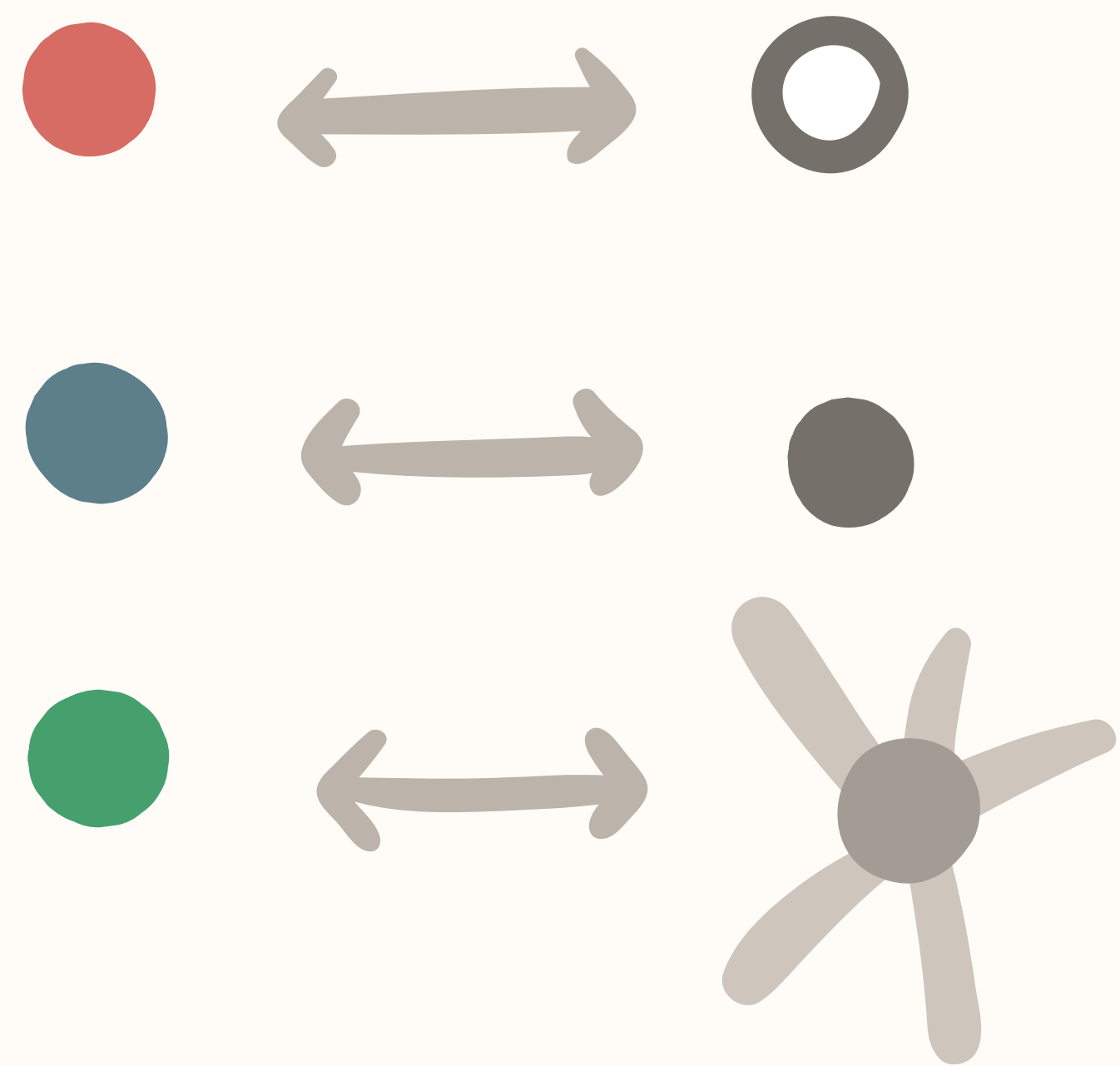
Polyhedral orientations



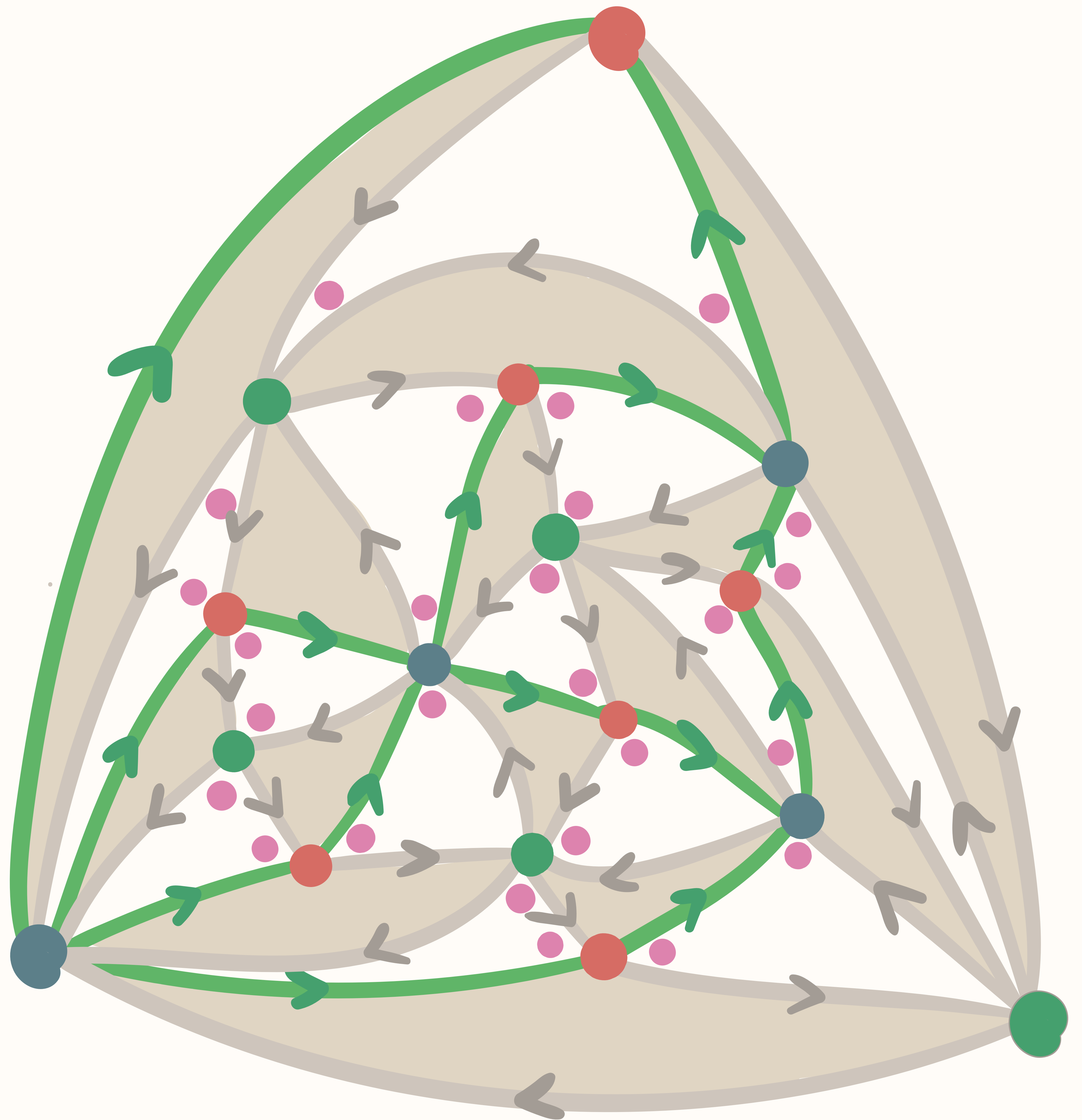
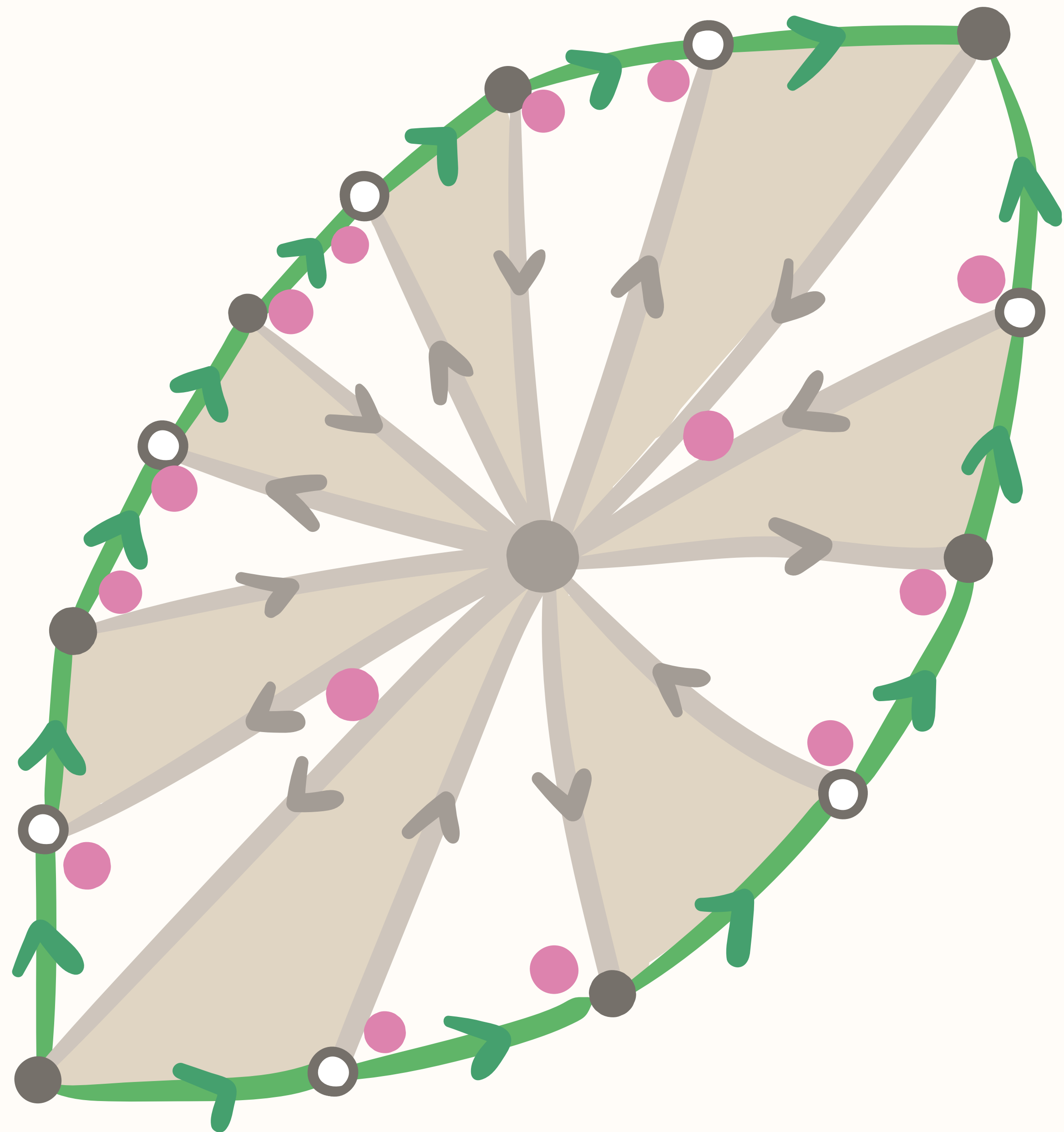
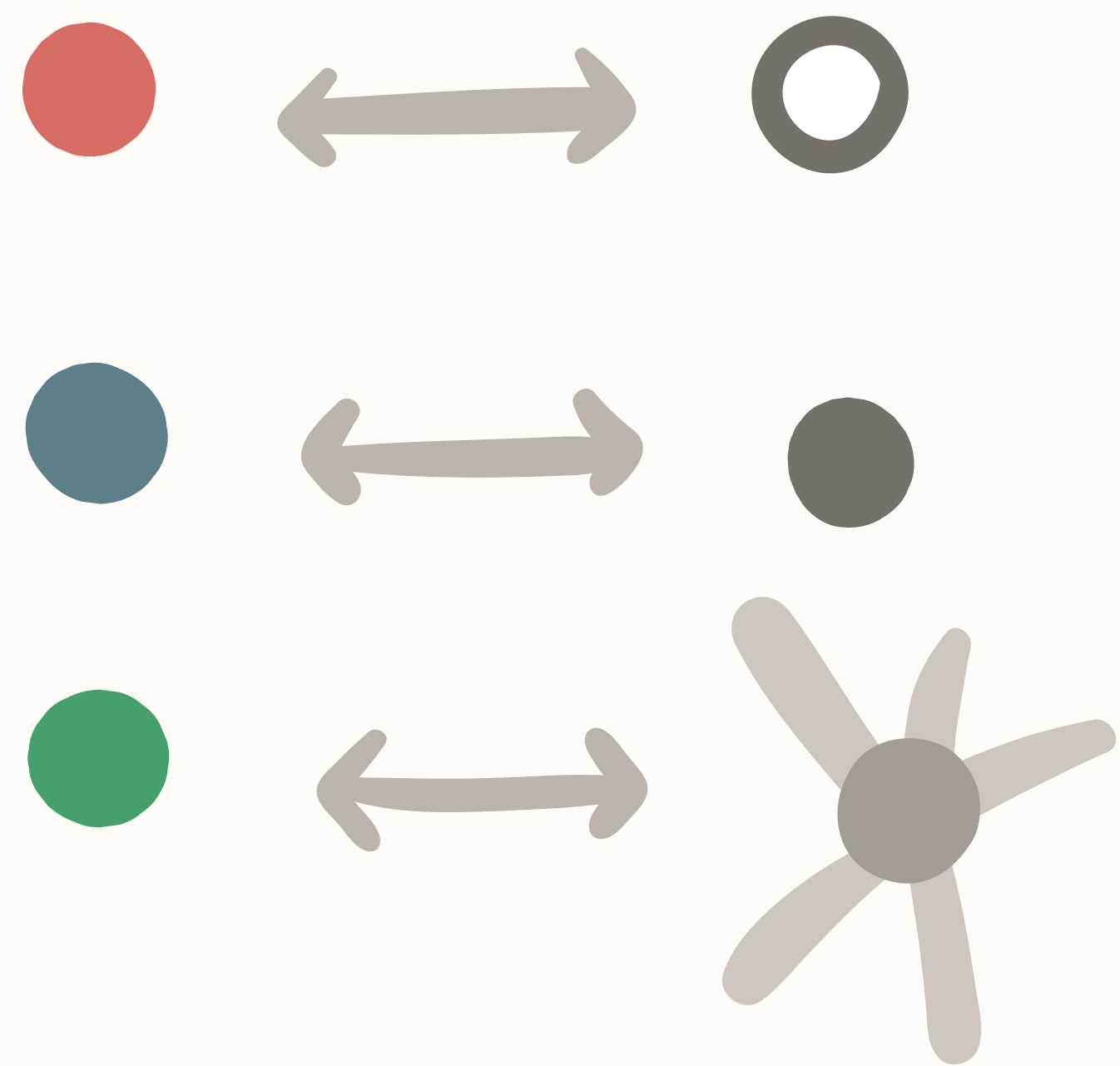
Polyhedral orientations



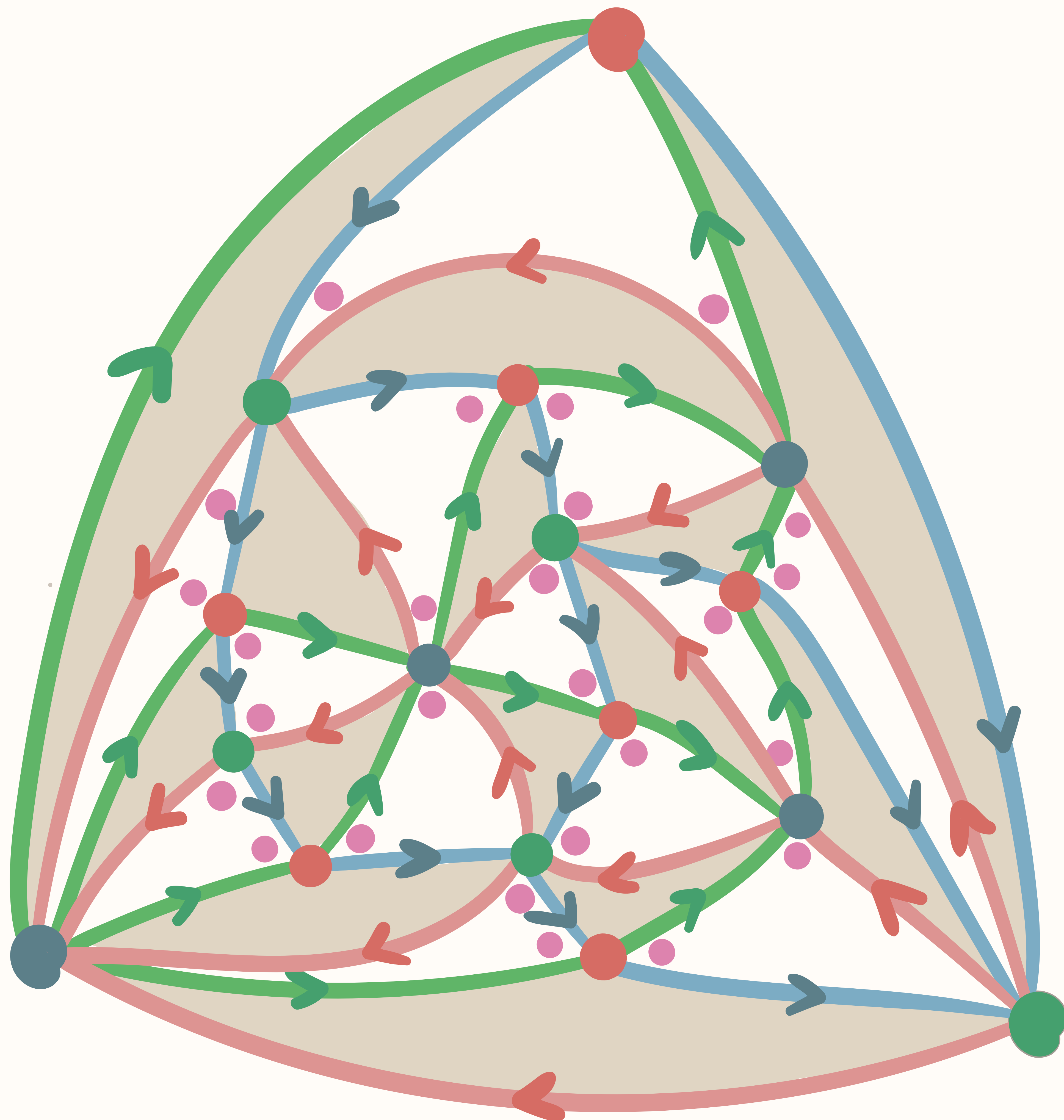
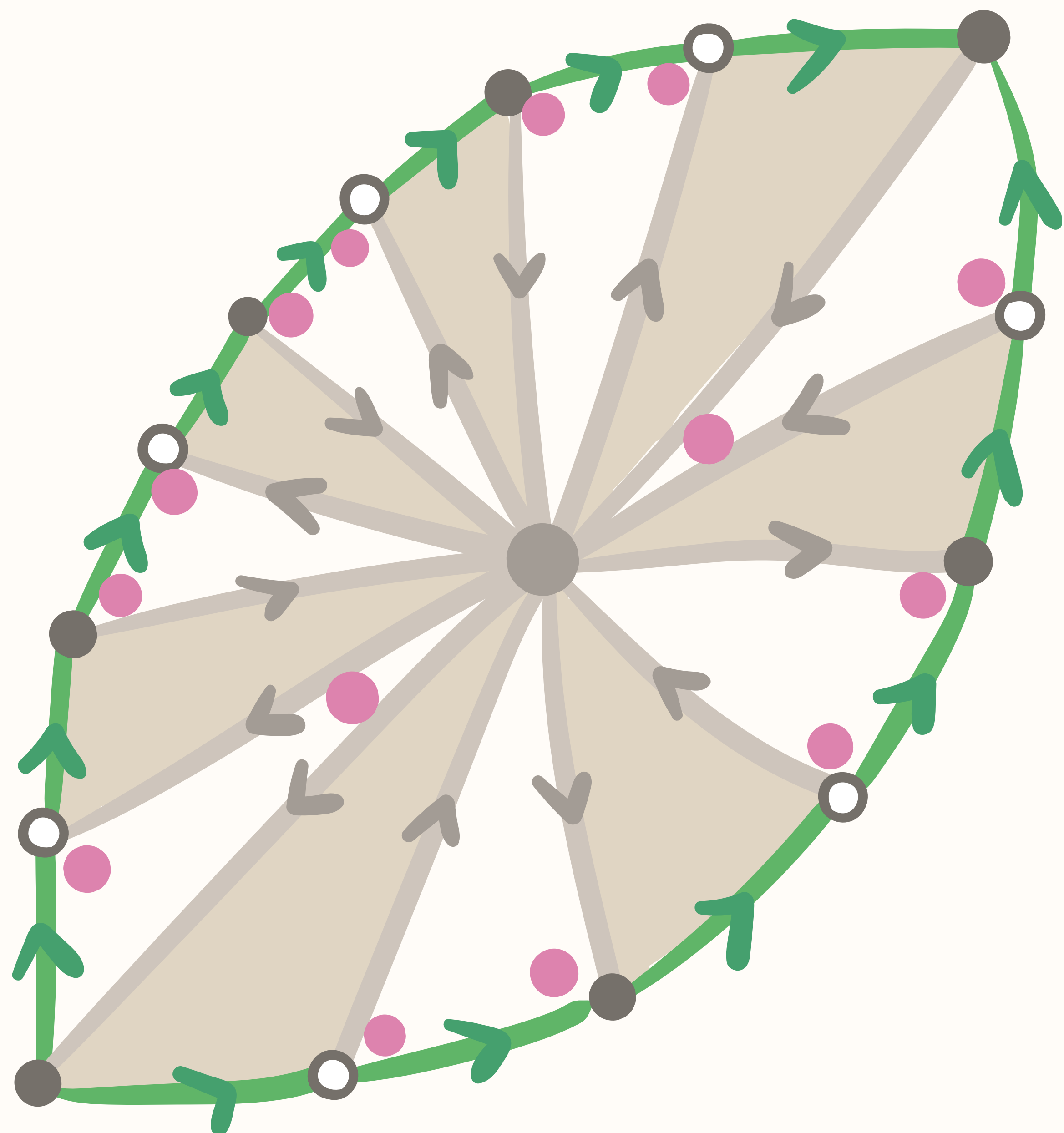
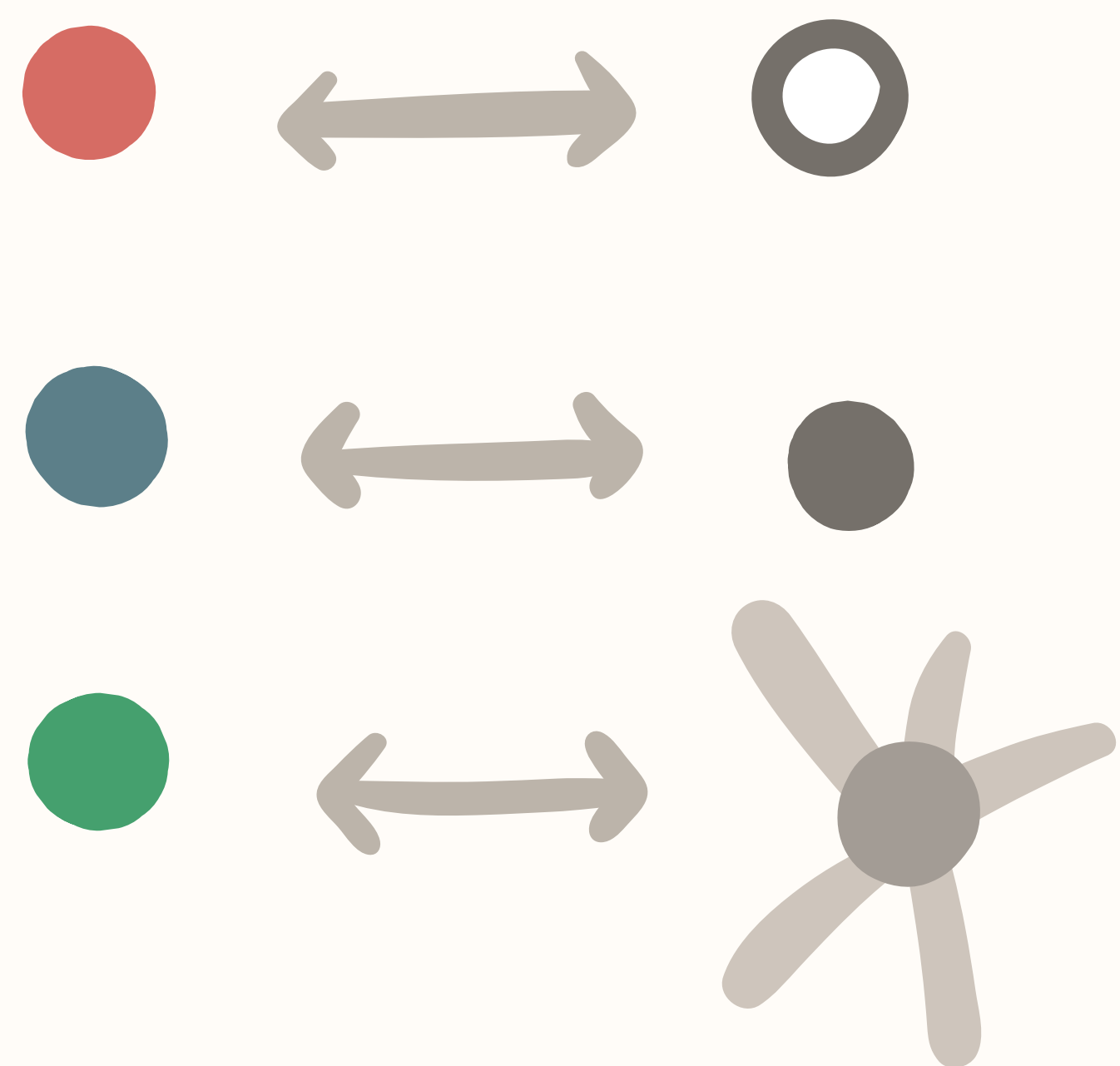
Polyhedral orientations



Polyhedral orientations

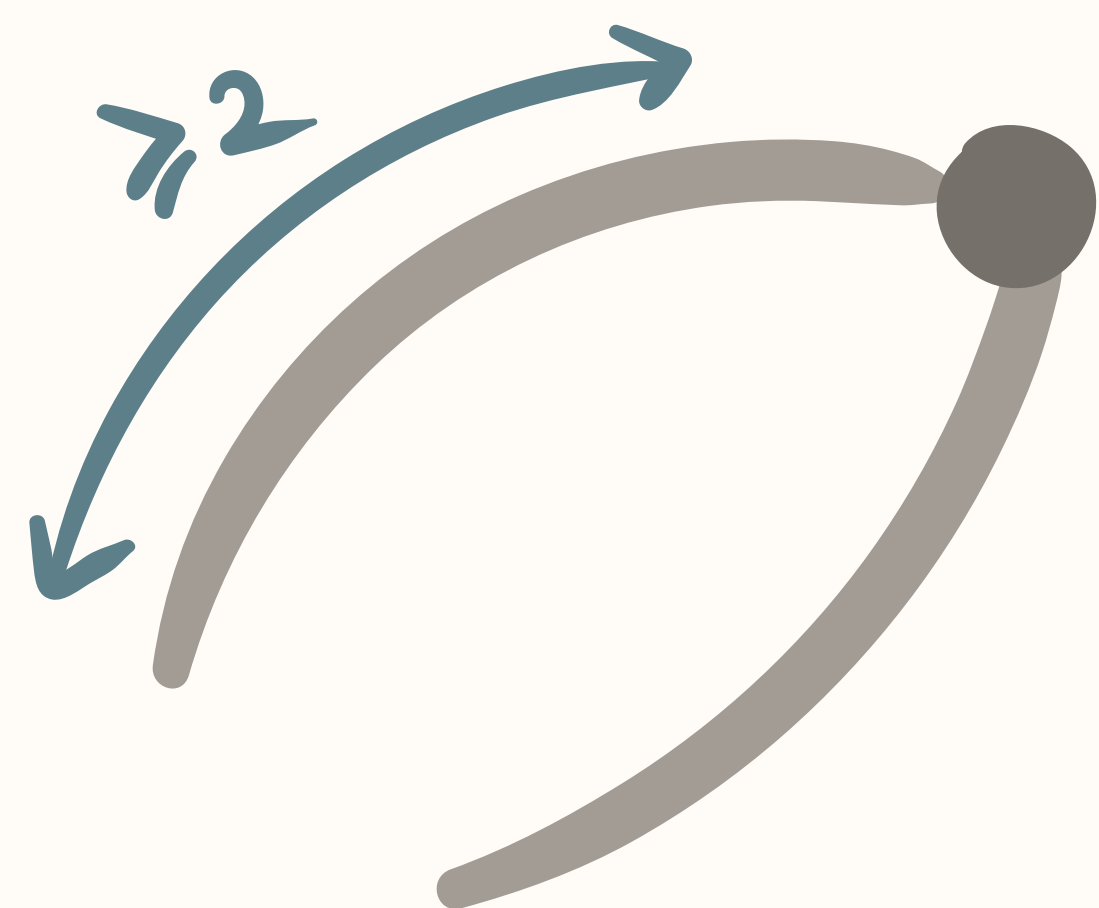
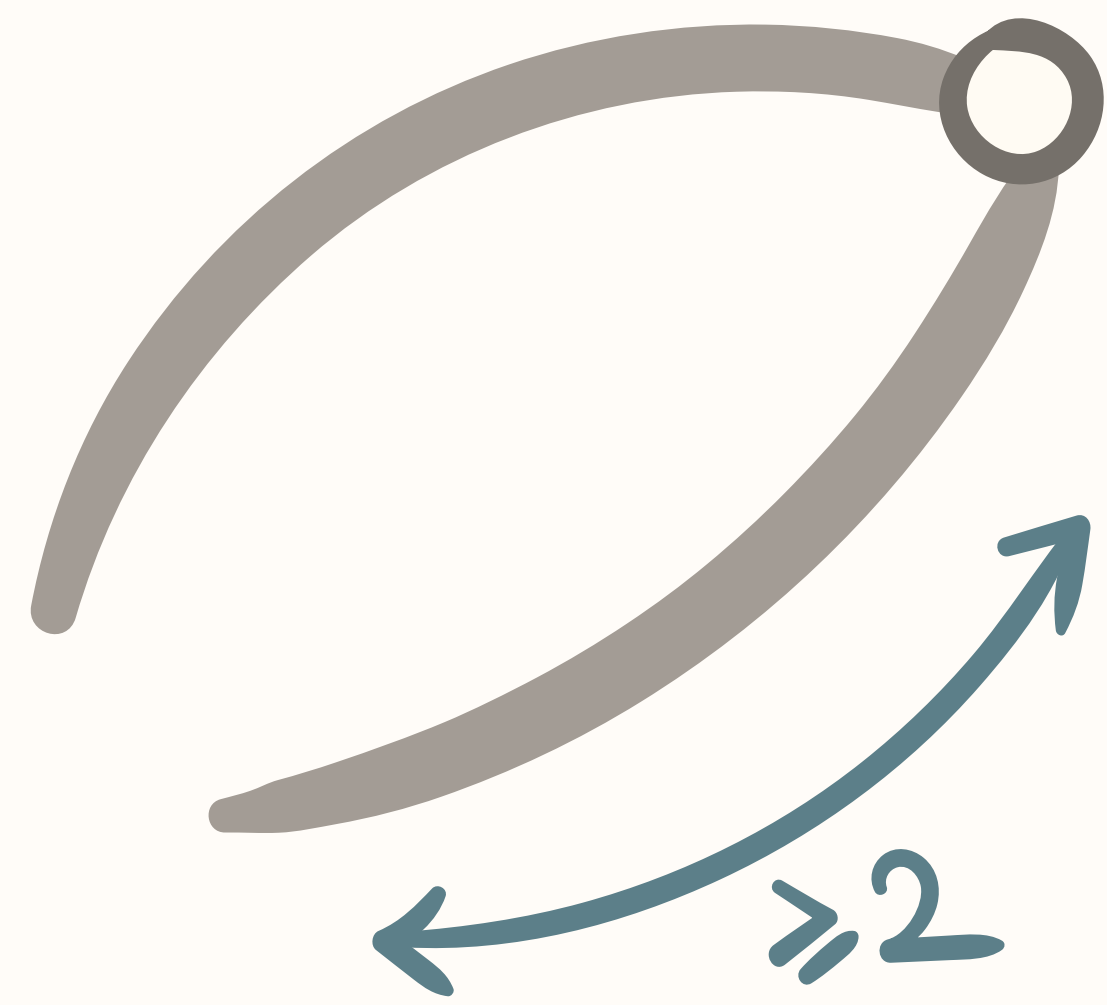
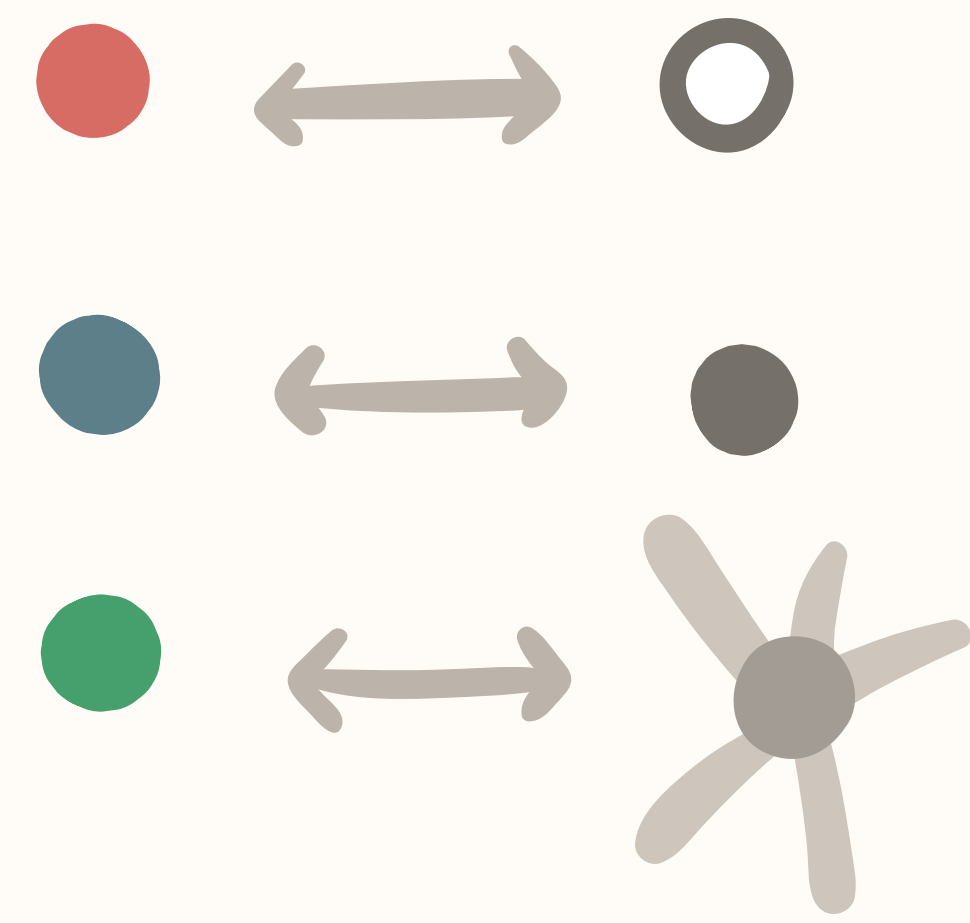
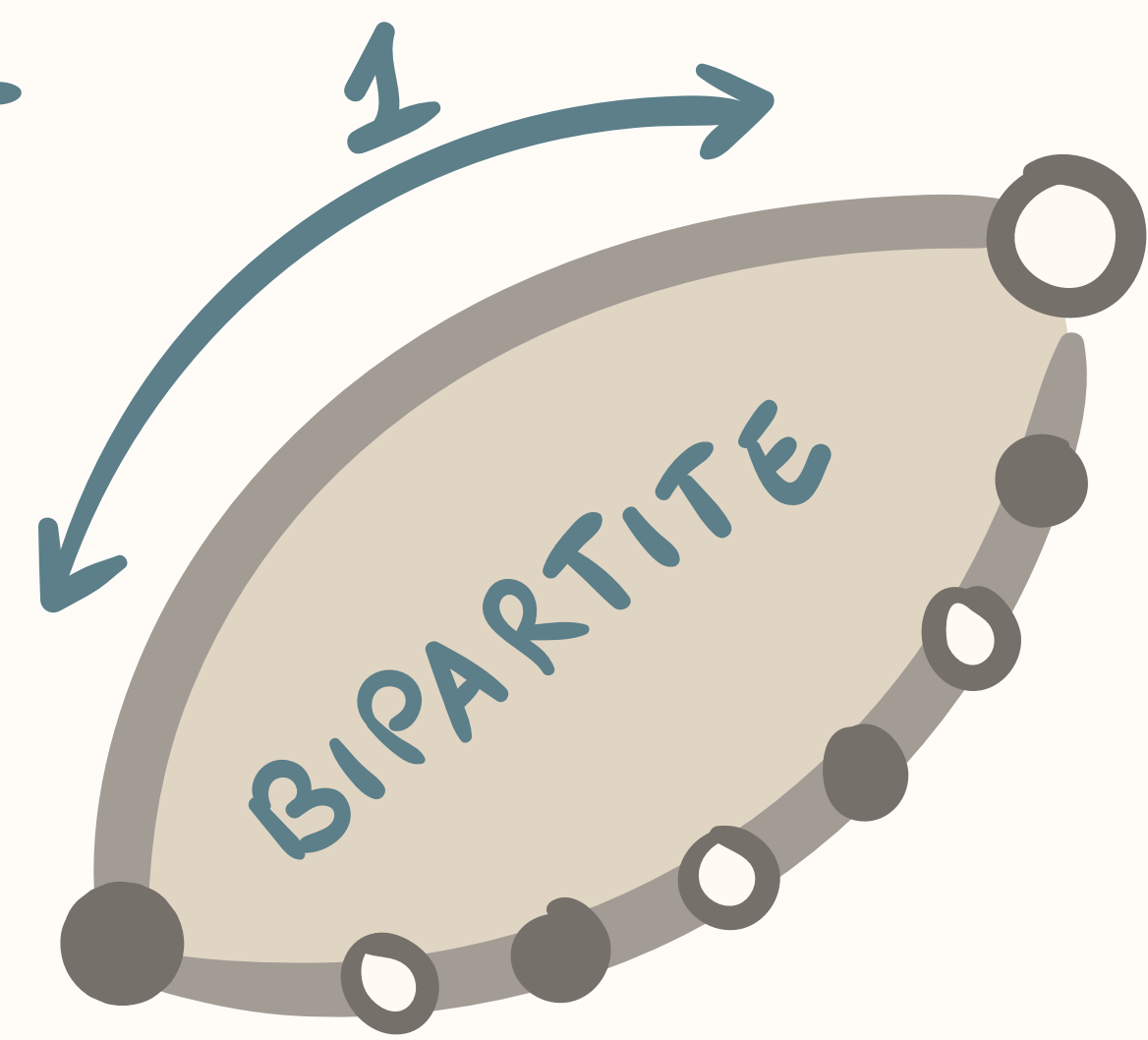


Polyhedral orientations

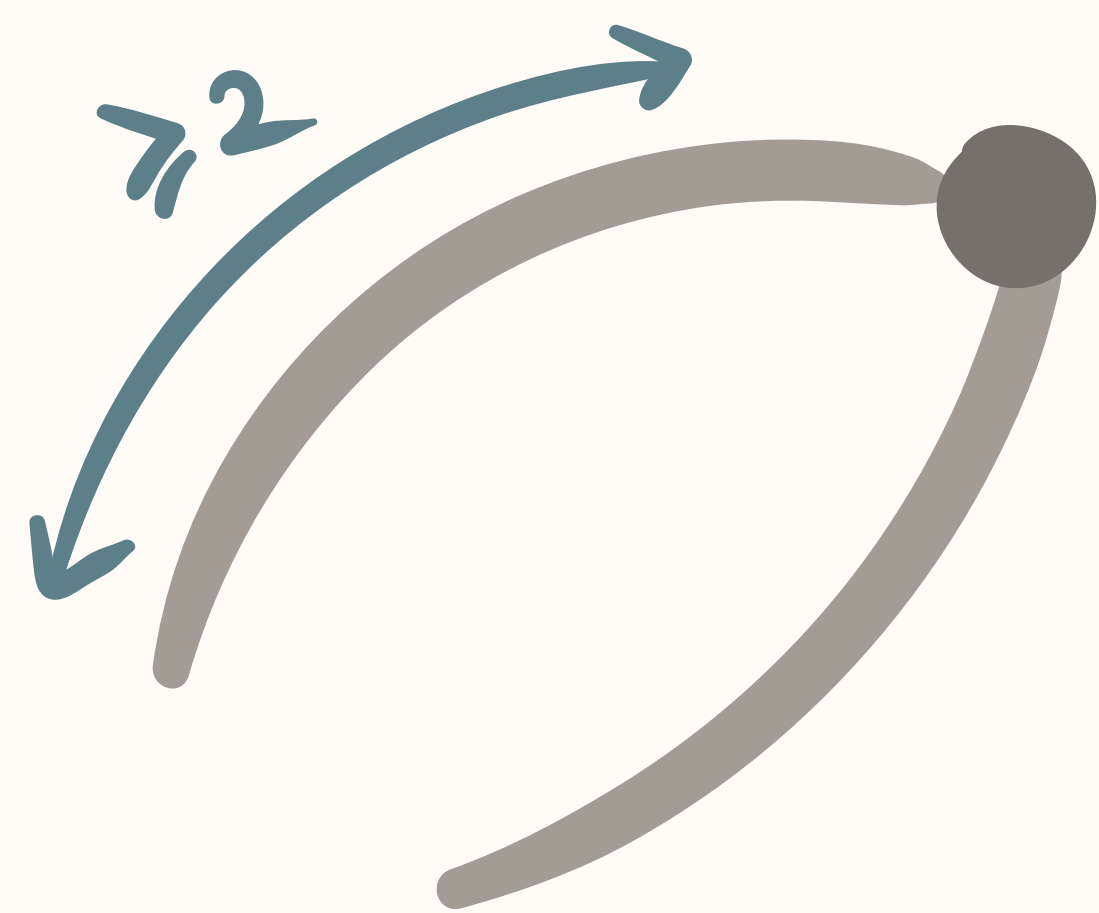
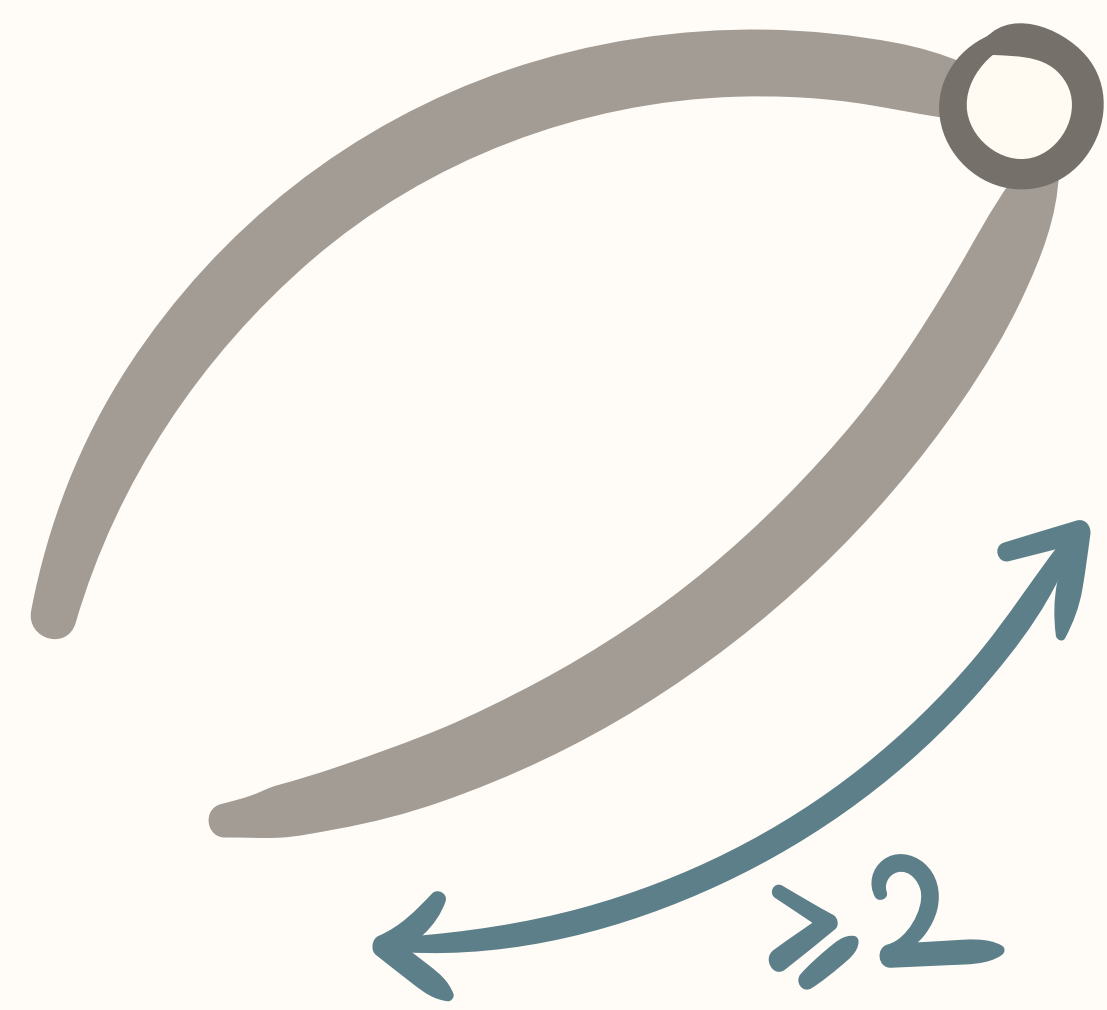
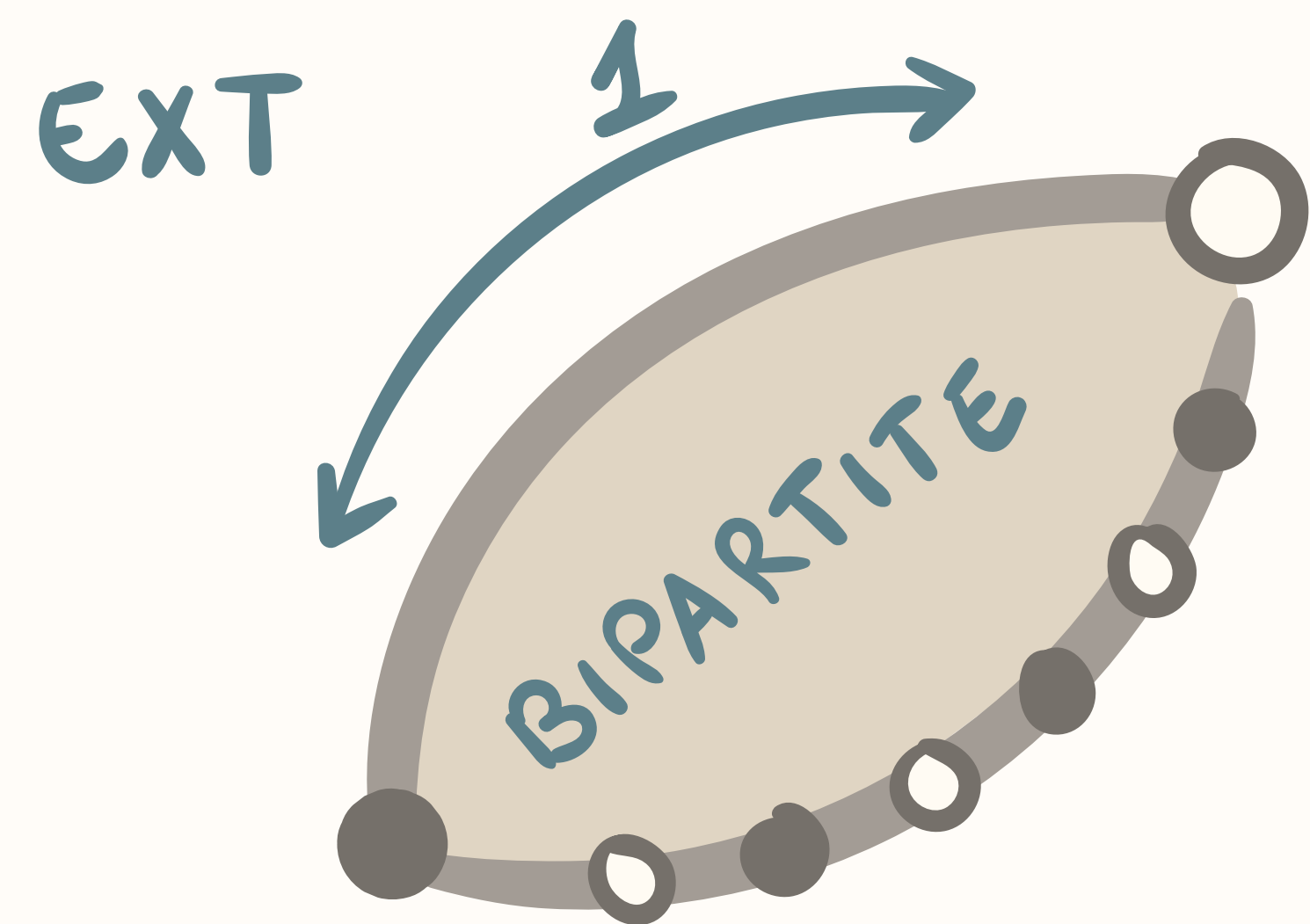


Polyhedral orientations

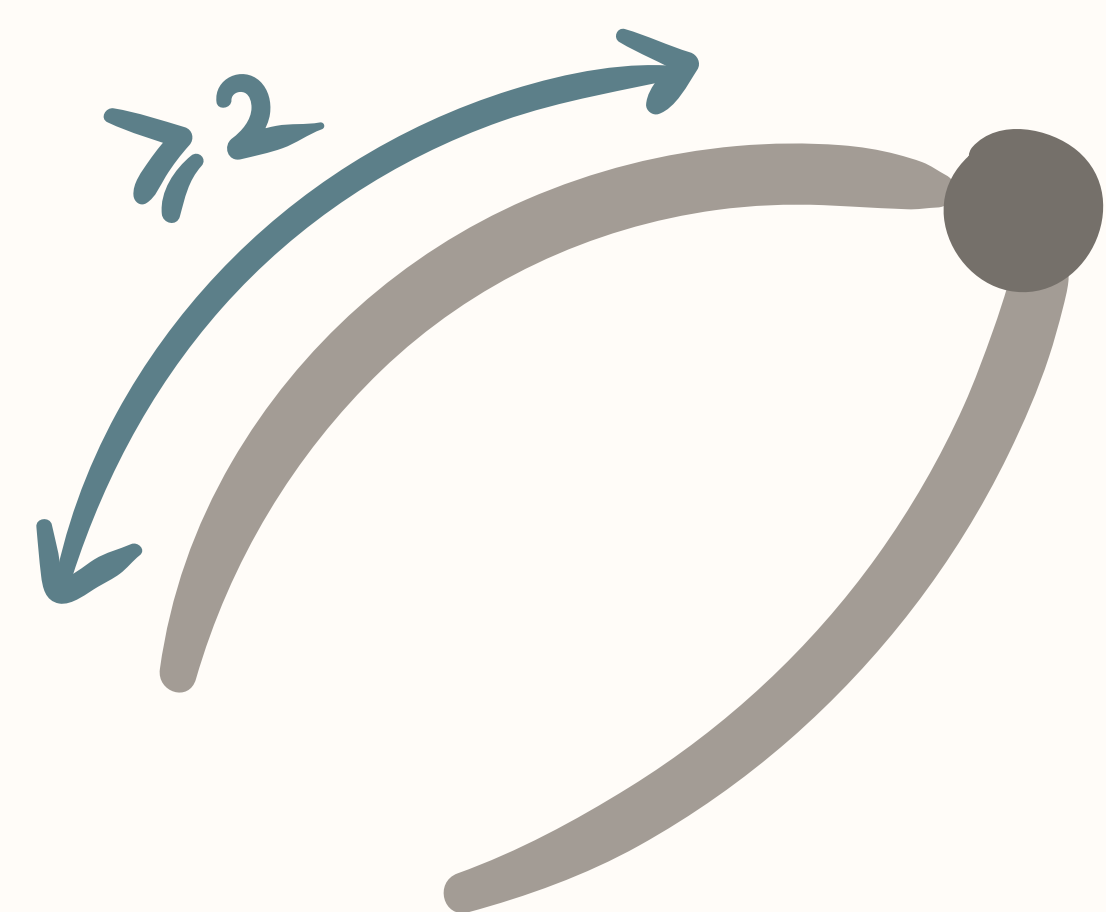
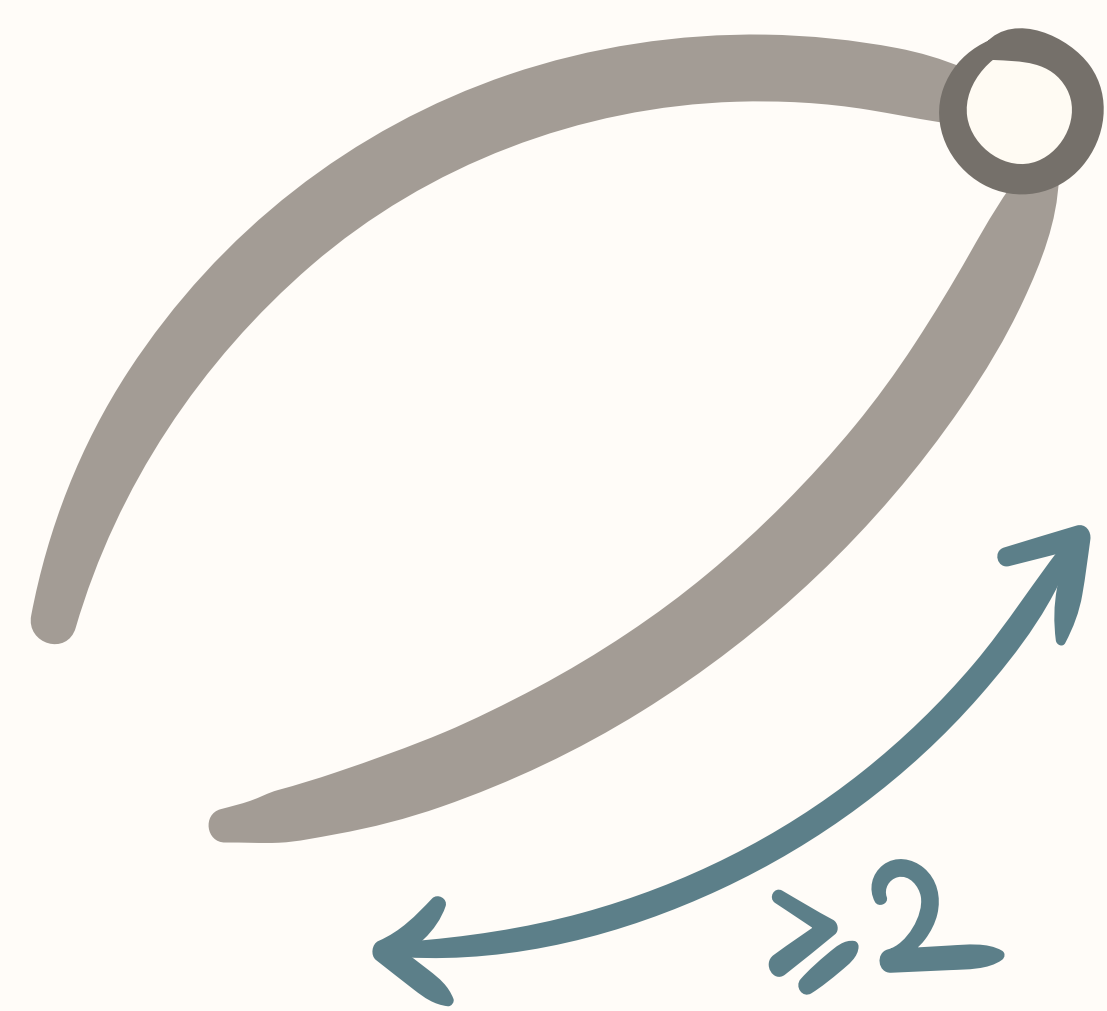
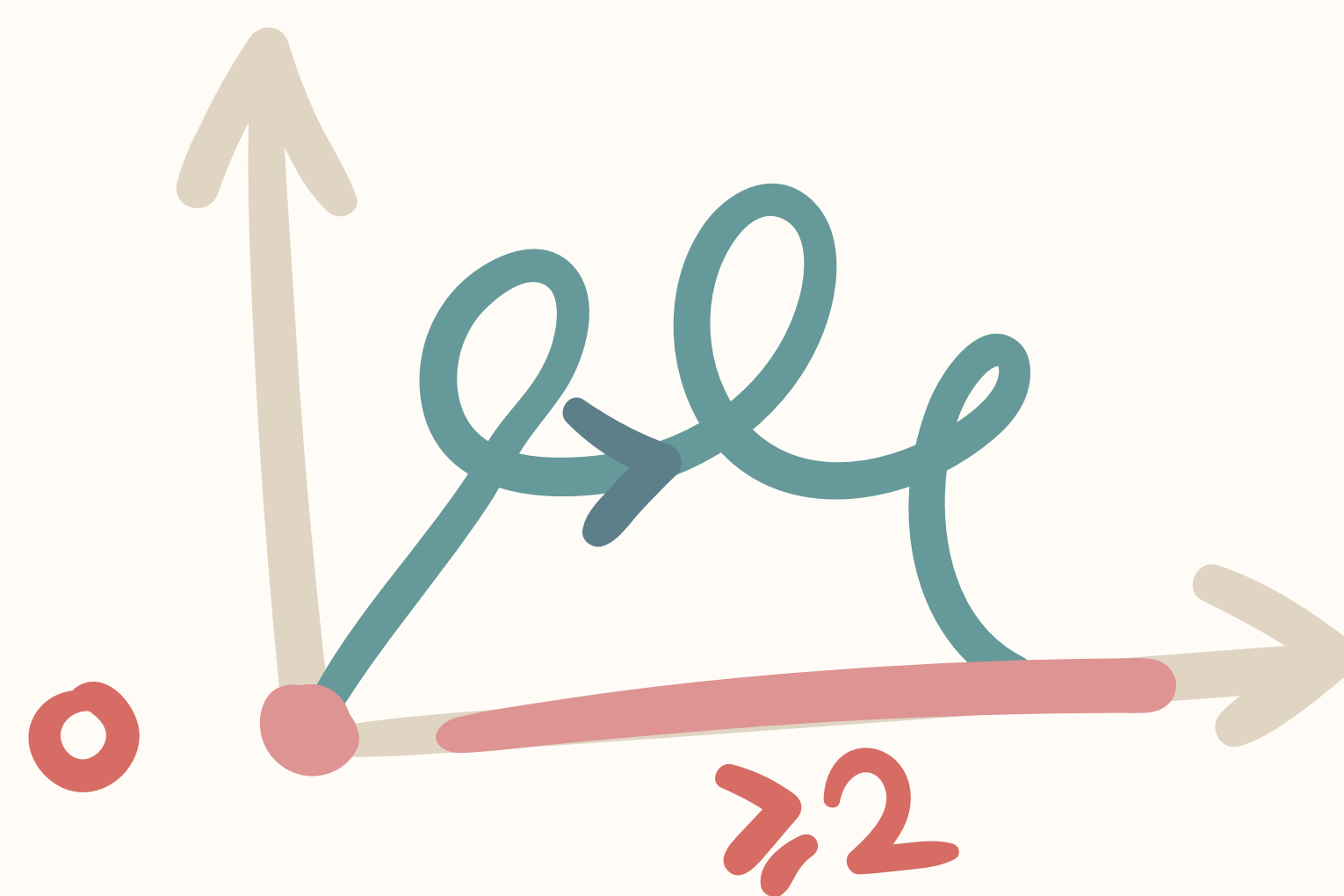
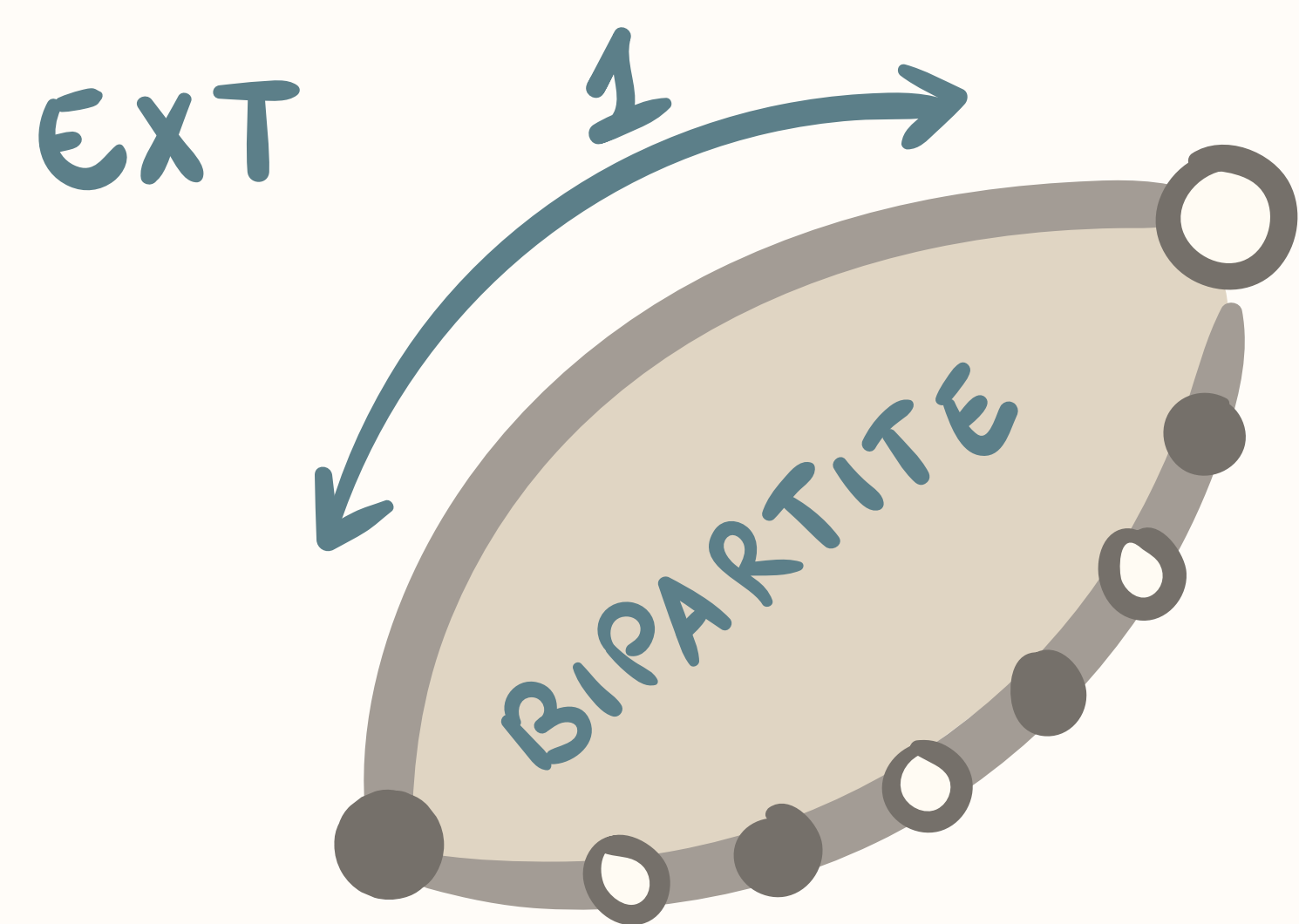
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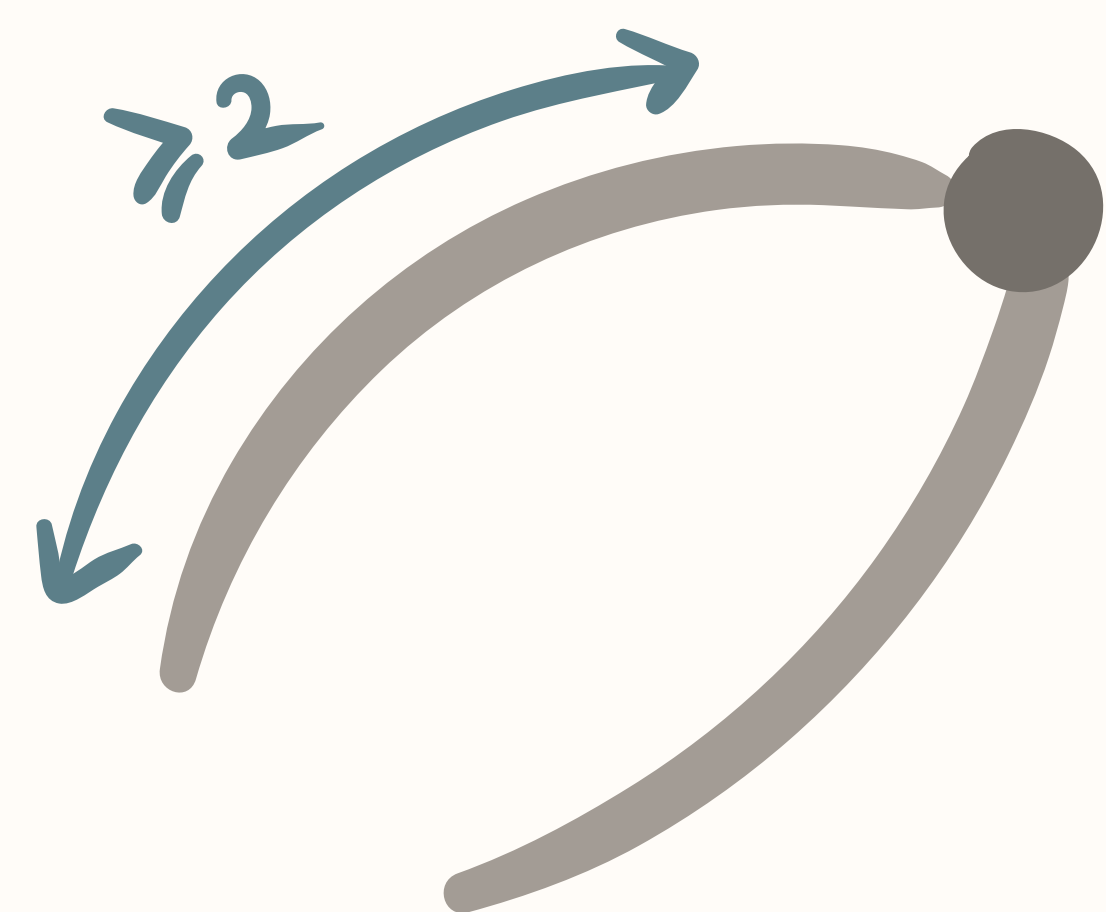
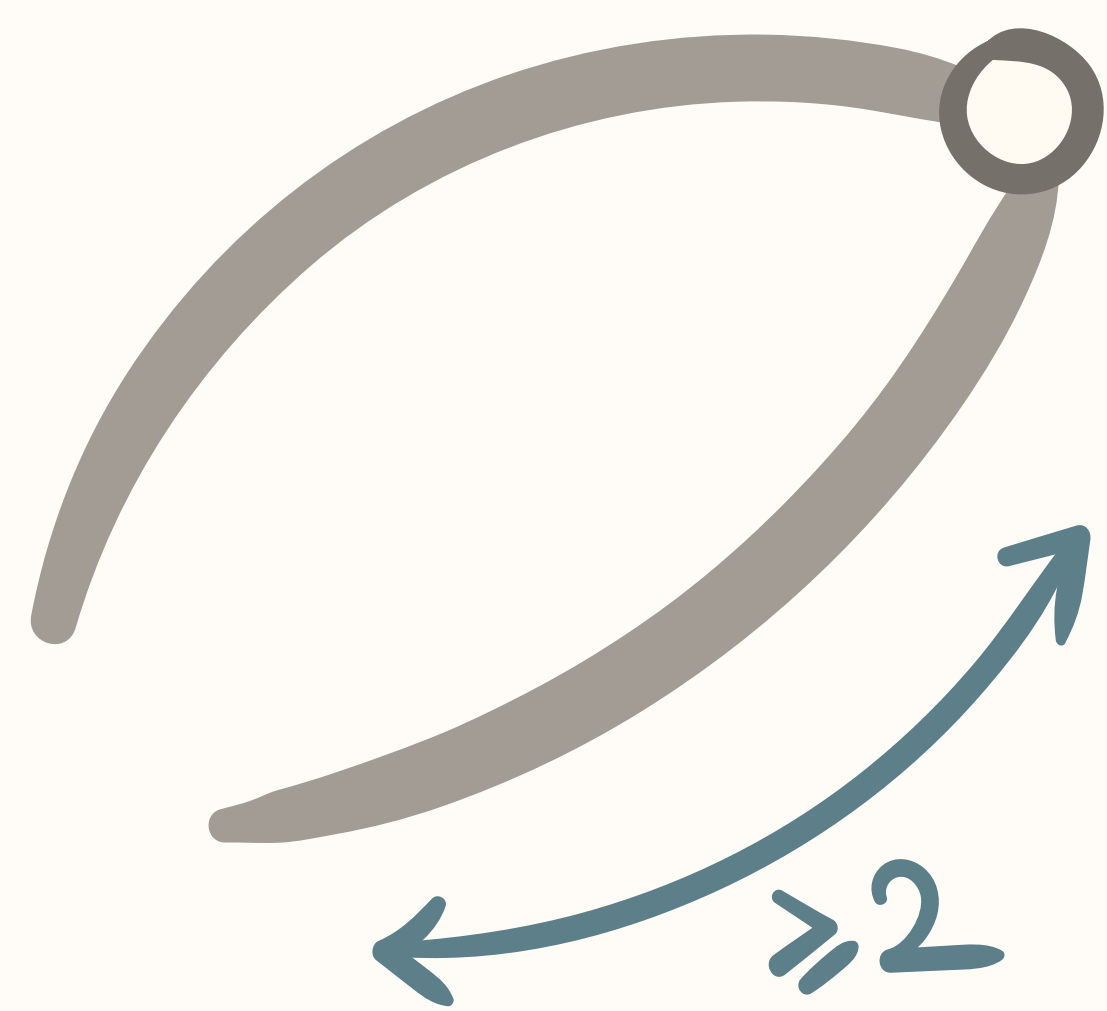
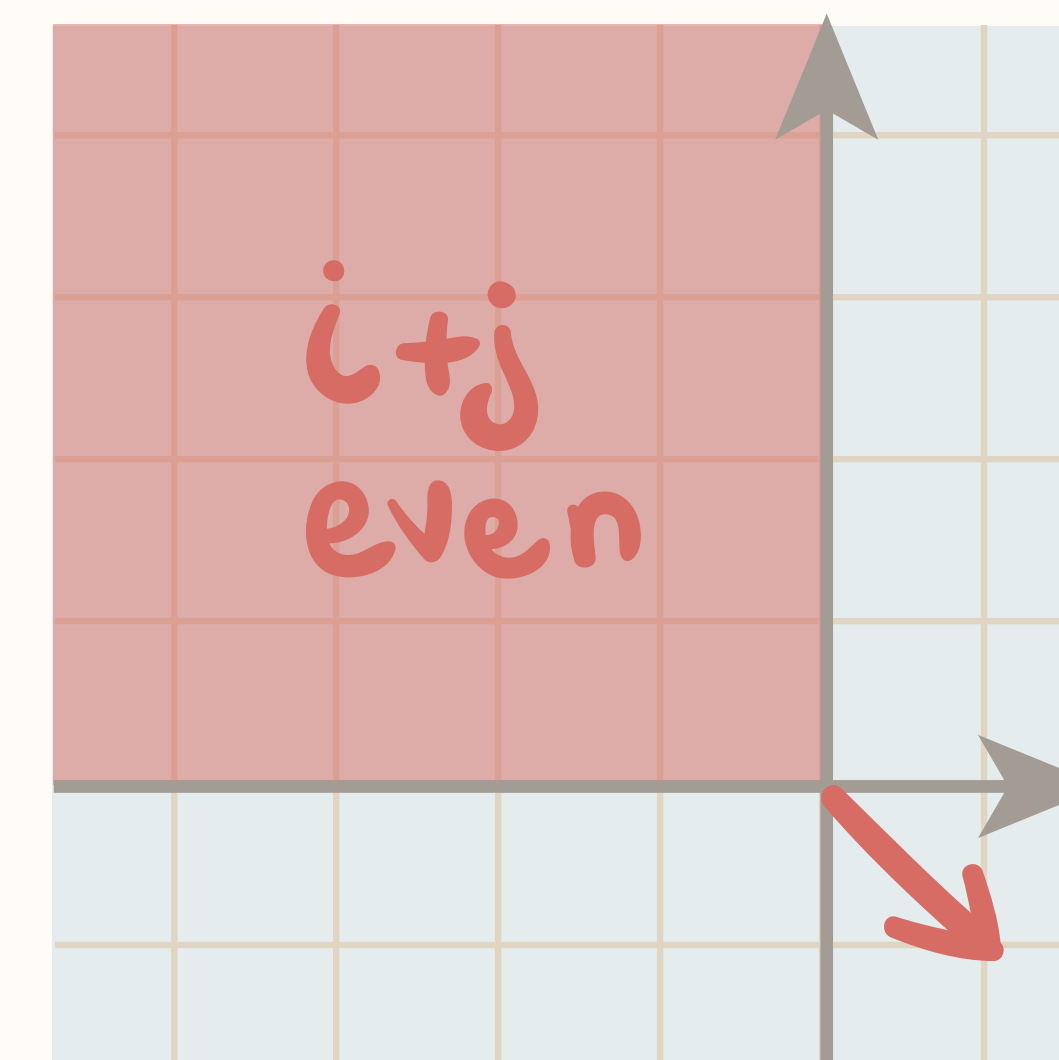
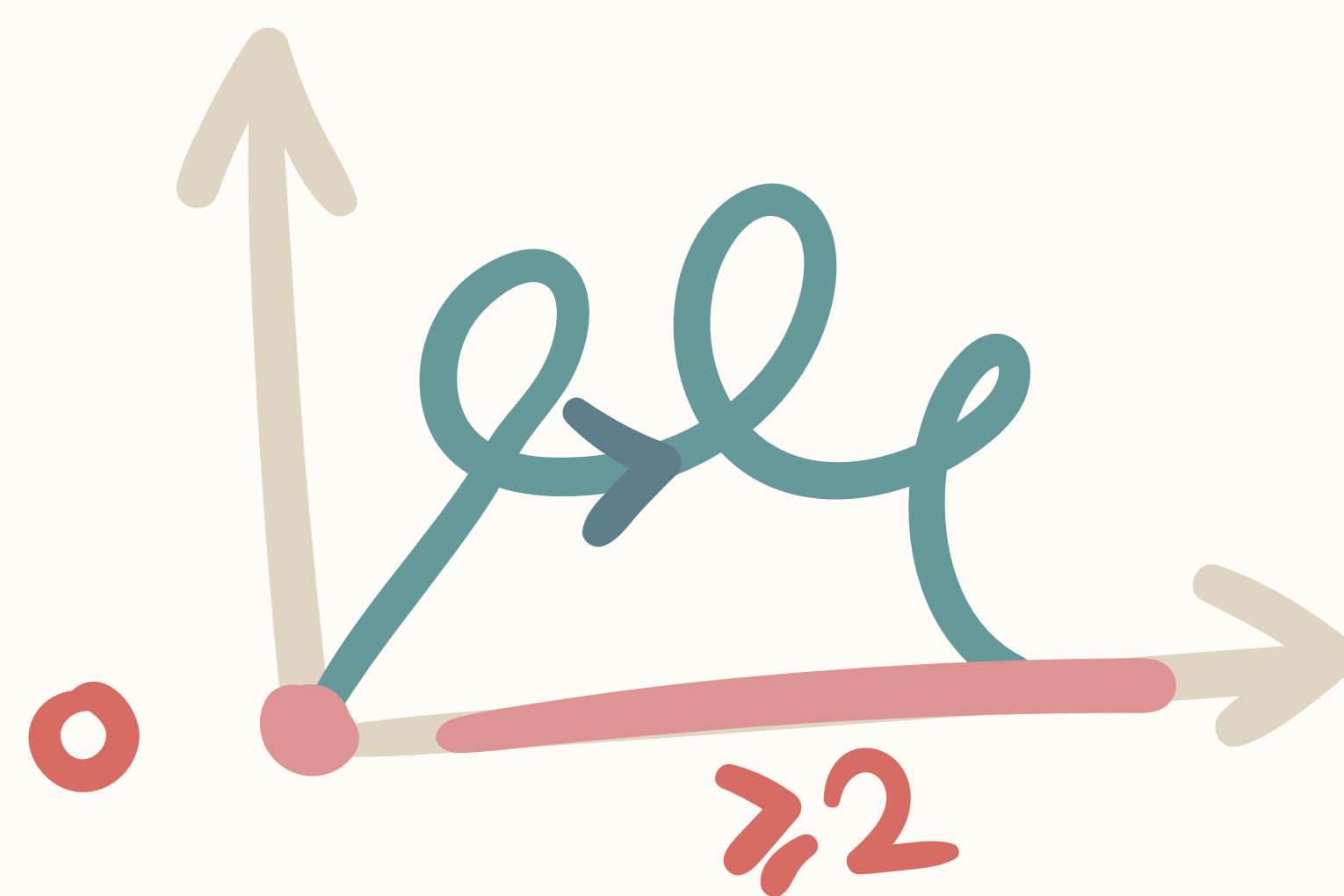
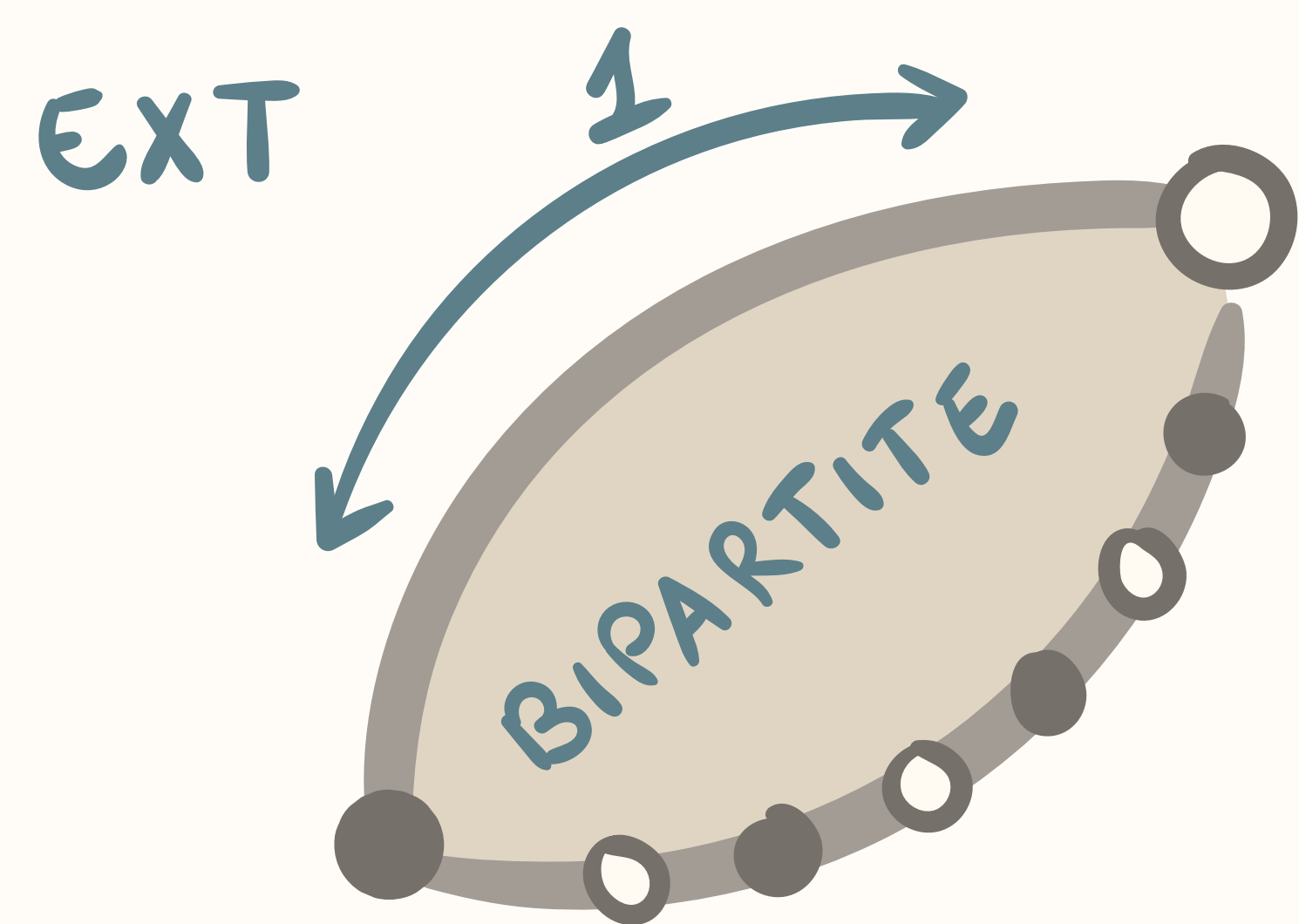
Polyhedral orientations via KMSW



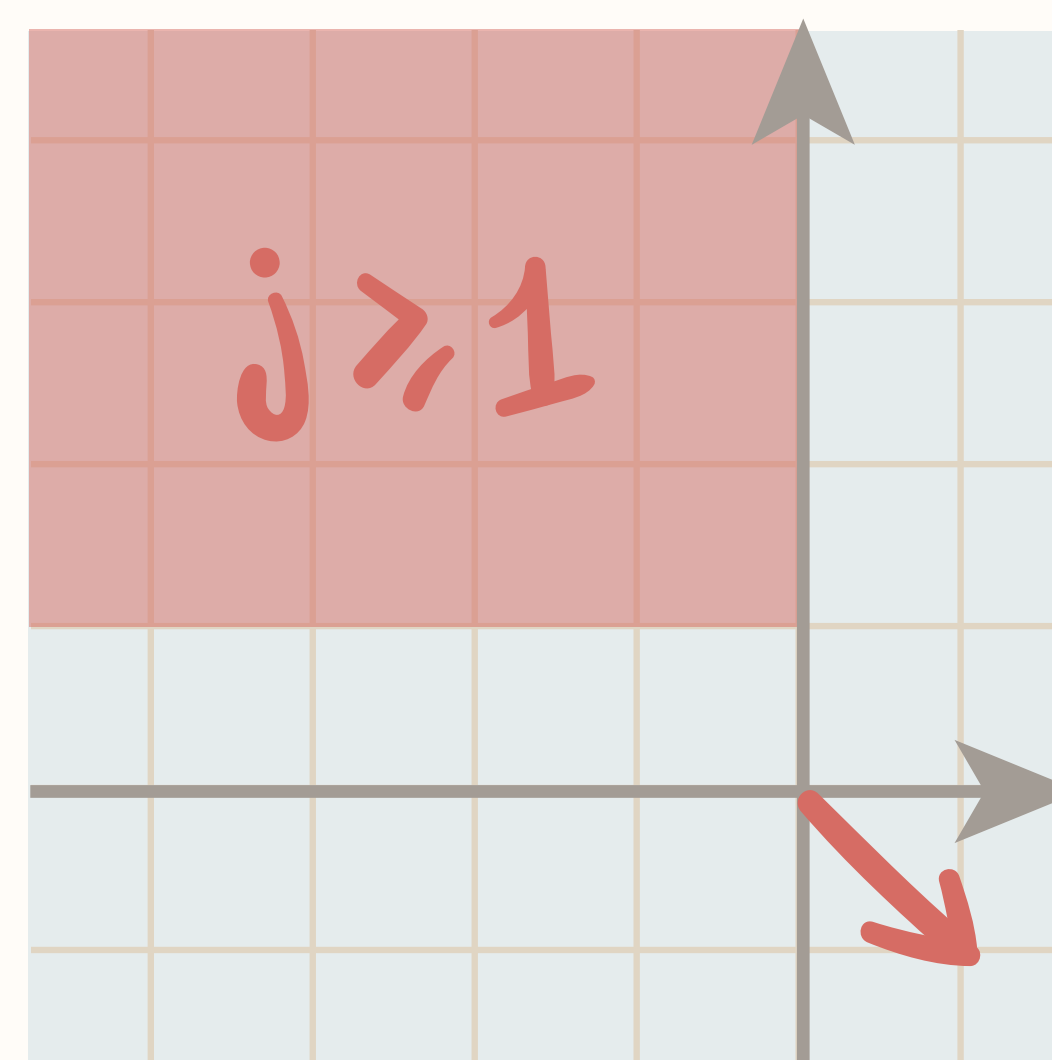
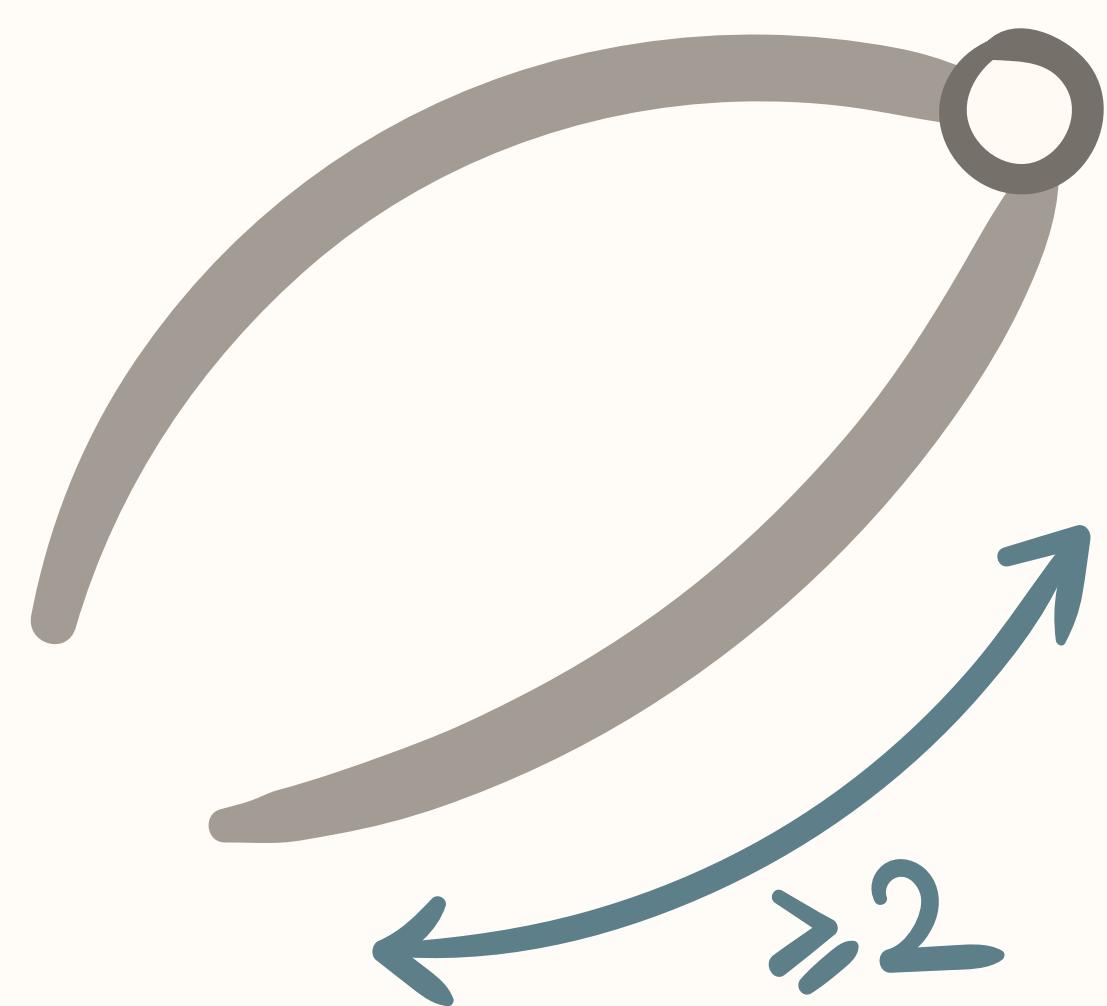
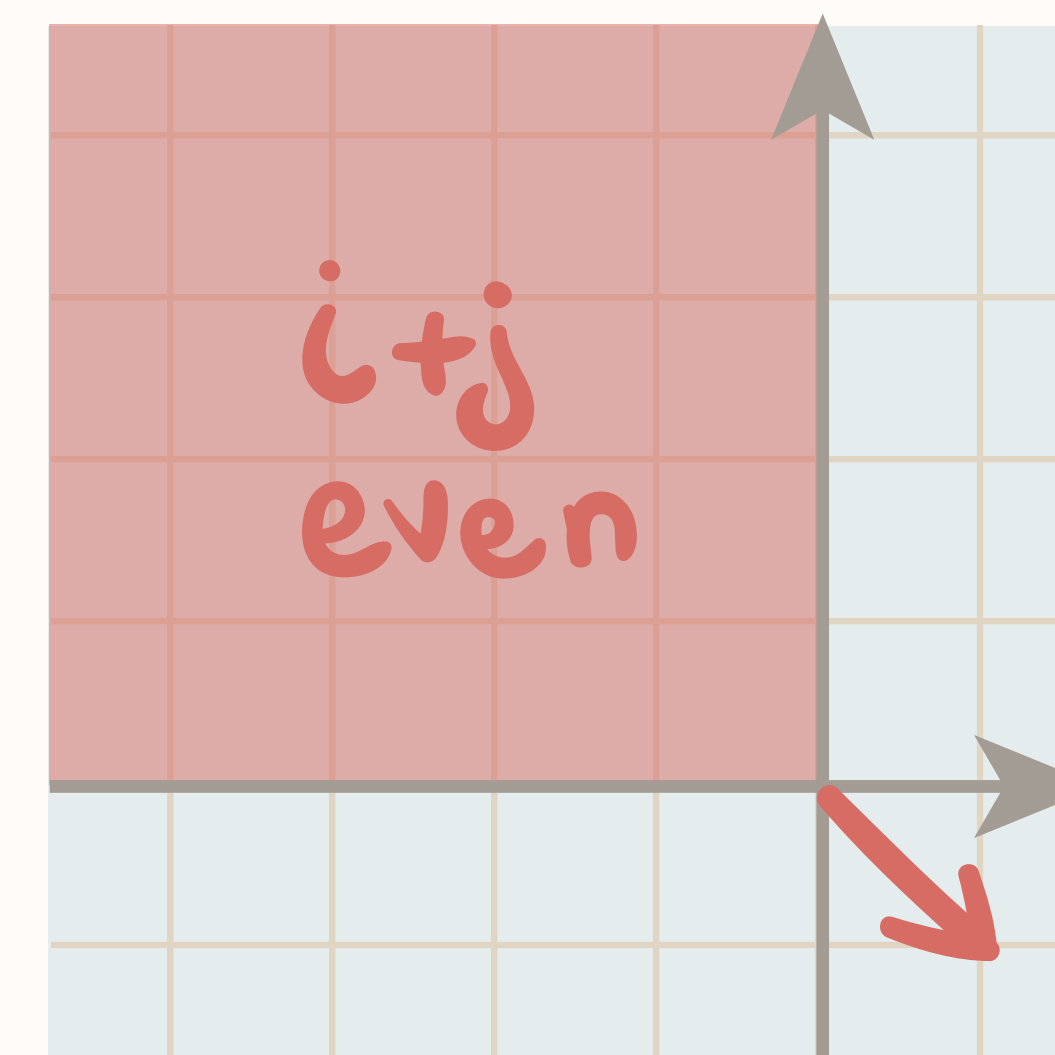
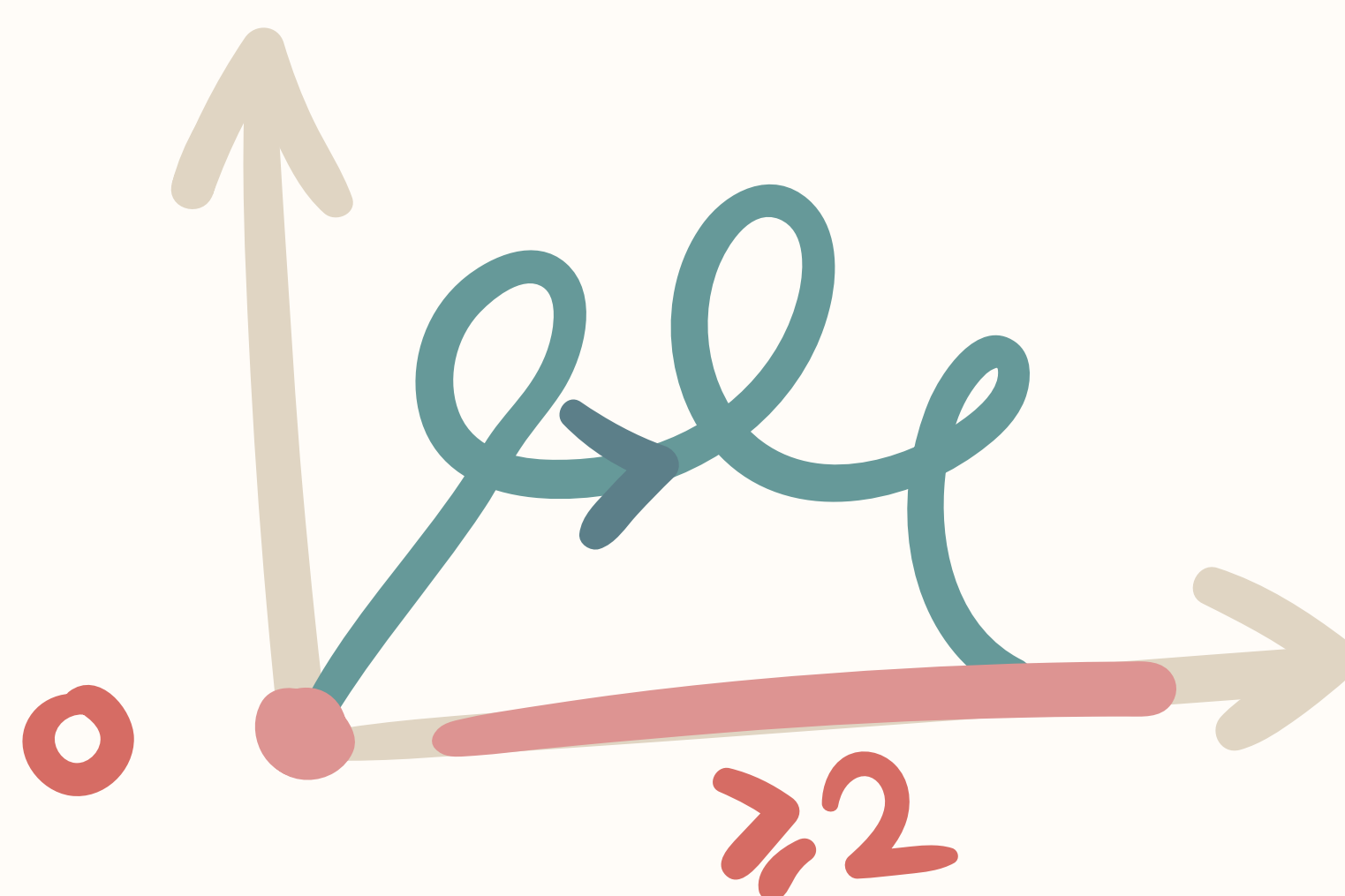
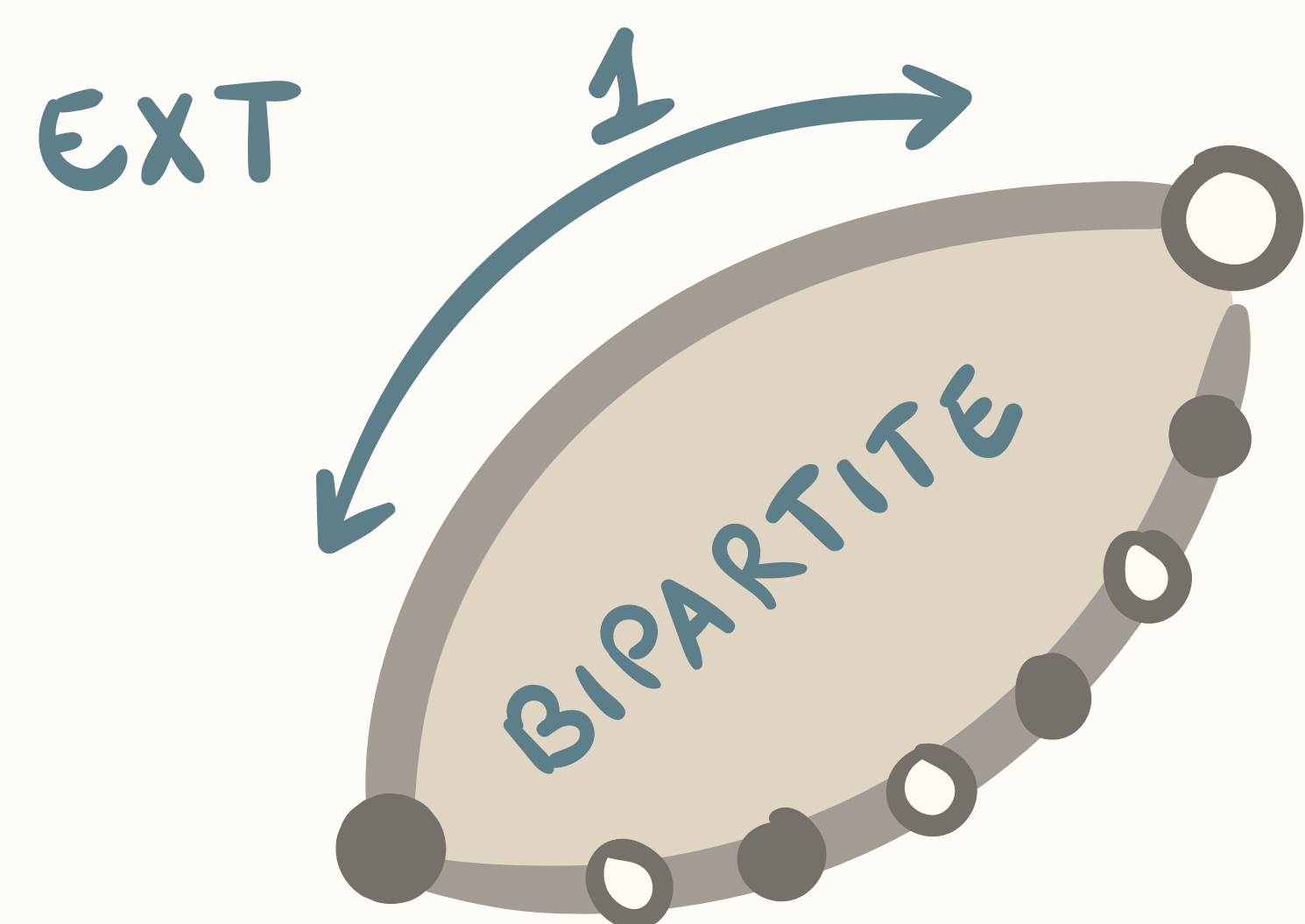
Polyhedral orientations via KMSW



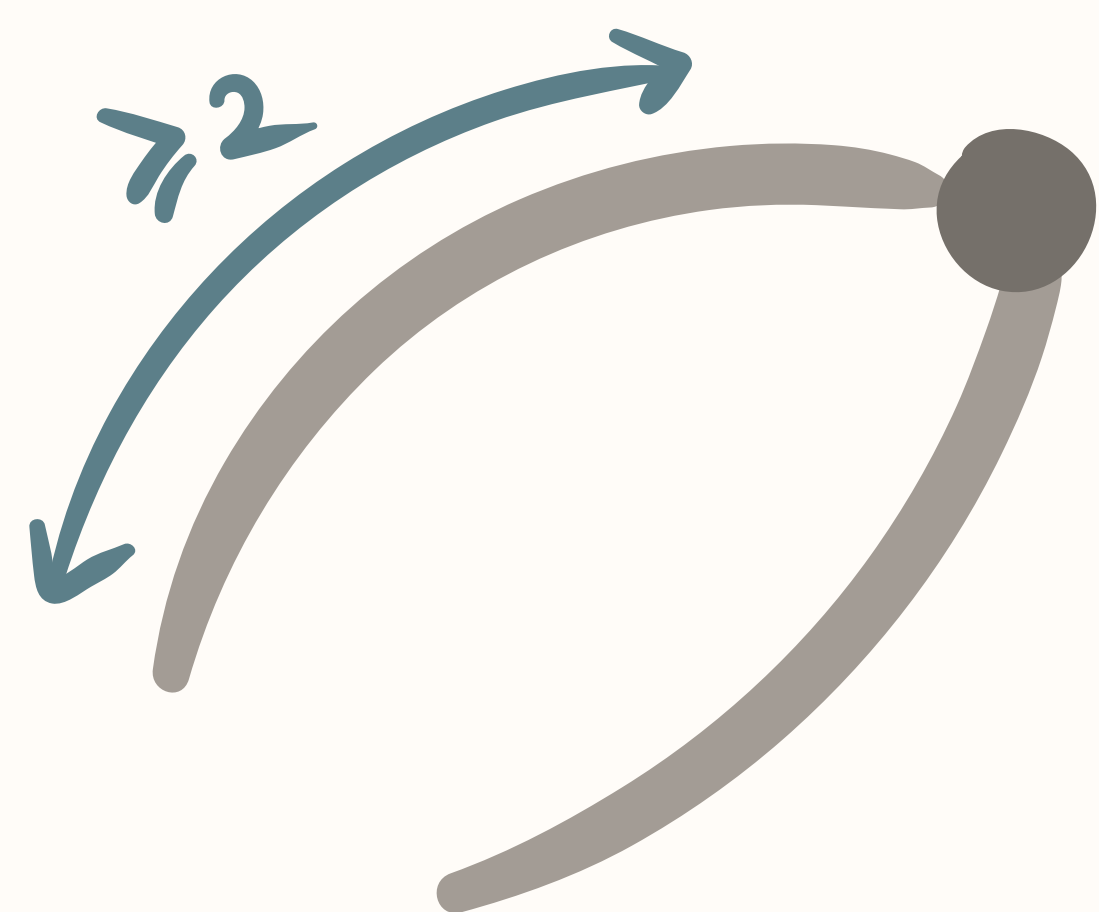
Polyhedral orientations via KMSW



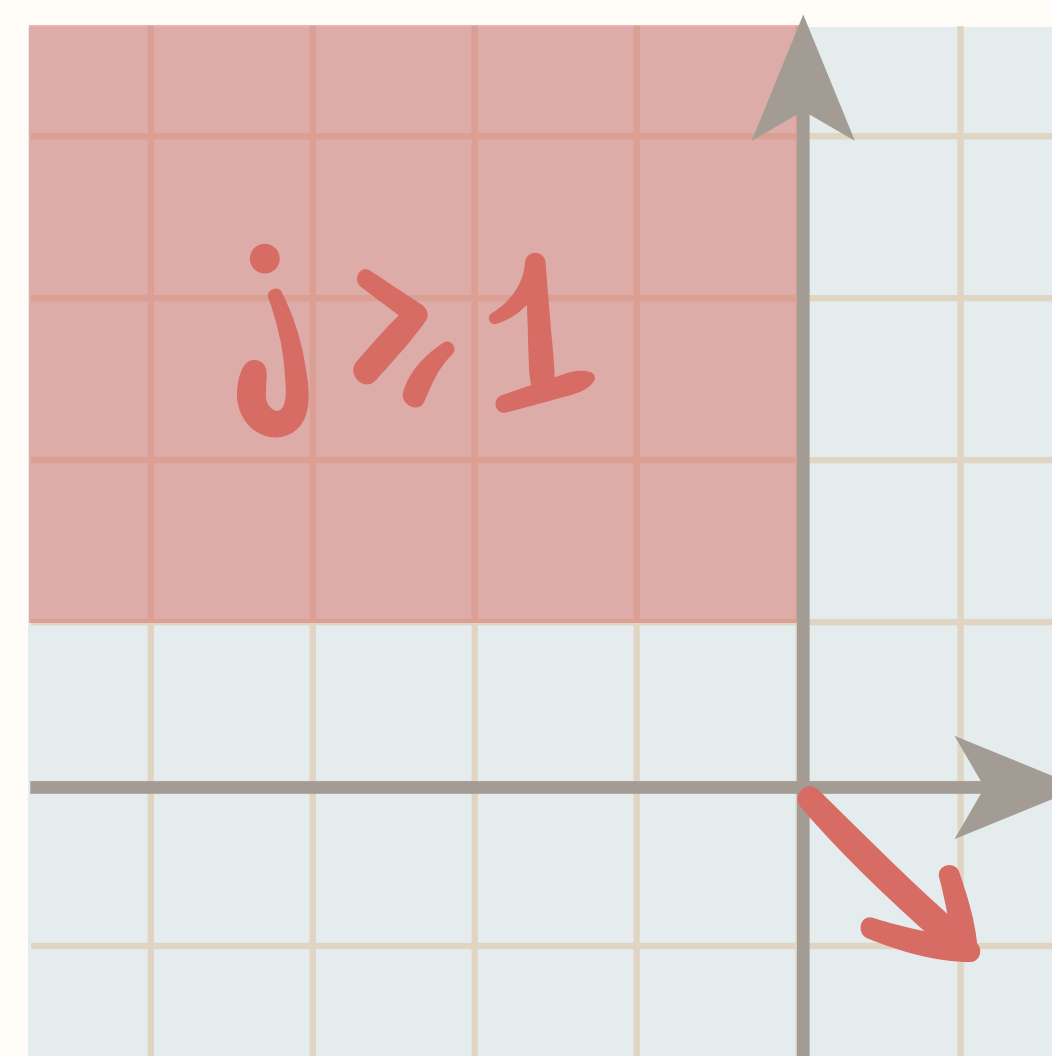
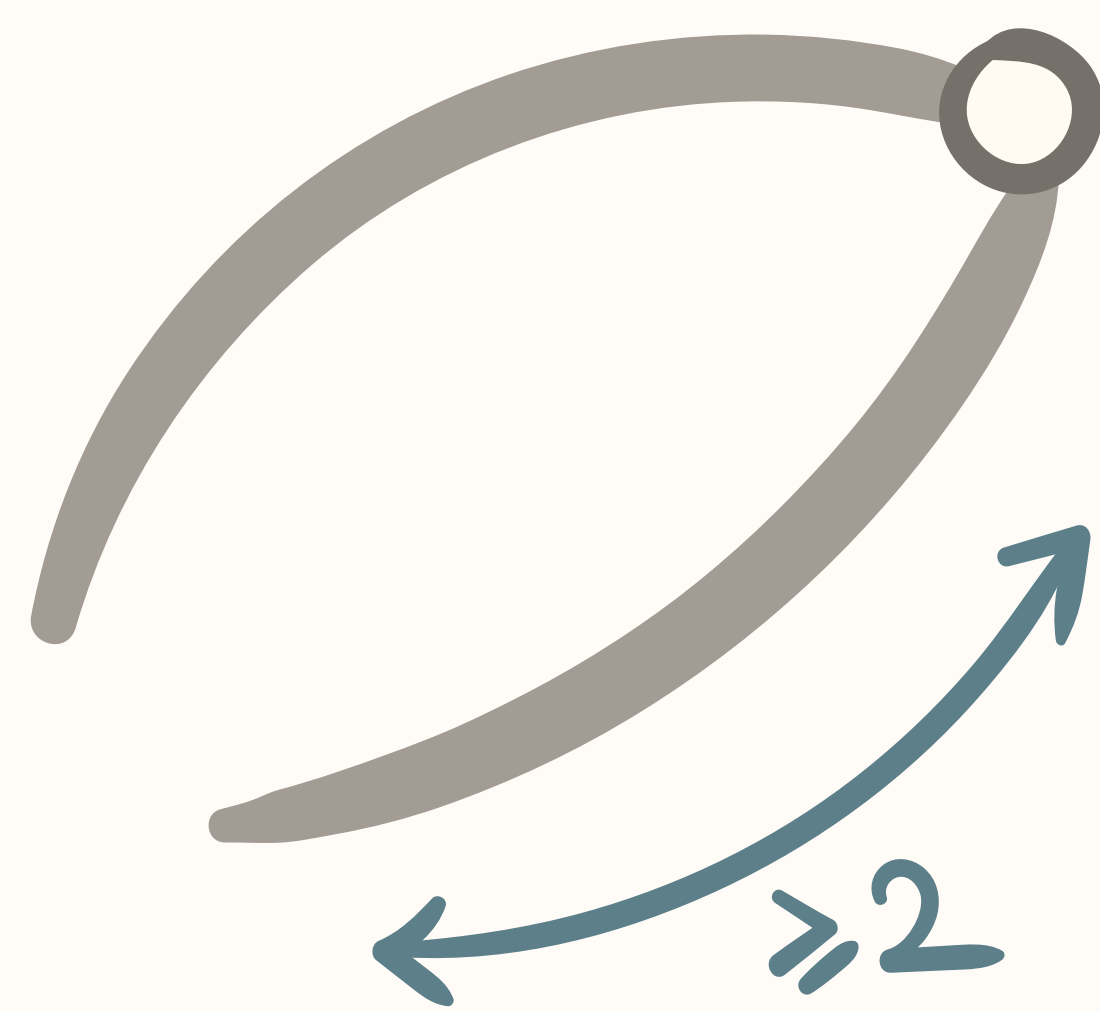
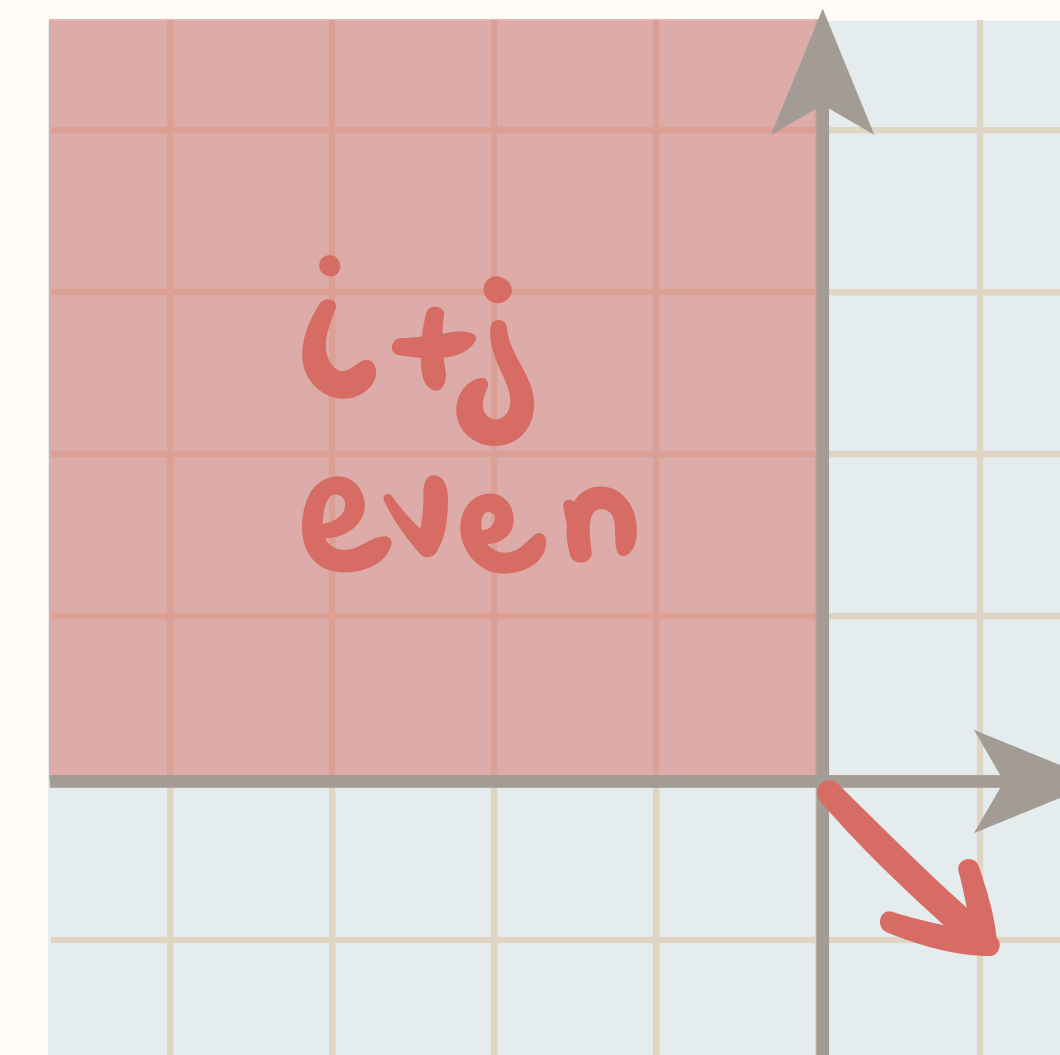
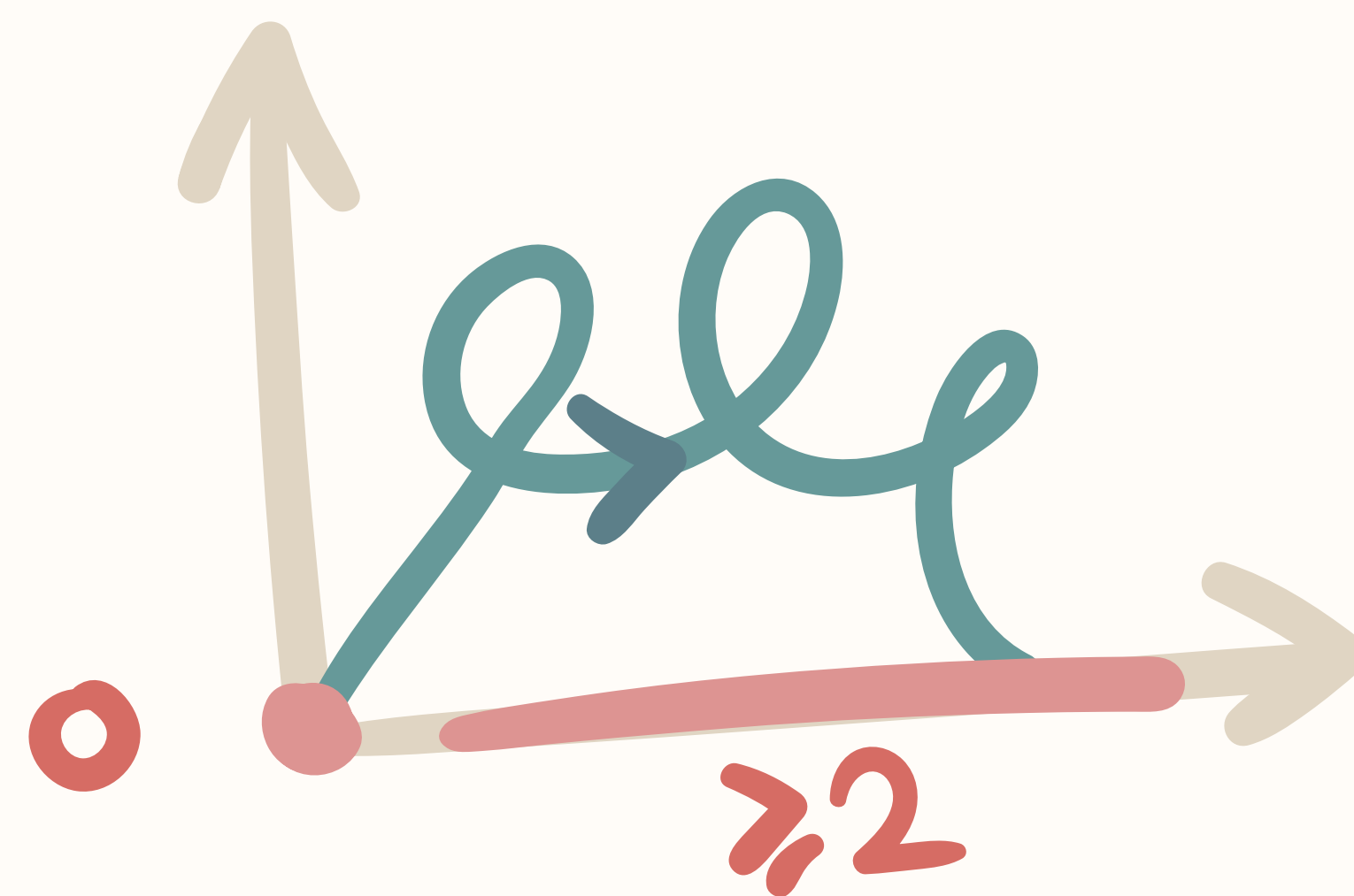
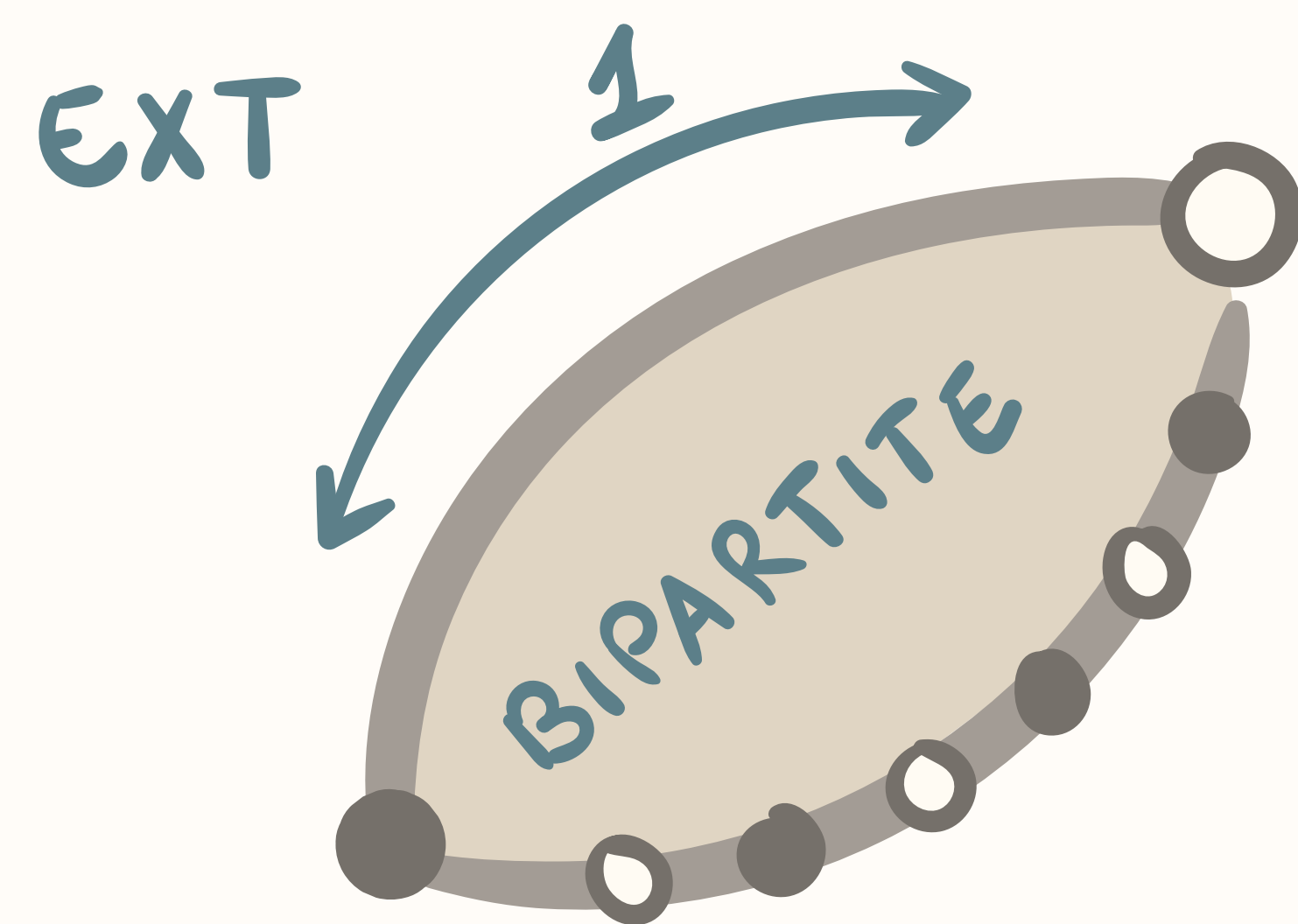
Polyhedral orientations via KMSW



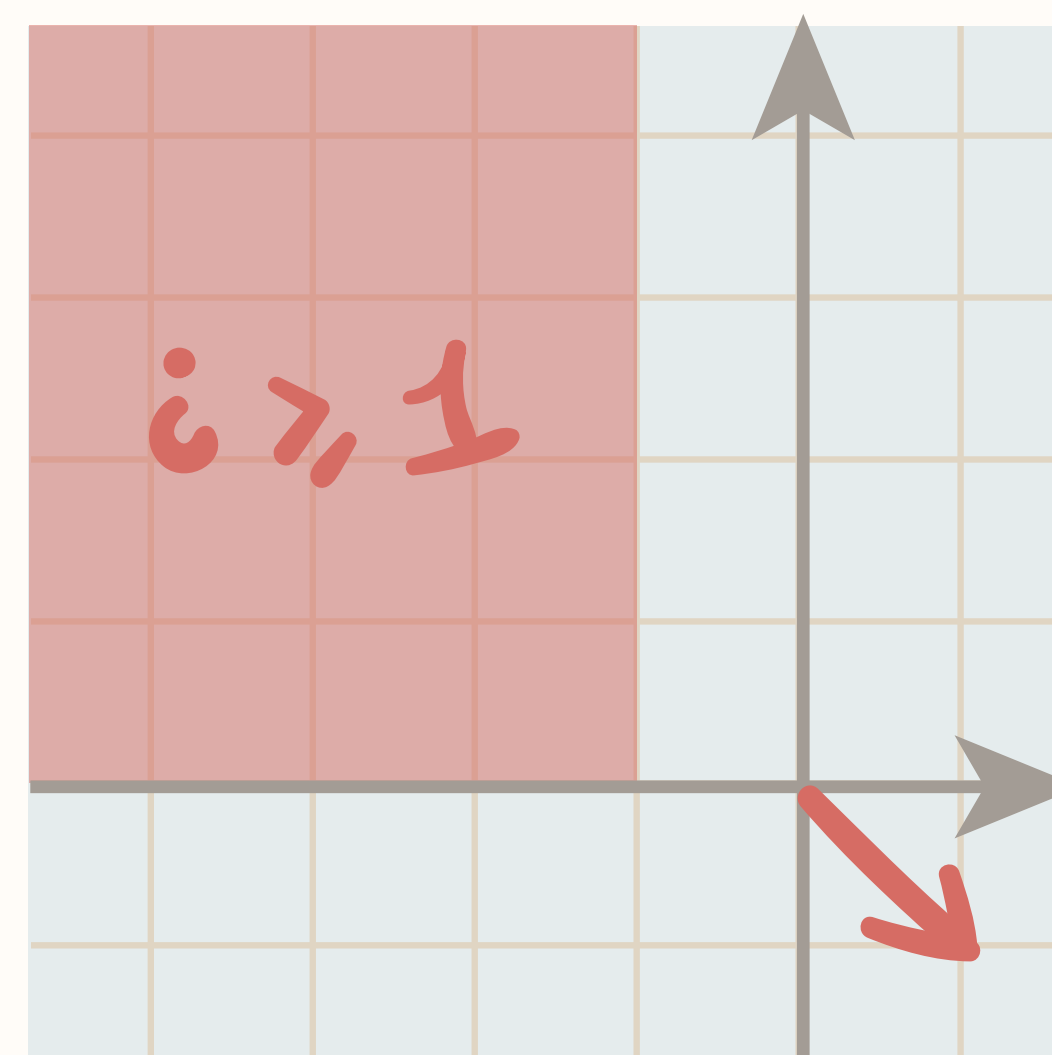
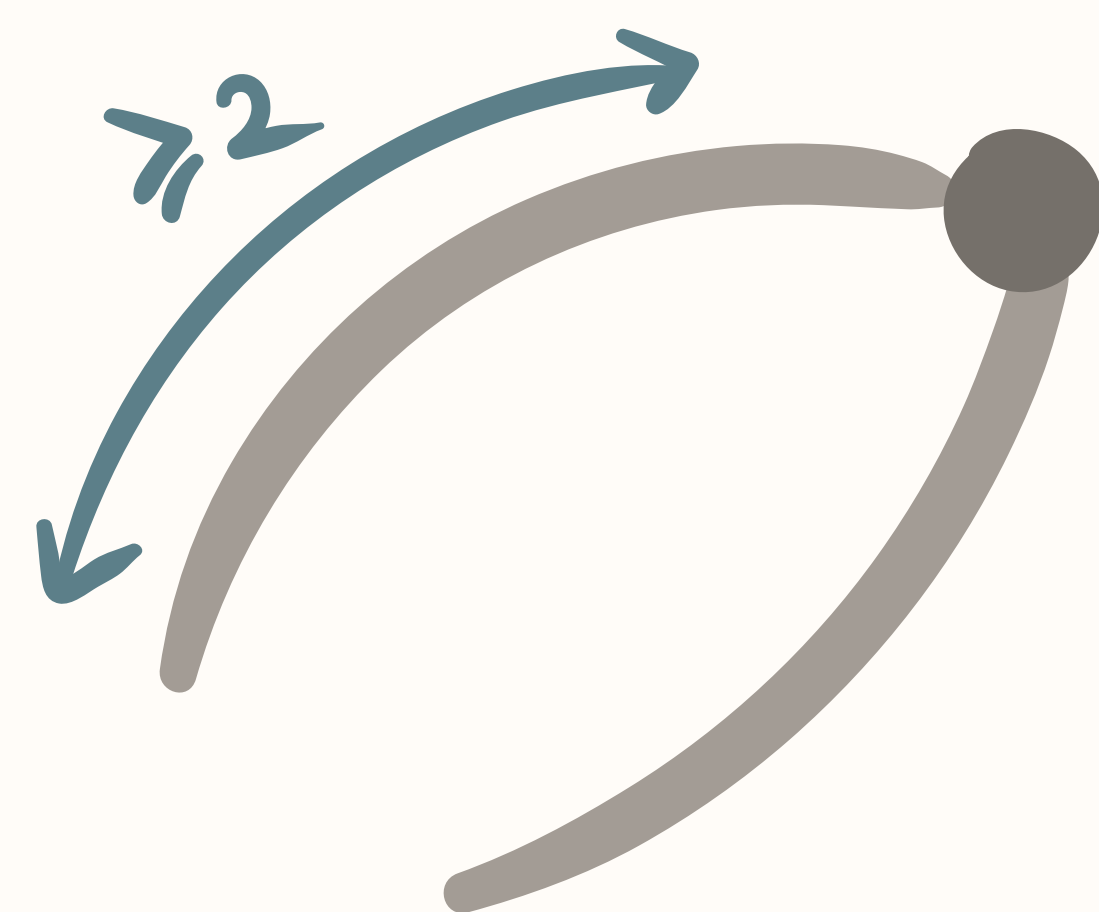
when x, y
are even



Polyhedral orientations via KMSW

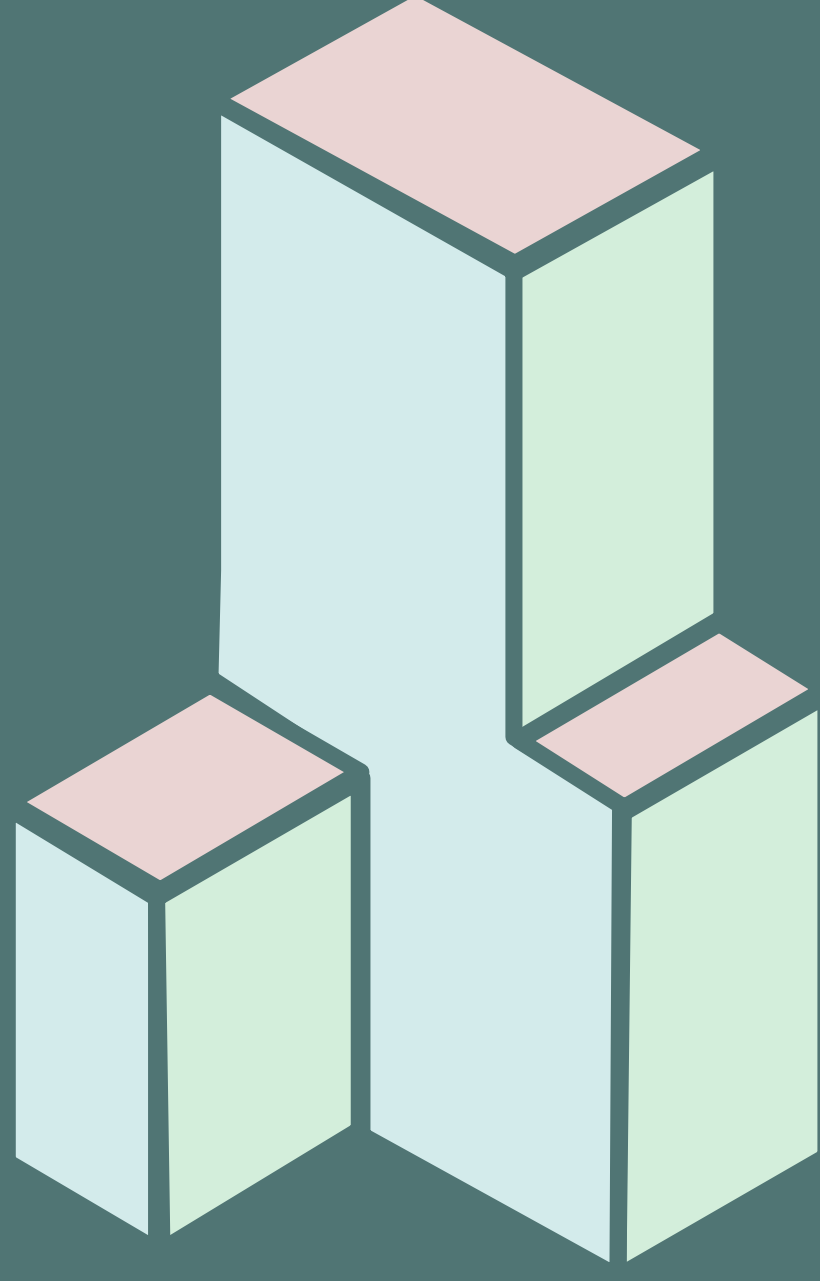


when x, y
are even

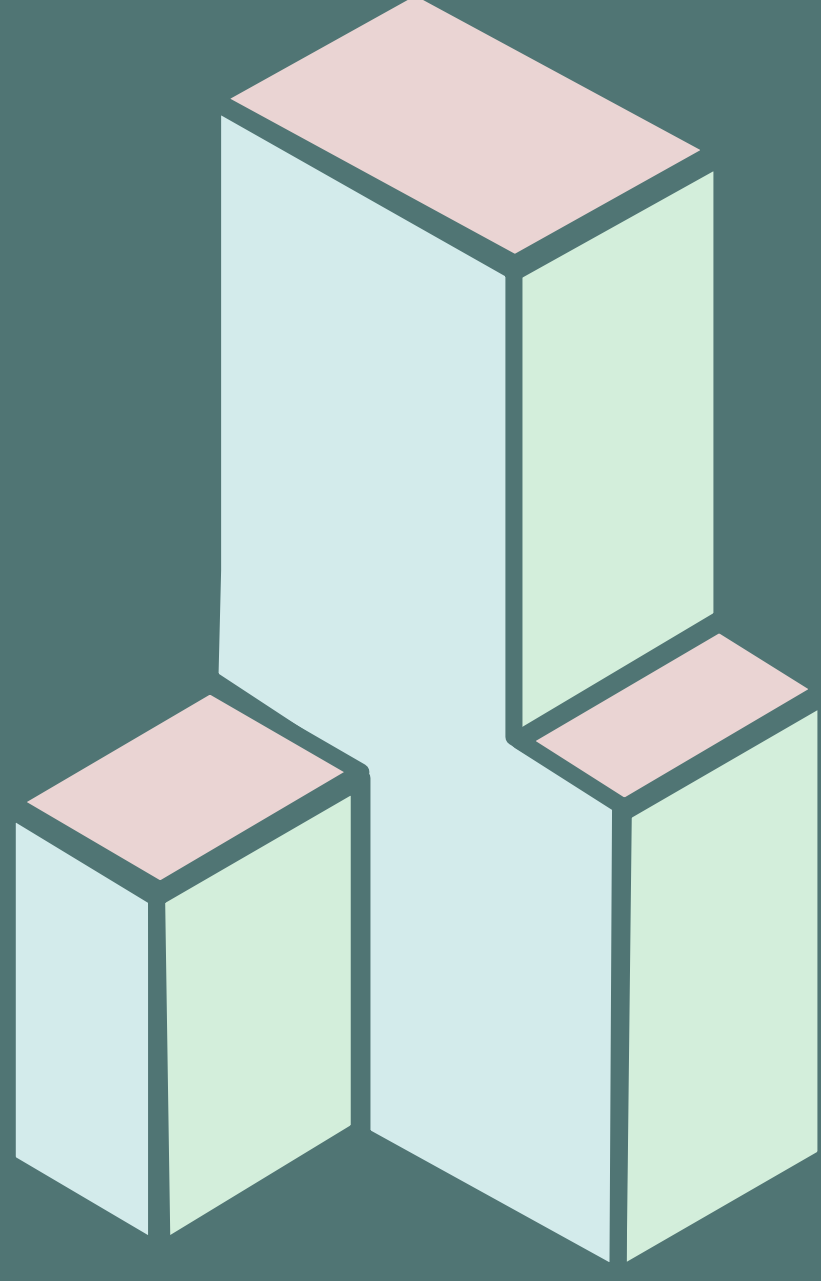



when x, y
are odd

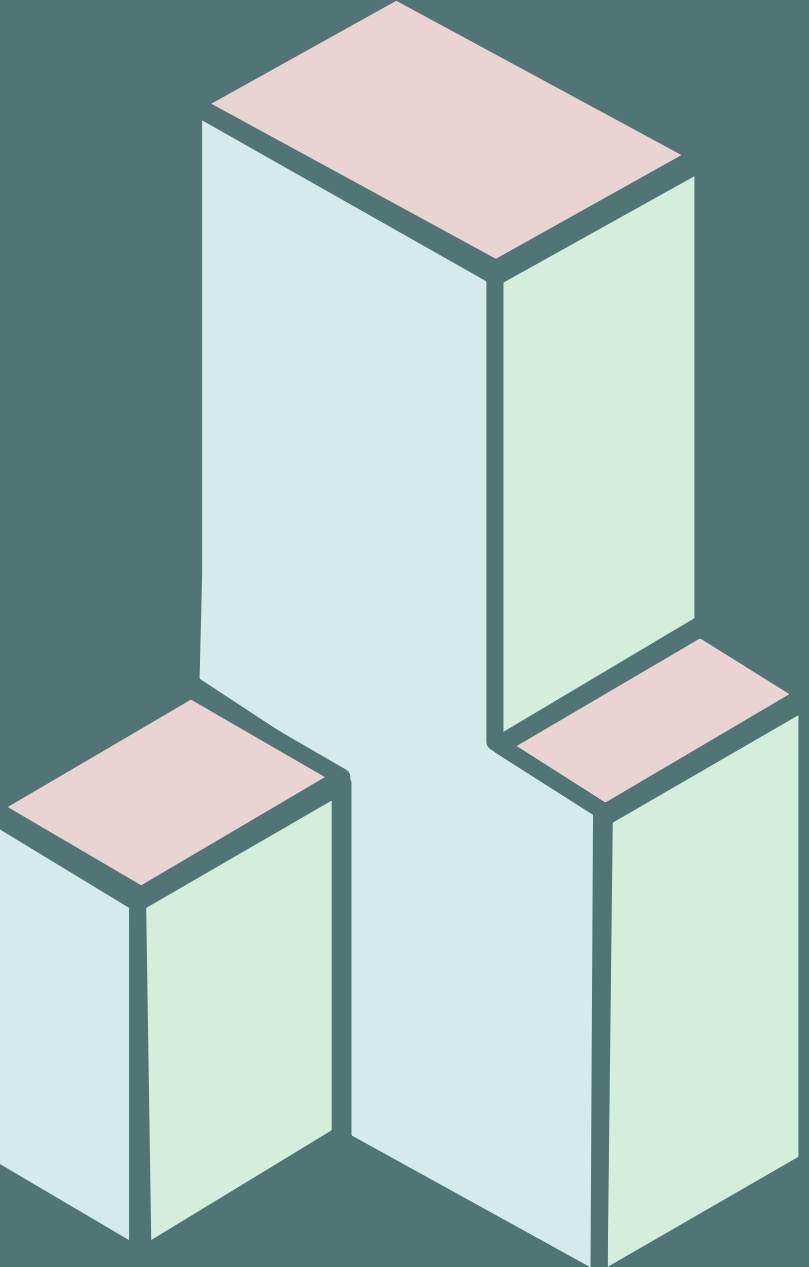


Bijections summary

<i>Model</i>	<i>Combinatorial type</i>	<i>Bipolar orient./Tandem walk</i>
 <p data-bbox="302 1078 533 1164"><i>n flats</i></p>		

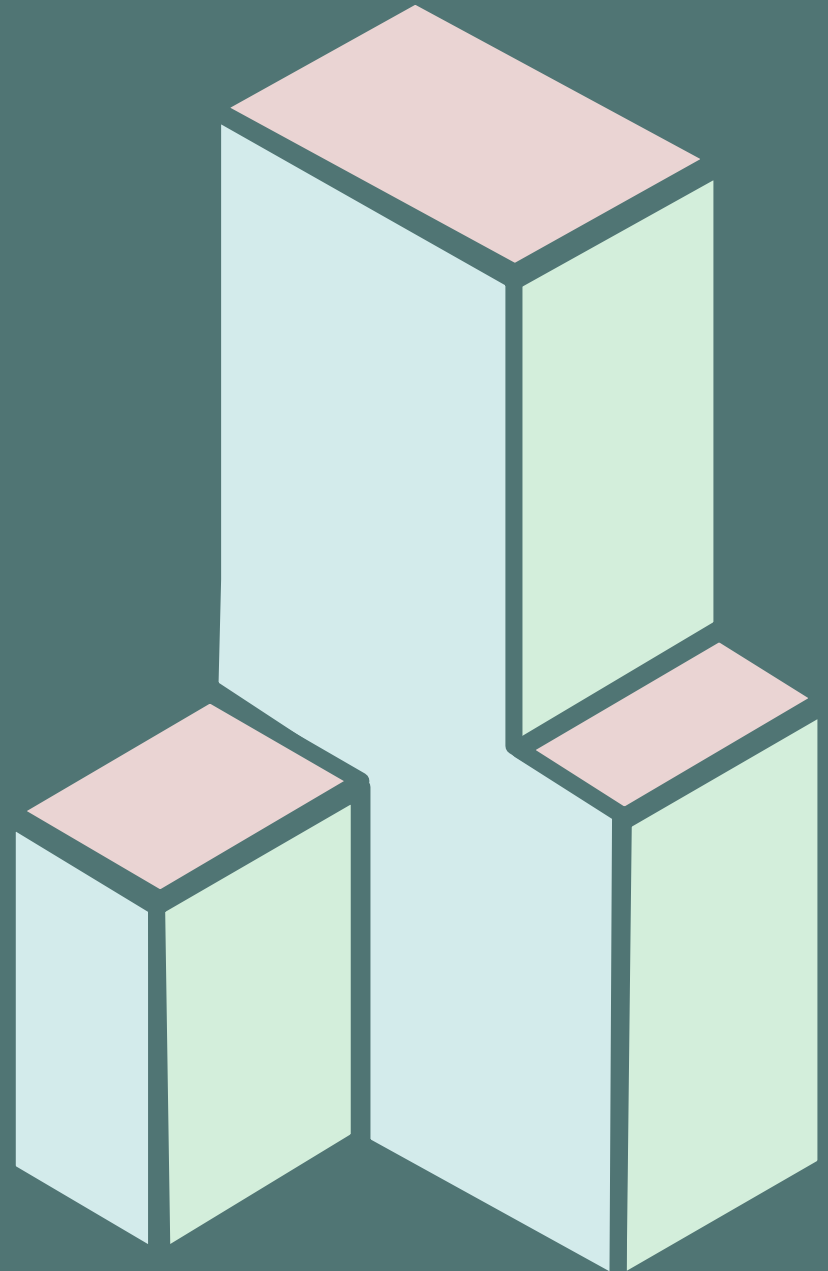

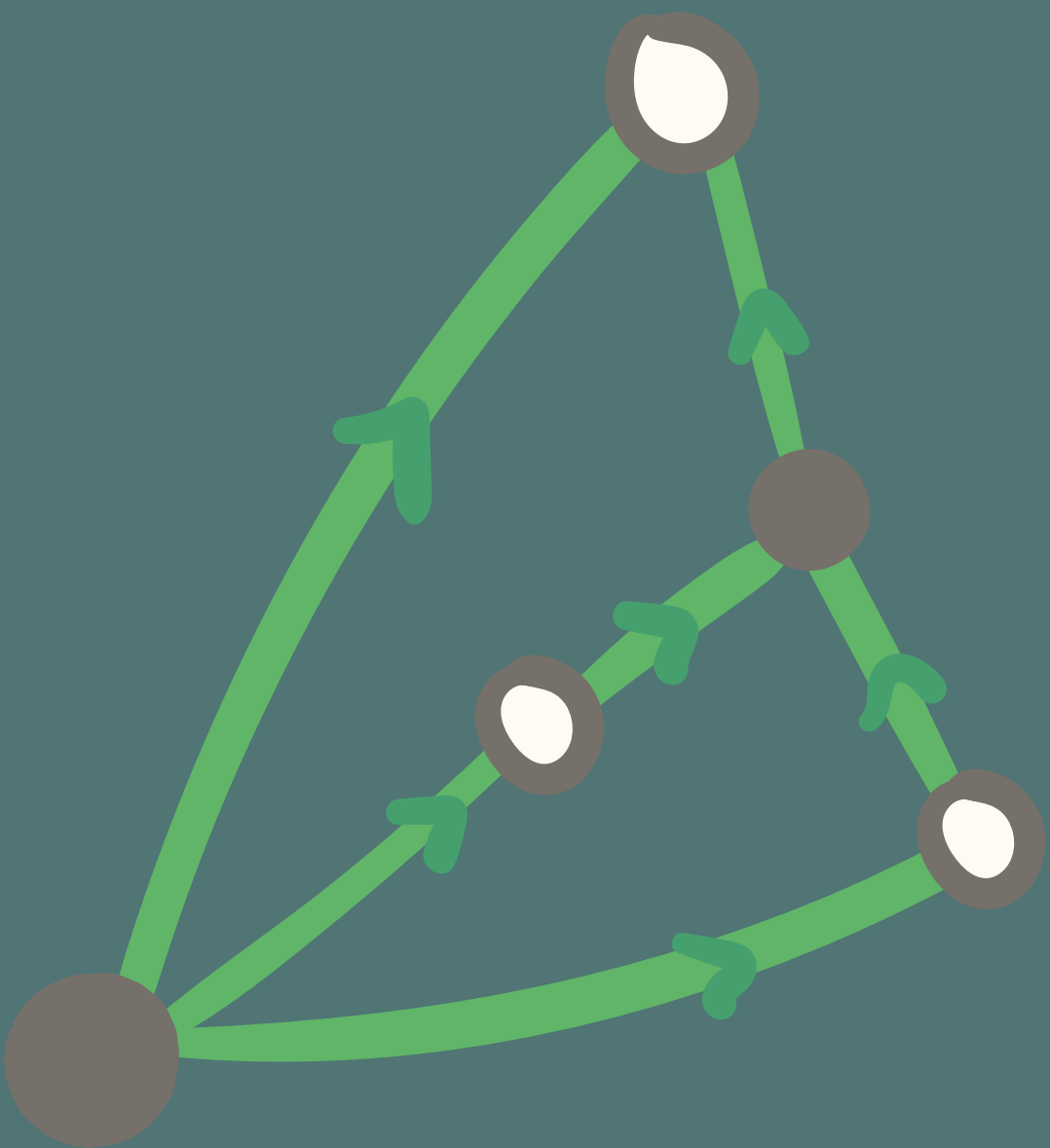
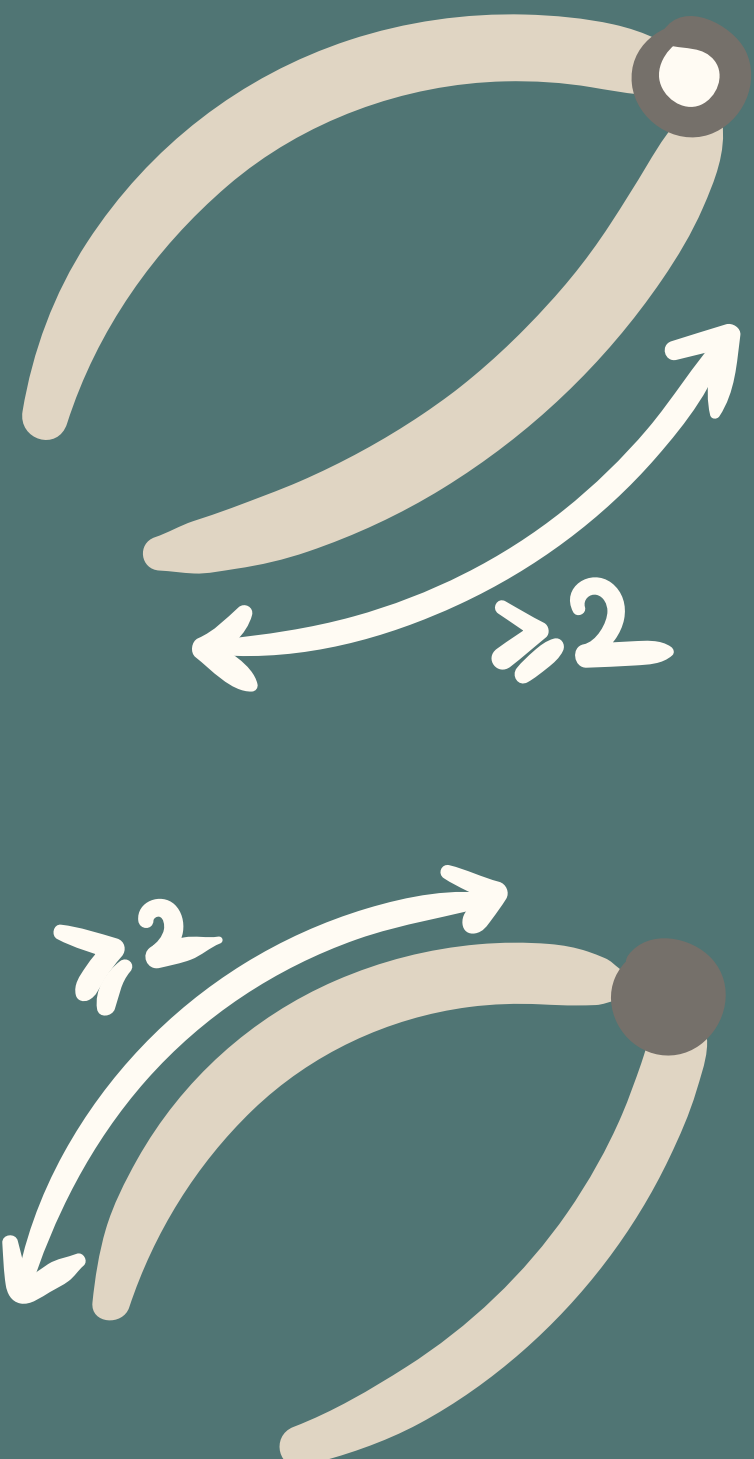

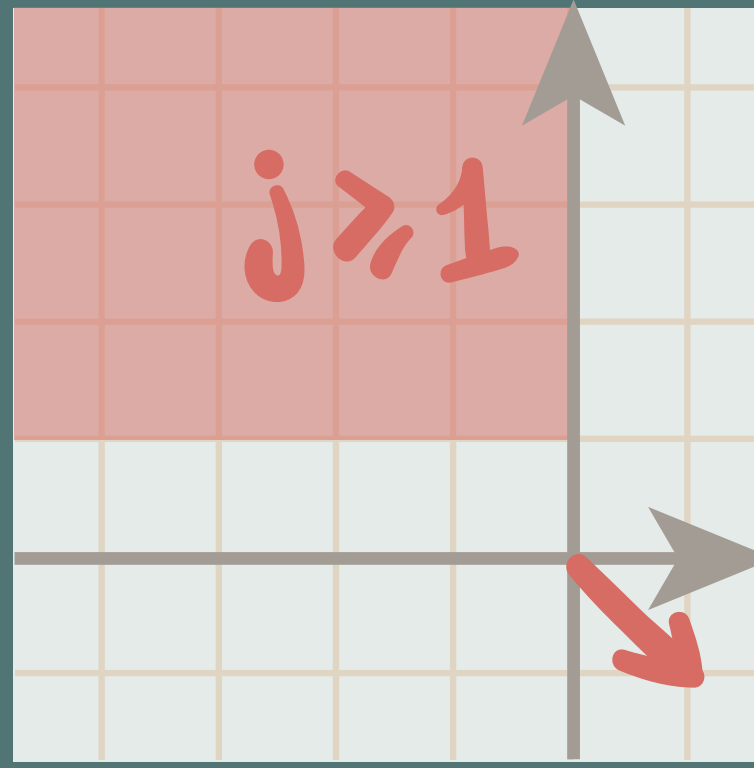

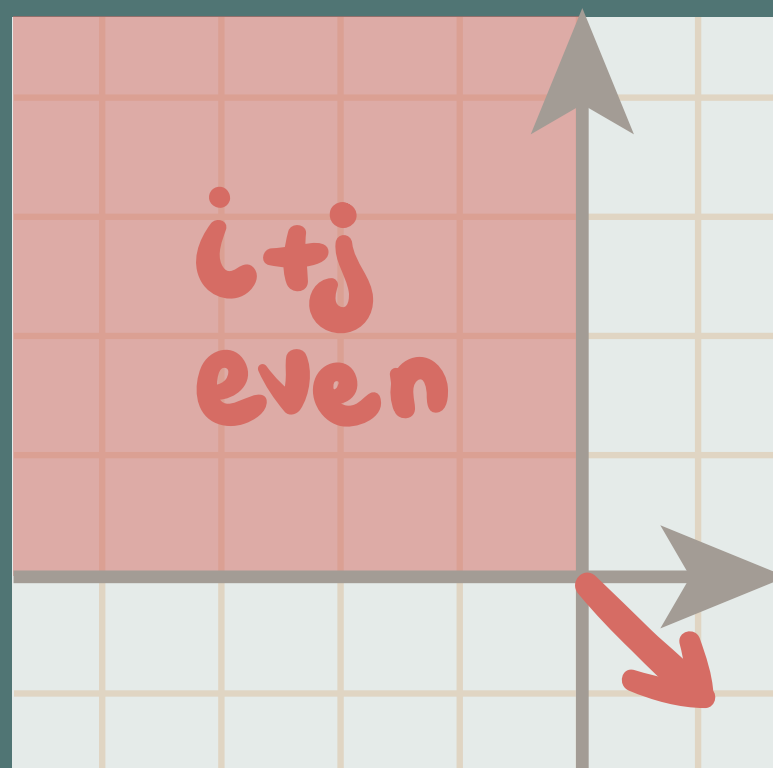
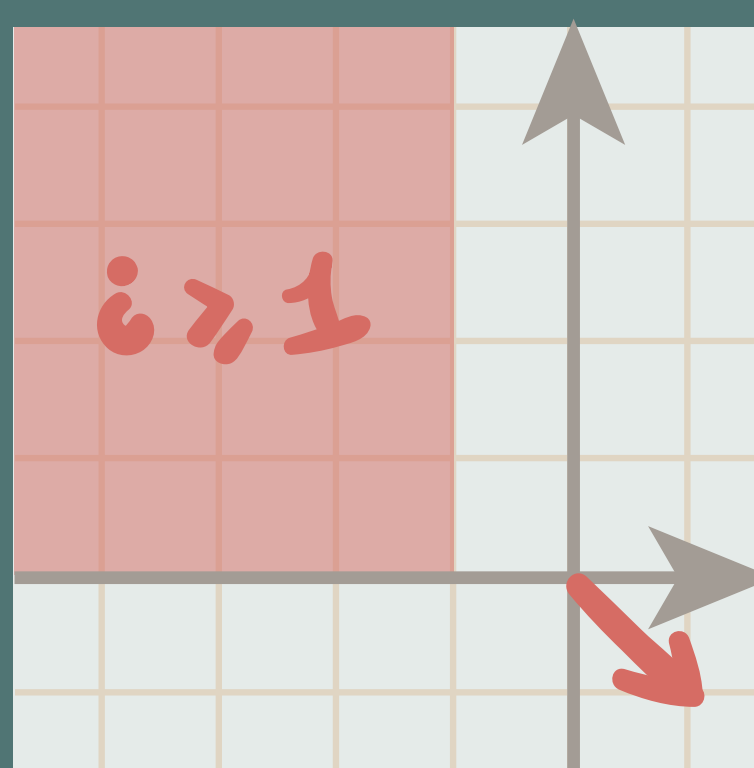

Bijections summary

<i>Model</i>	<i>Combinatorial type</i>	<i>Bipolar orient./Tandem walk</i>
 <p data-bbox="302 1078 533 1164"><i>n flats</i></p>	 <p data-bbox="821 1078 1384 1164"><i>n inner vertices</i></p>	

Bijections summary

<i>Model</i>	<i>Combinatorial type</i>	<i>Bipolar orient./Tandem walk</i>
 <p data-bbox="311 1086 530 1159"><i>n flats</i></p>	 <p data-bbox="830 1086 1378 1159"><i>n inner vertices</i></p>	 <p data-bbox="1655 1086 2001 1159"><i>n+1 edges</i></p>

Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk
 <p data-bbox="302 1080 533 1159">n flats</p>	 <p data-bbox="830 1080 1378 1159">n inner vertices</p>	<div style="display: flex; justify-content: space-around;"> <div data-bbox="1638 519 2093 1015">  <p data-bbox="1652 1080 1998 1159">$n+1$ edges</p> </div> <div data-bbox="2266 476 2584 1090">  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div data-bbox="1522 1274 2021 1597">  </div> <div data-bbox="2165 1274 2483 1597">  </div> <div data-bbox="2526 1392 2598 1457">  </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div data-bbox="1623 1662 1949 1985">  </div> <div data-bbox="2165 1662 2483 1985">  </div> <div data-bbox="2526 1802 2598 1867">  </div> </div>

Sommaire

The KMSW Bijection

1. Application to three maps model

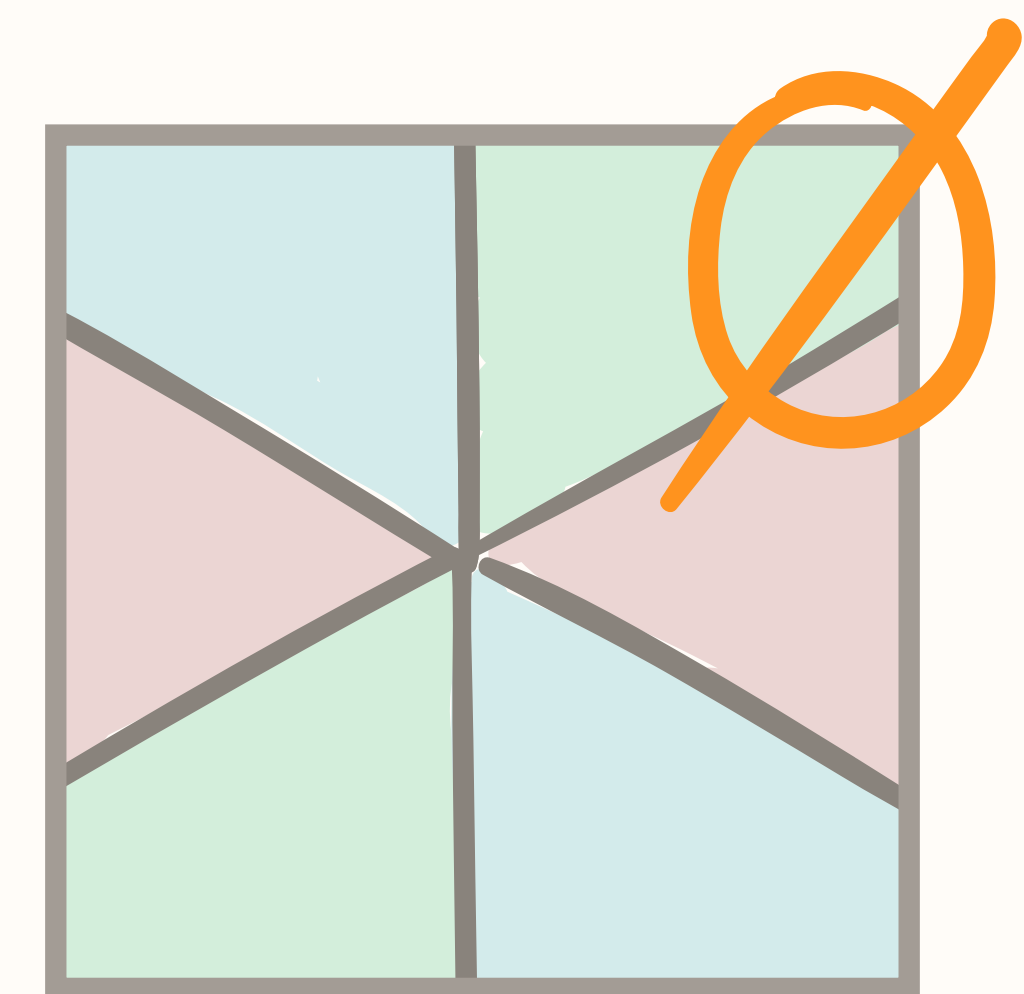
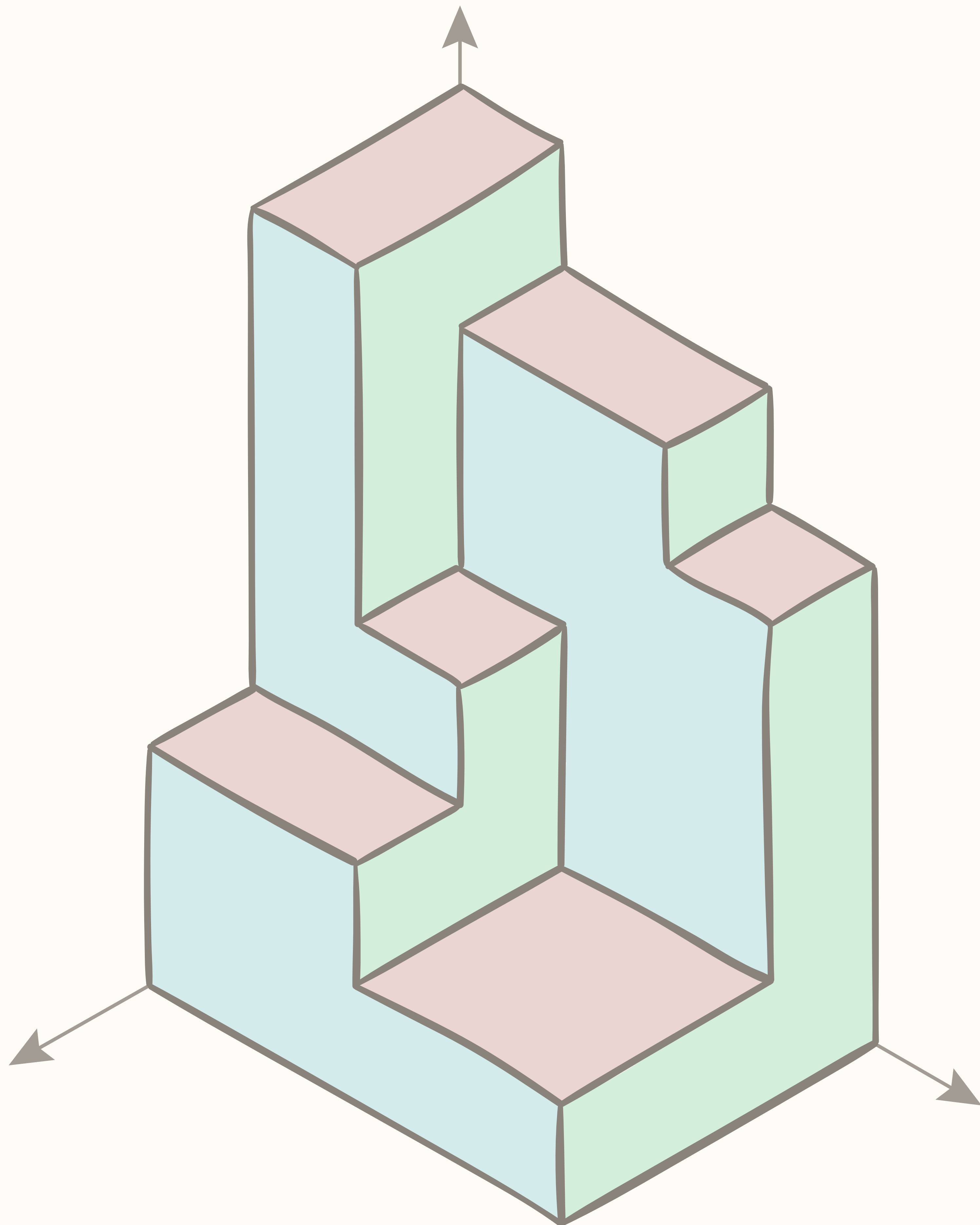
- a. Plane bipolar posets*
- b. Plane bipolar posets by vertices*
- c. Transversal structures*
- d. Asymtotoxic*

2. Interlude : plane permutations

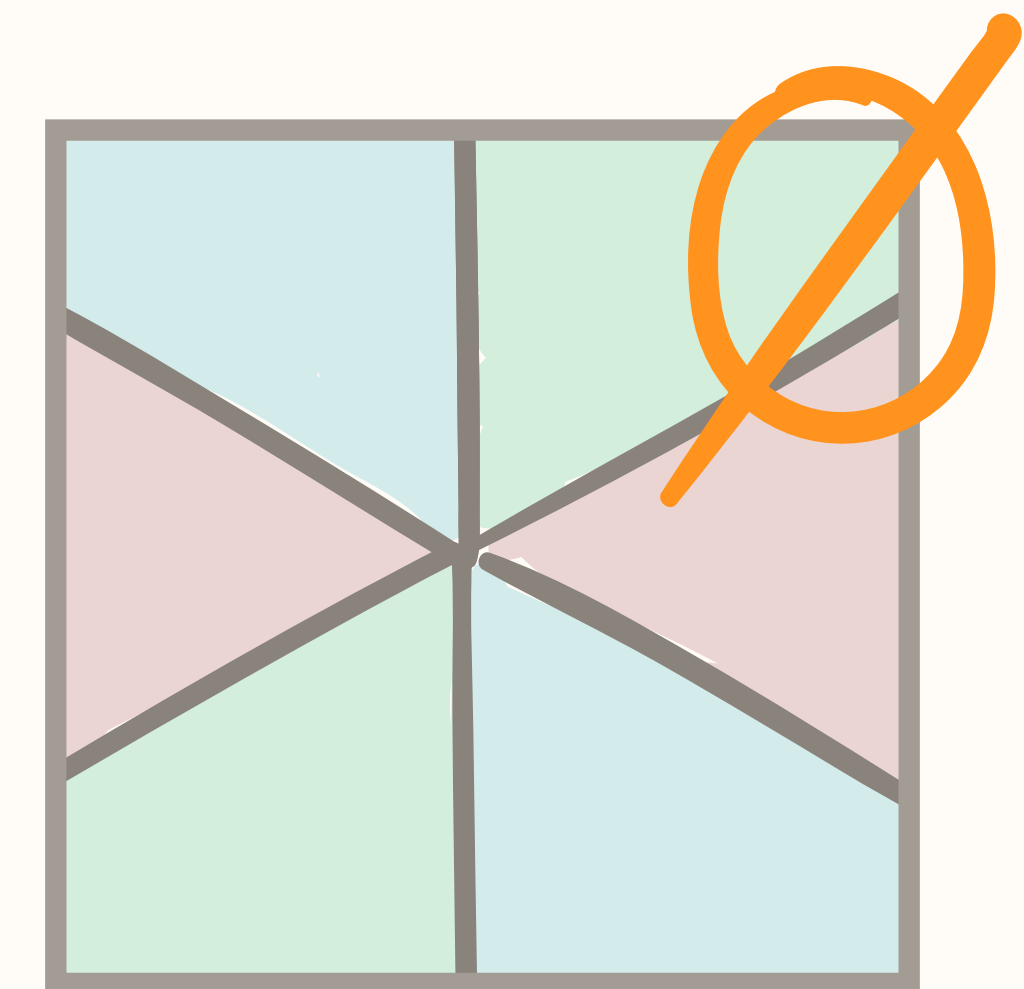
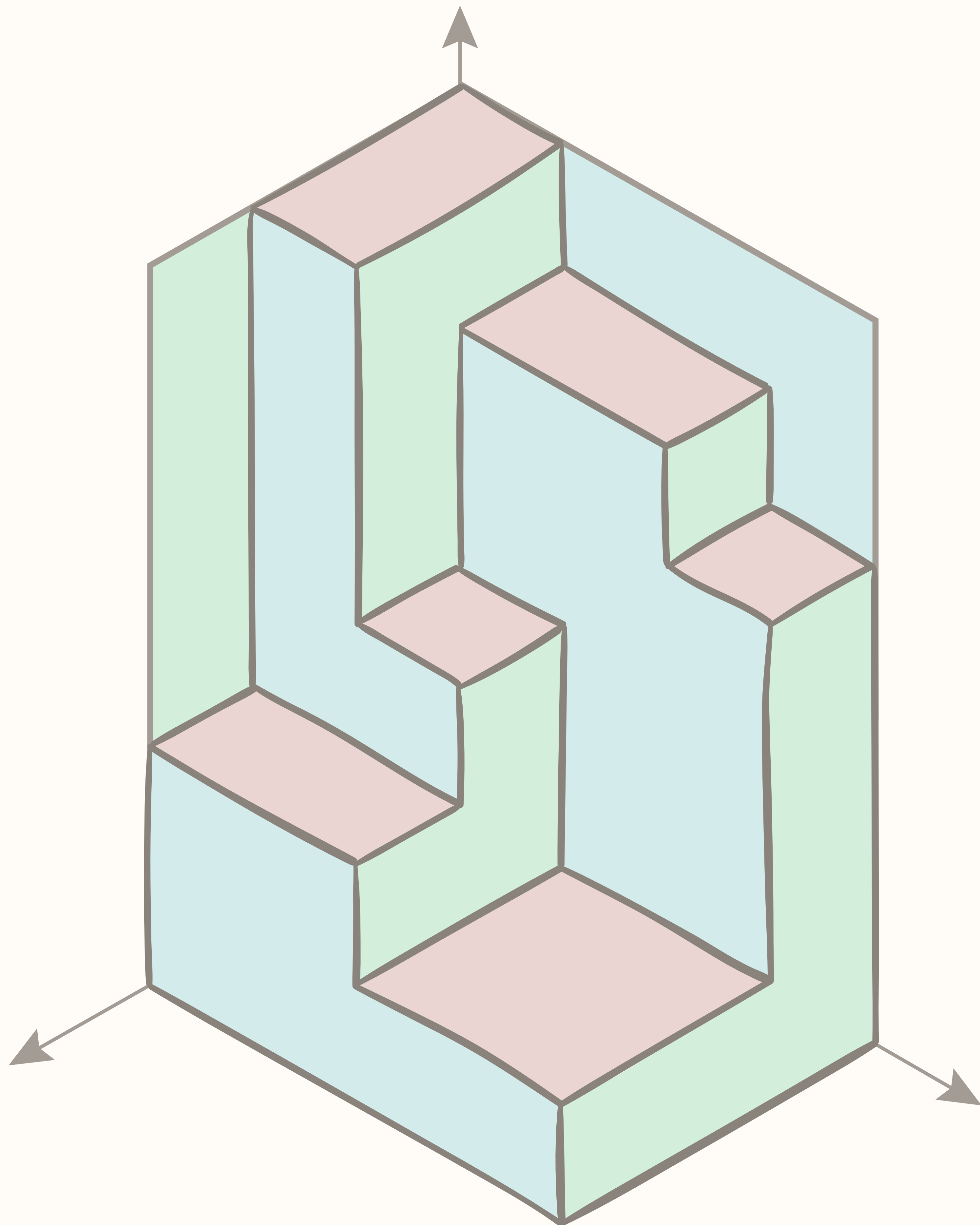
3. Application to corner polyhedra

- a. Via polyhedral orientations*
- b. Via Schnyder colorings*
- c. Asymtotoxic*

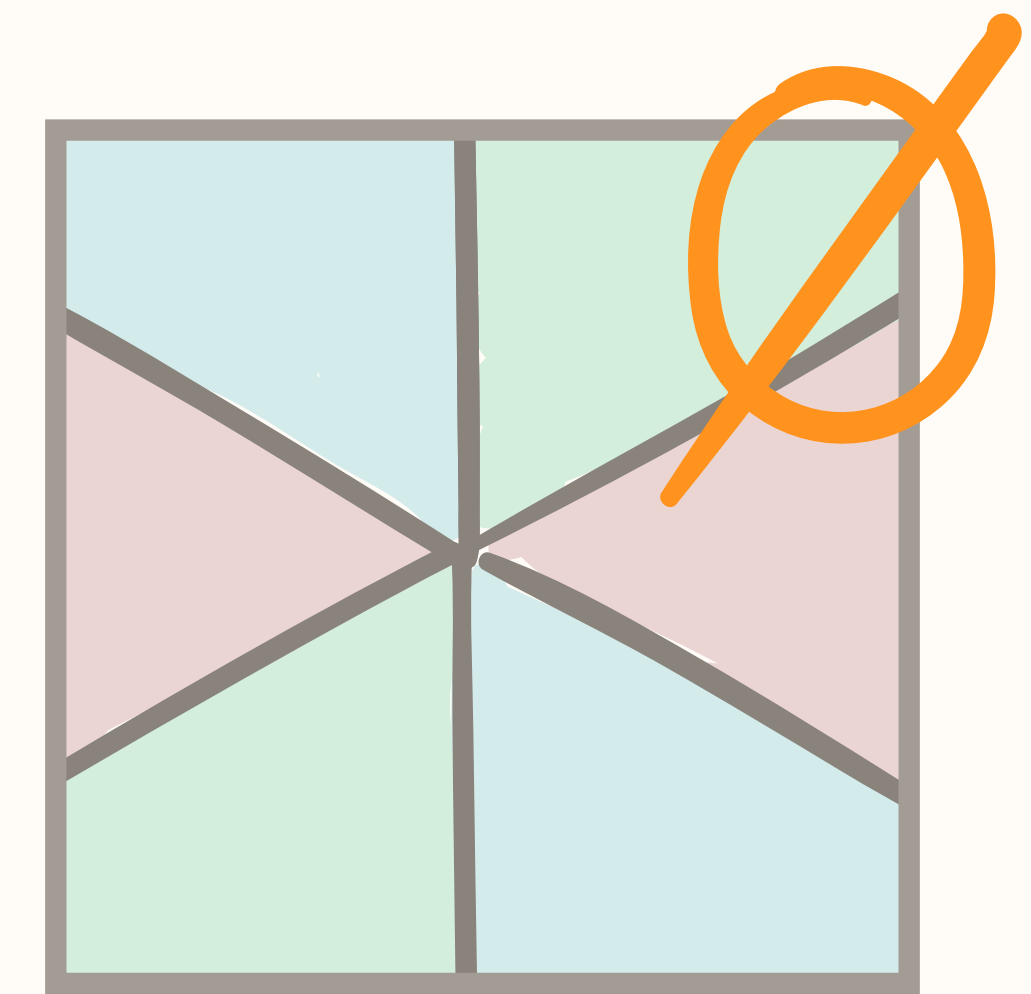
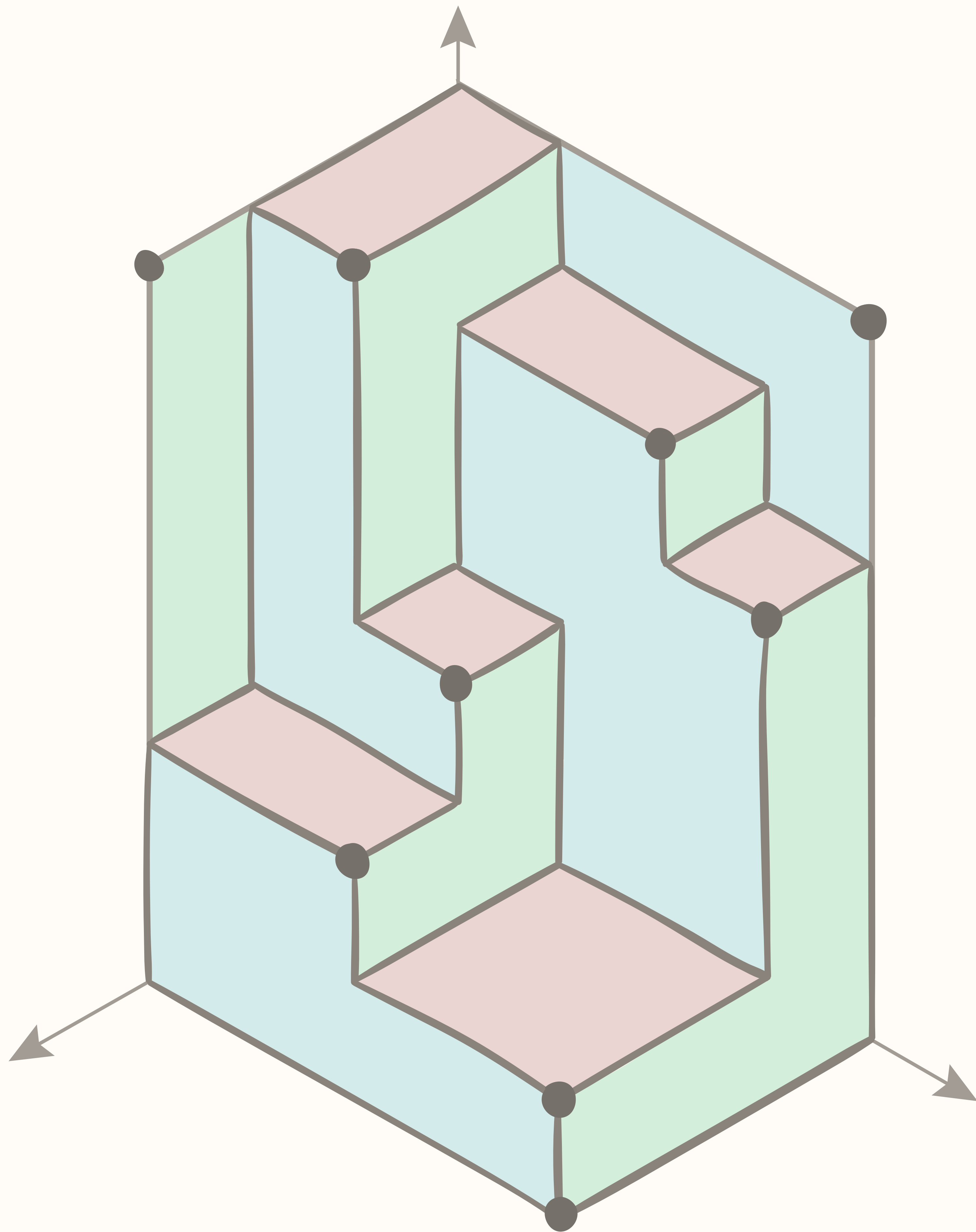
Rigid corner polyhedra



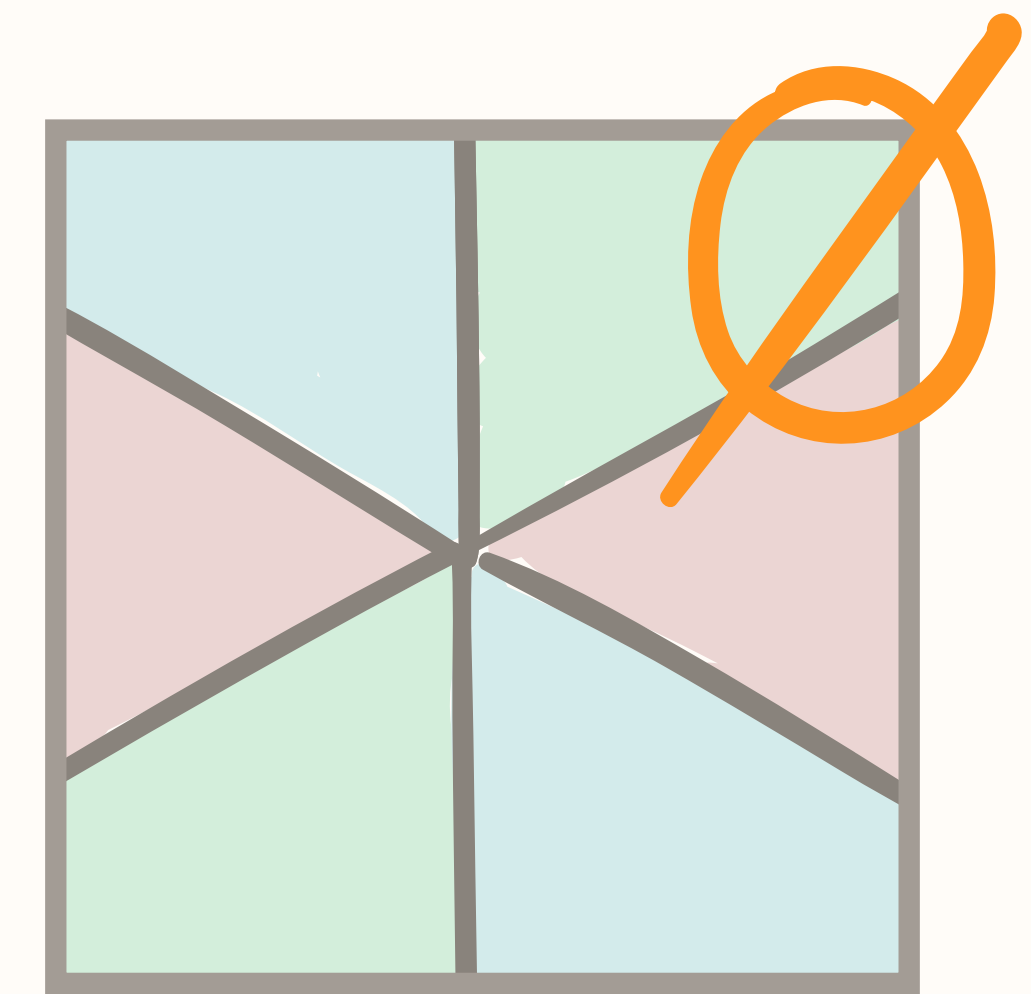
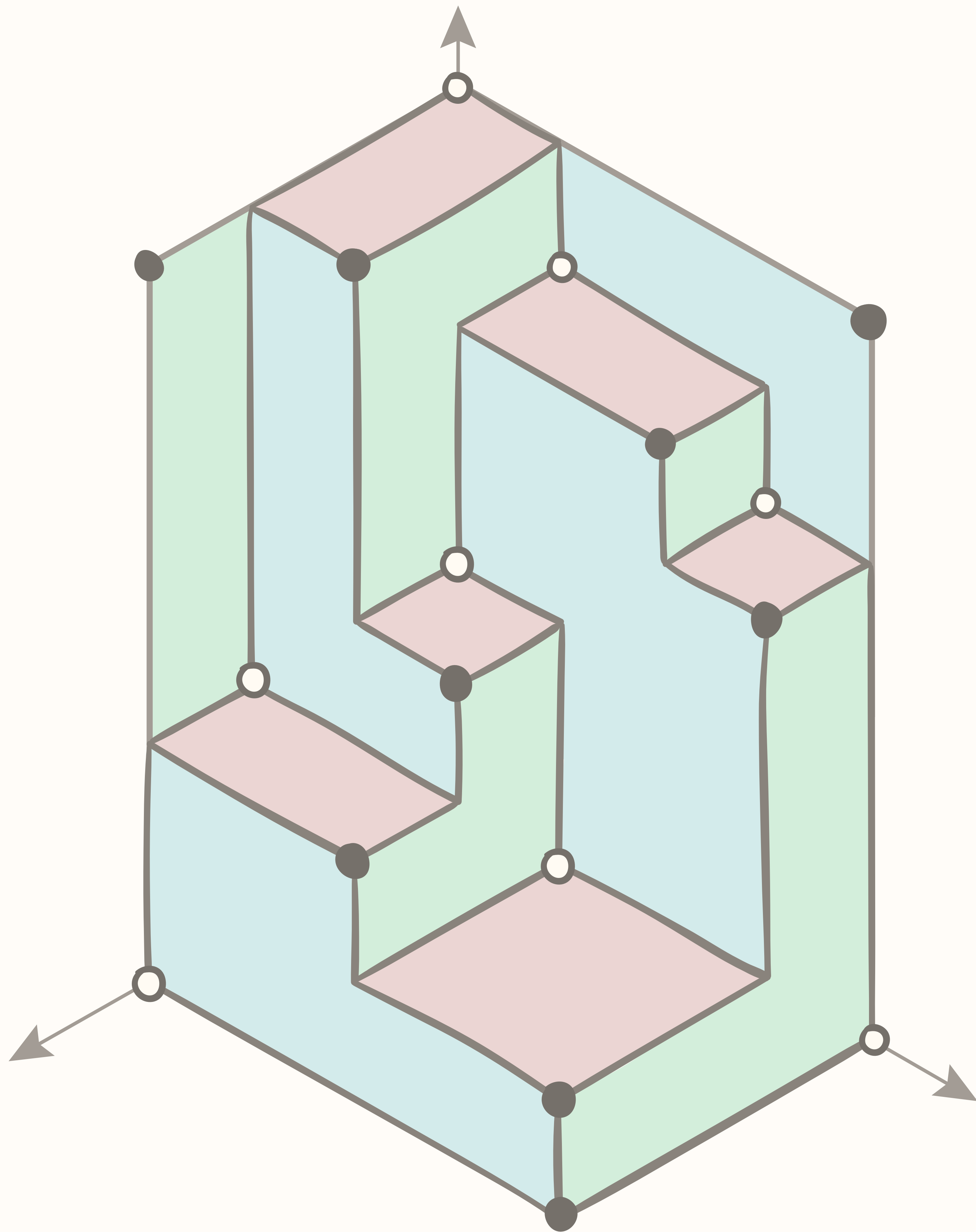
Rigid corner polyhedra



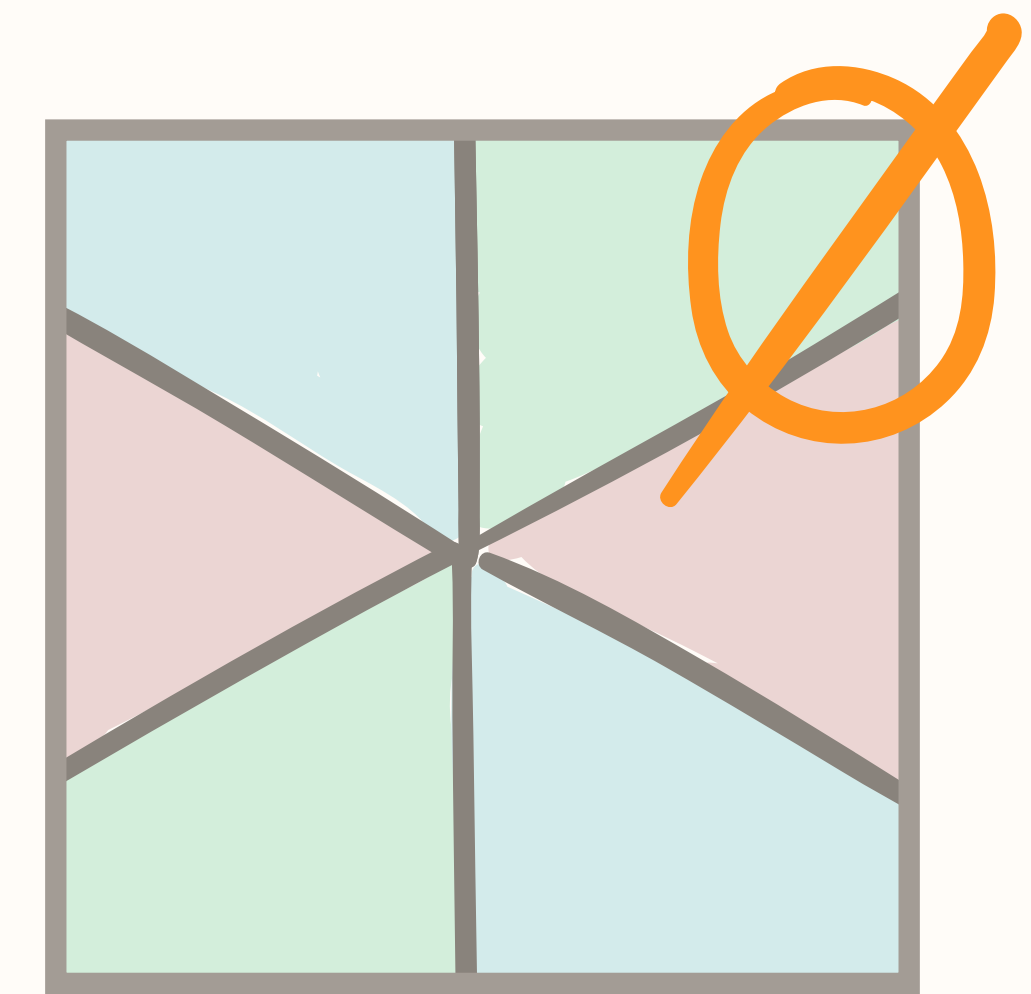
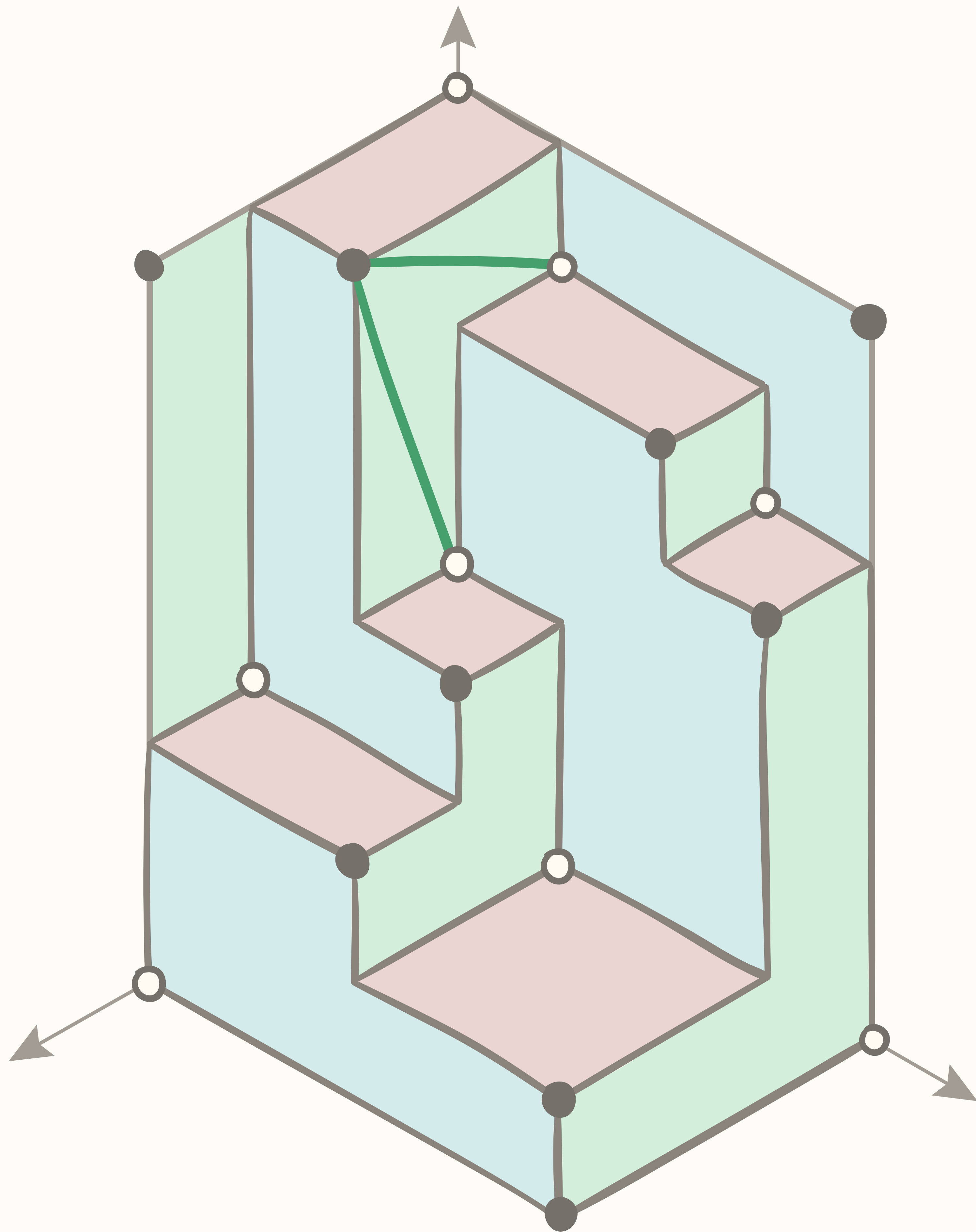
Rigid corner polyhedra



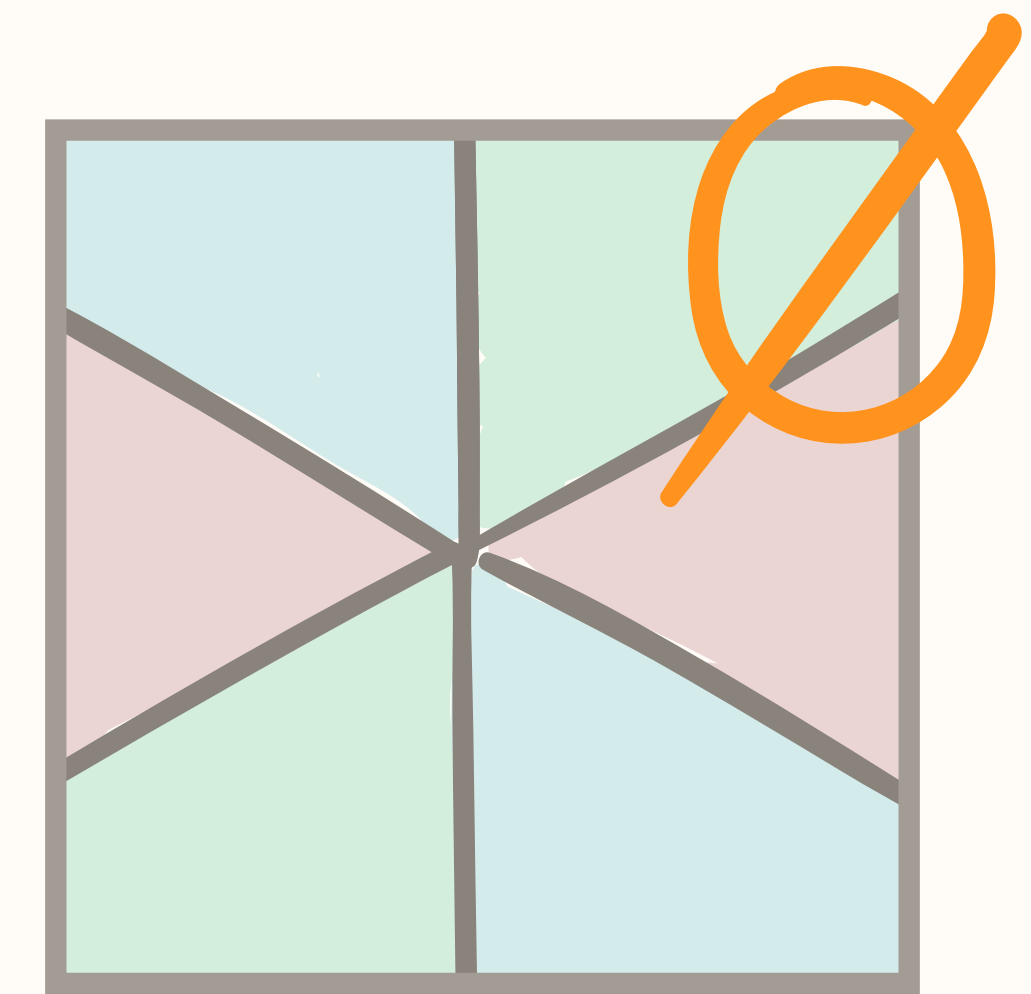
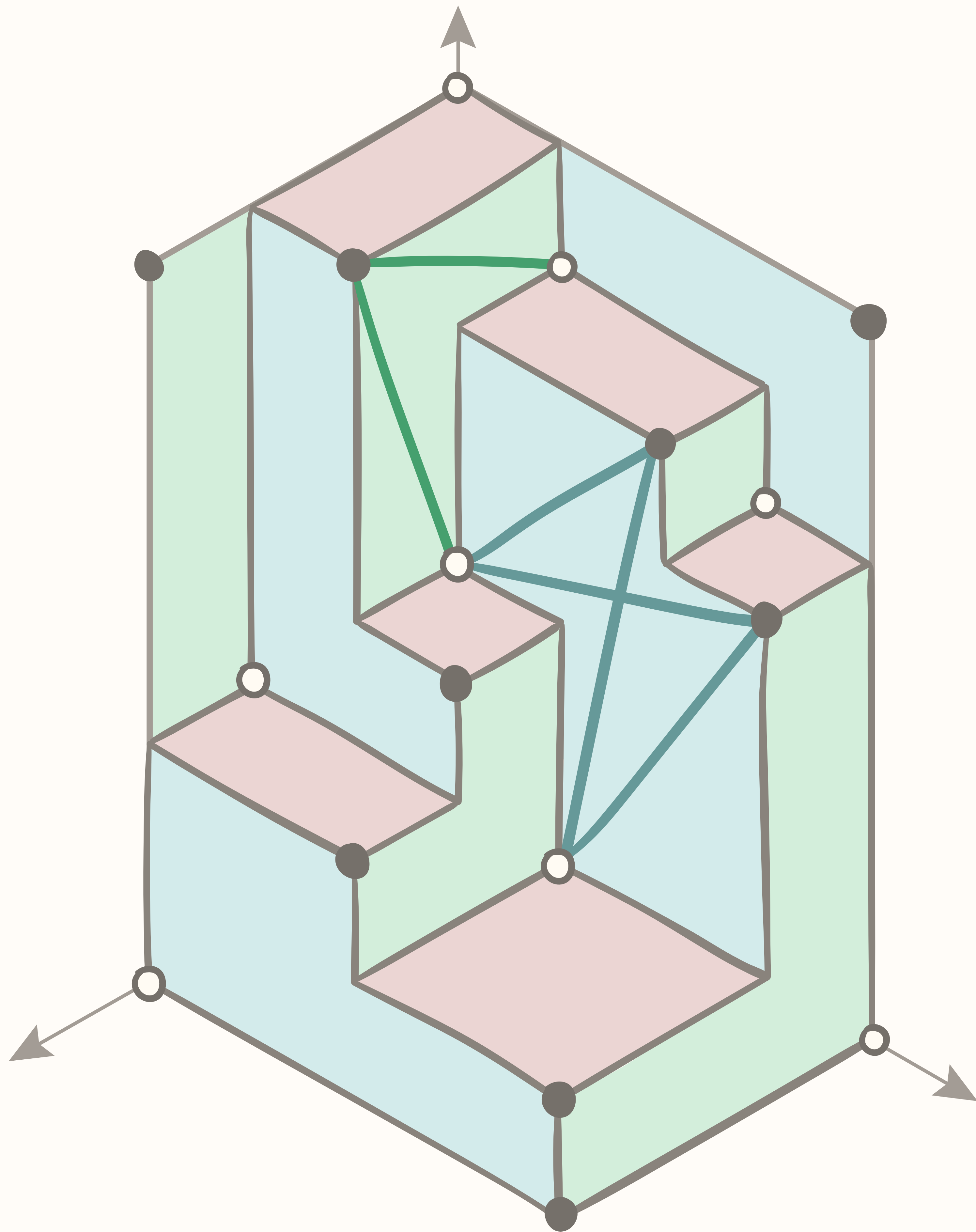
Rigid corner polyhedra



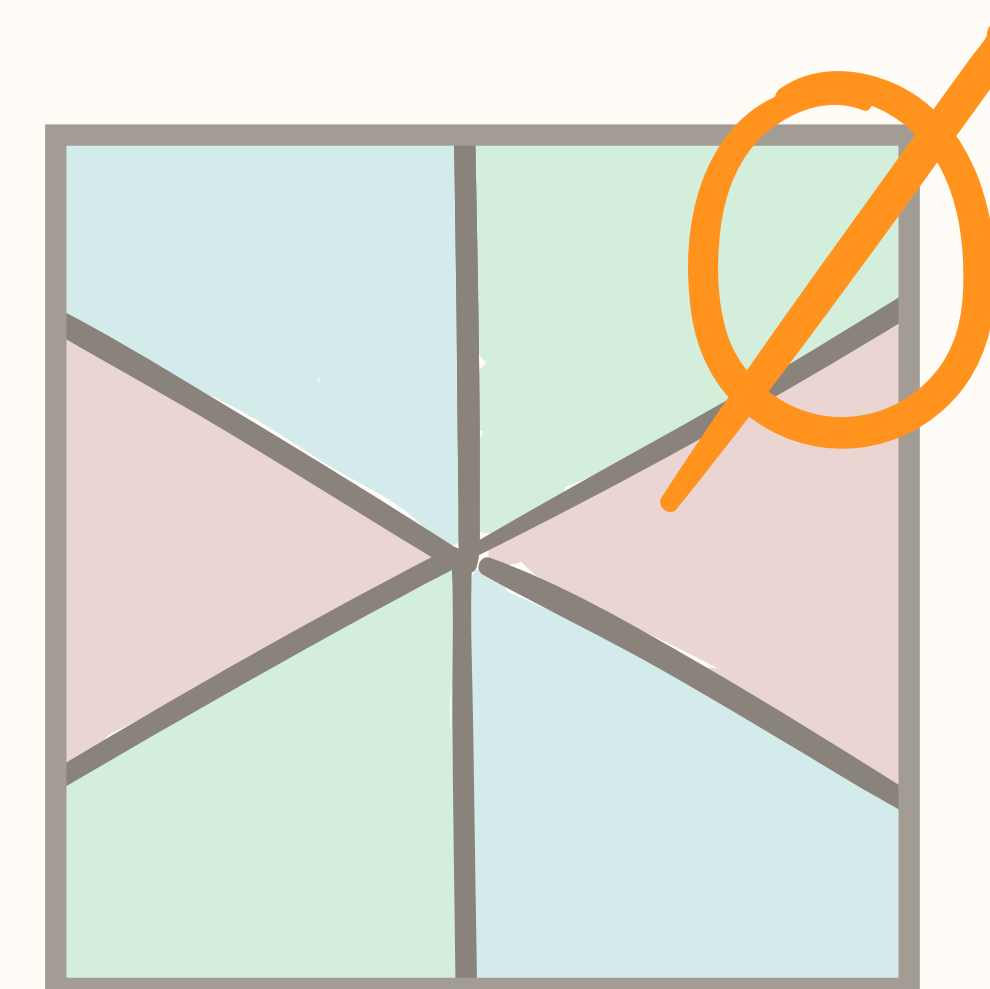
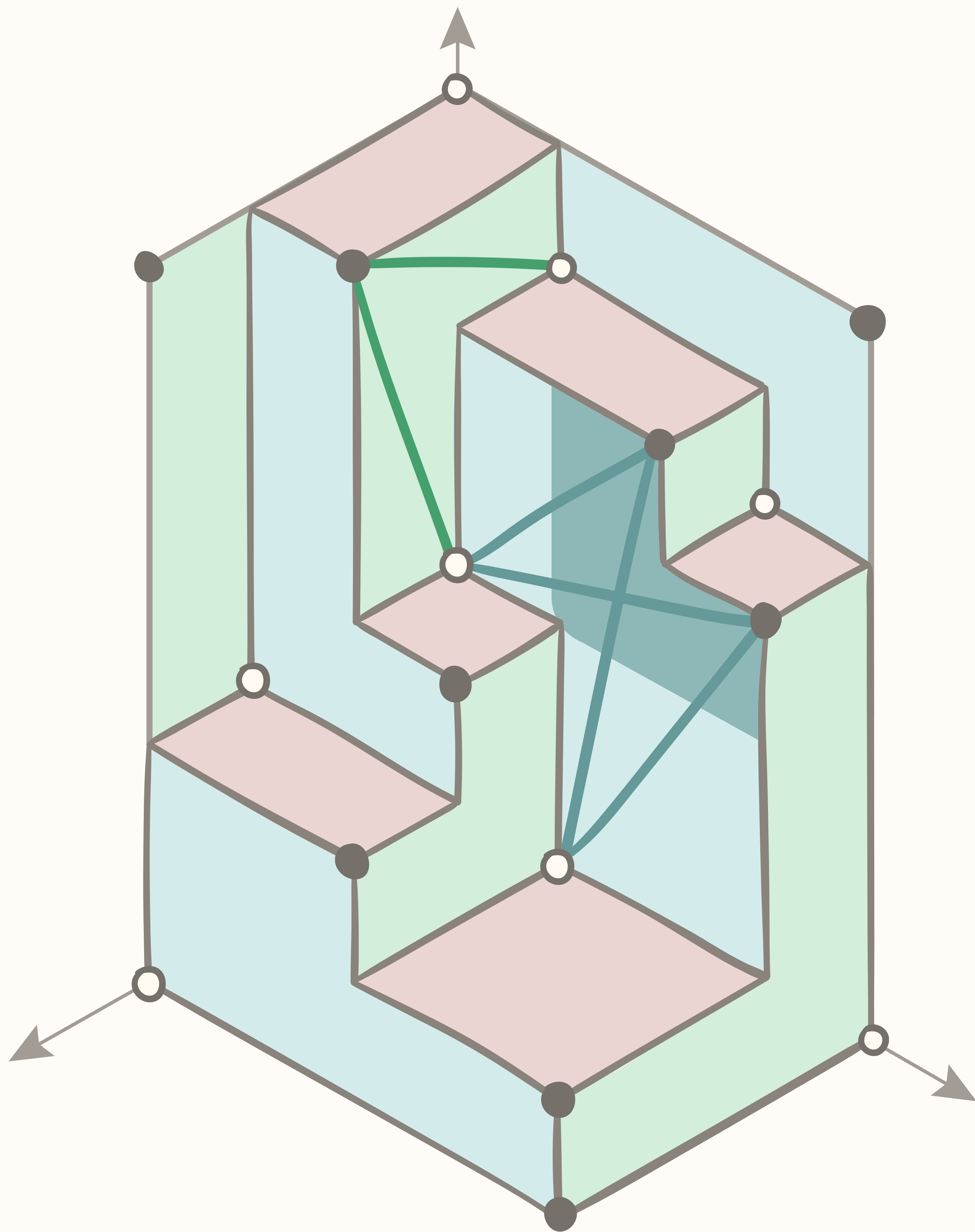
Rigid corner polyhedra



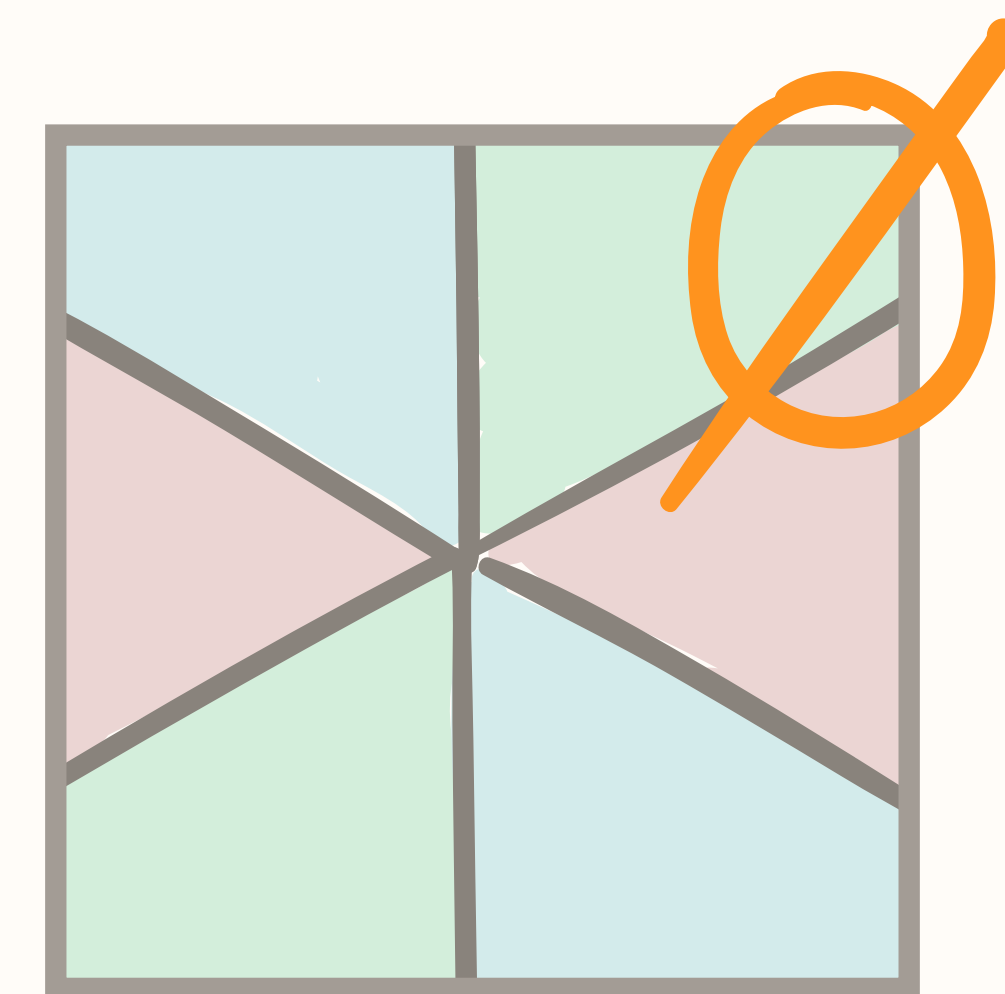
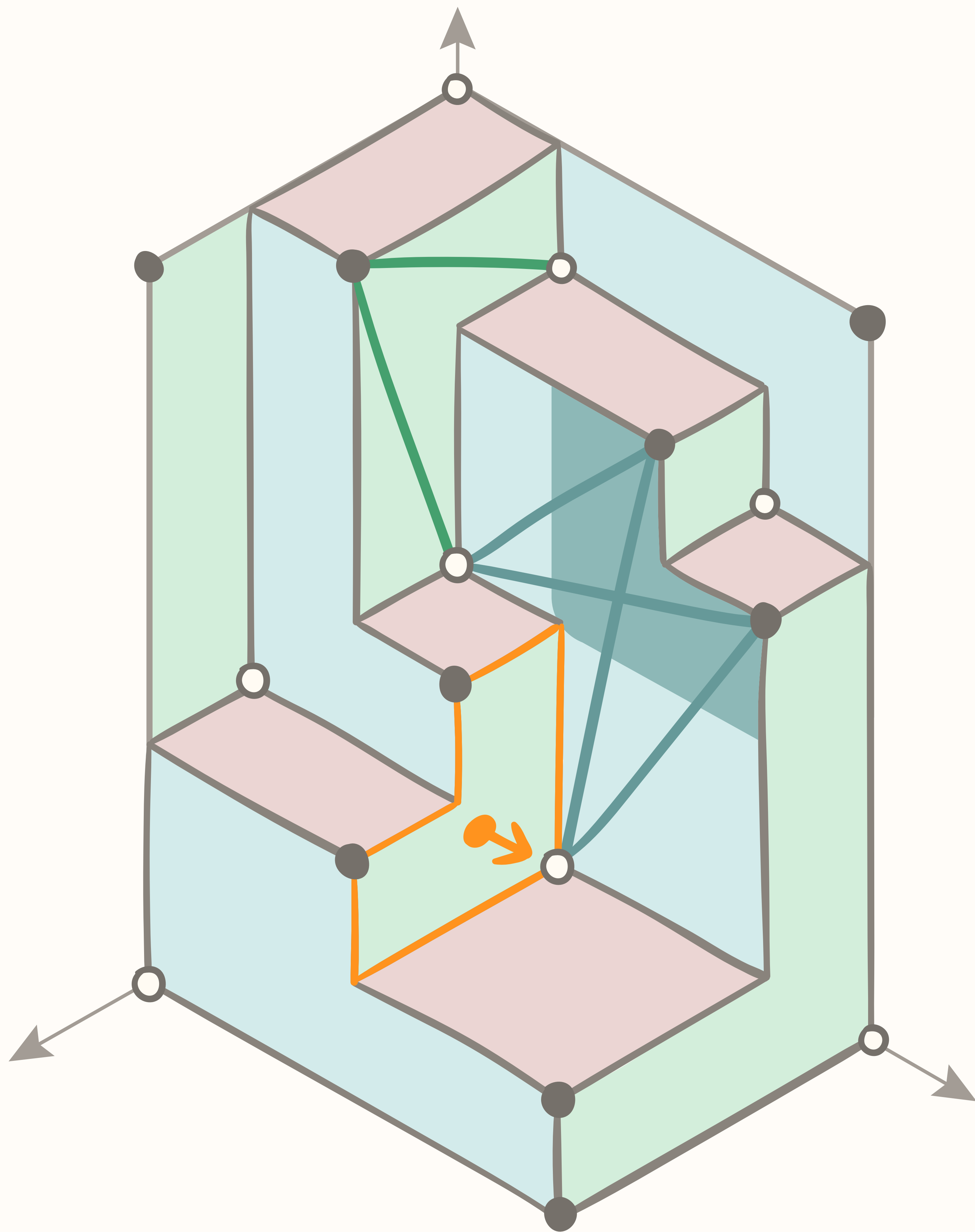
Rigid corner polyhedra



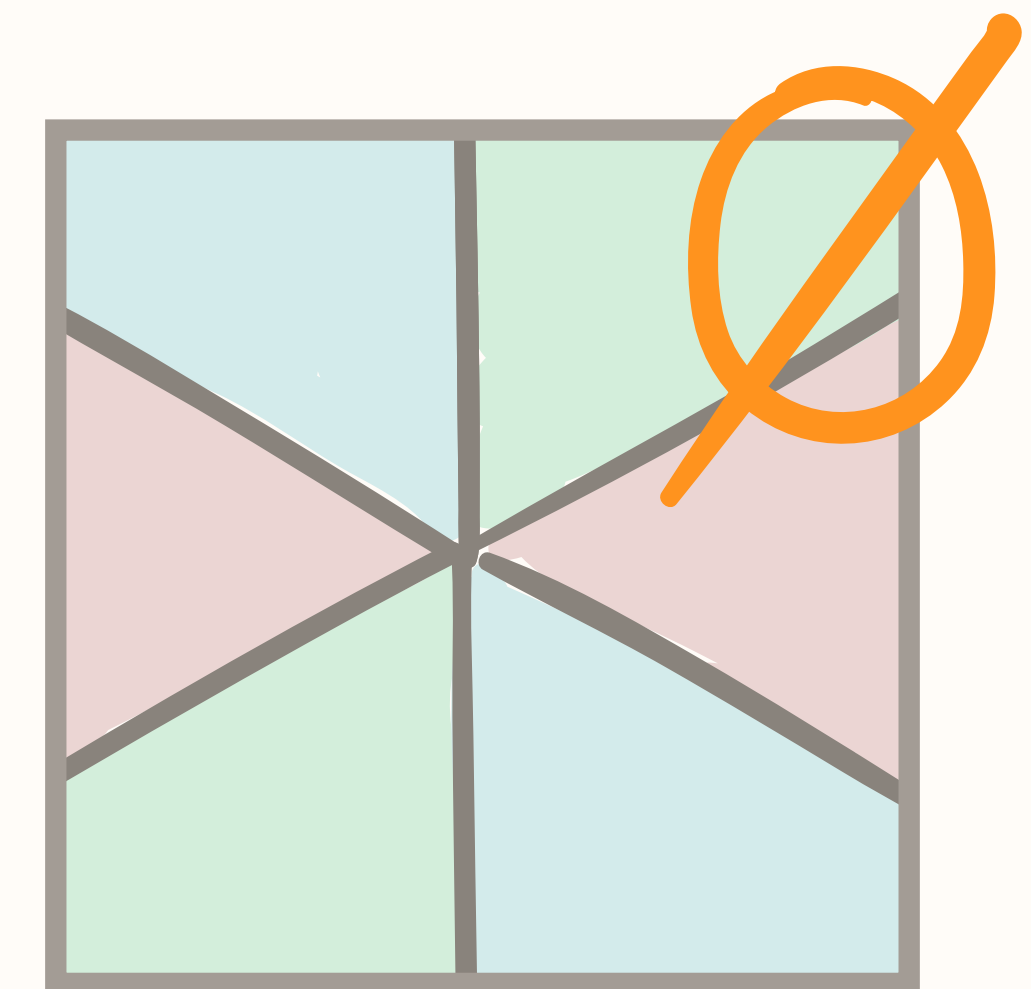
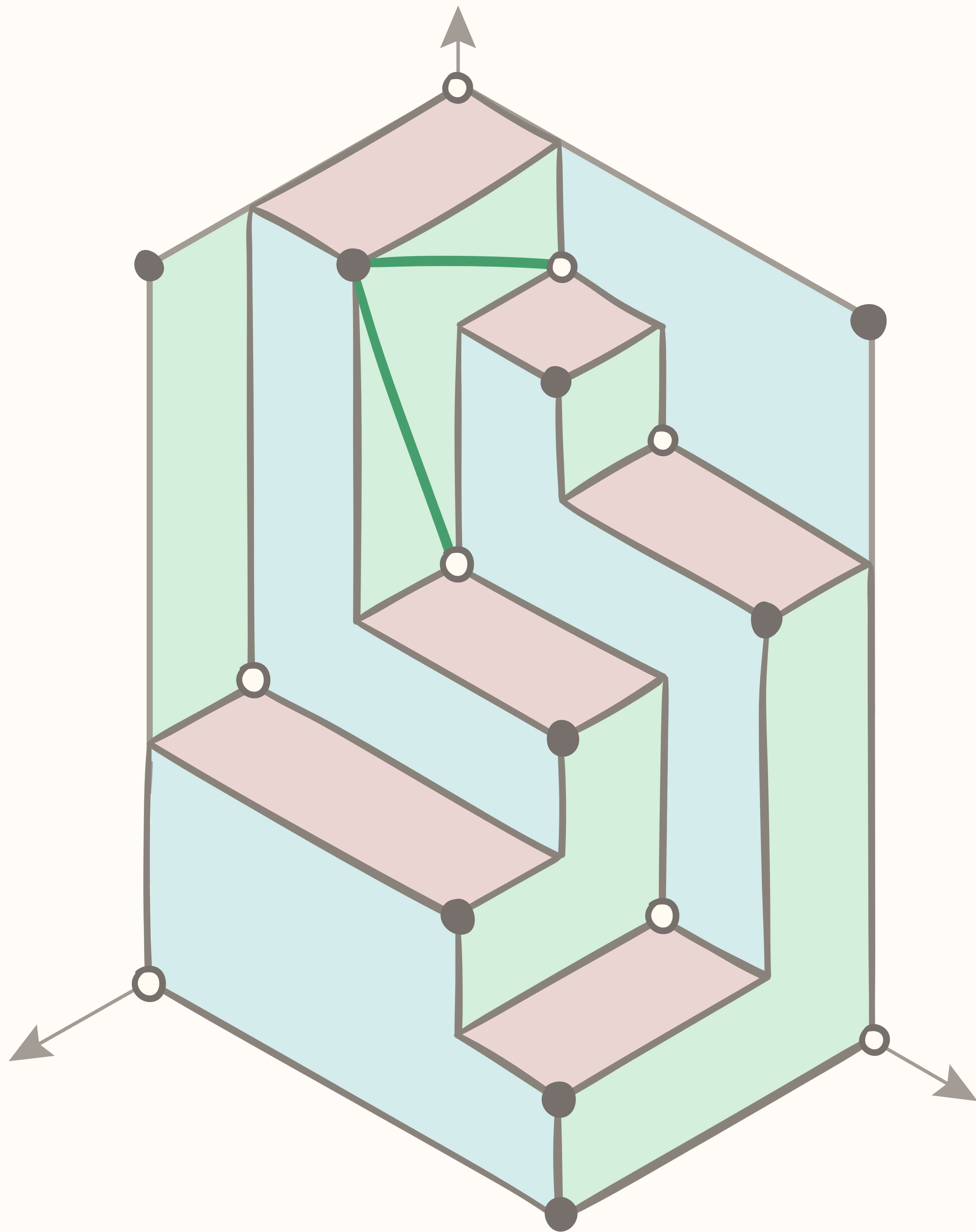
Rigid corner polyhedra



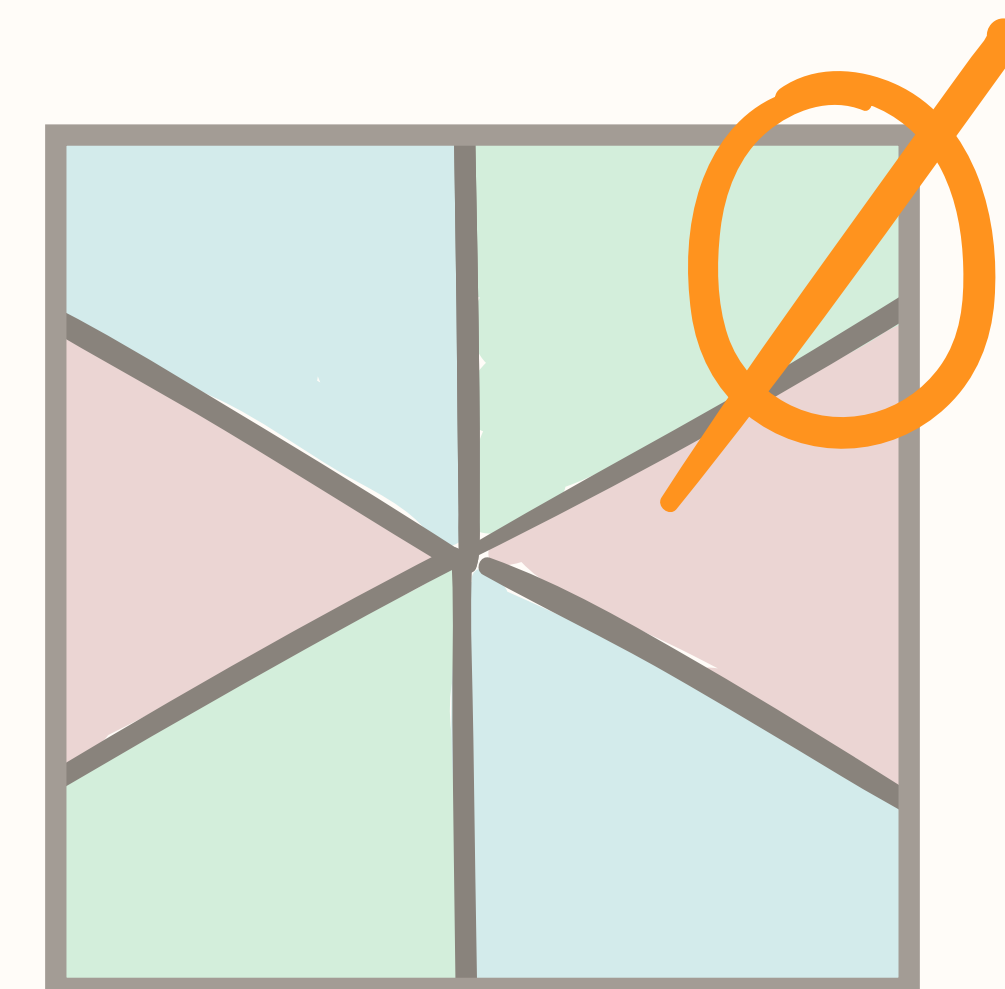
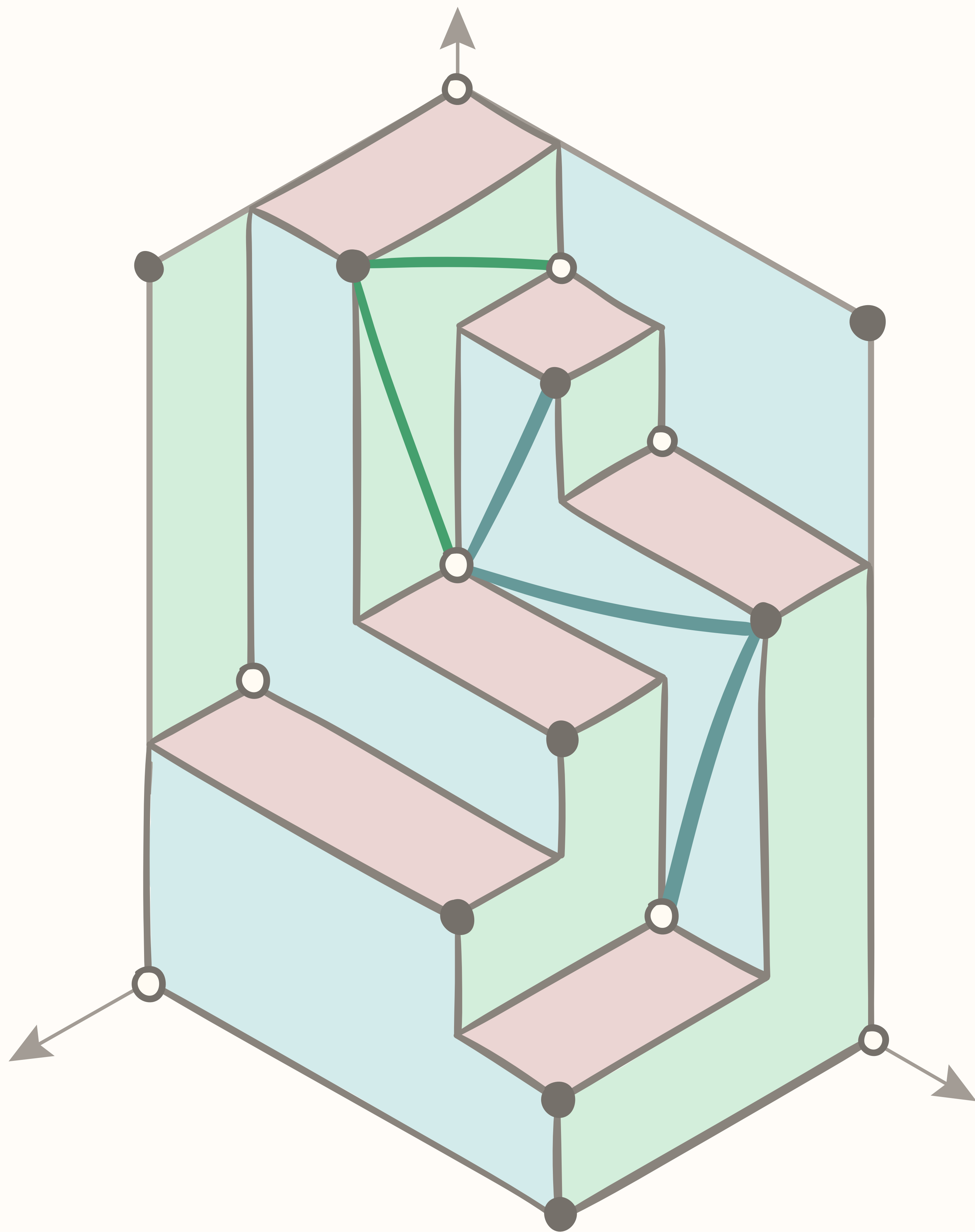
Rigid corner polyhedra



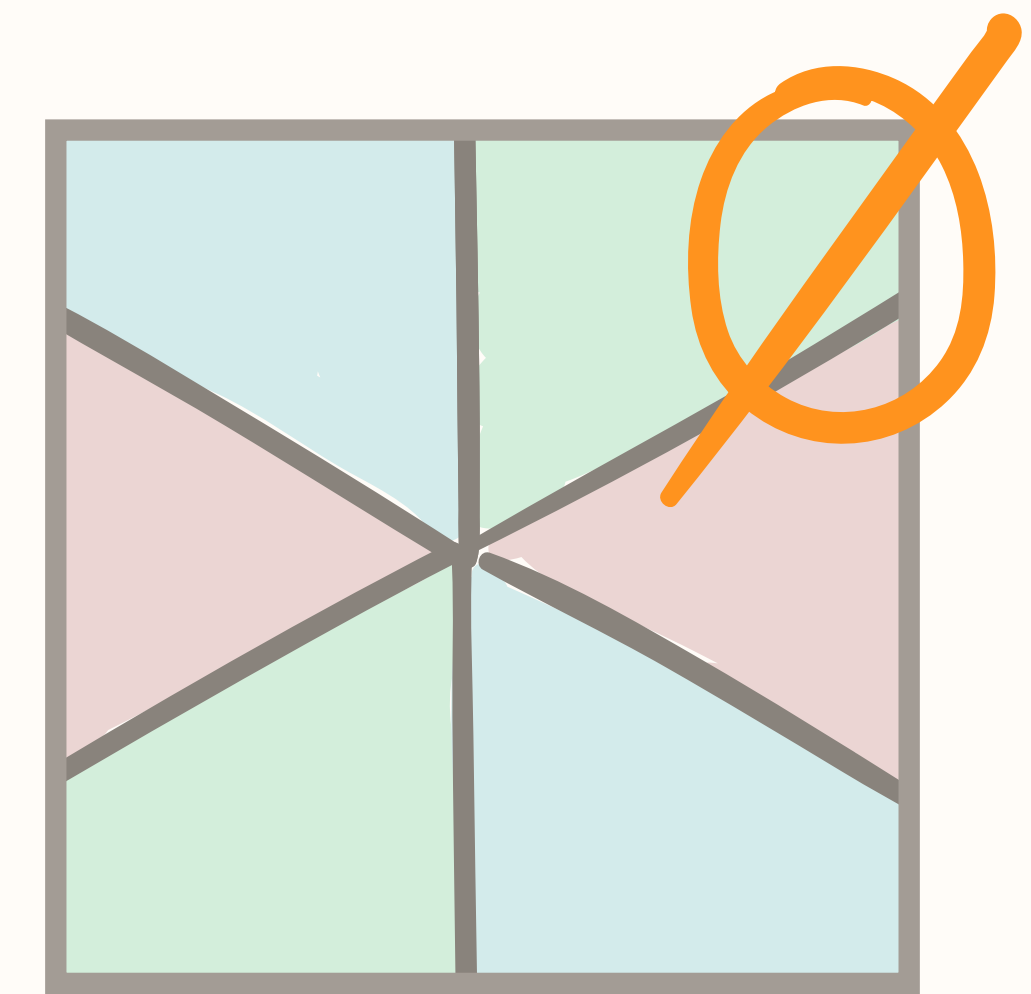
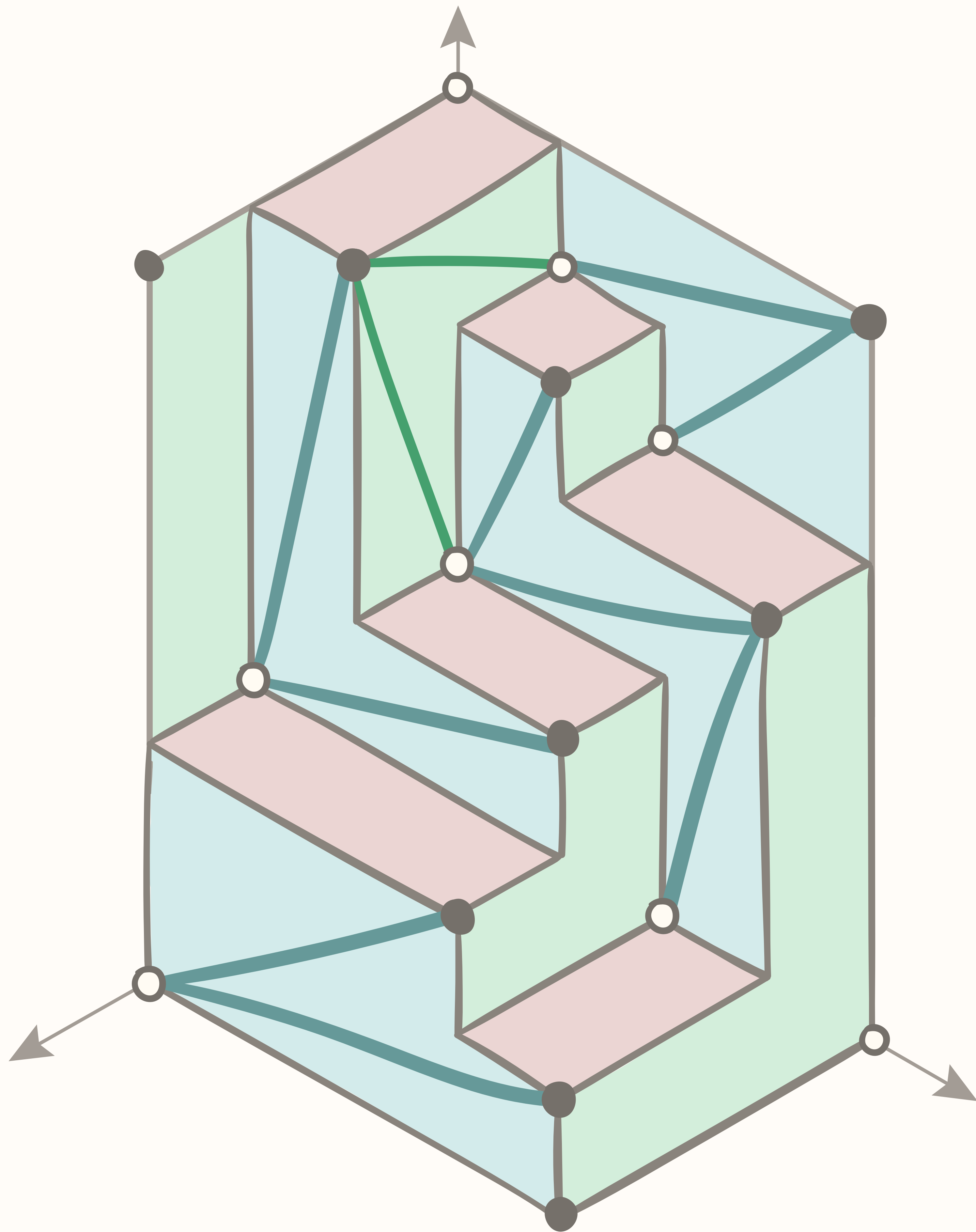
Rigid corner polyhedra



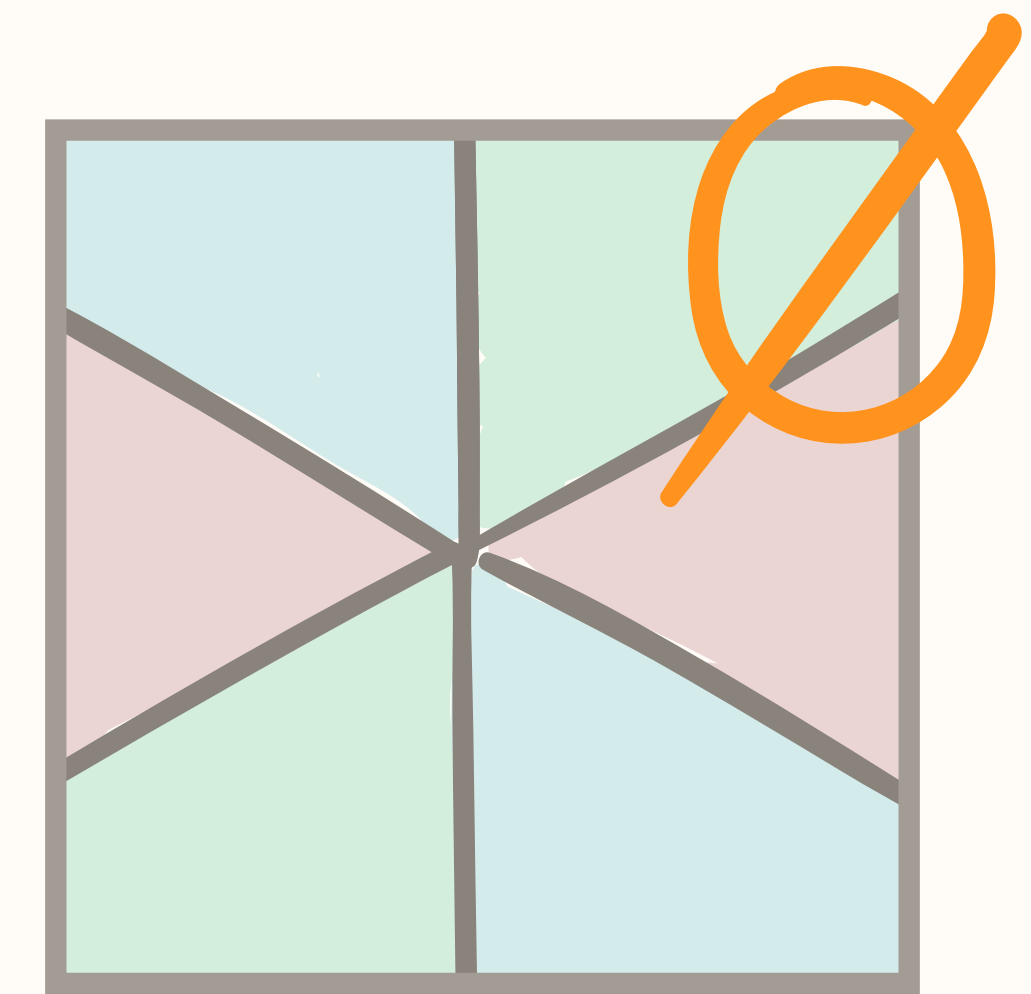
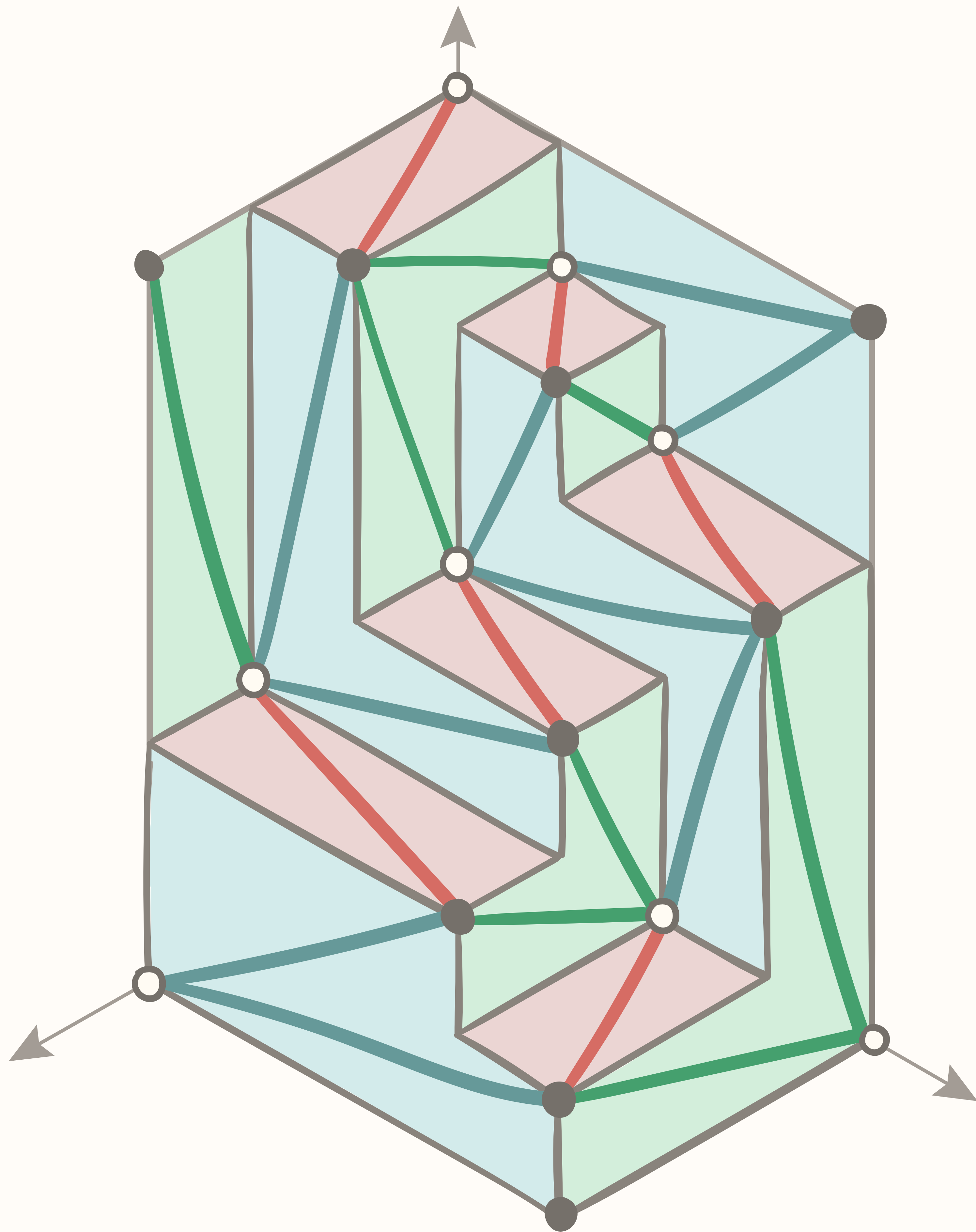
Rigid corner polyhedra



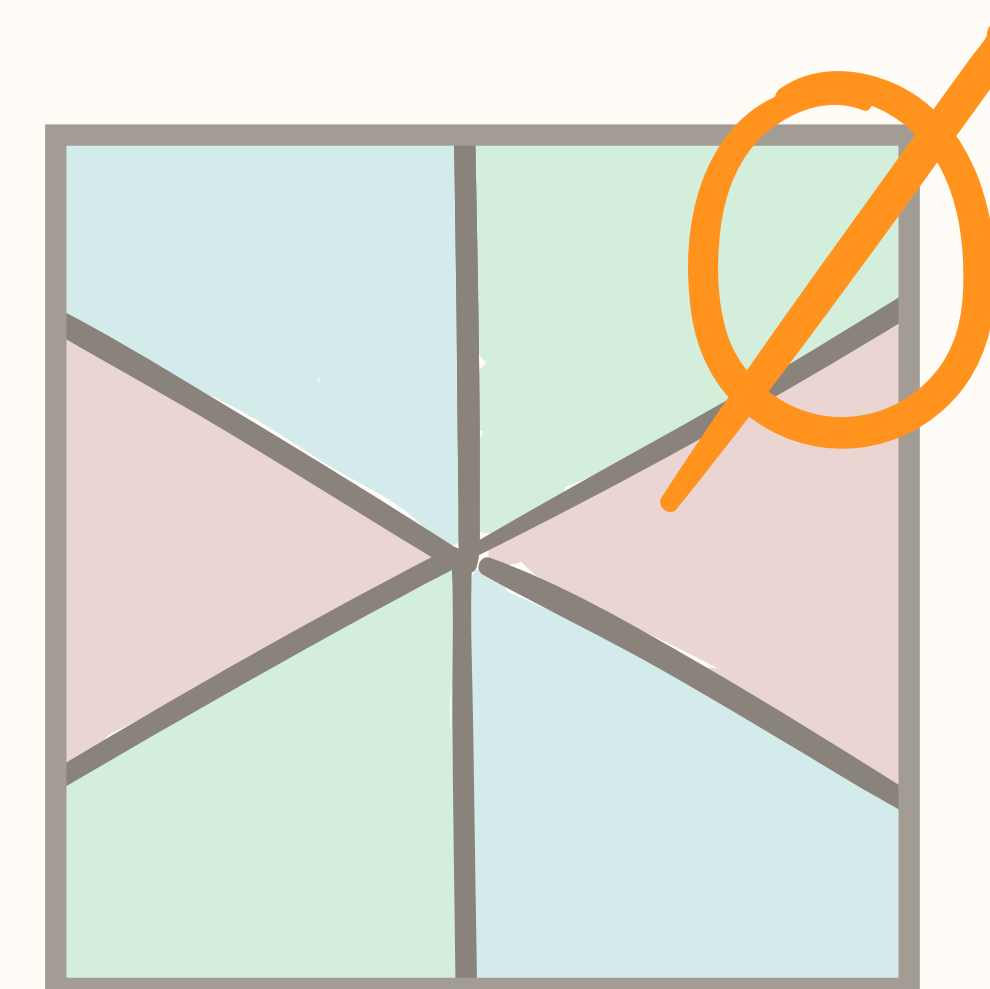
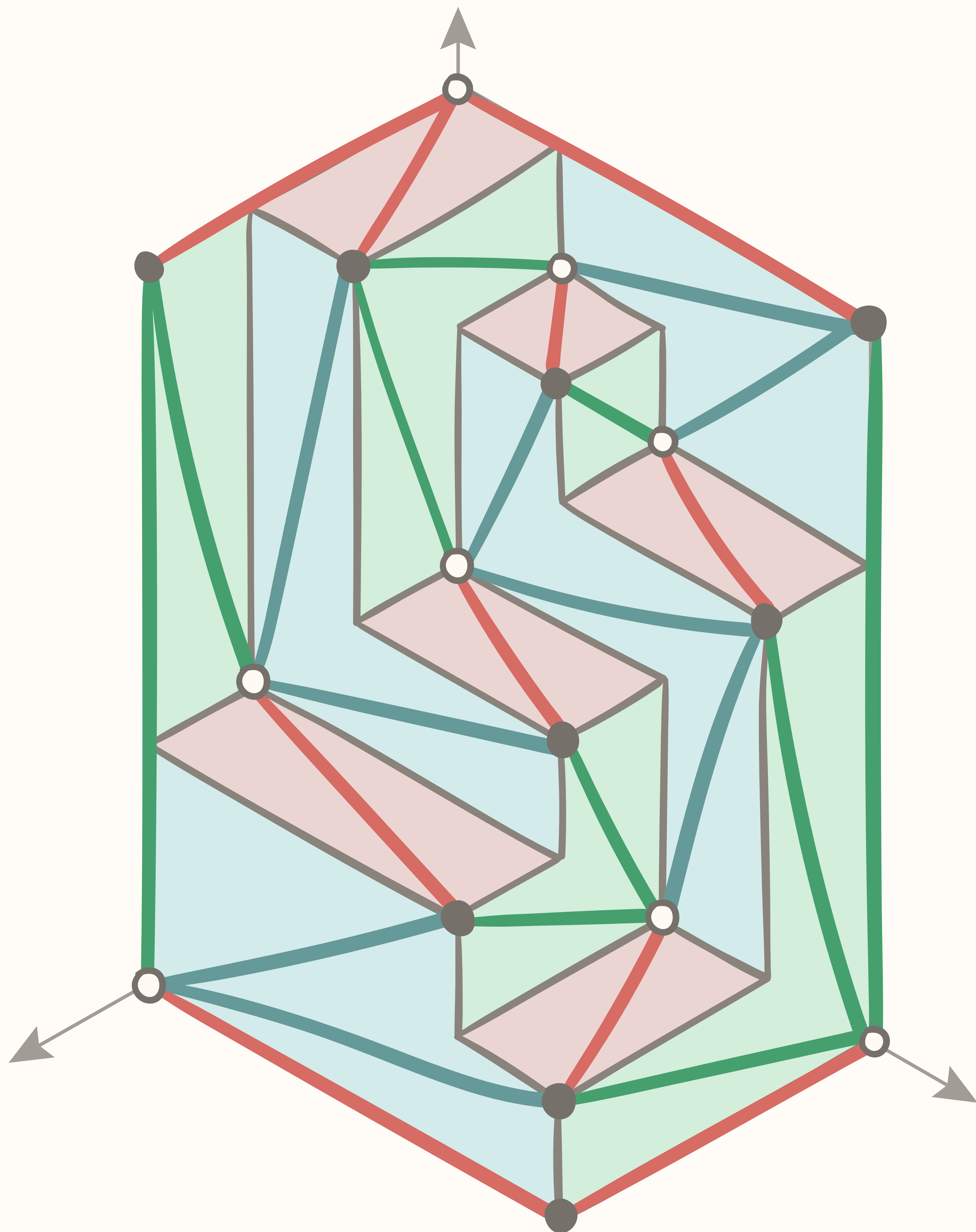
Rigid corner polyhedra



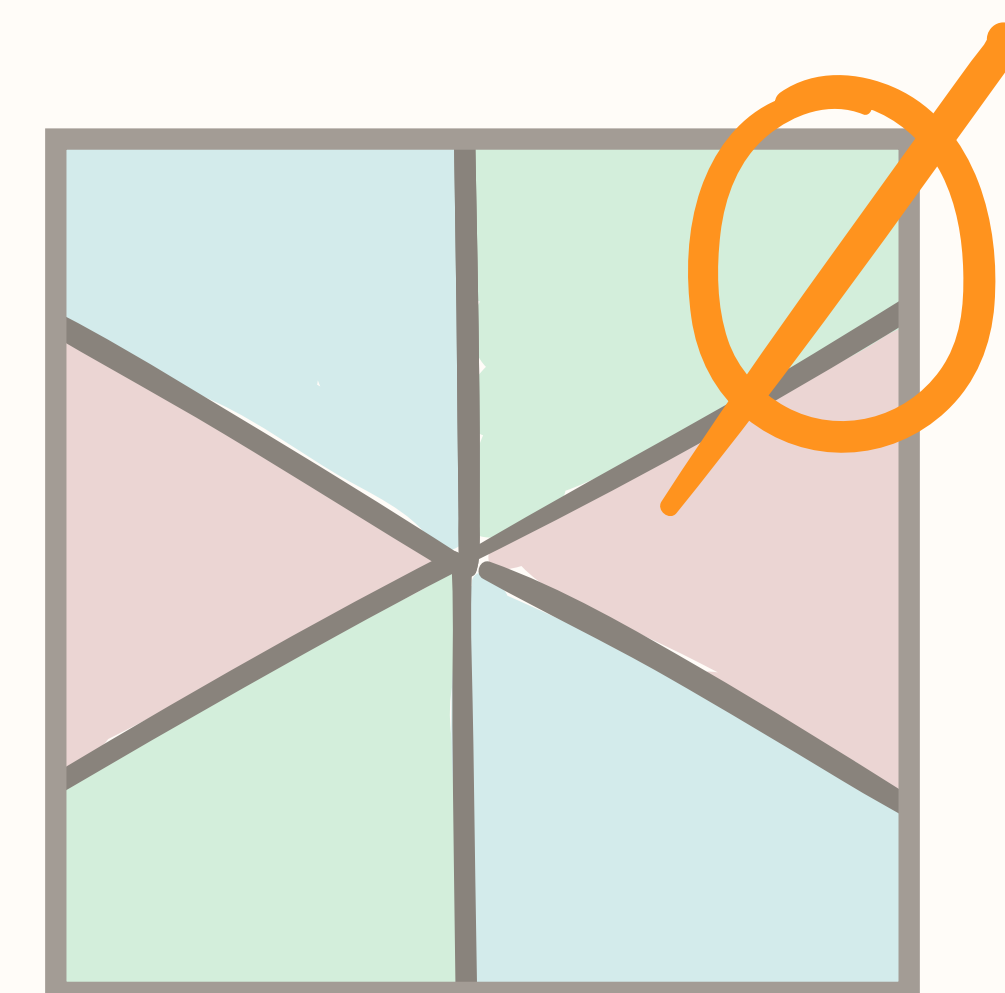
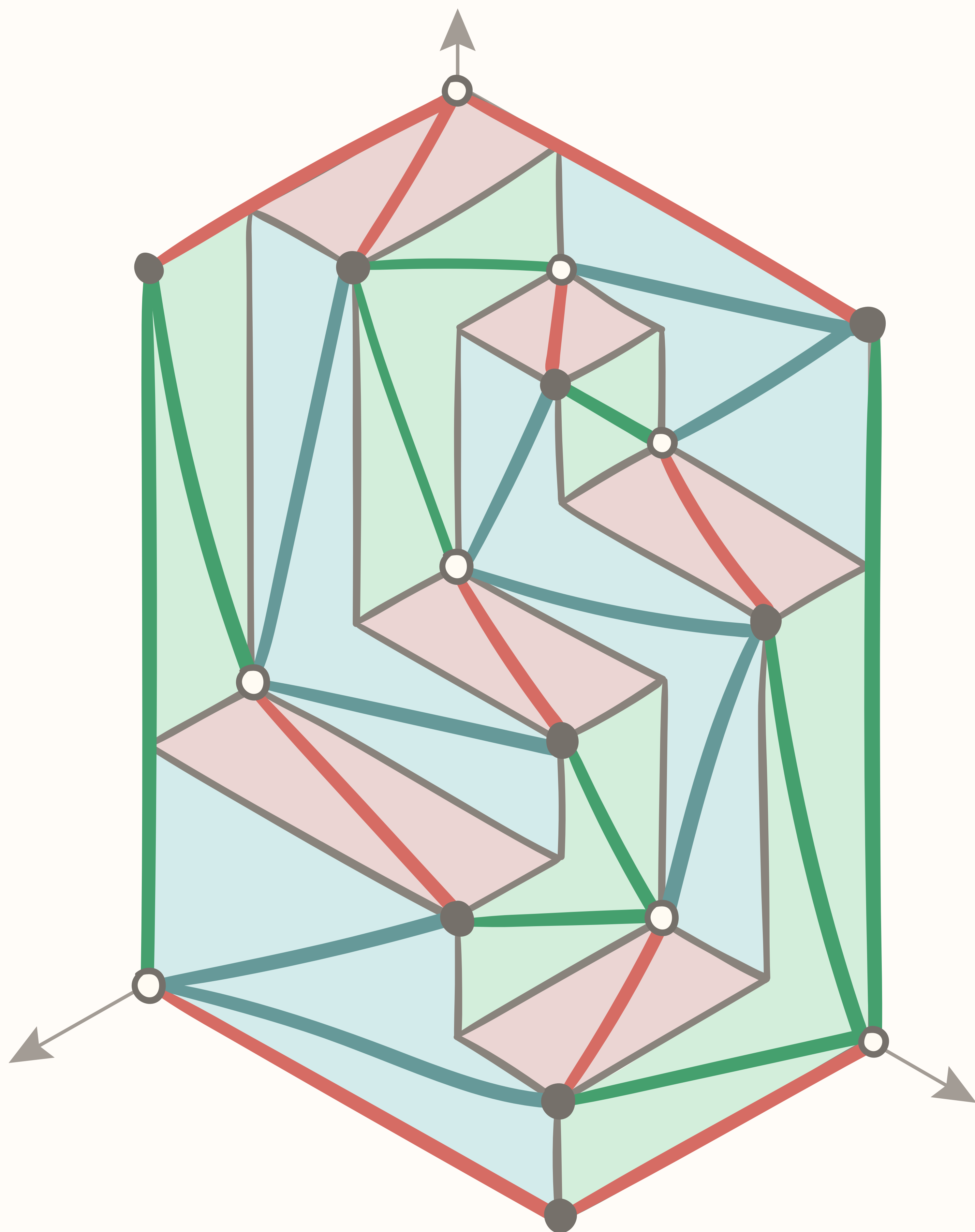
Rigid corner polyhedra



Rigid corner polyhedra



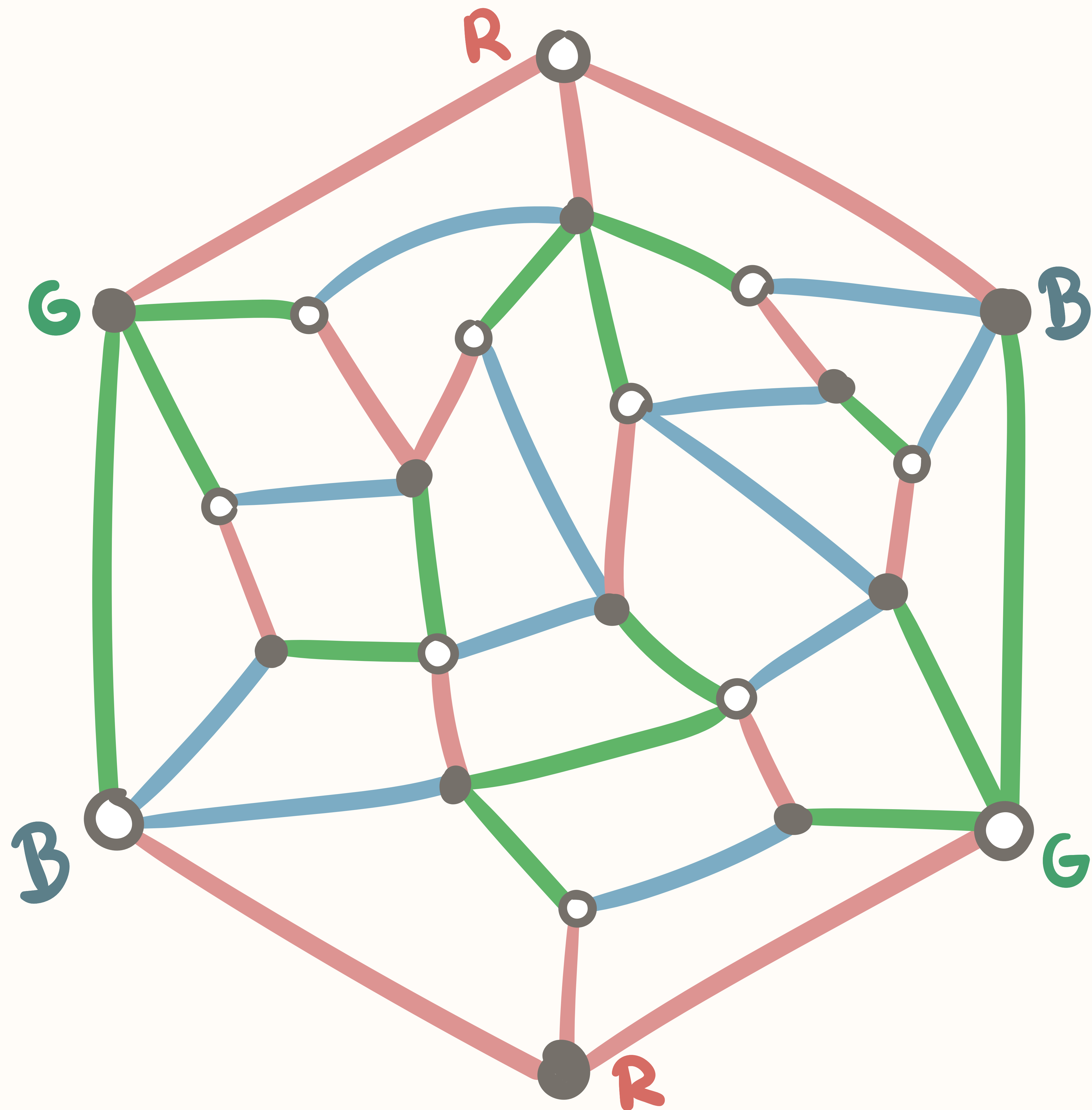
Rigid corner polyhedra



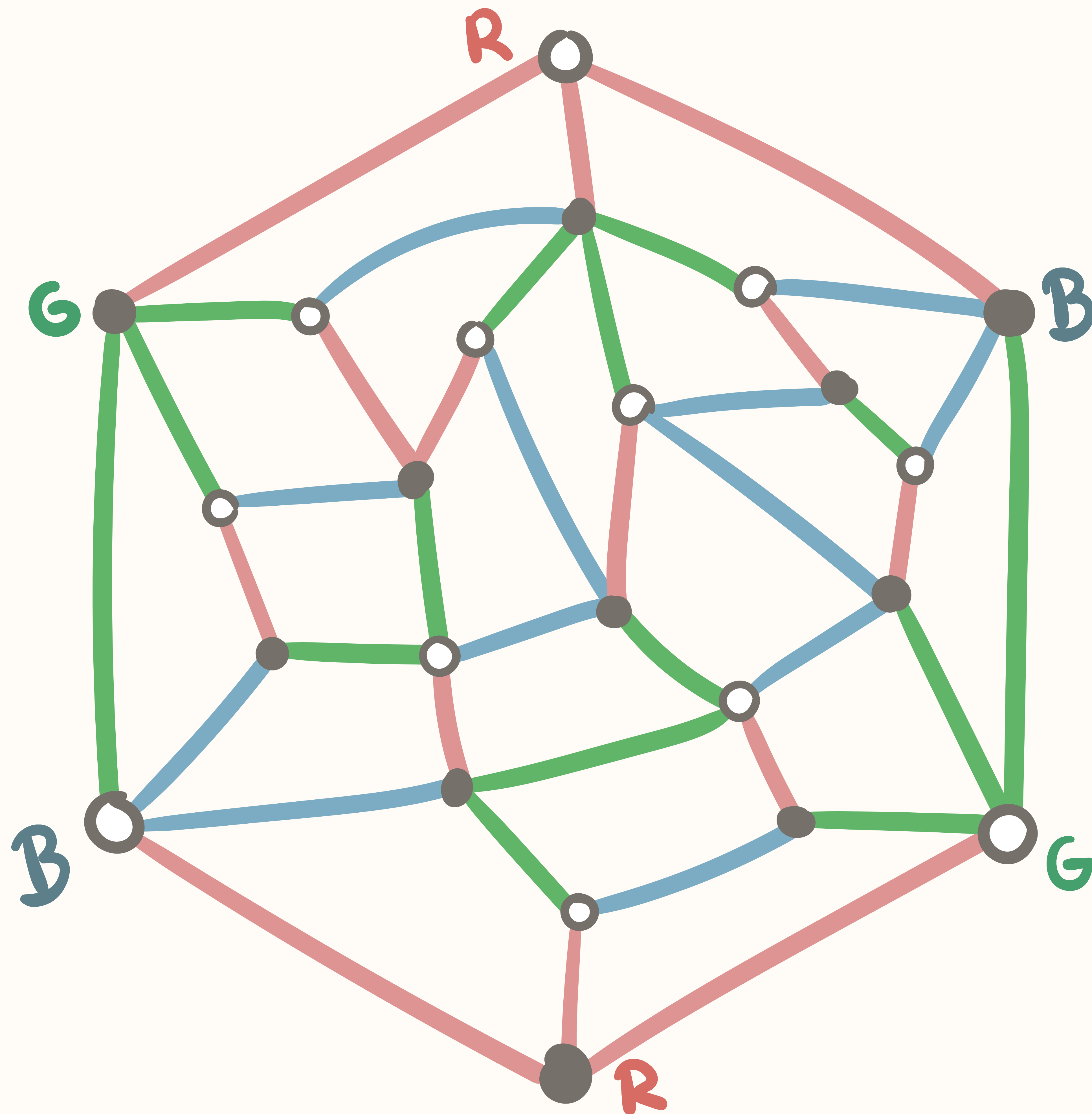
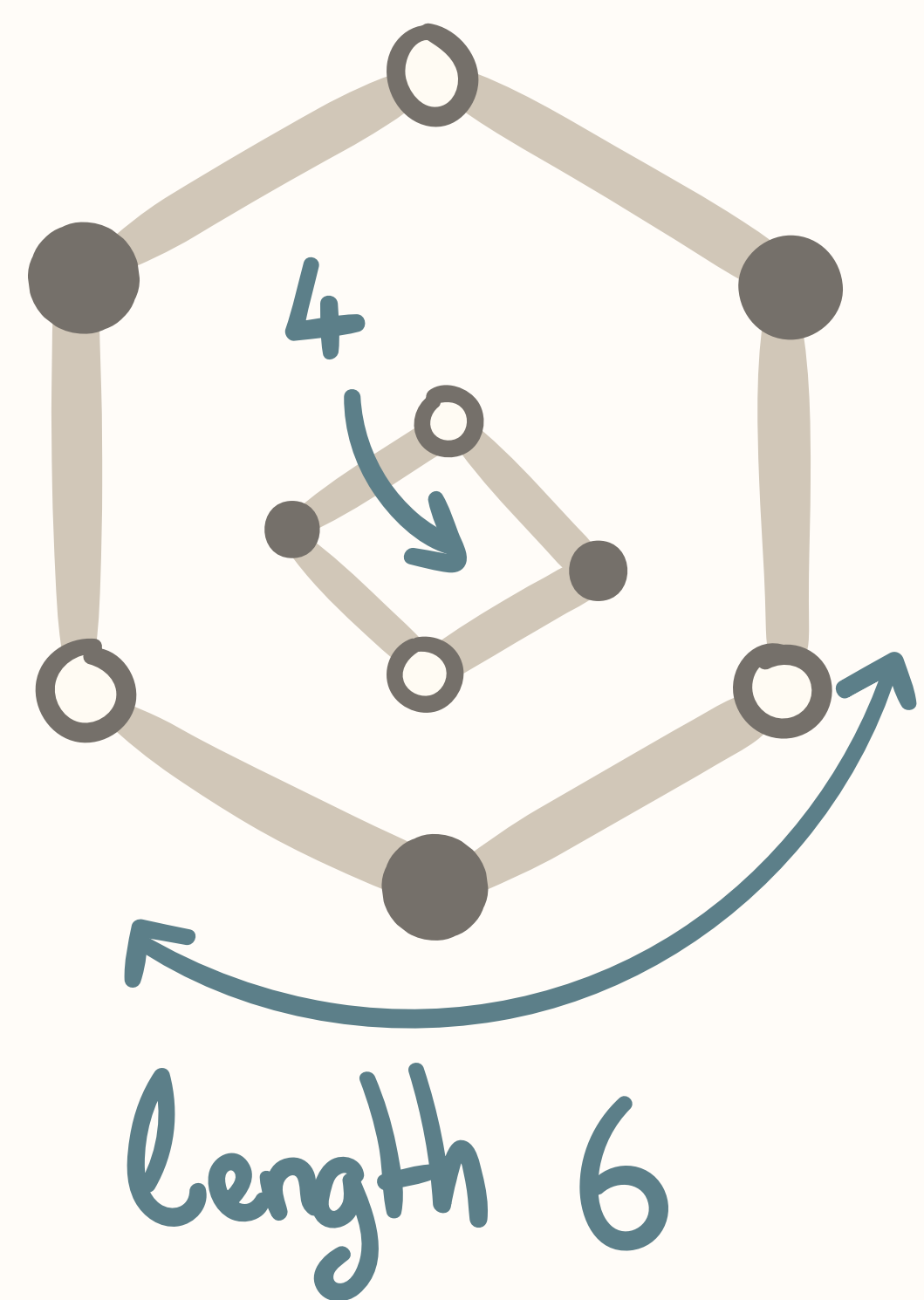
rigid corner polyhedra \longleftrightarrow *Schnyder labellings*

\Rightarrow *Geodesic embeddings and planar graphs,*
S. Felsner (2003)

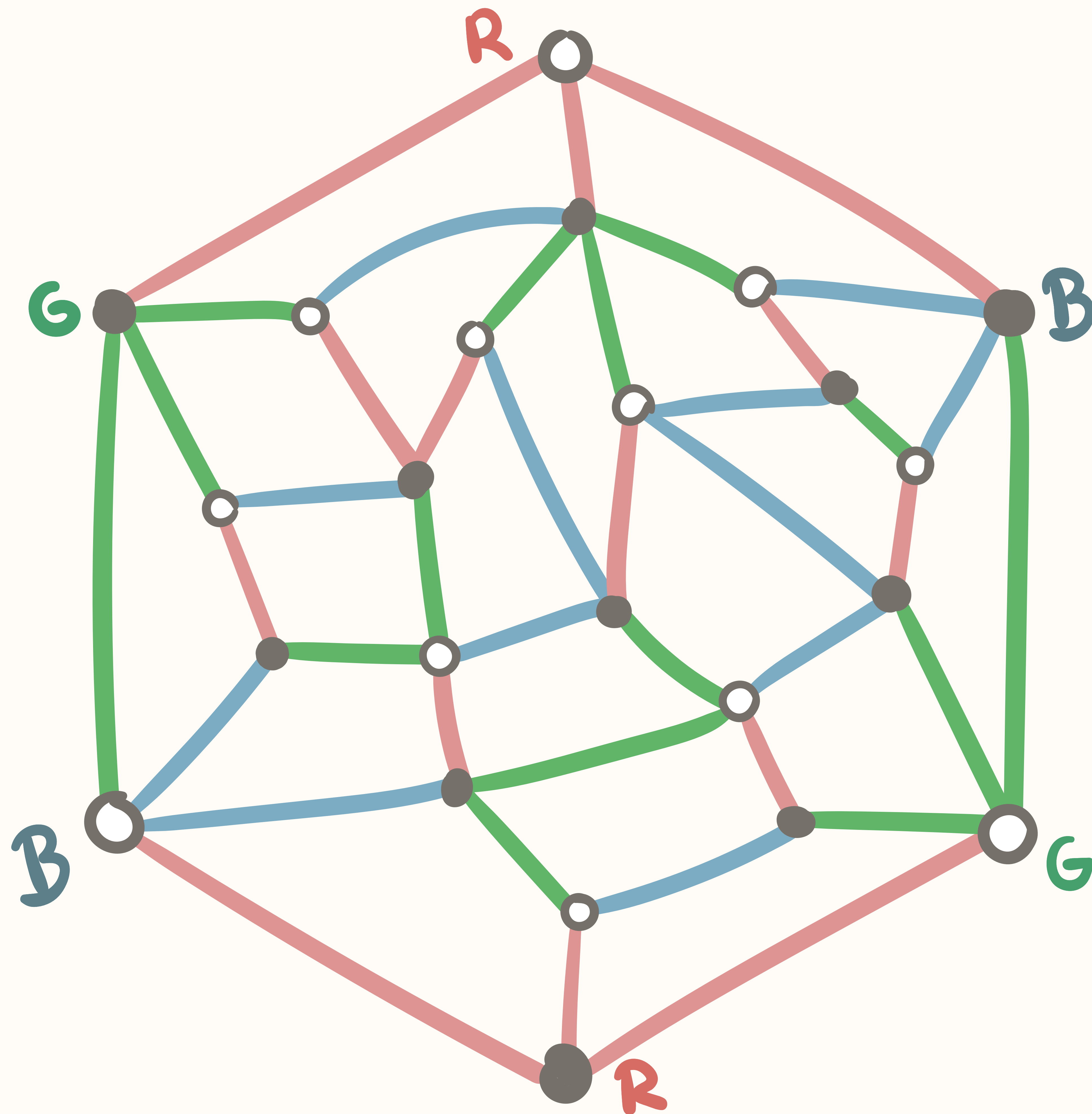
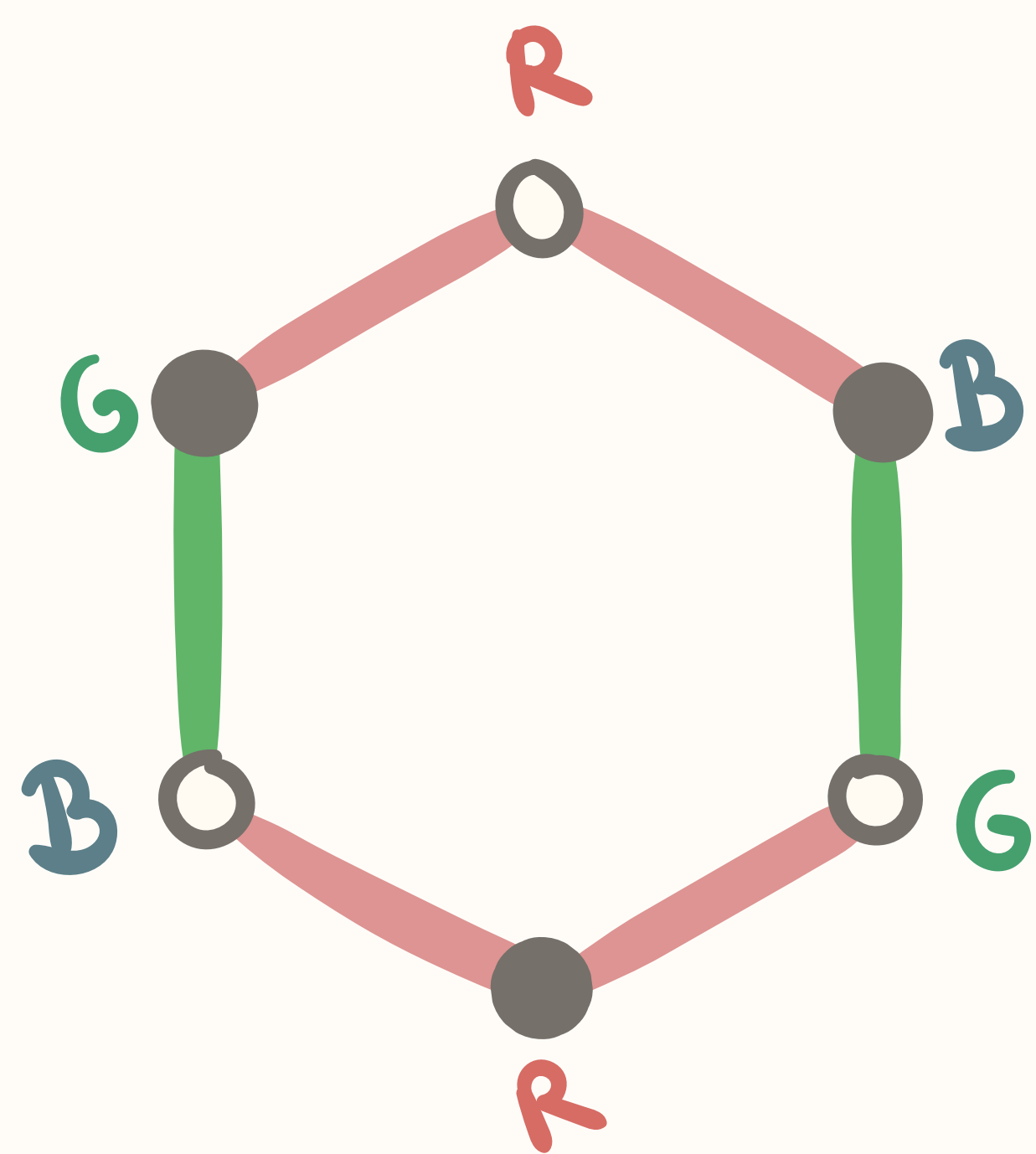
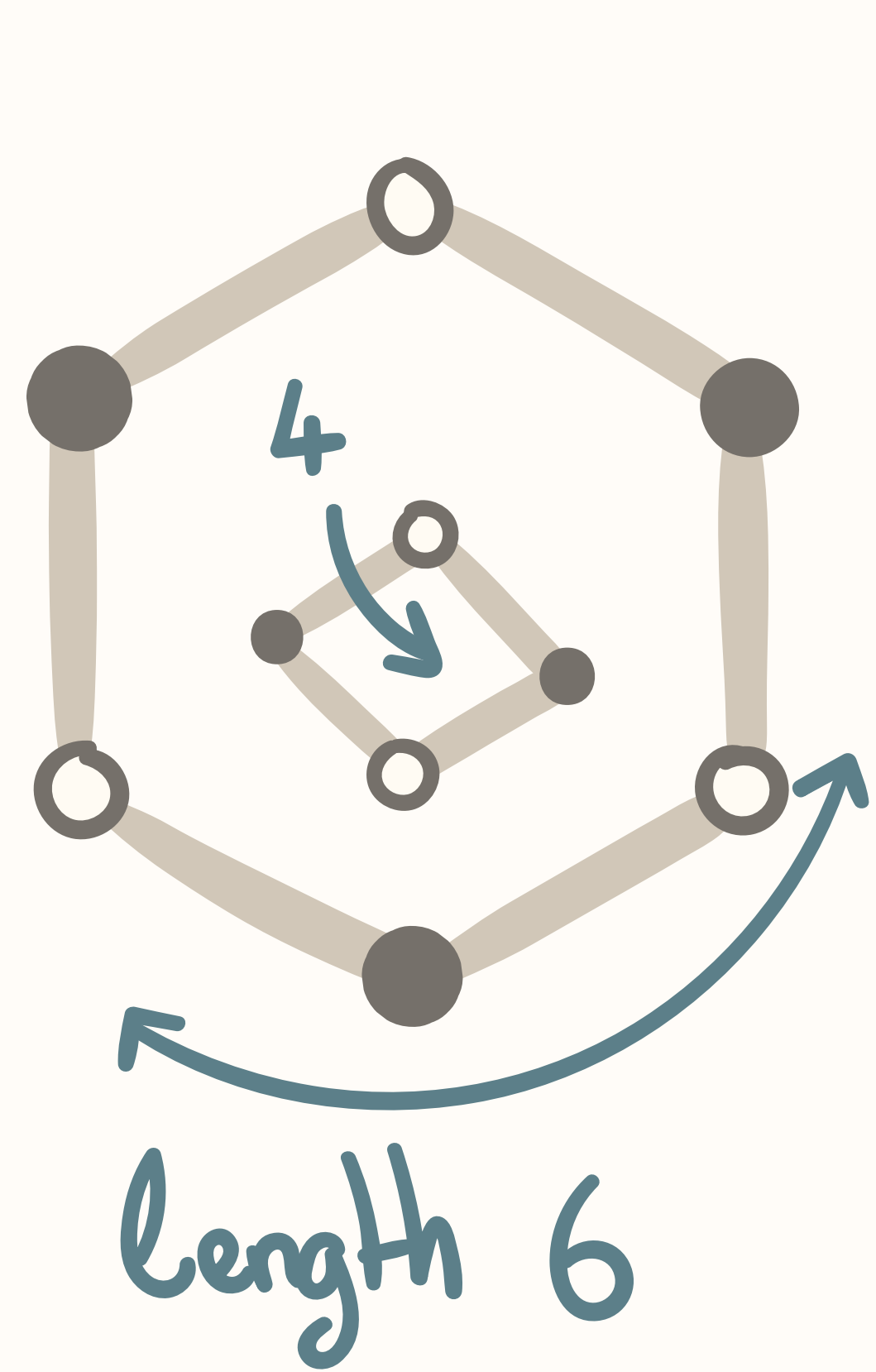
Schnyder labellings



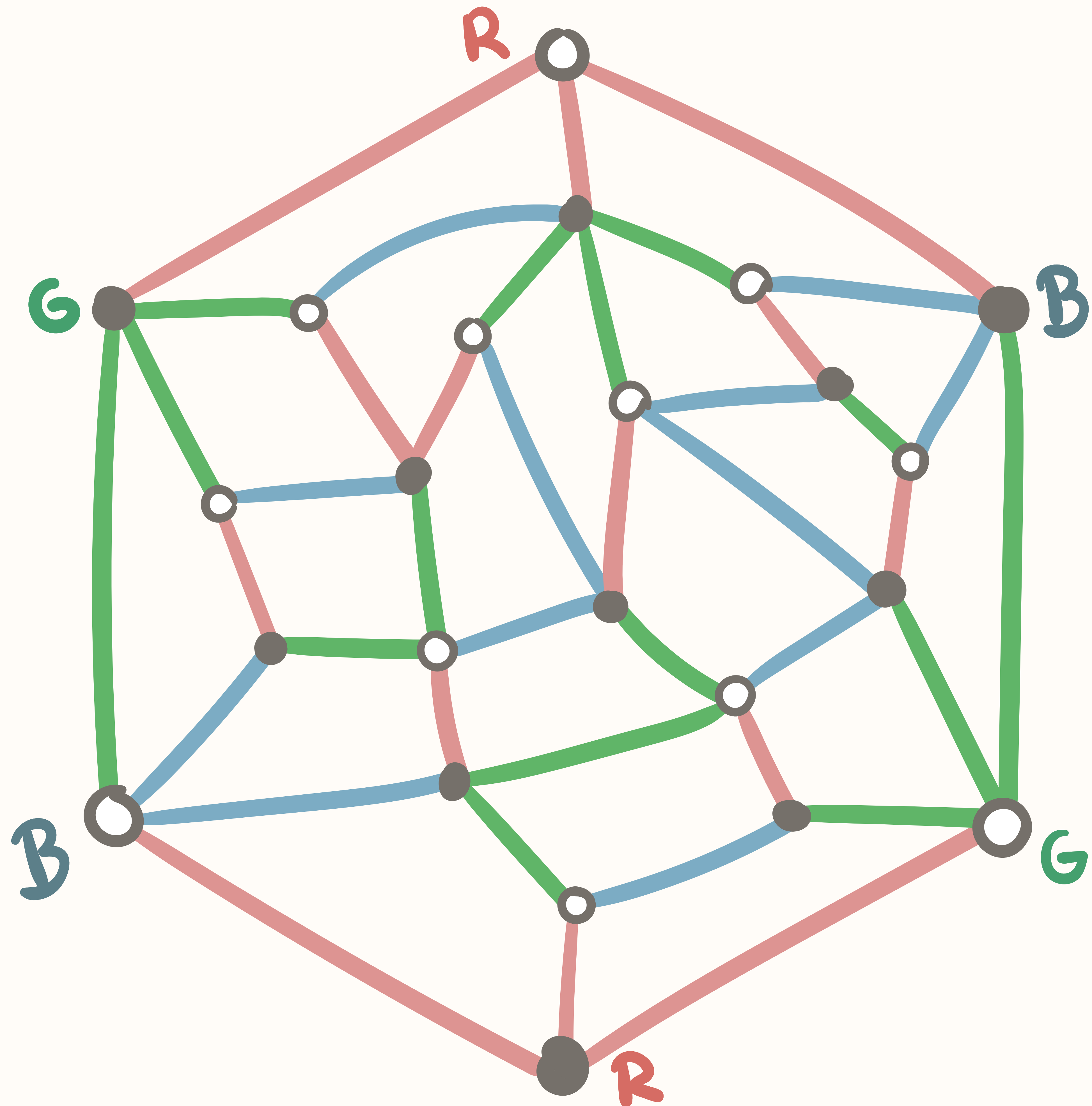
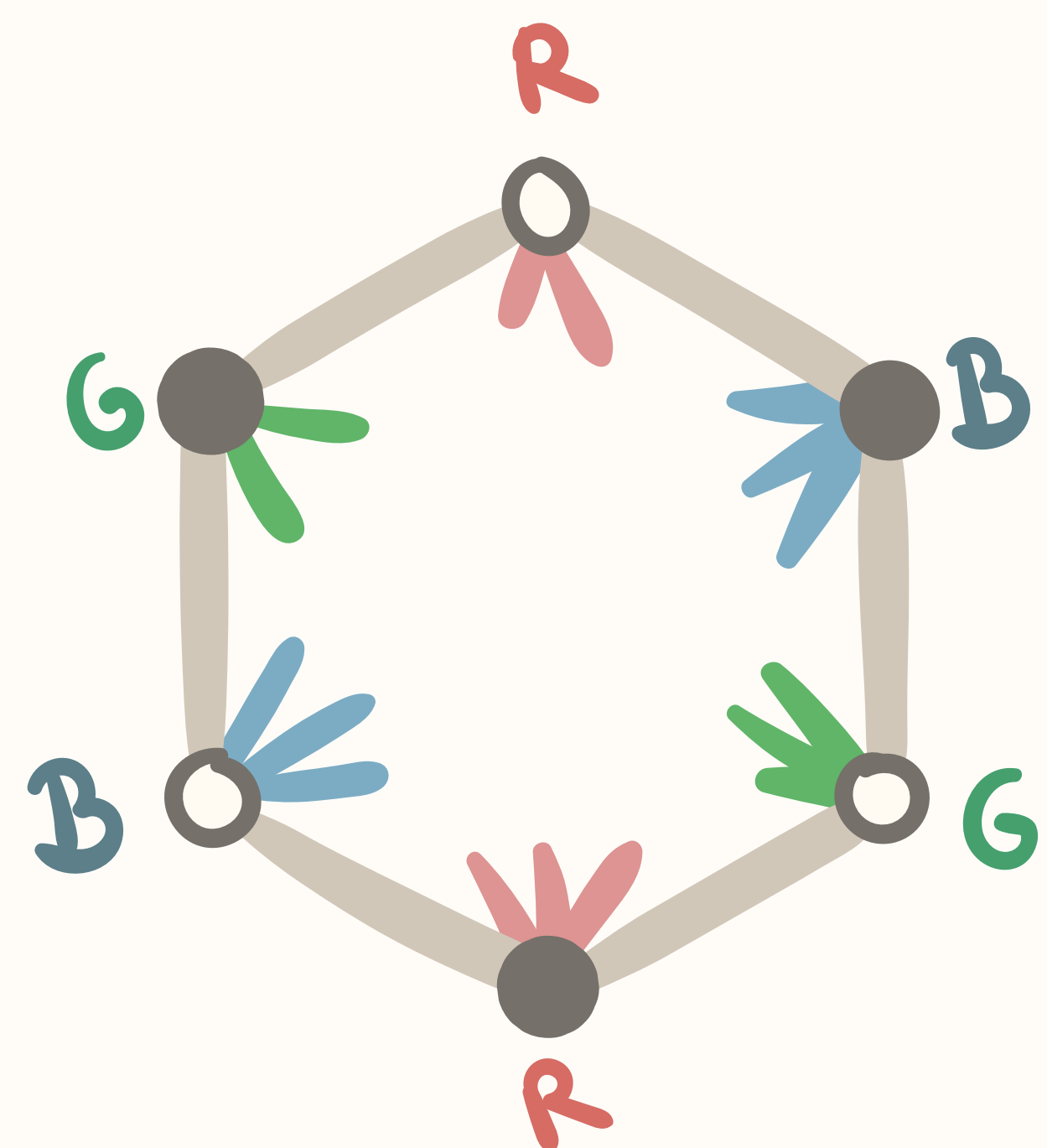
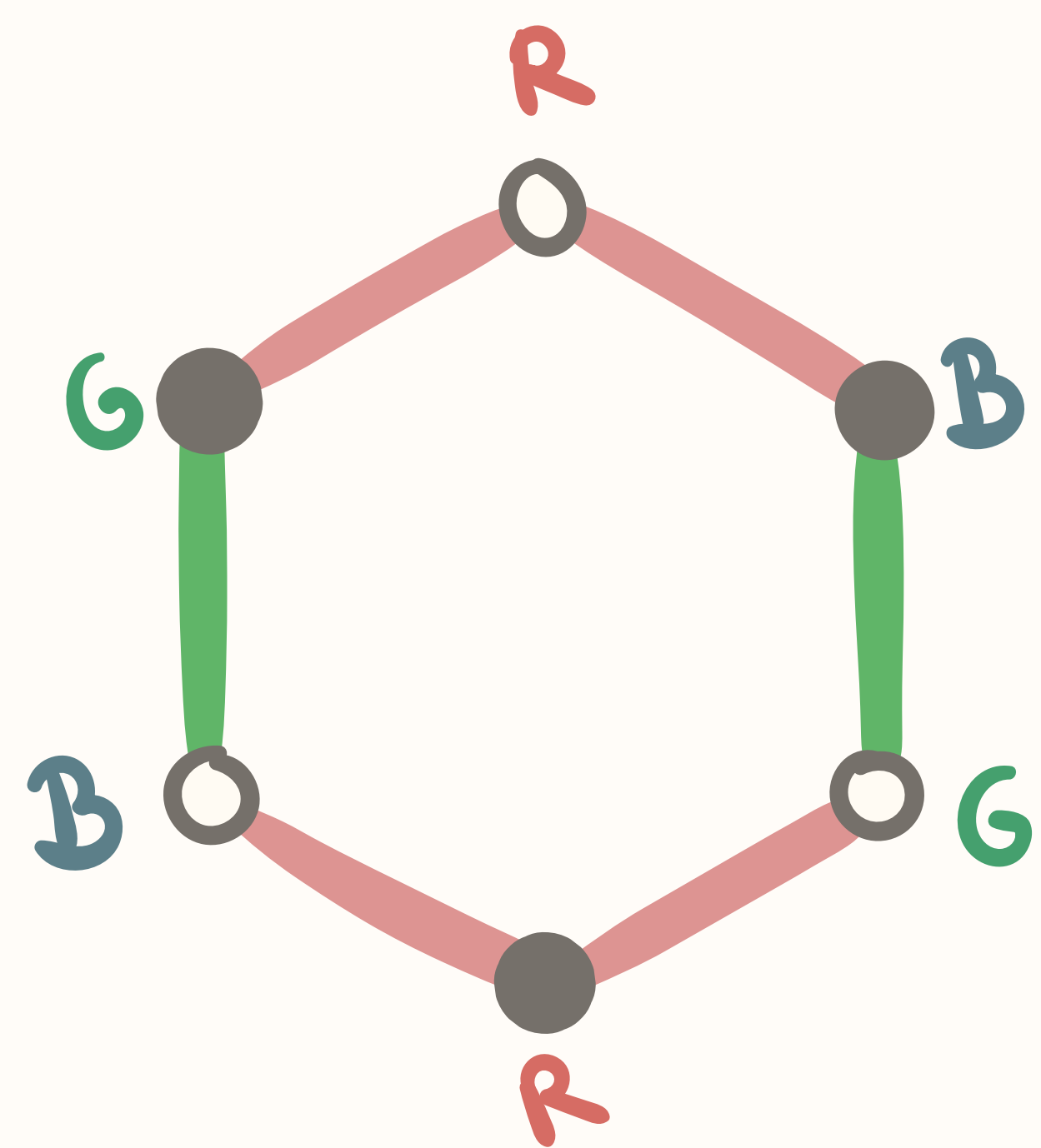
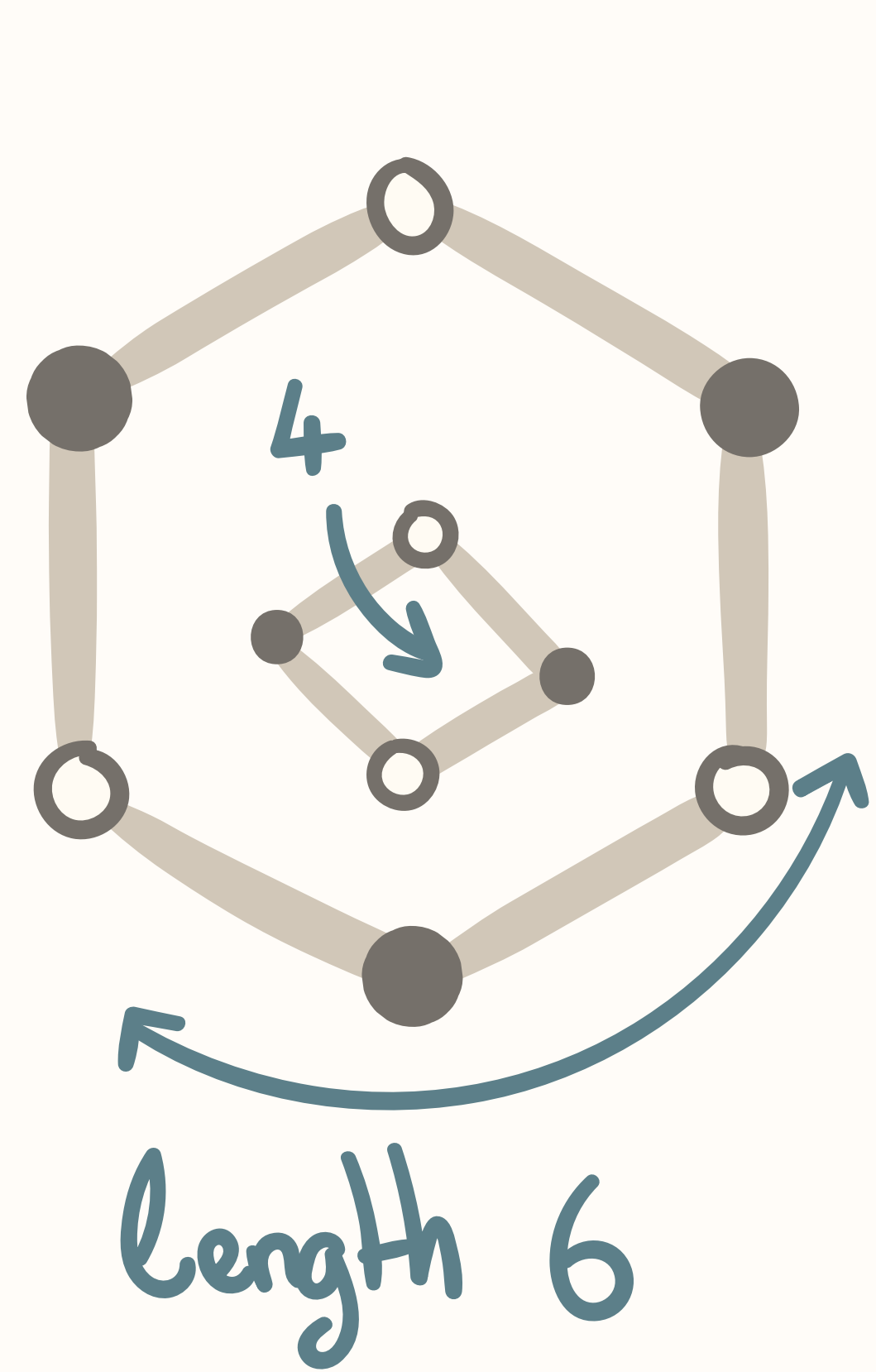
Schnyder labellings



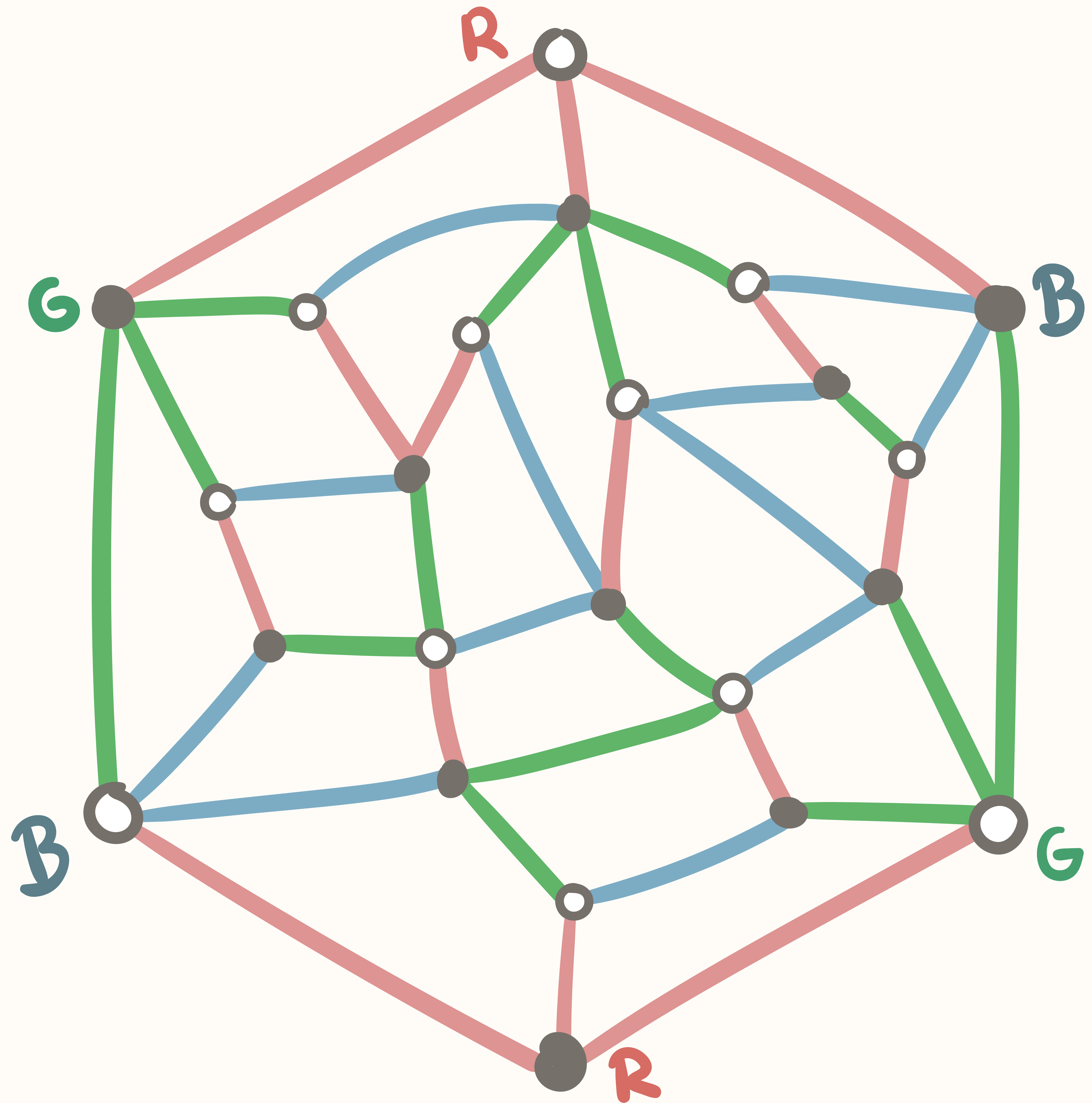
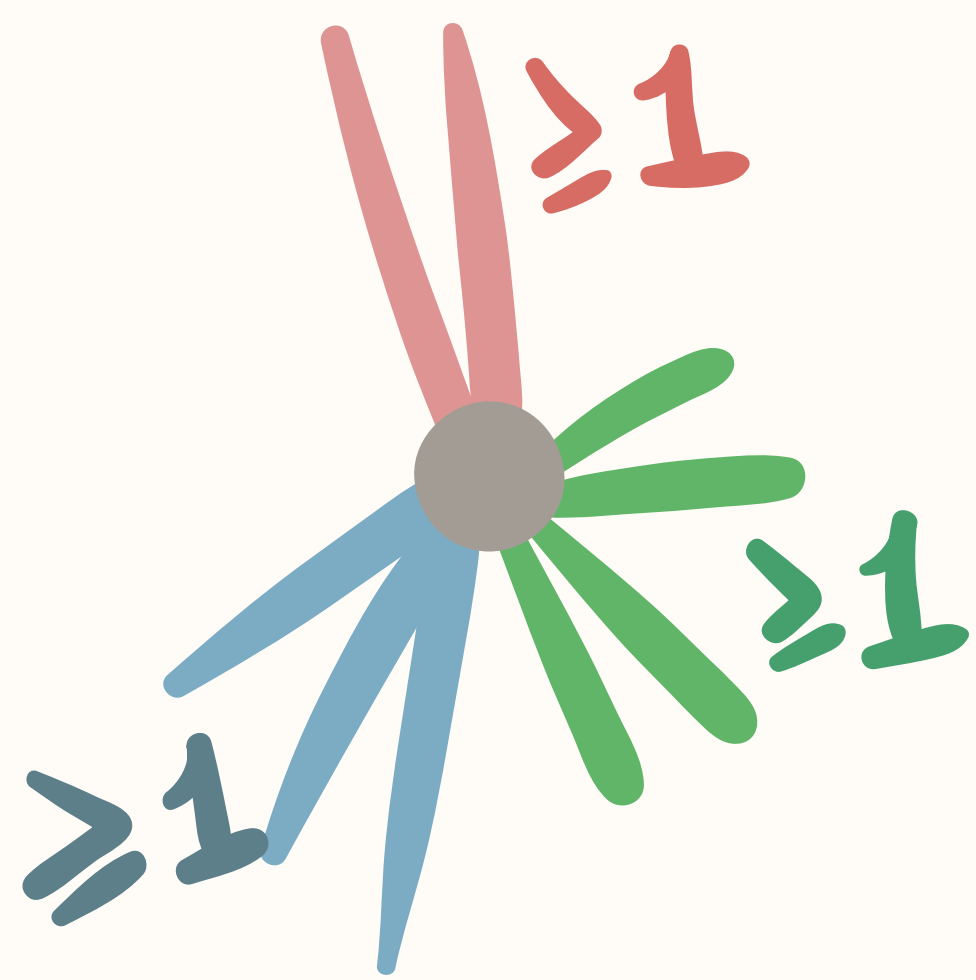
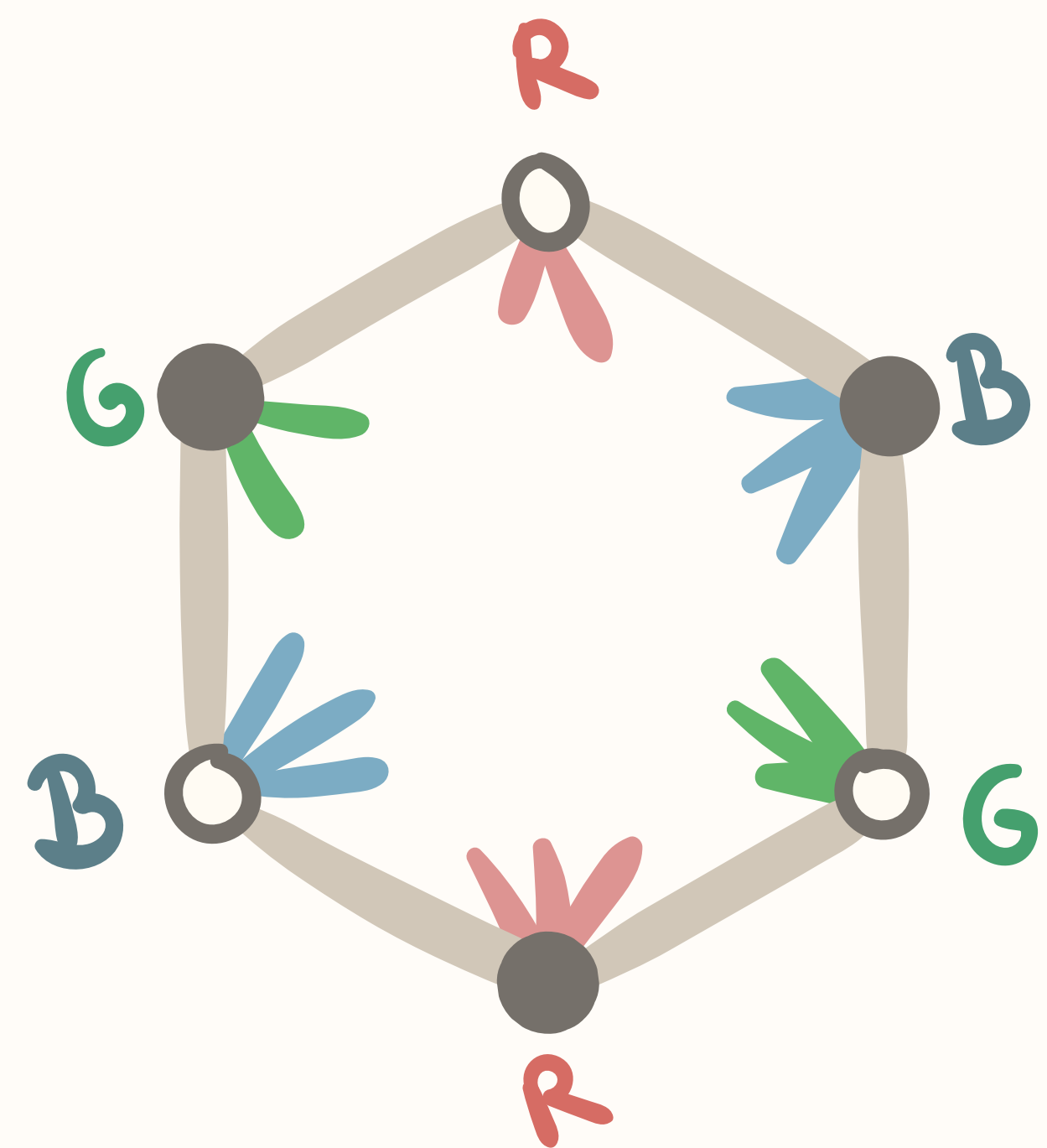
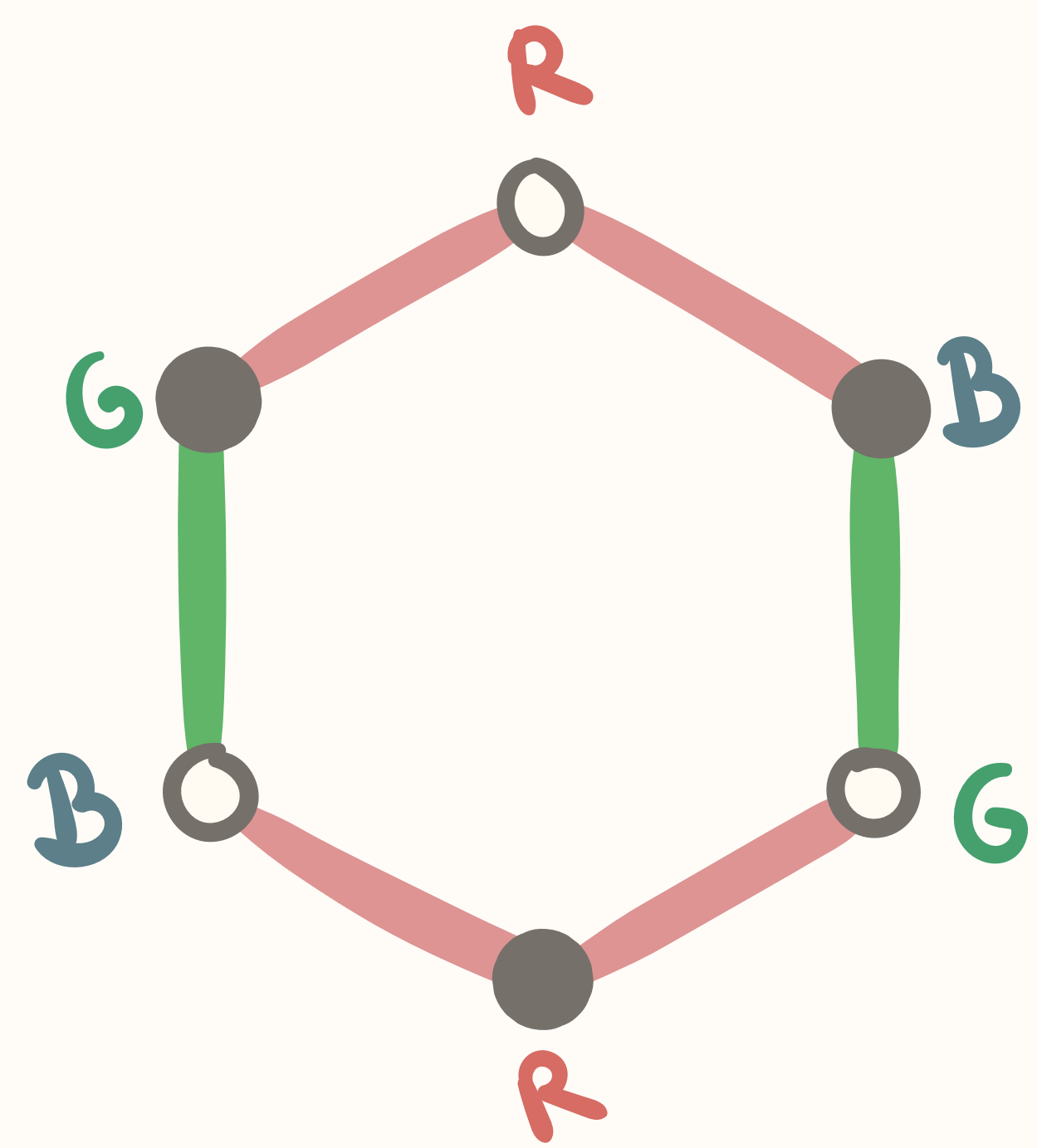
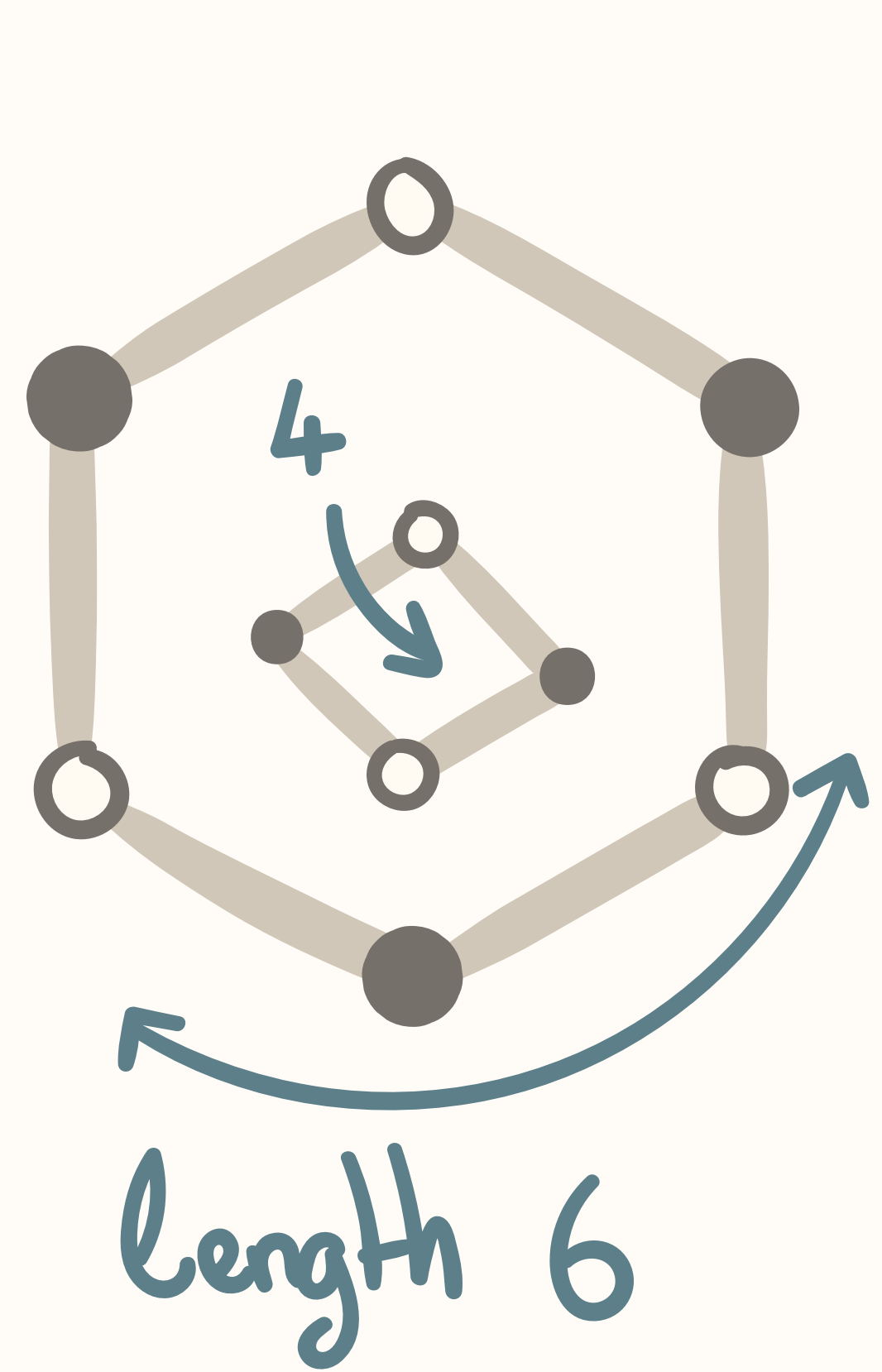
Schnyder labellings



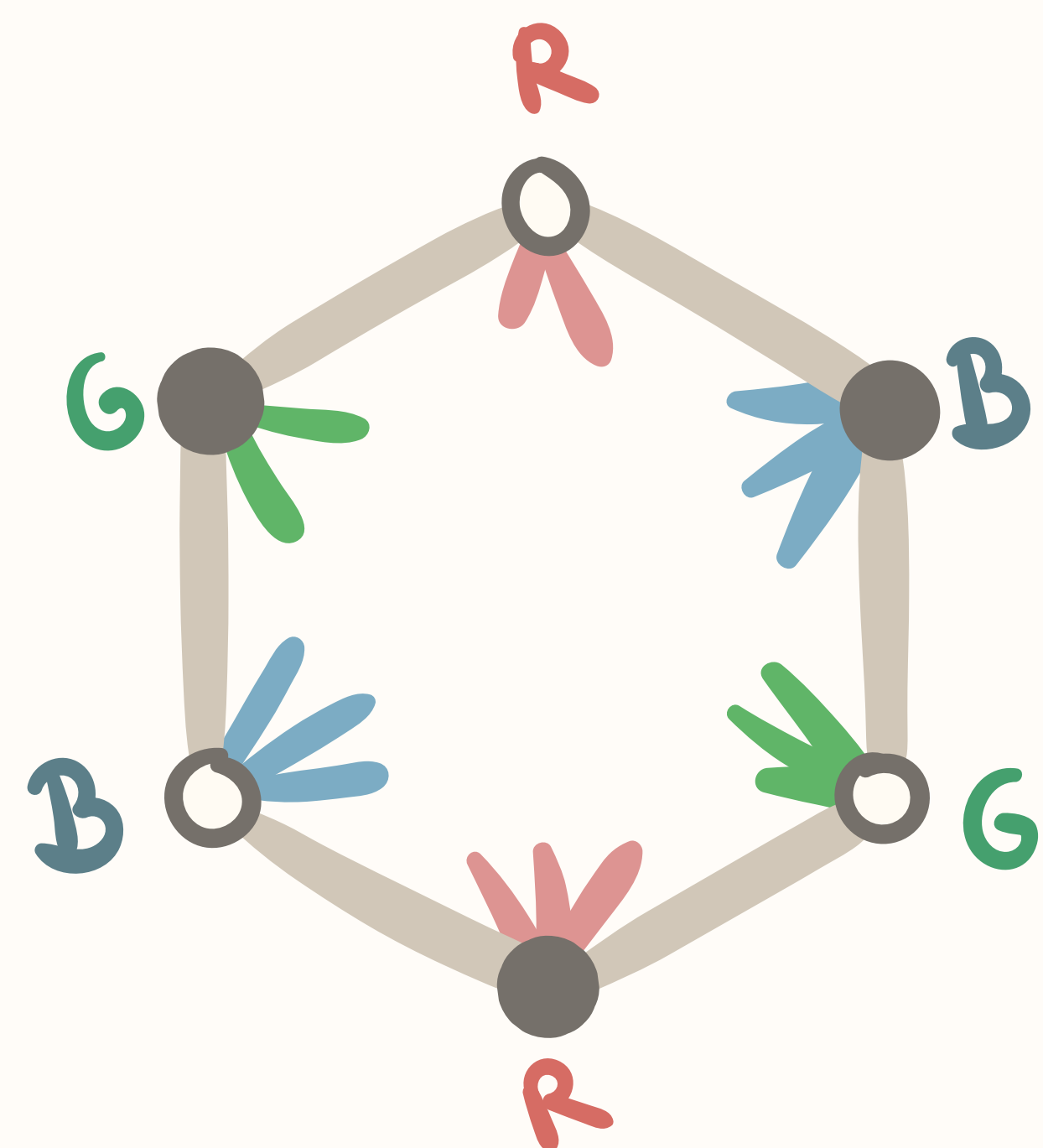
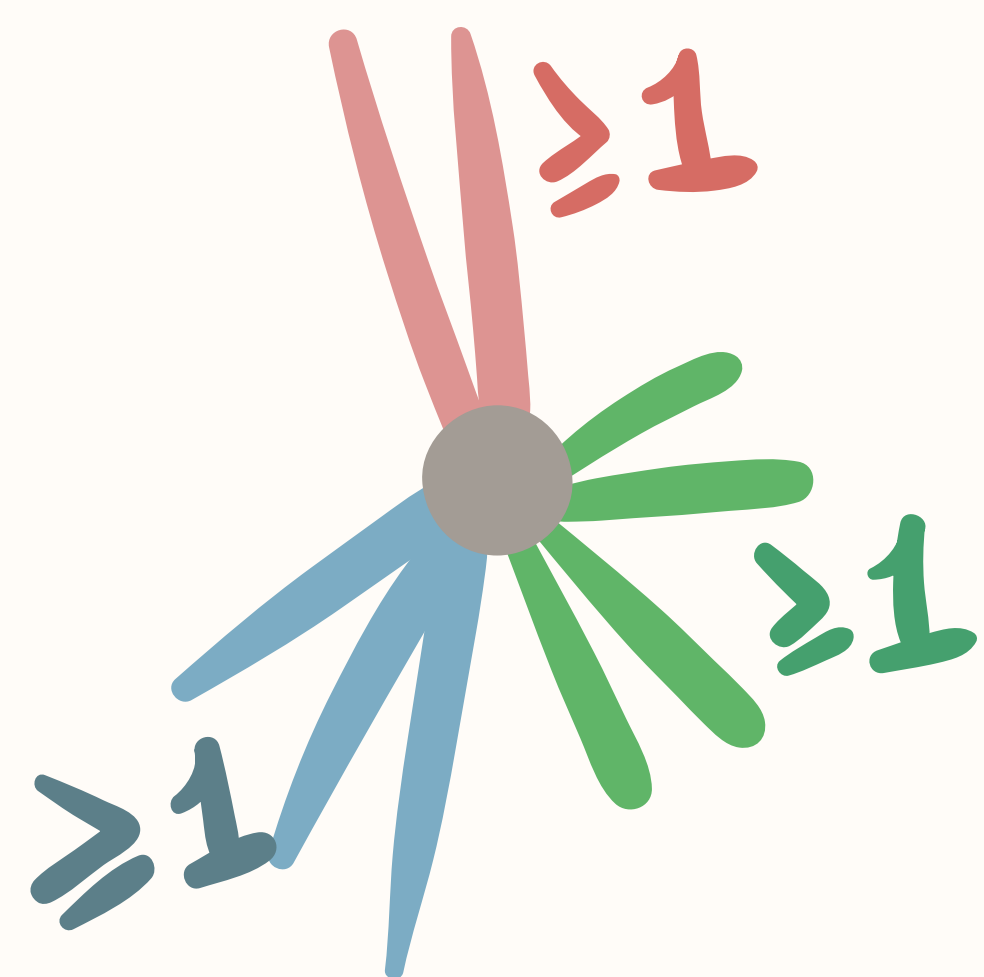
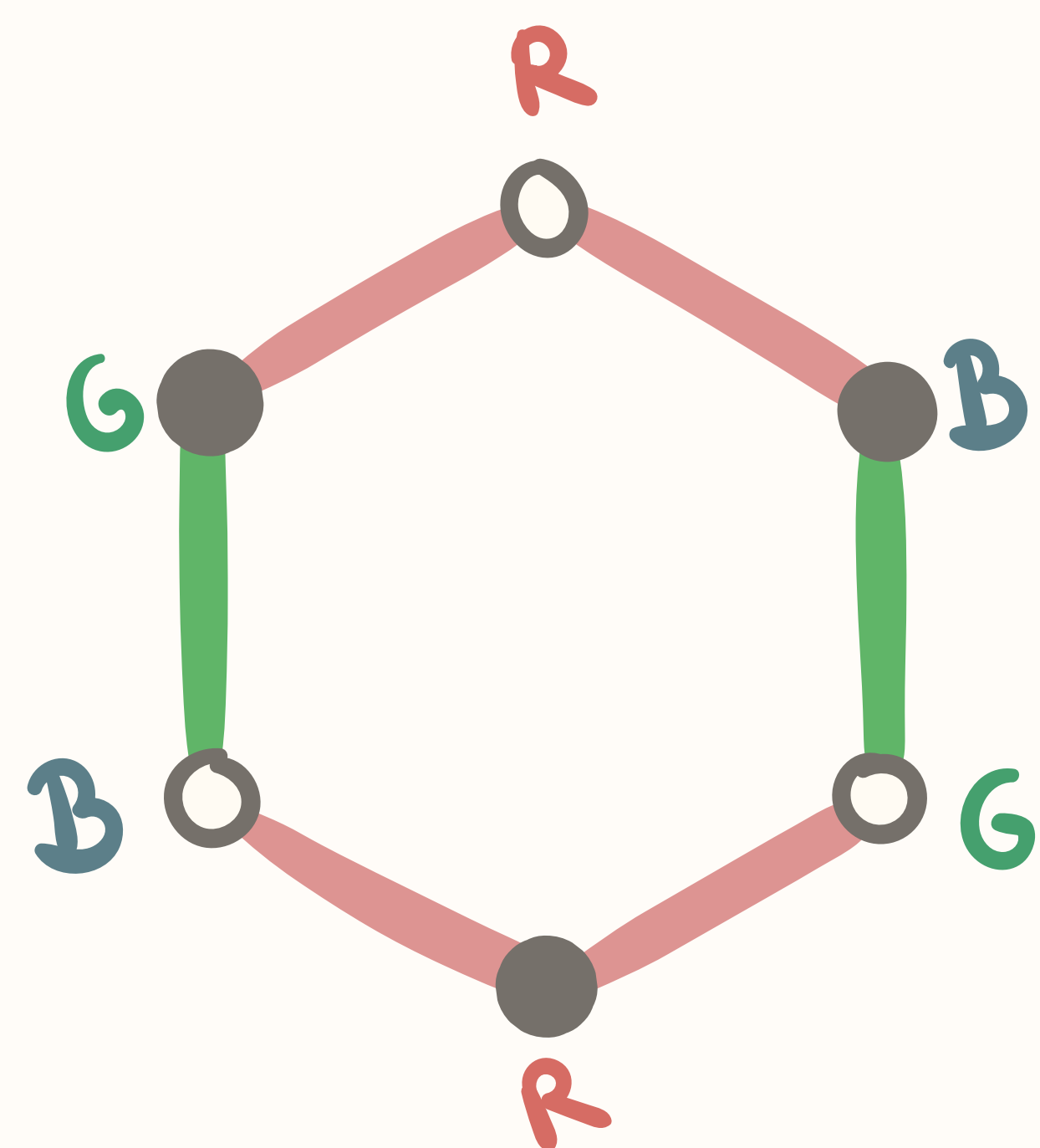
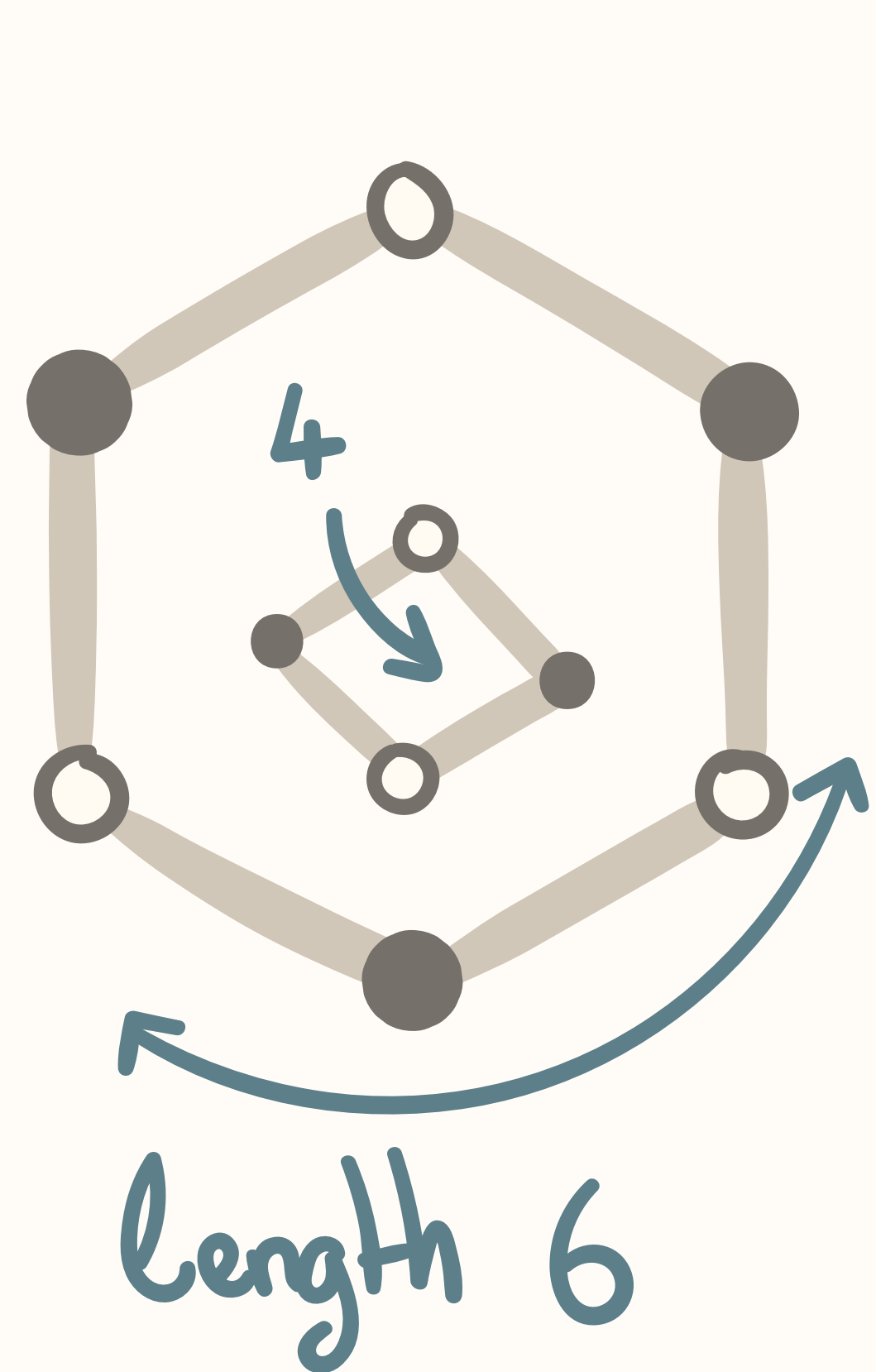
Schnyder labellings



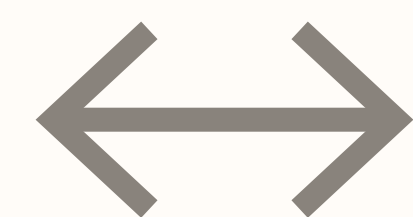
Schnyder labellings



Schnyder labellings

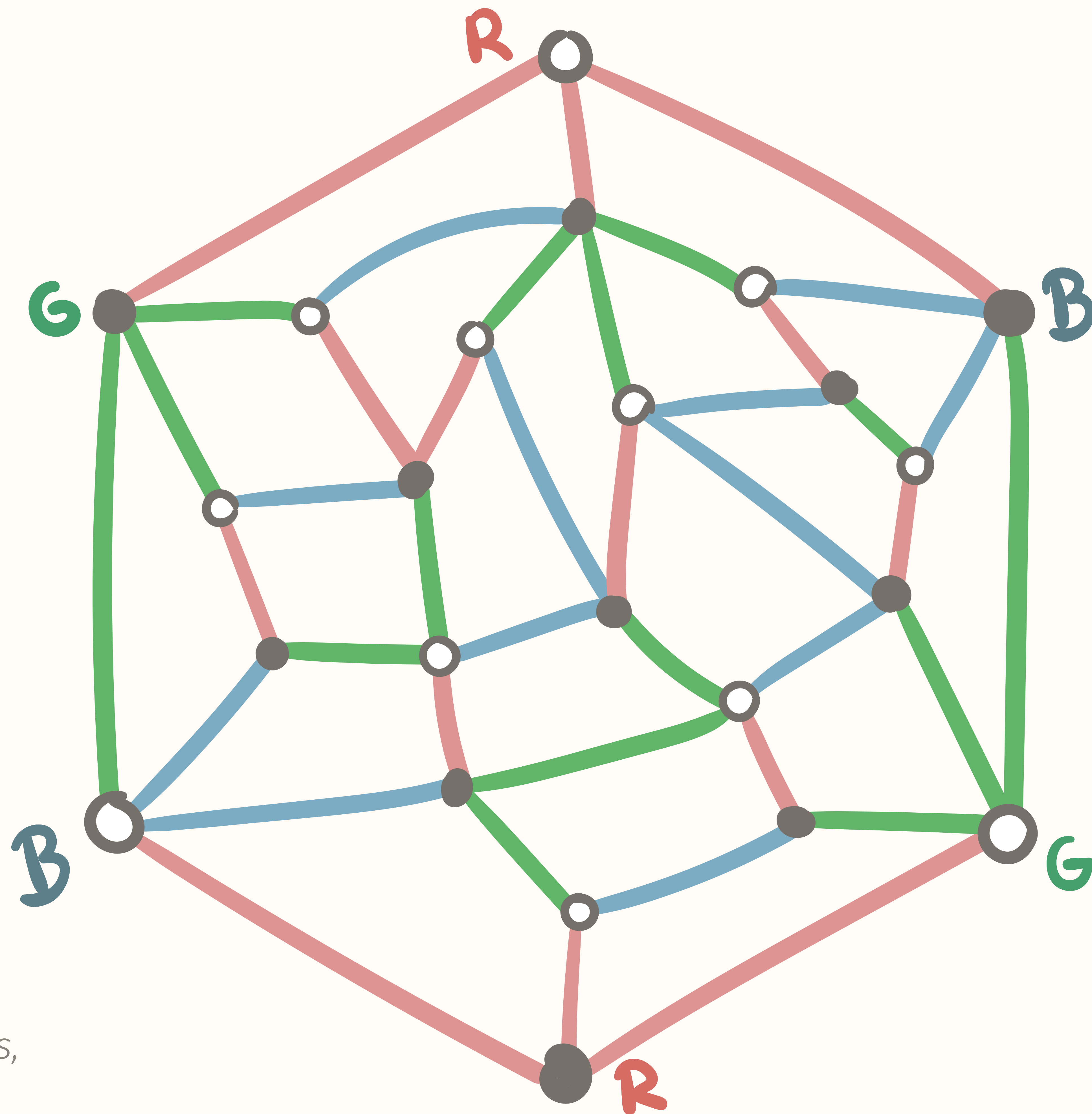


rigid corner
polyhedra

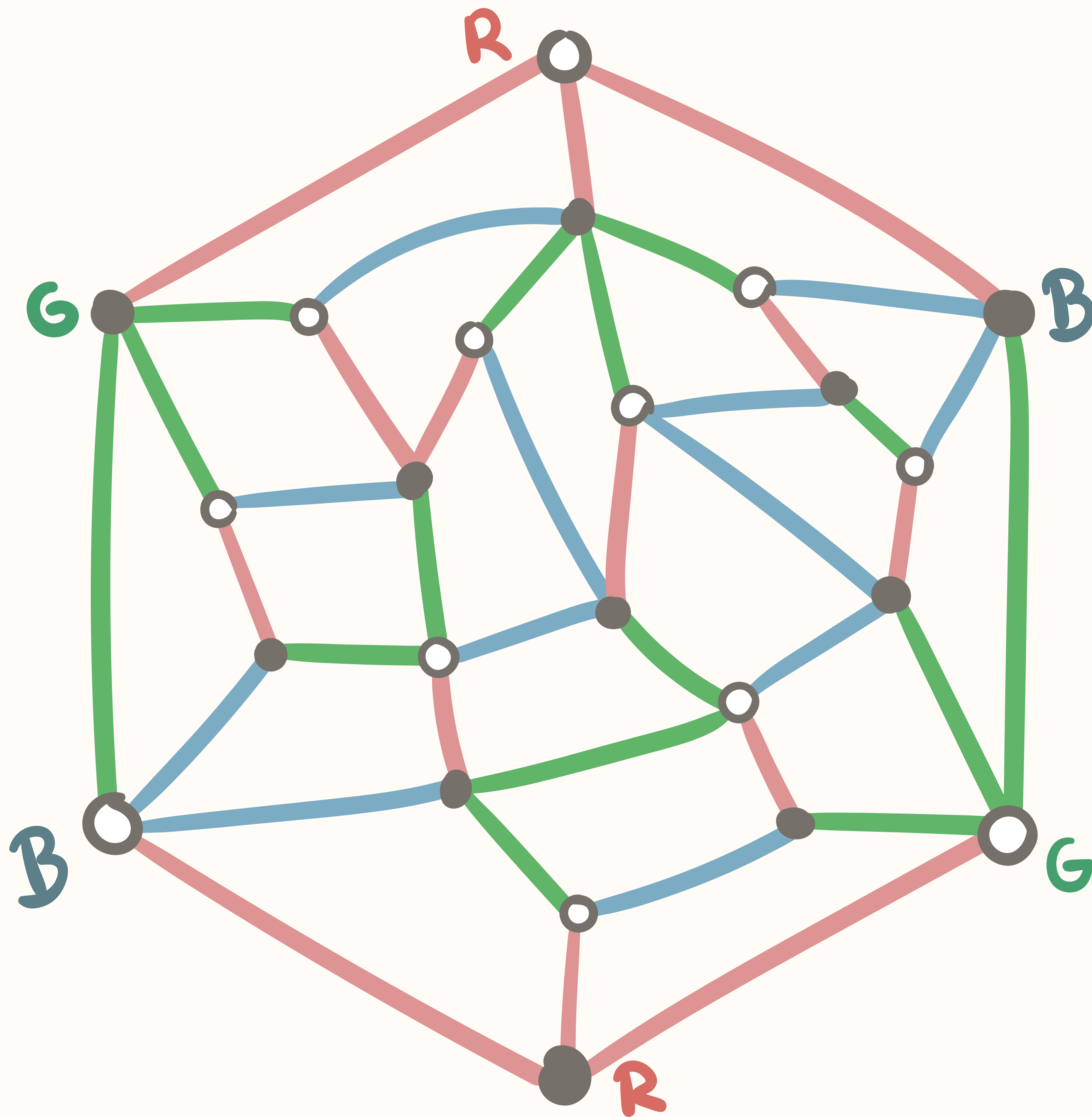
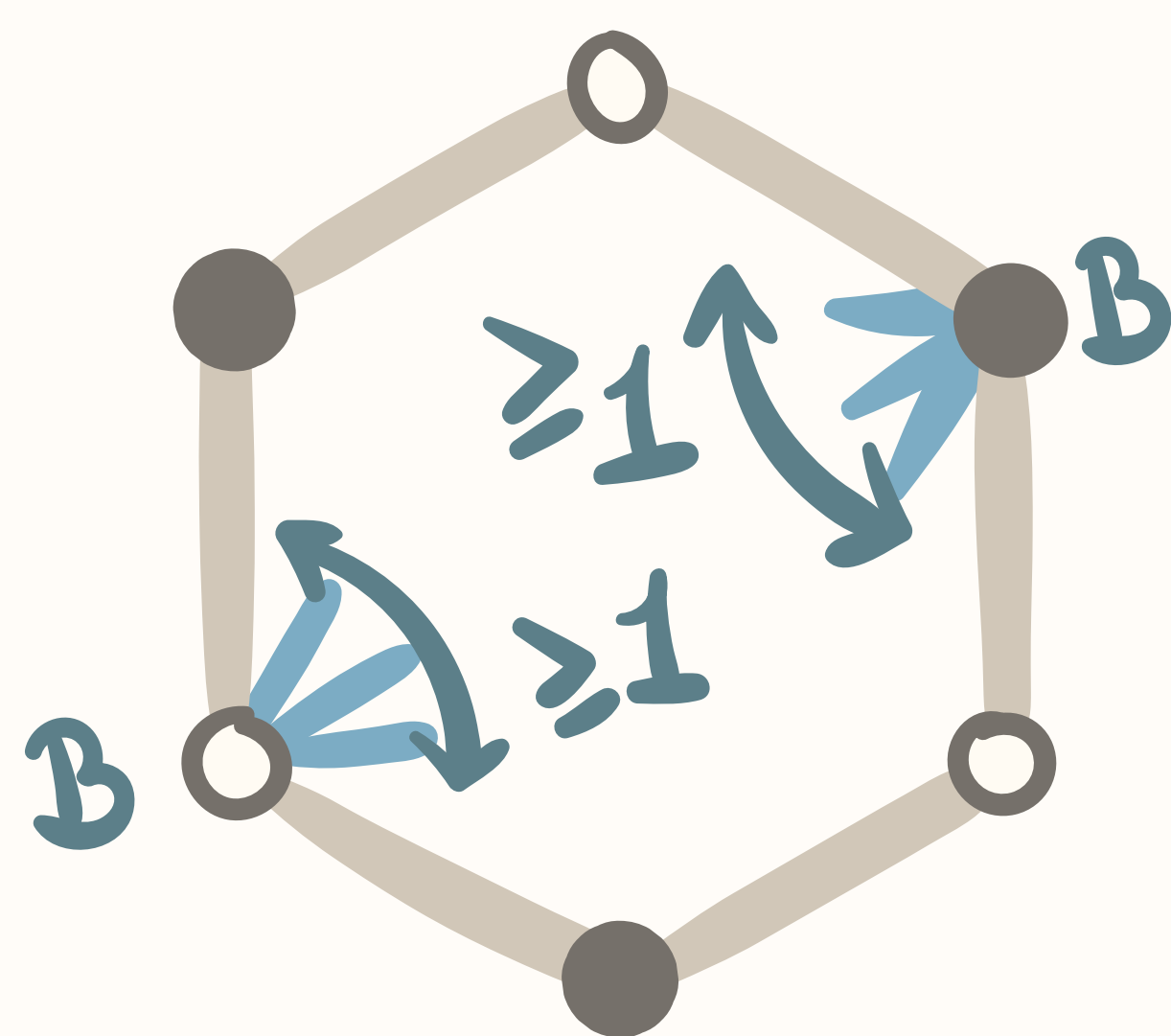
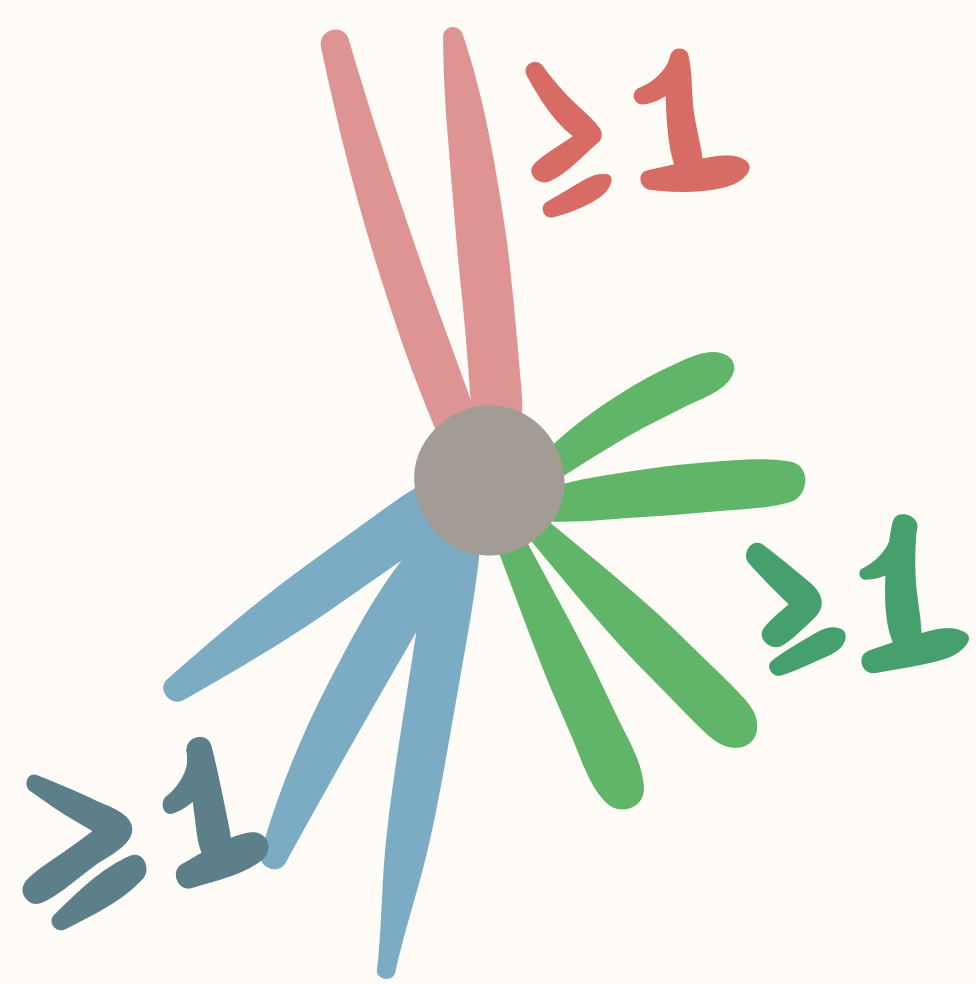
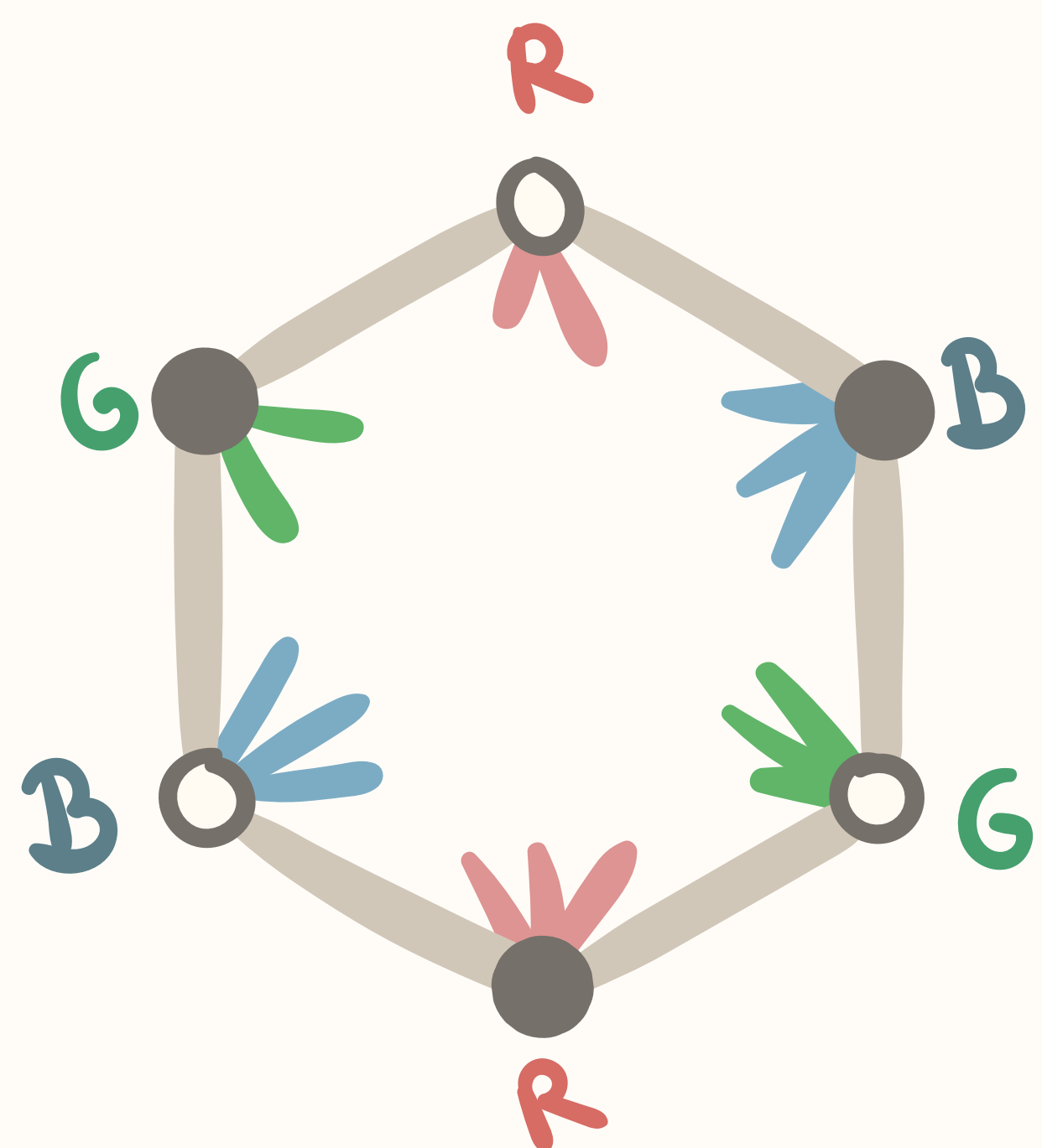
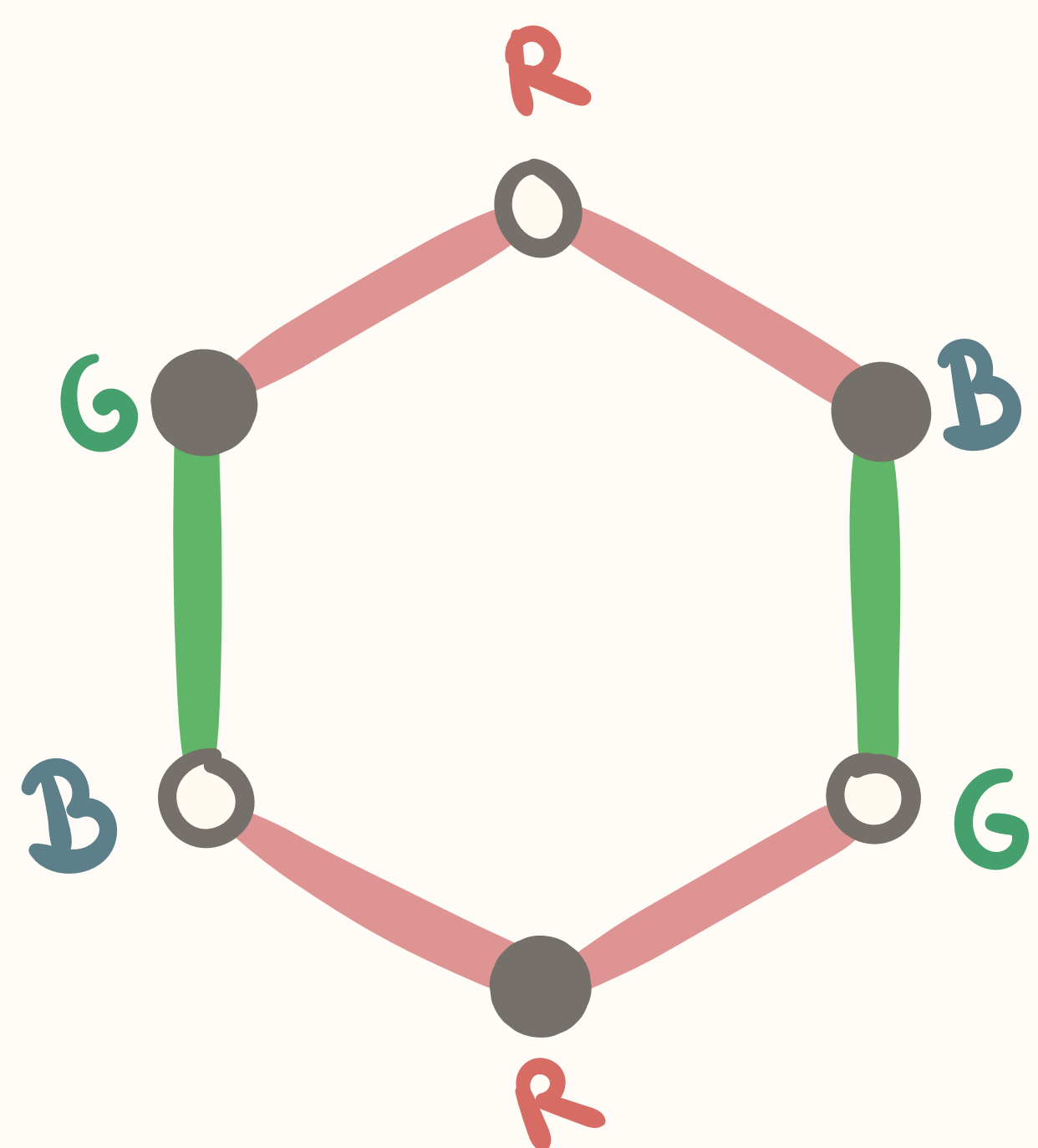
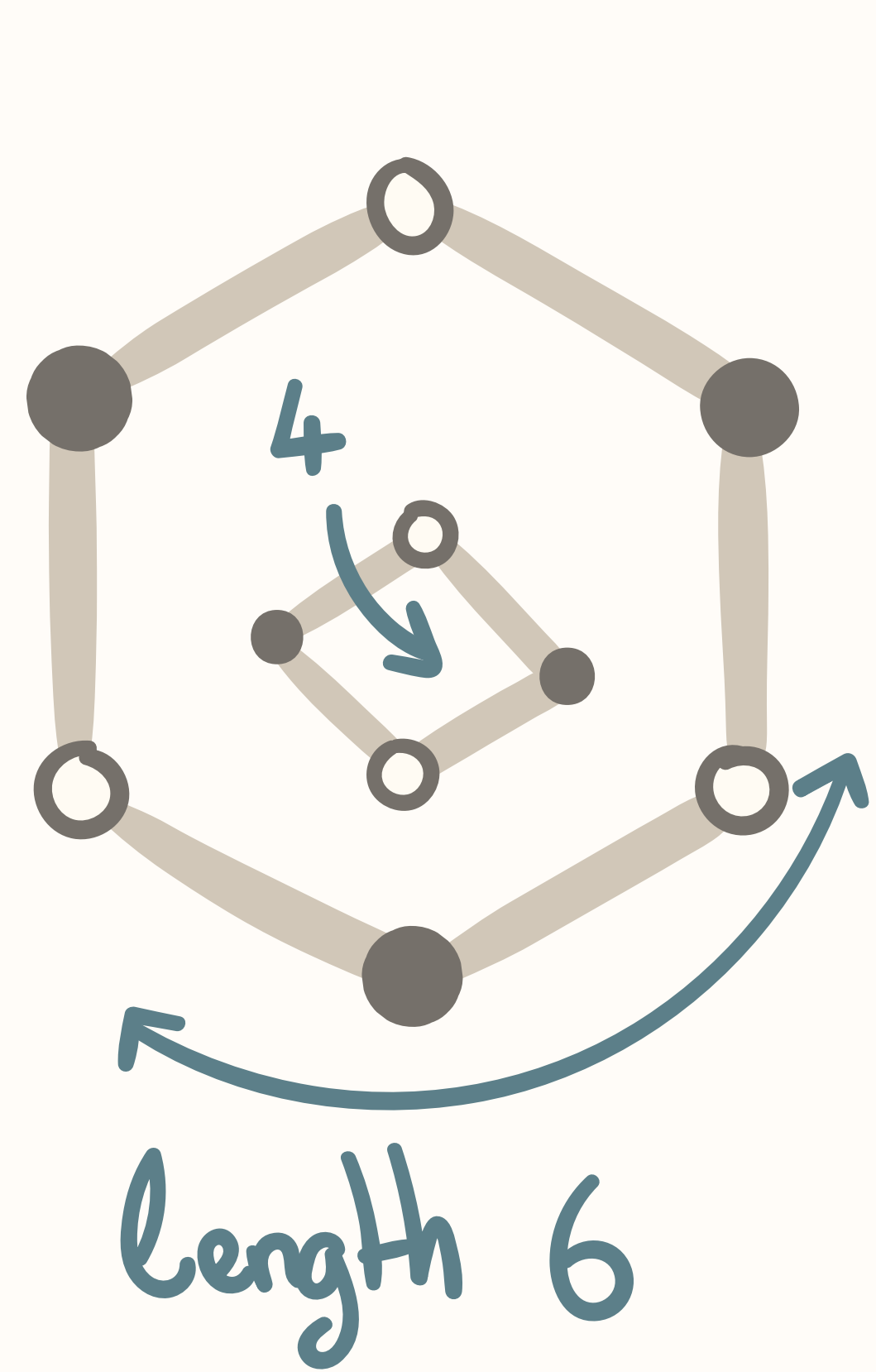


Schnyder
labellings

⇒ Geodesic embeddings and planar graphs,
S. Felsner (2003)

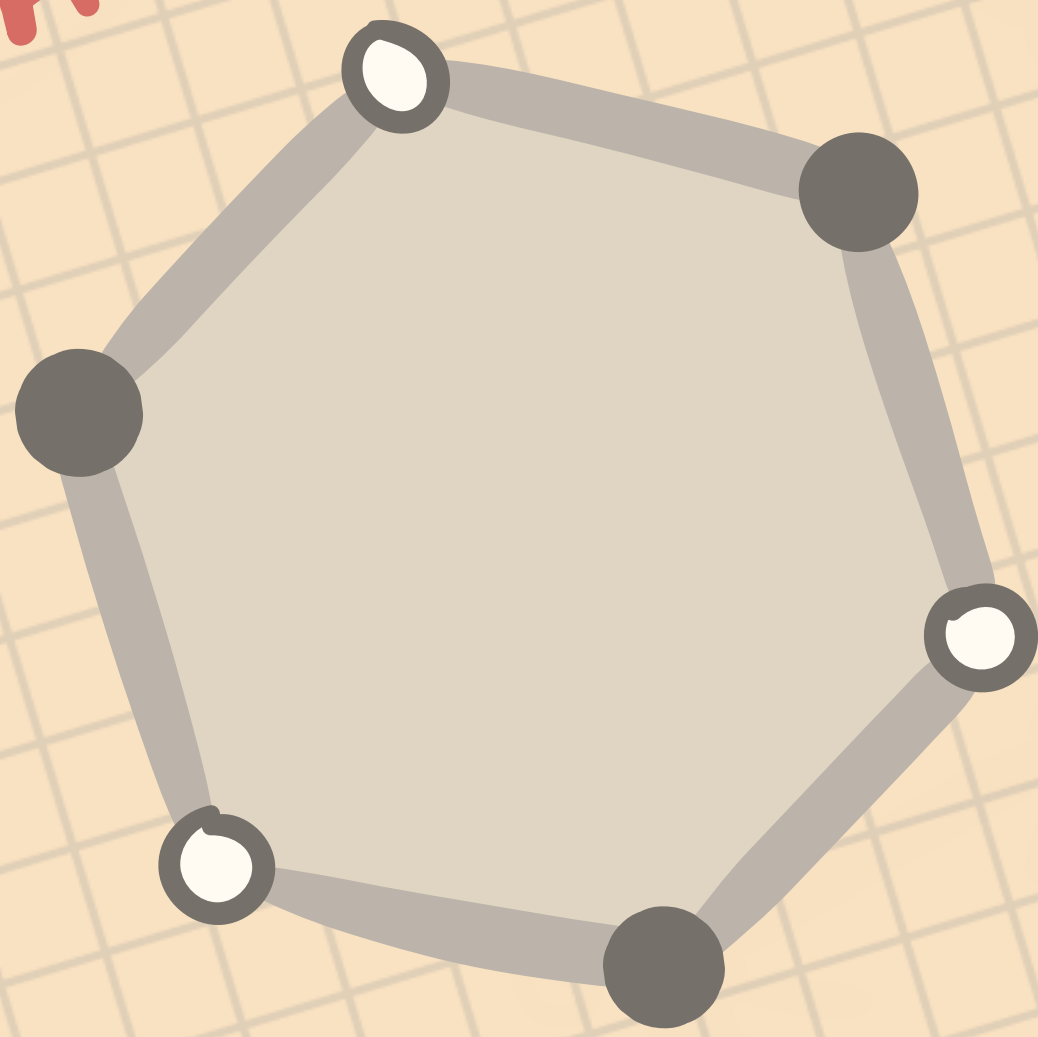


Schnyder labellings



Schnyder labellings

ALL
SCHNYDER
LABELLINGS



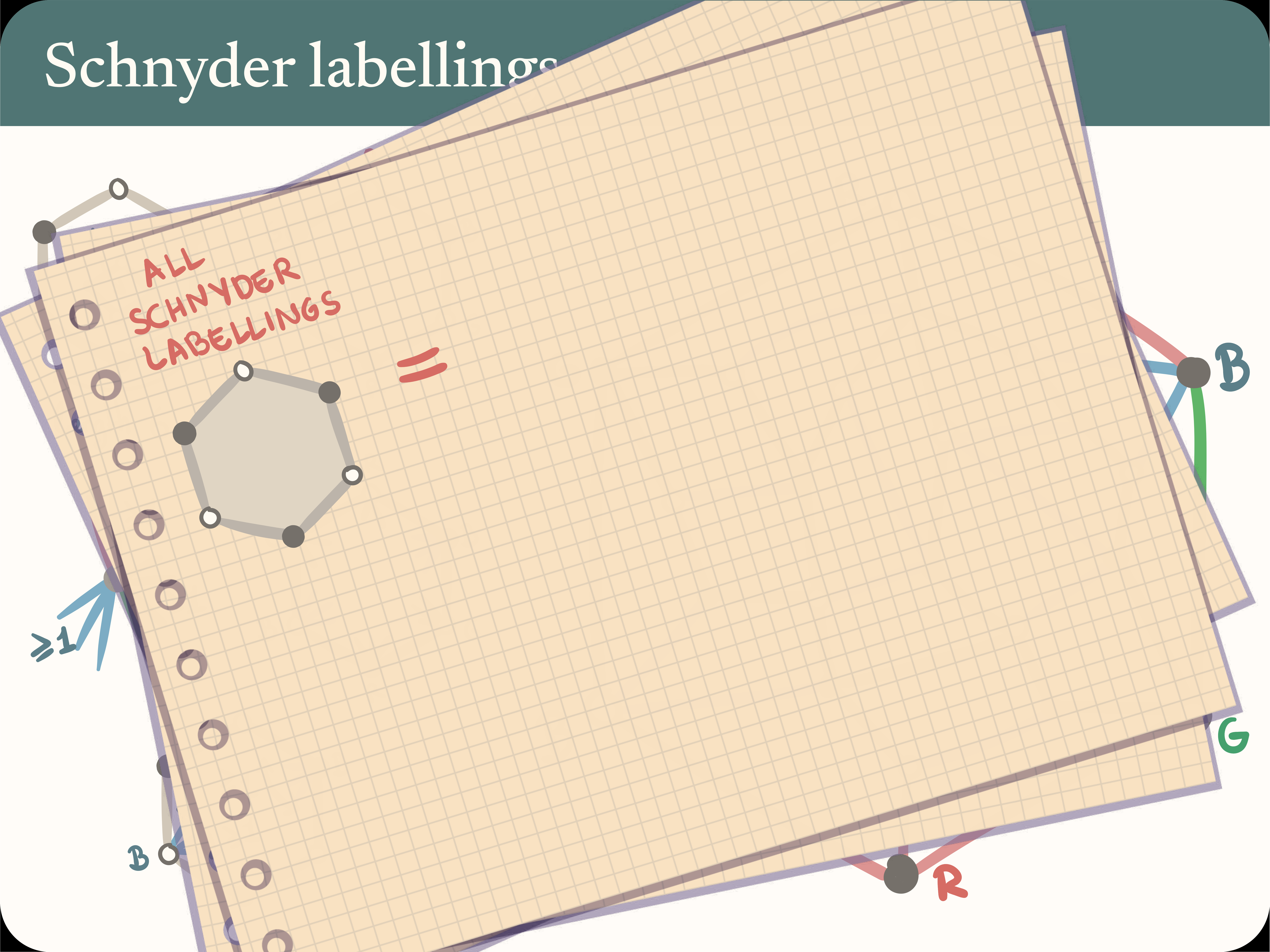
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B

B

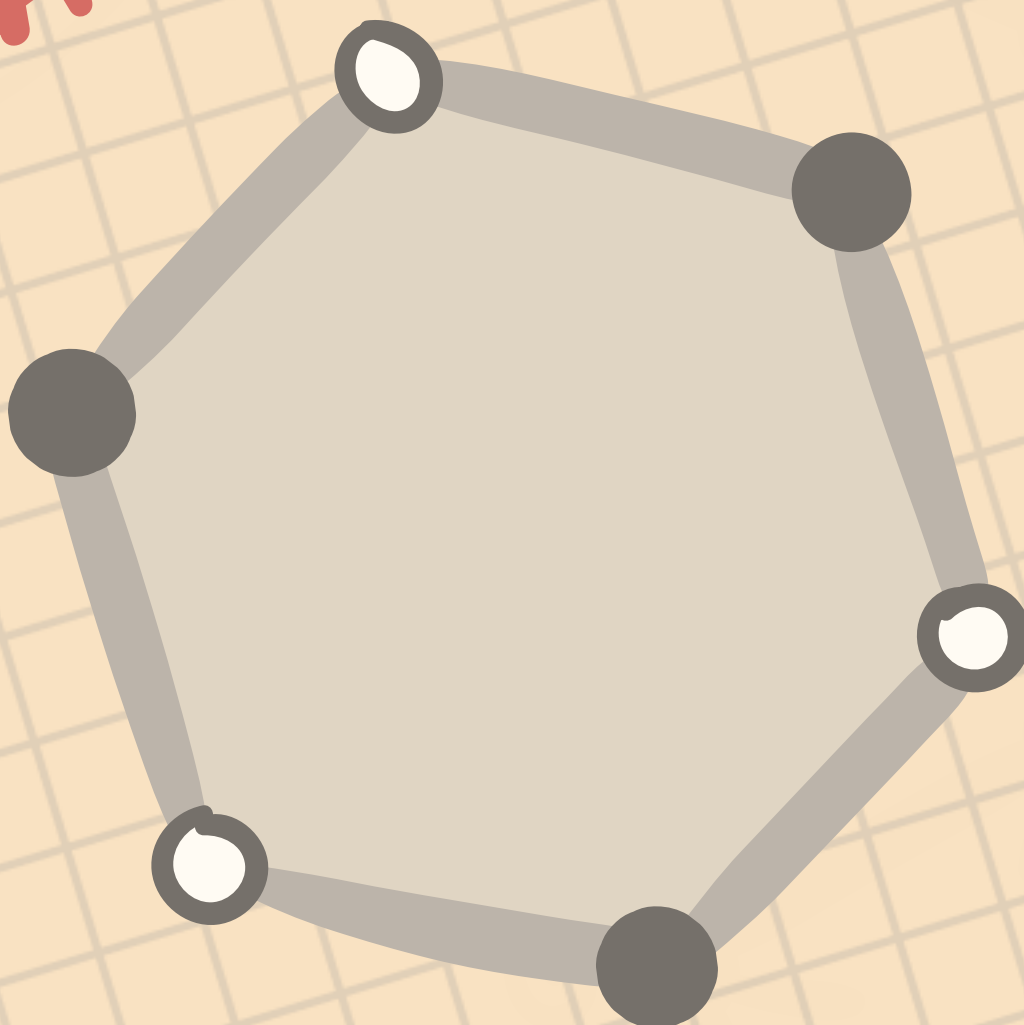
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R

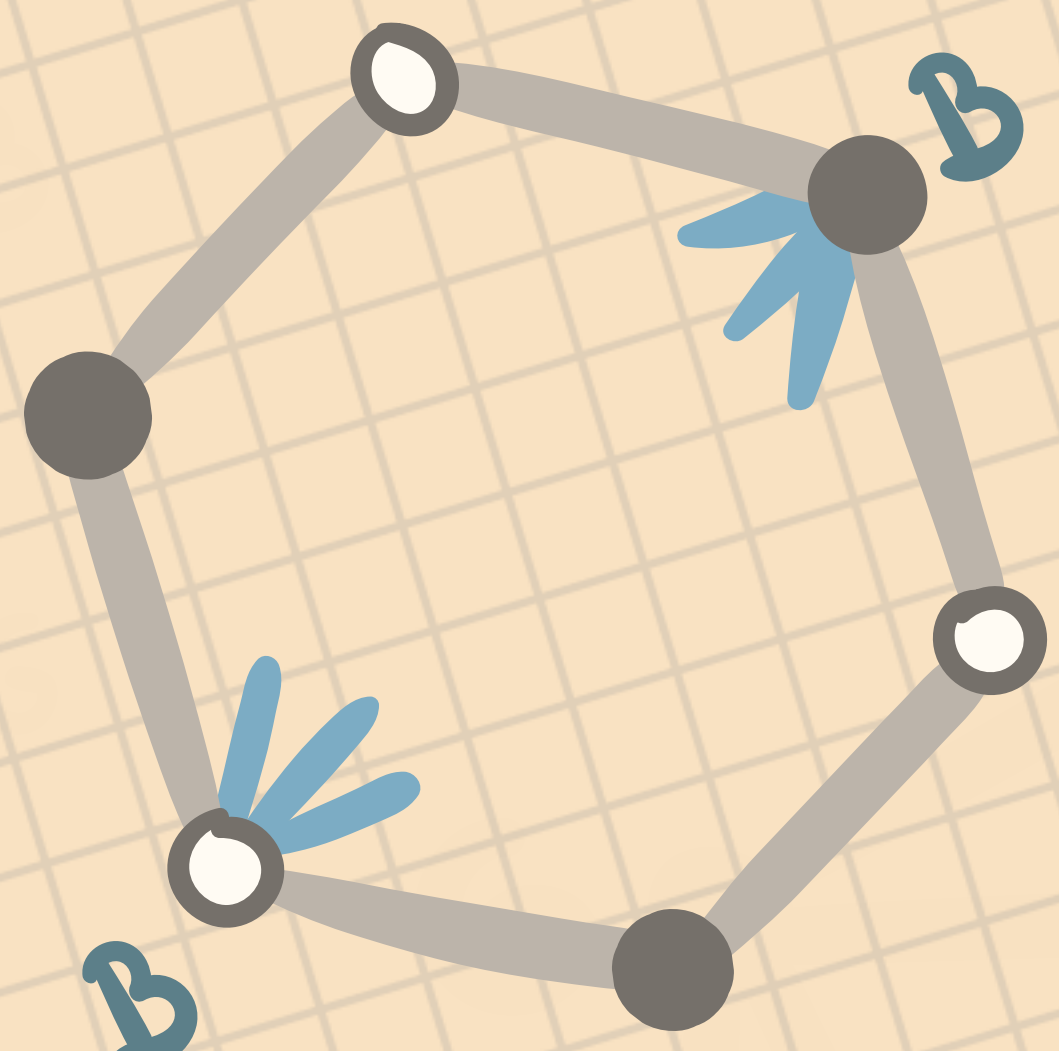


Schnyder labellings

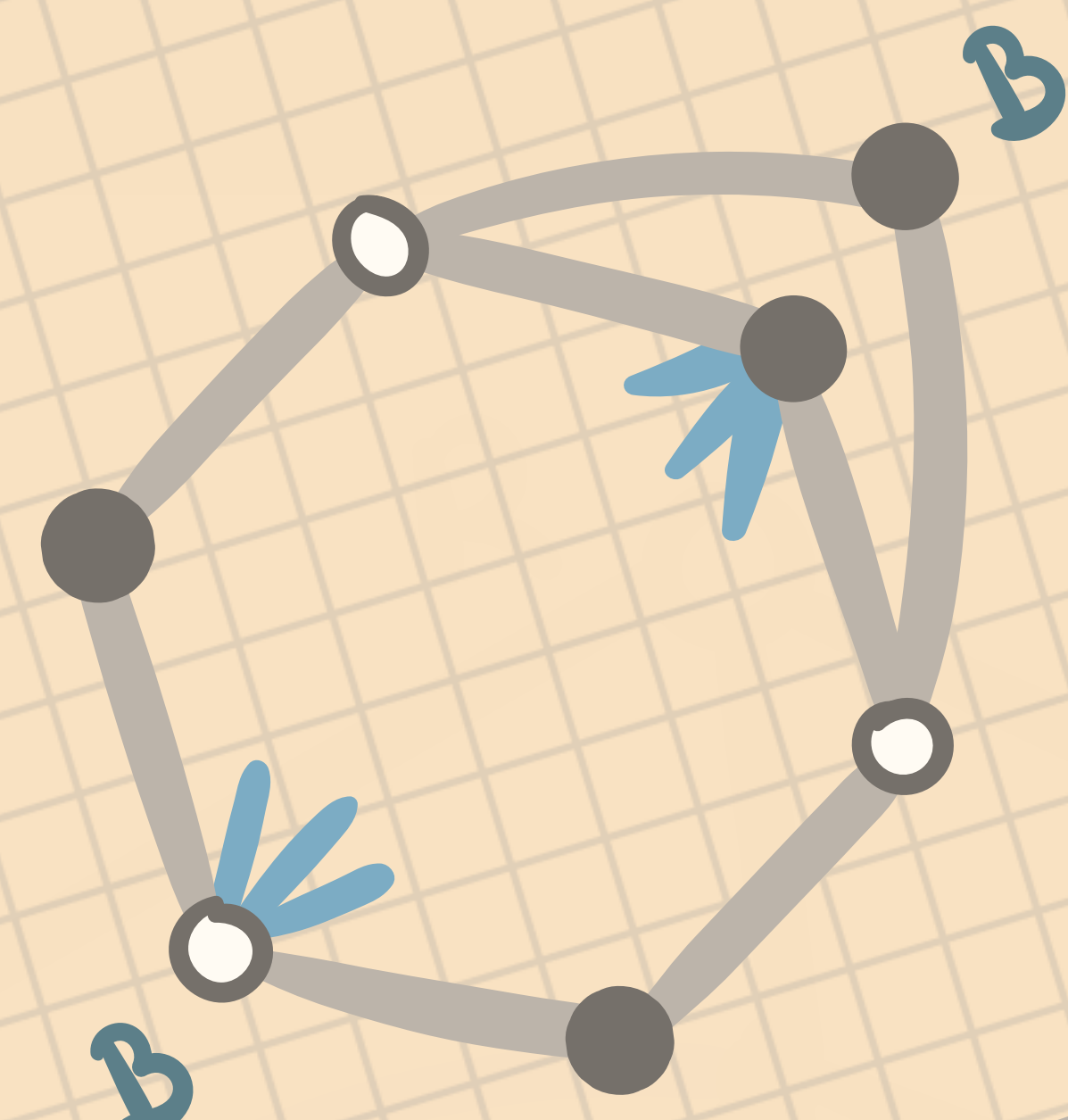
ALL SCHNYDER LABELLINGS



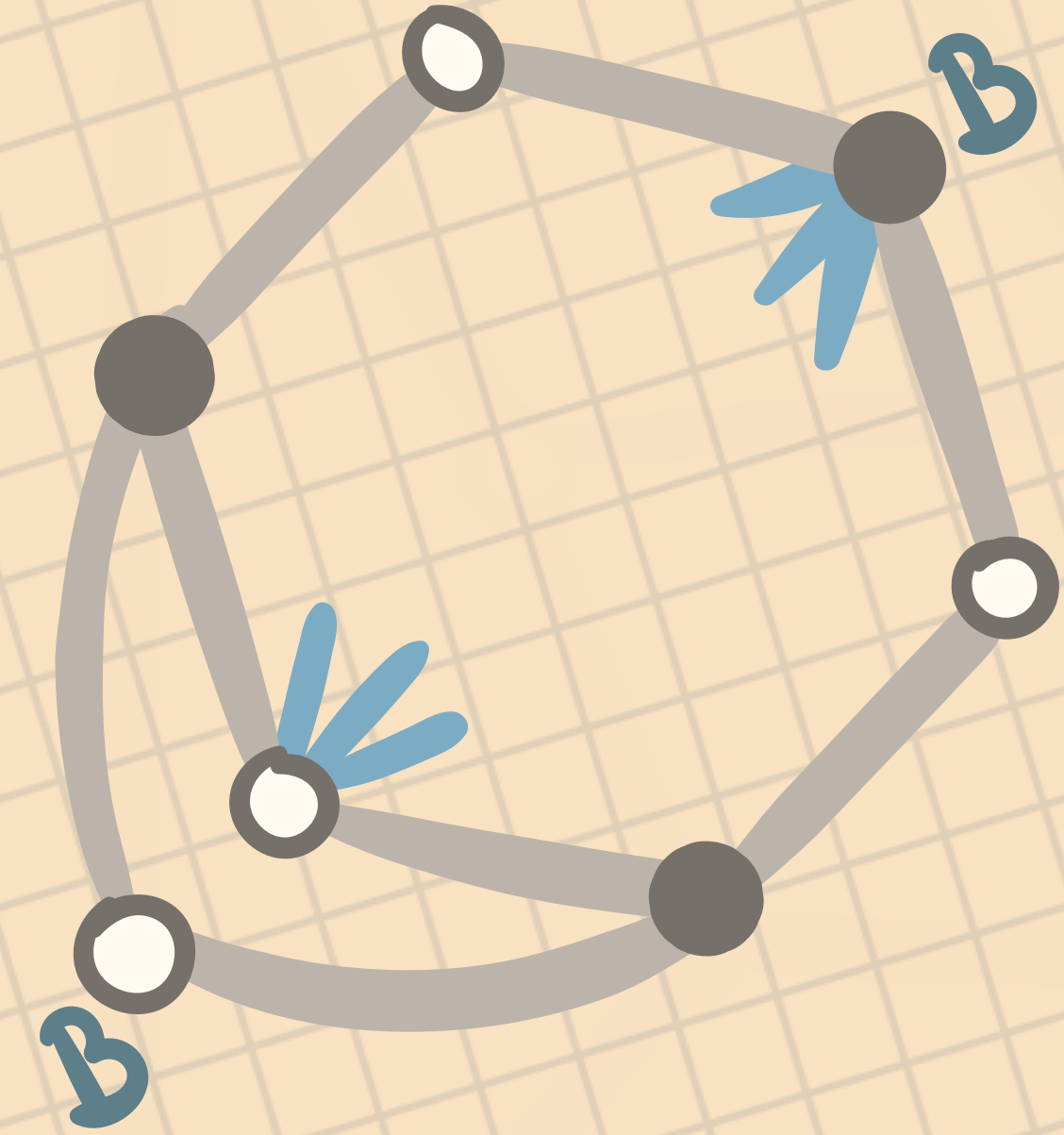
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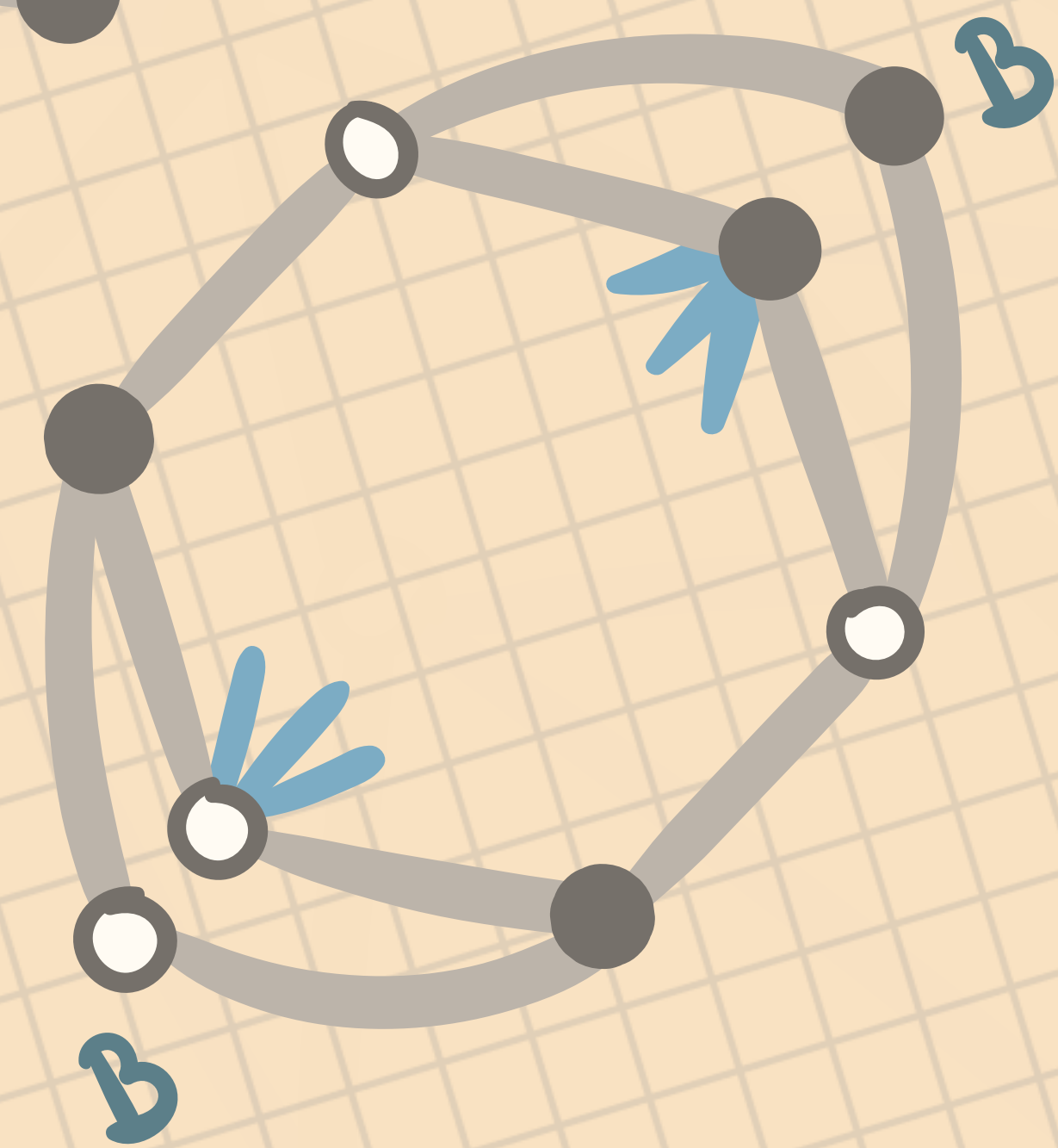
+



+



+



G

B

R

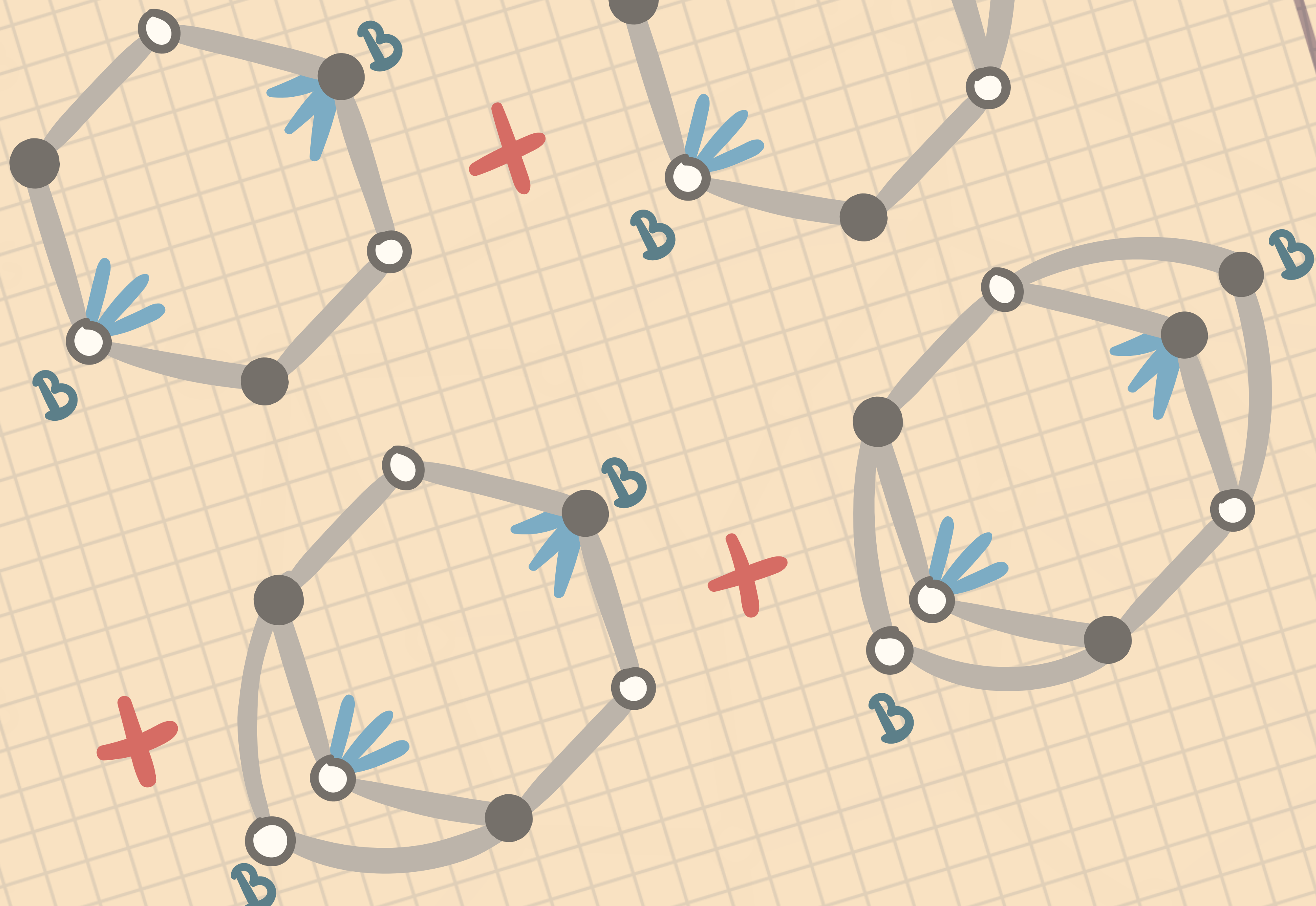
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Schnyder labellings

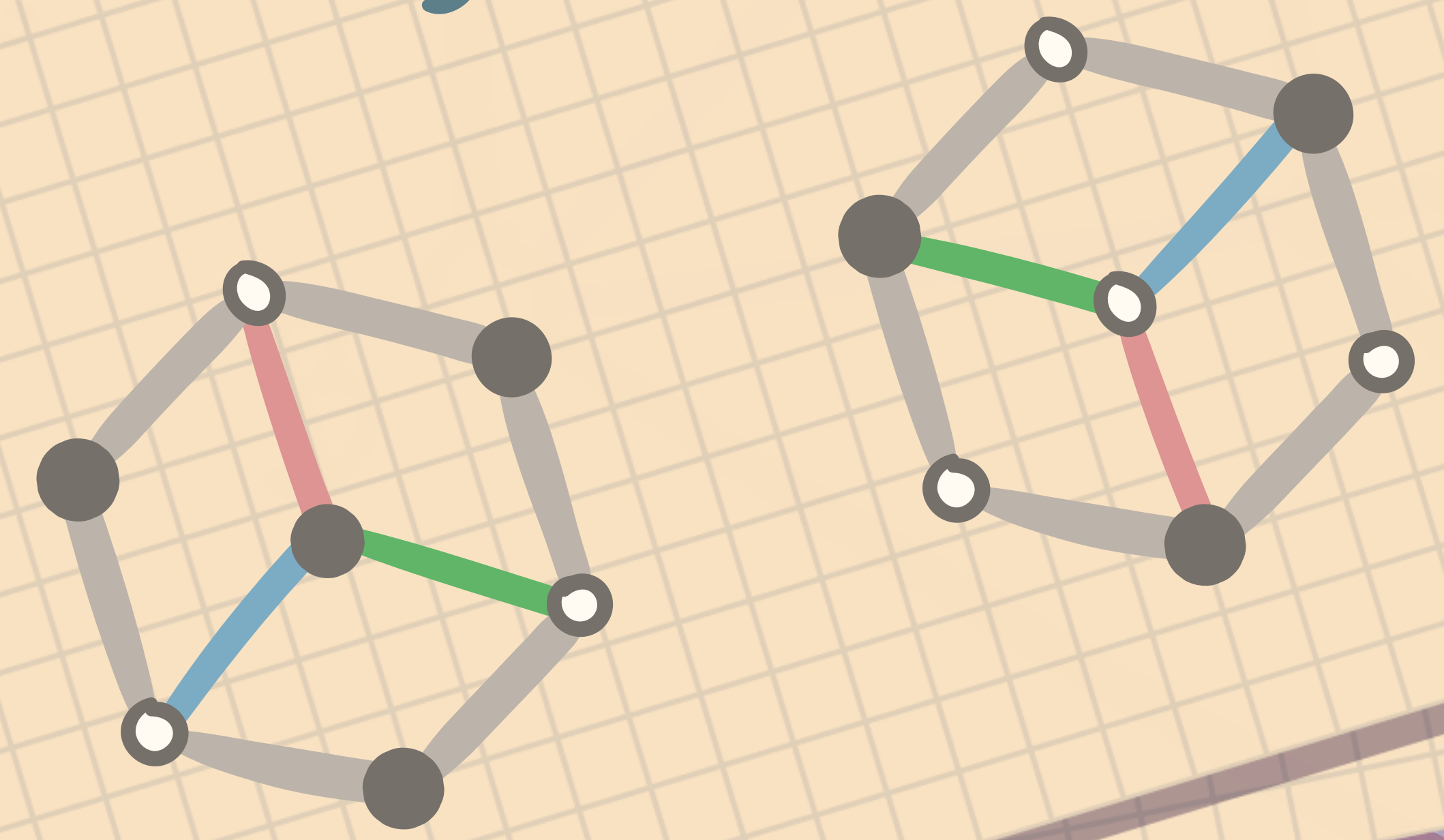
ALL SCHNYDER LABELLINGS

=

≥ 1



EXEPT:



G

R

B

B

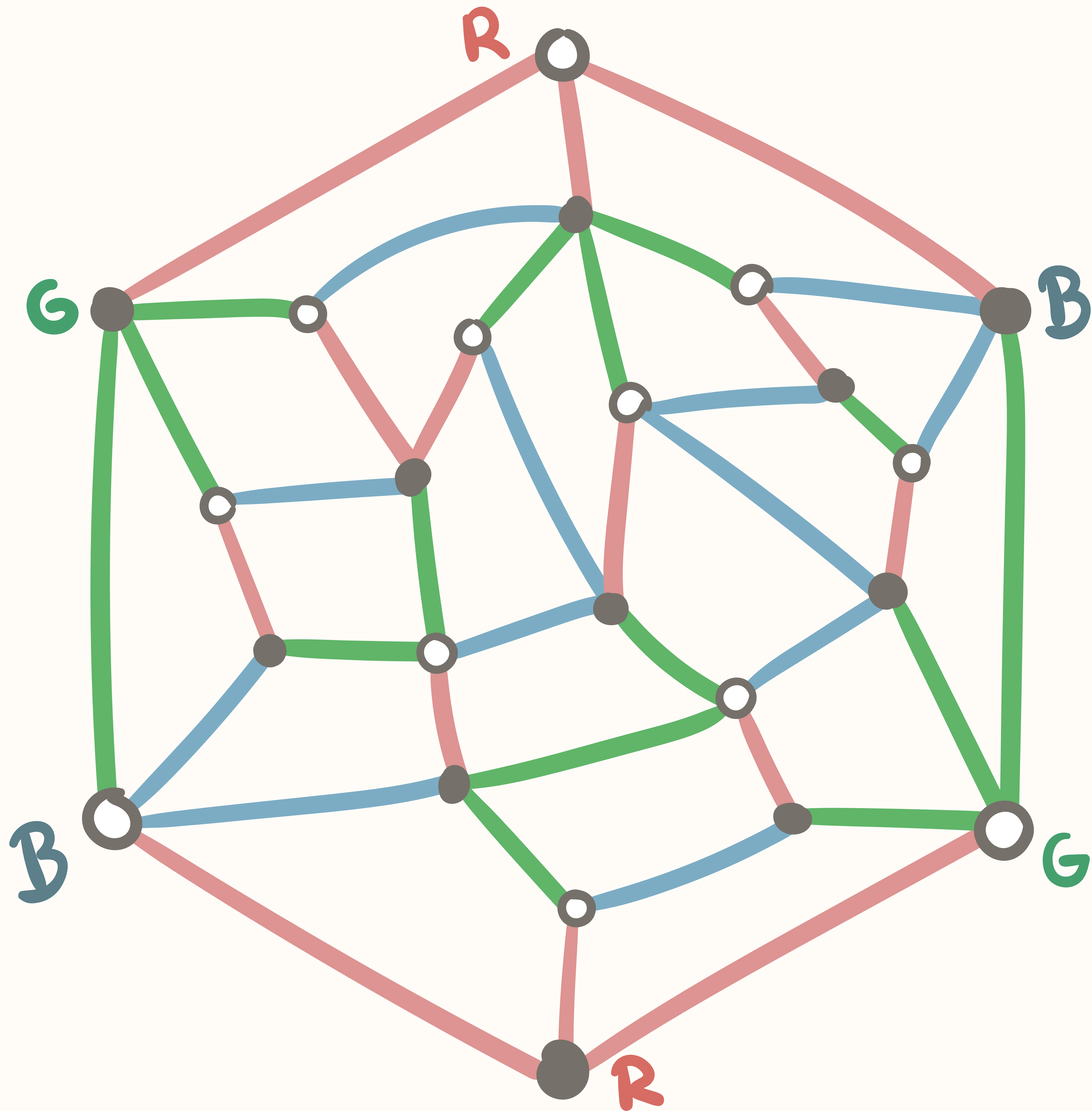
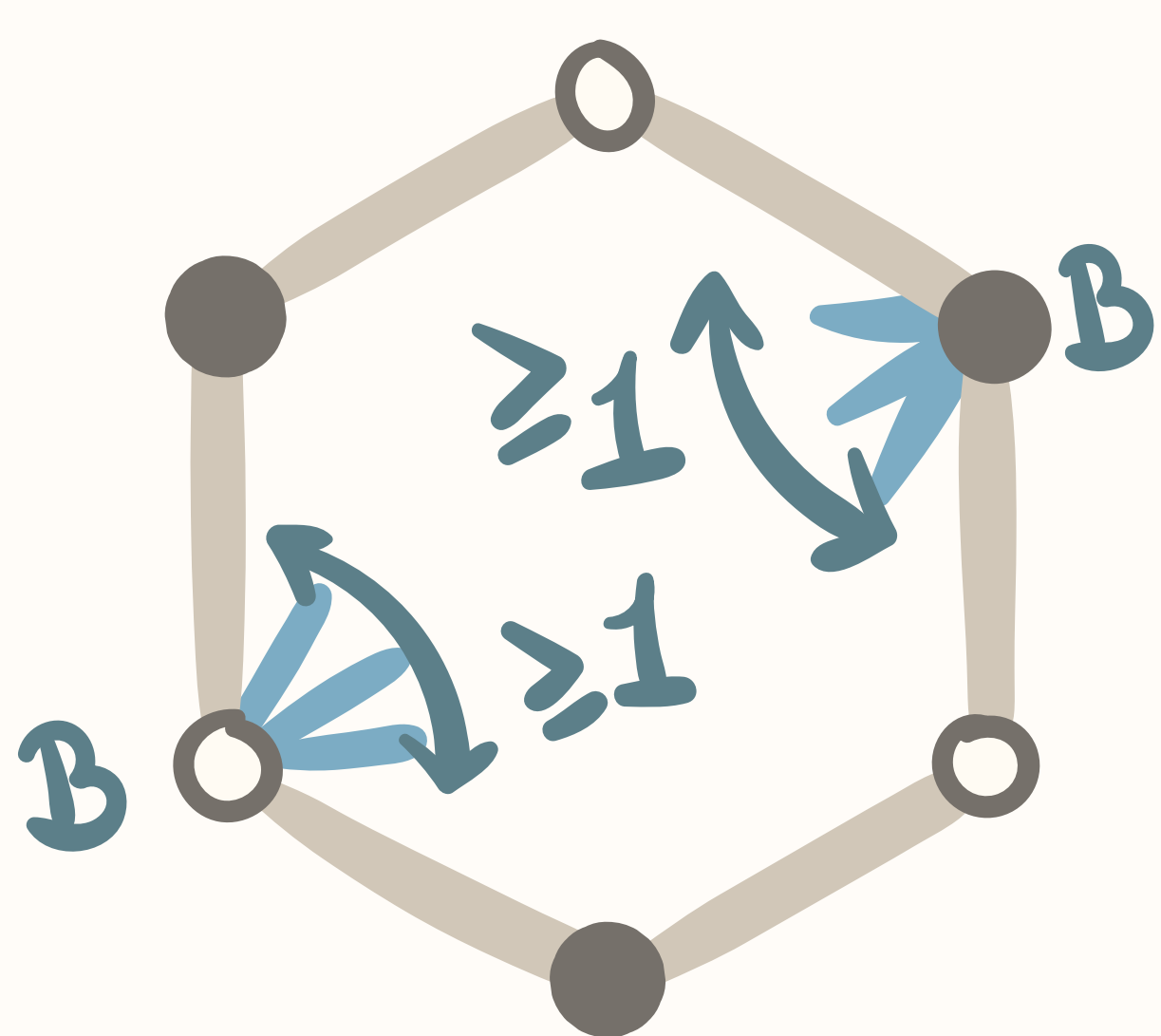
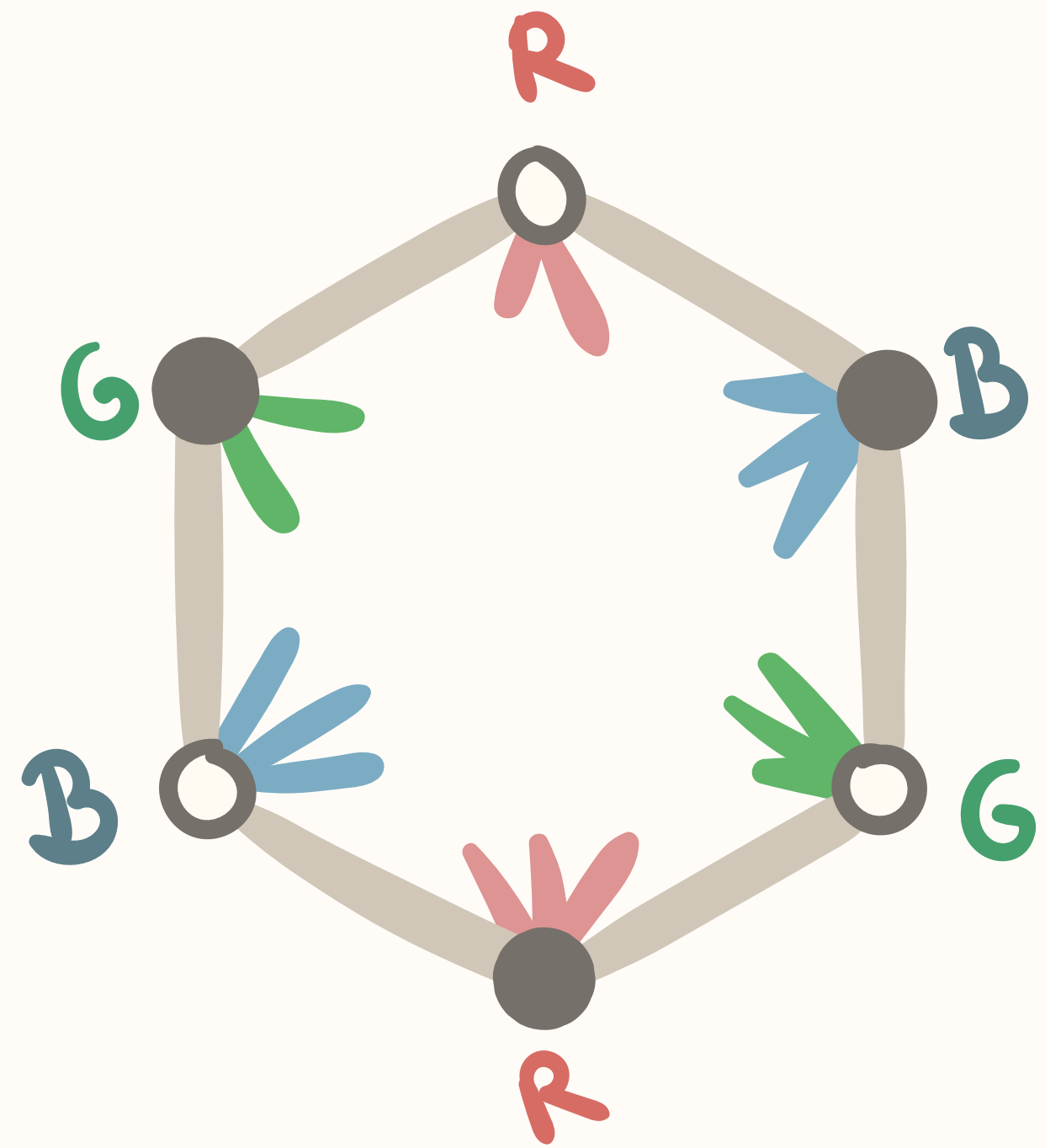
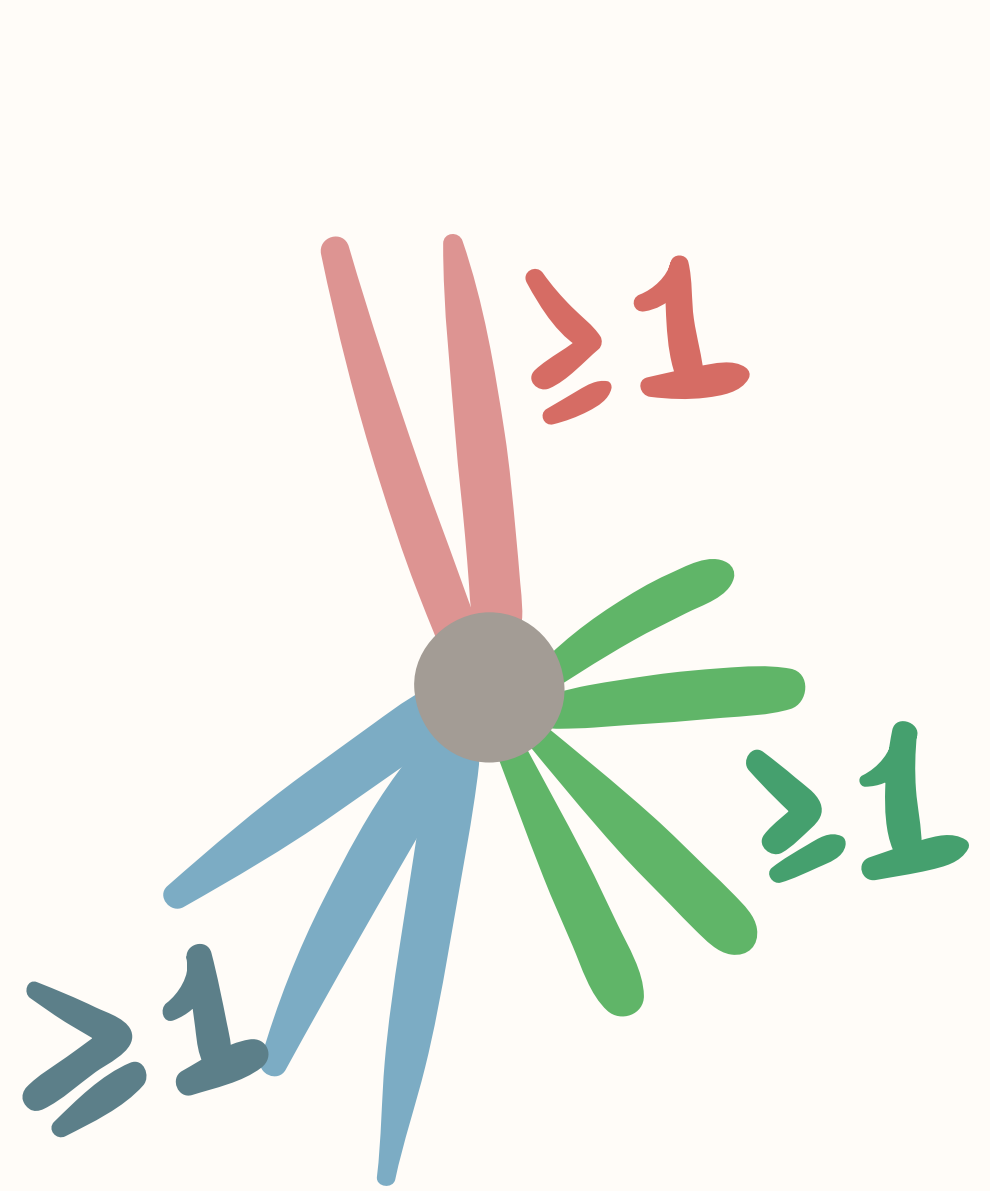
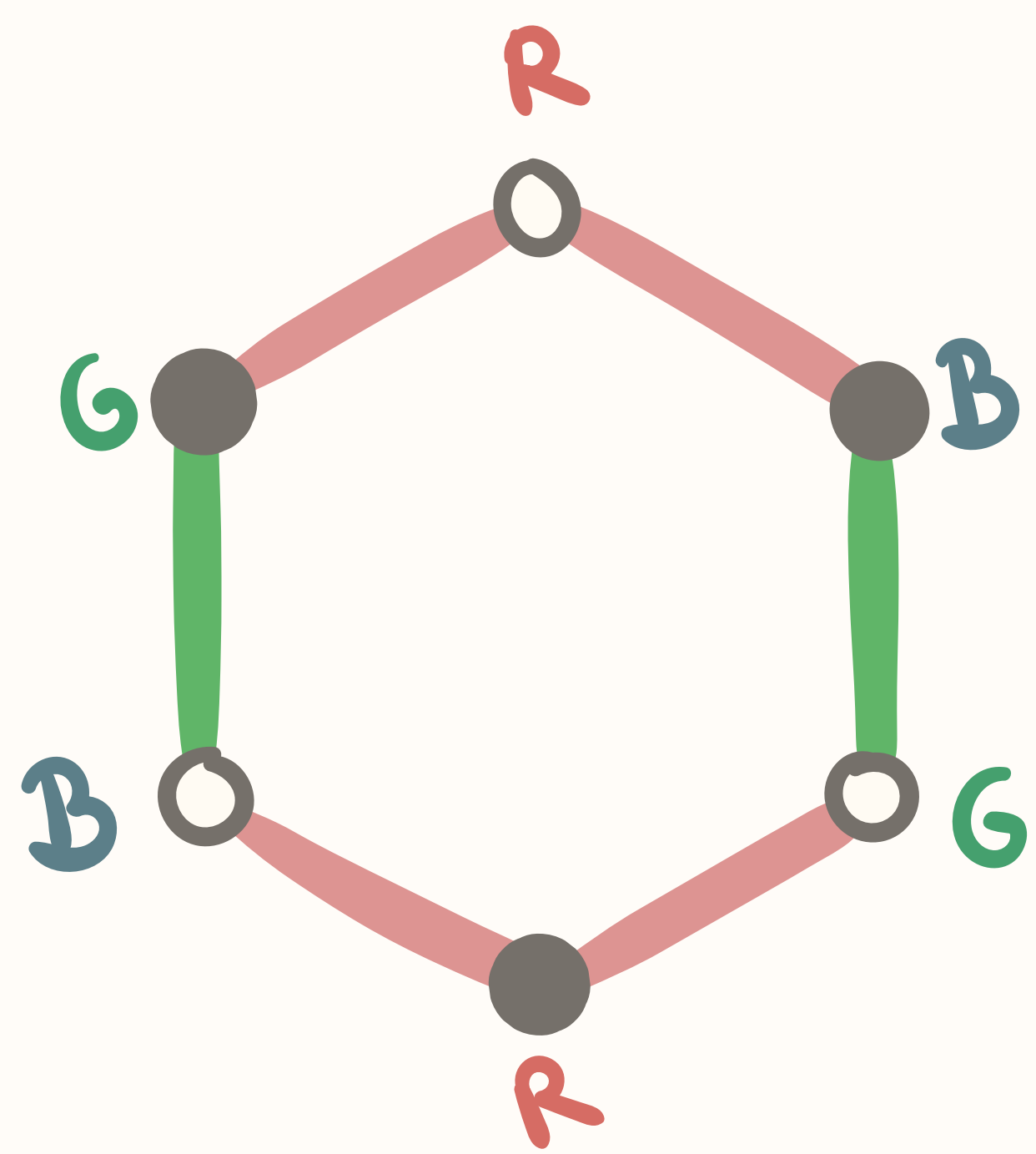
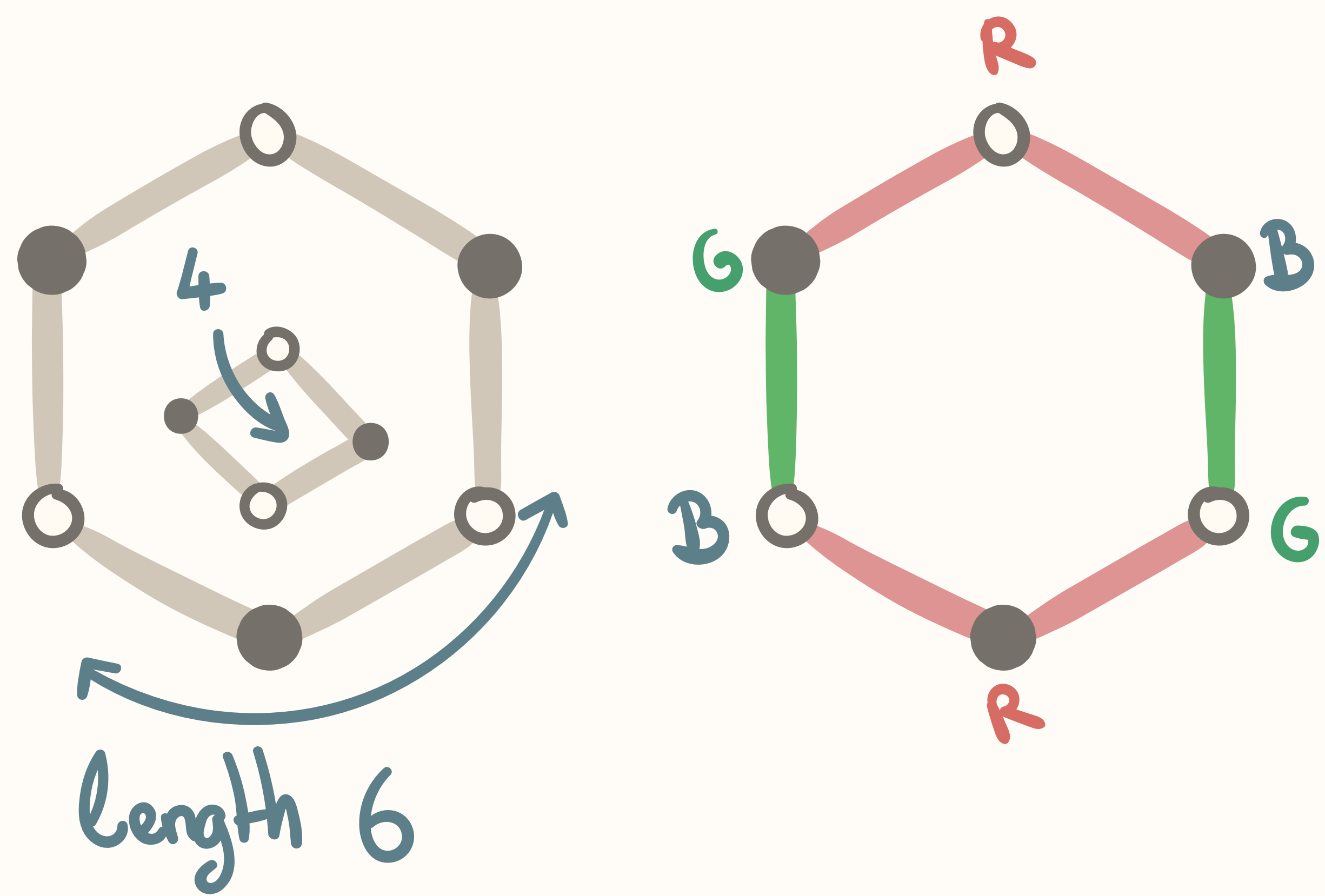
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B

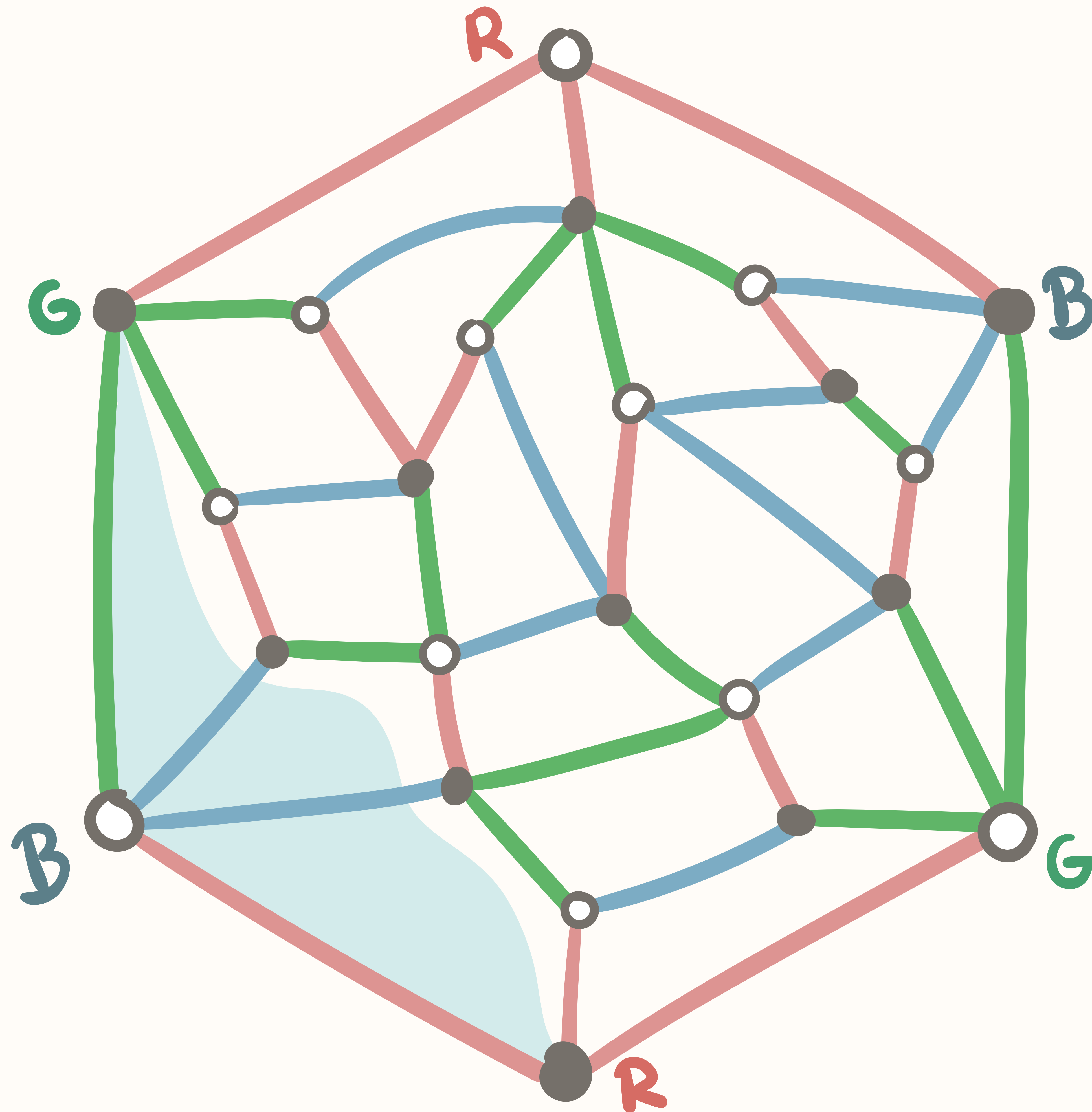
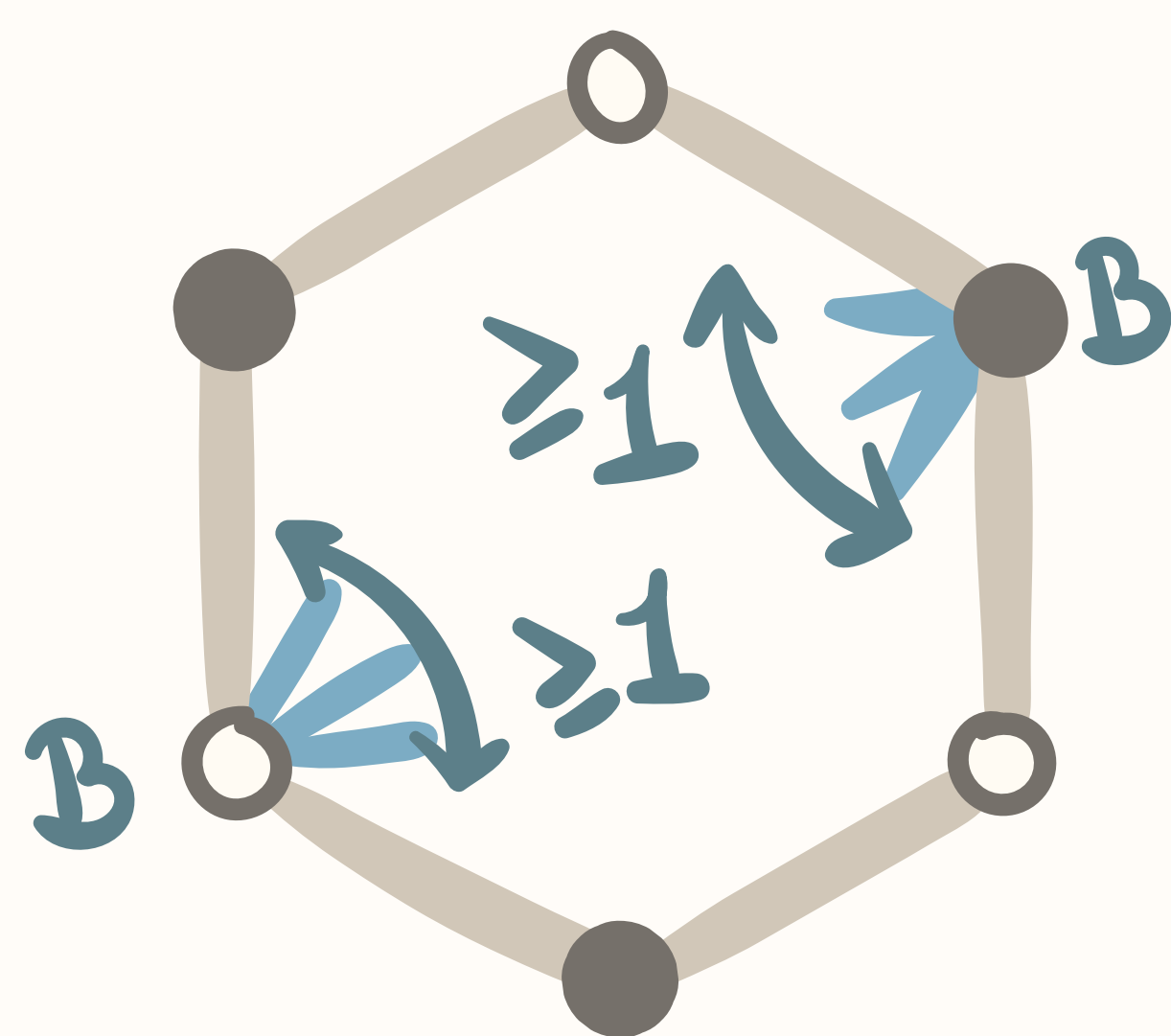
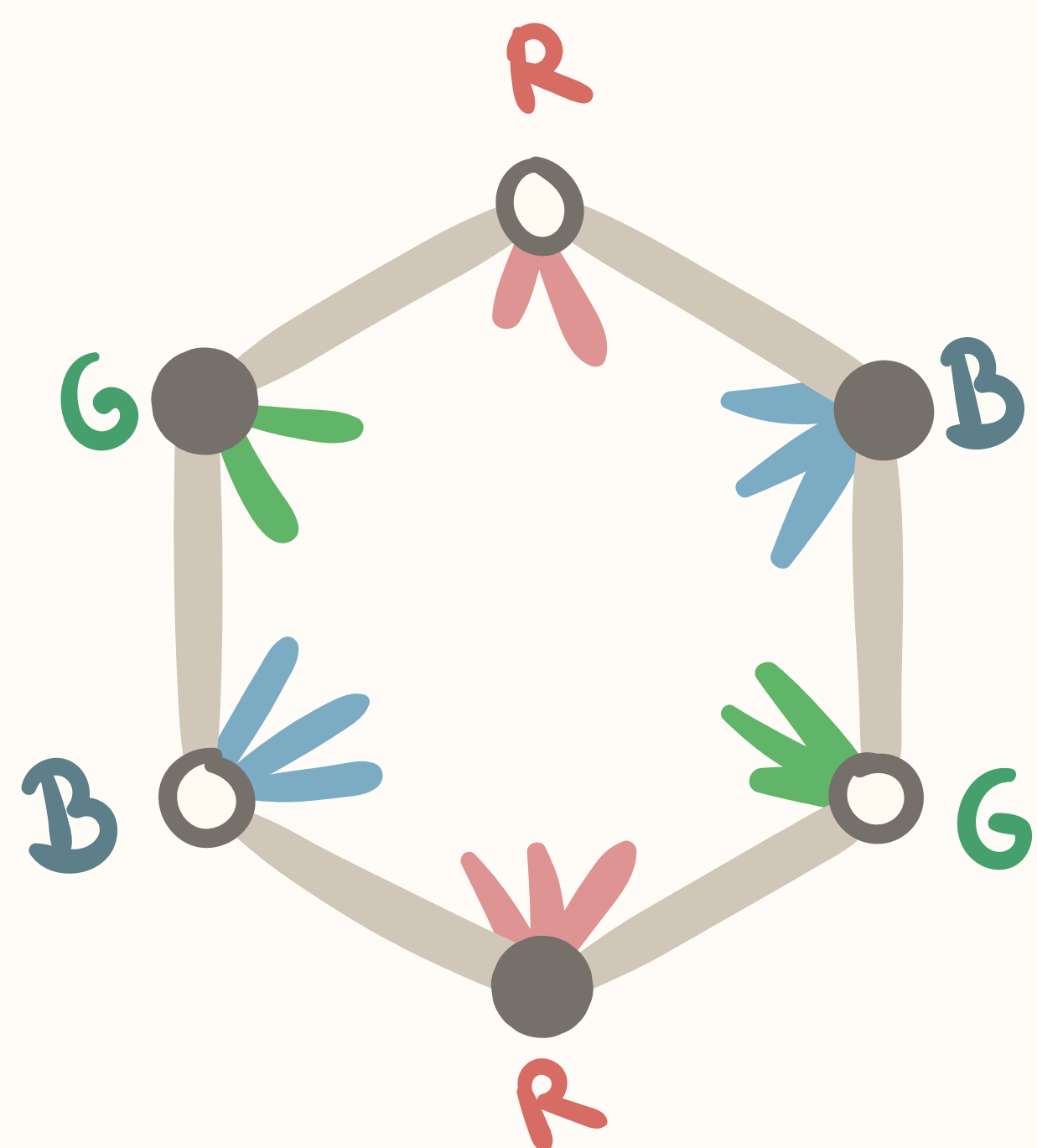
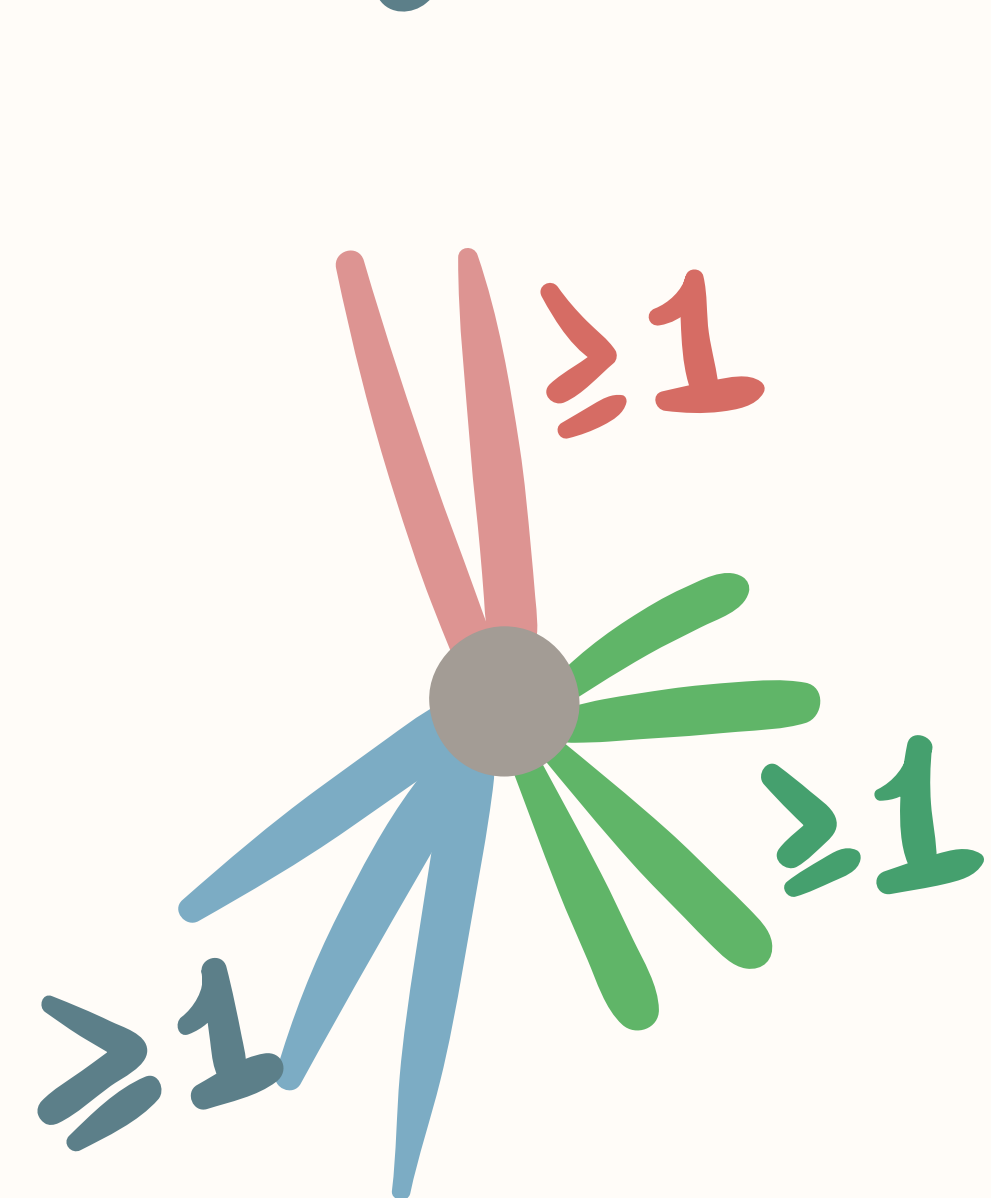
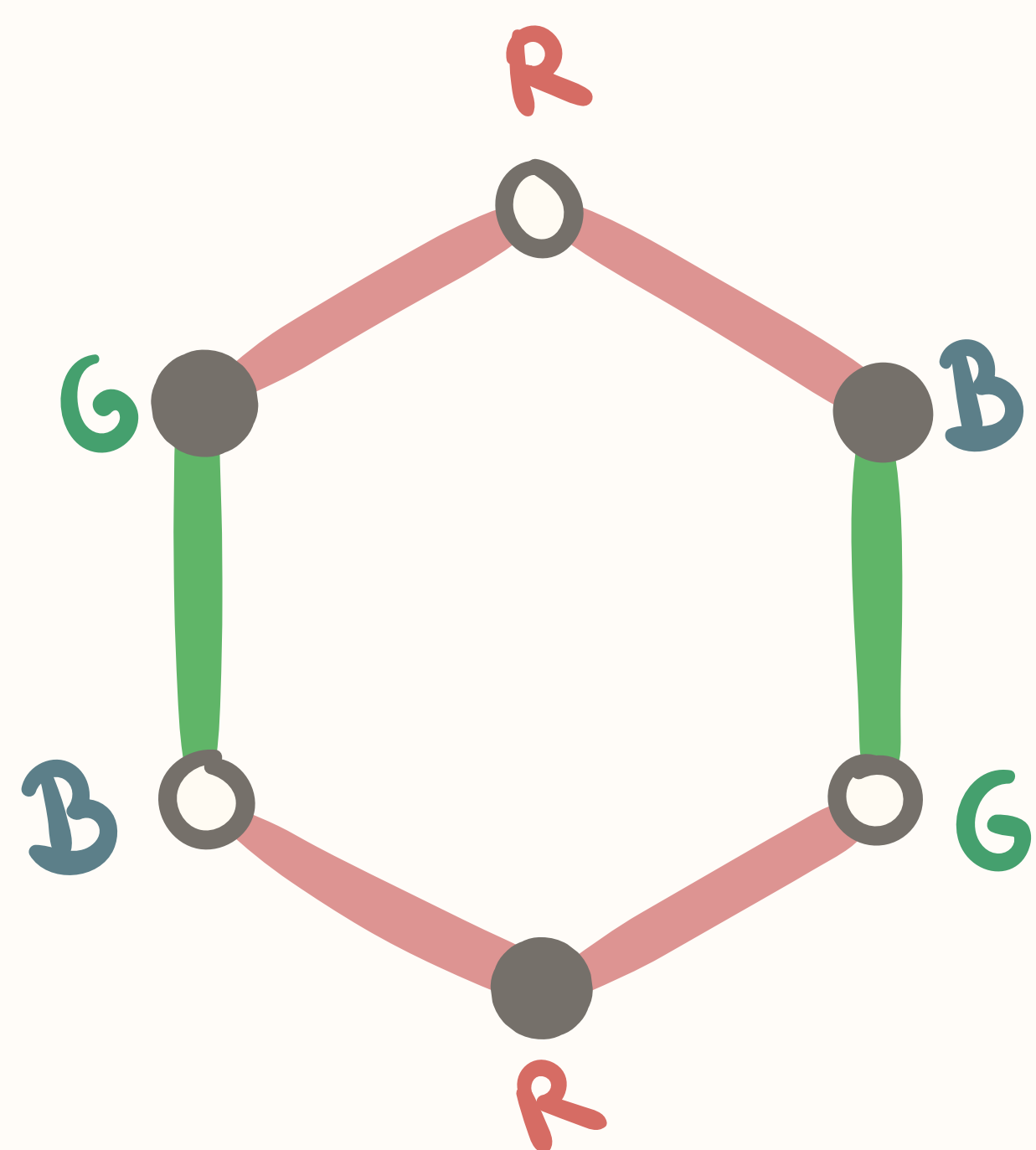
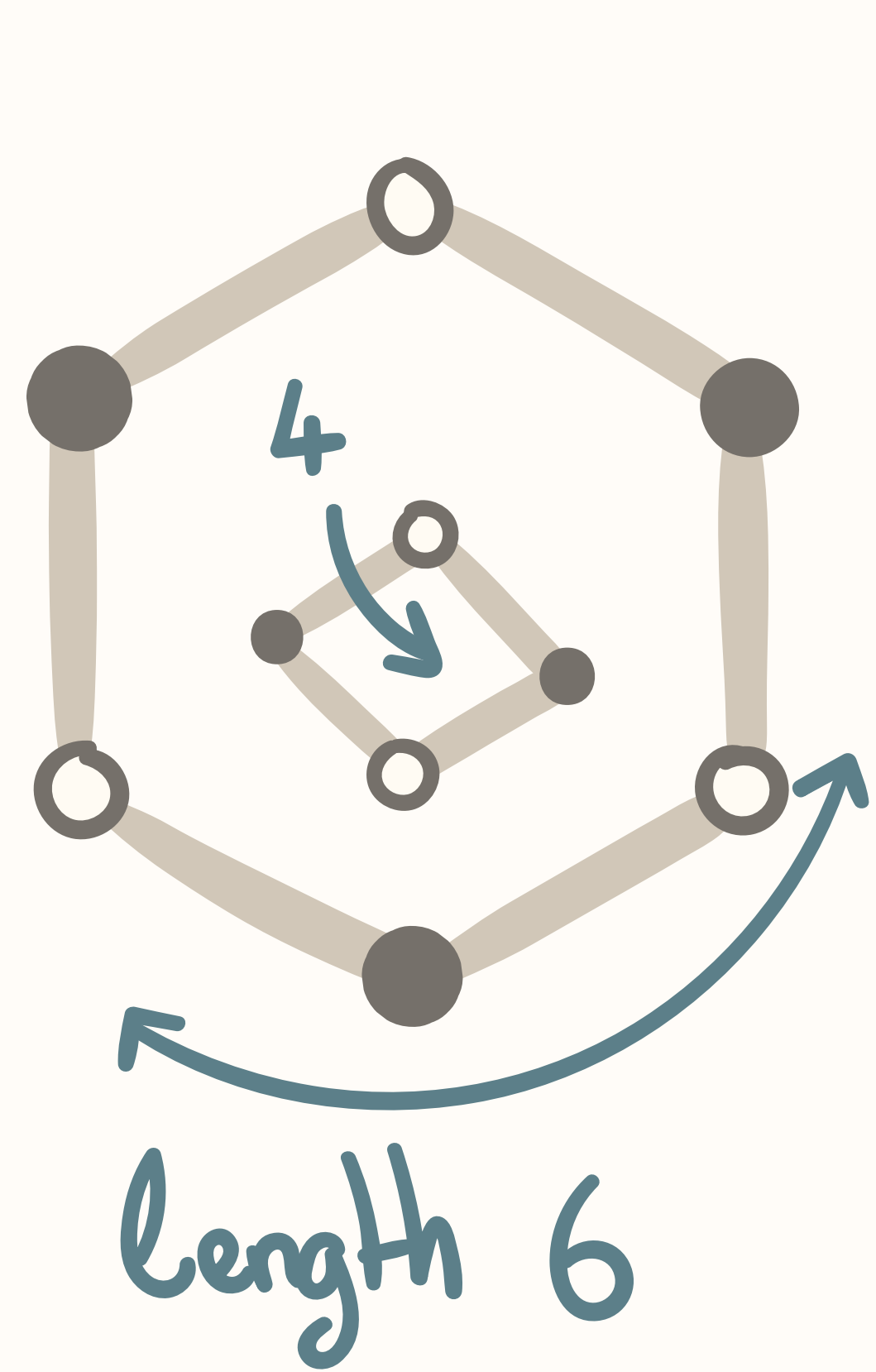
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B

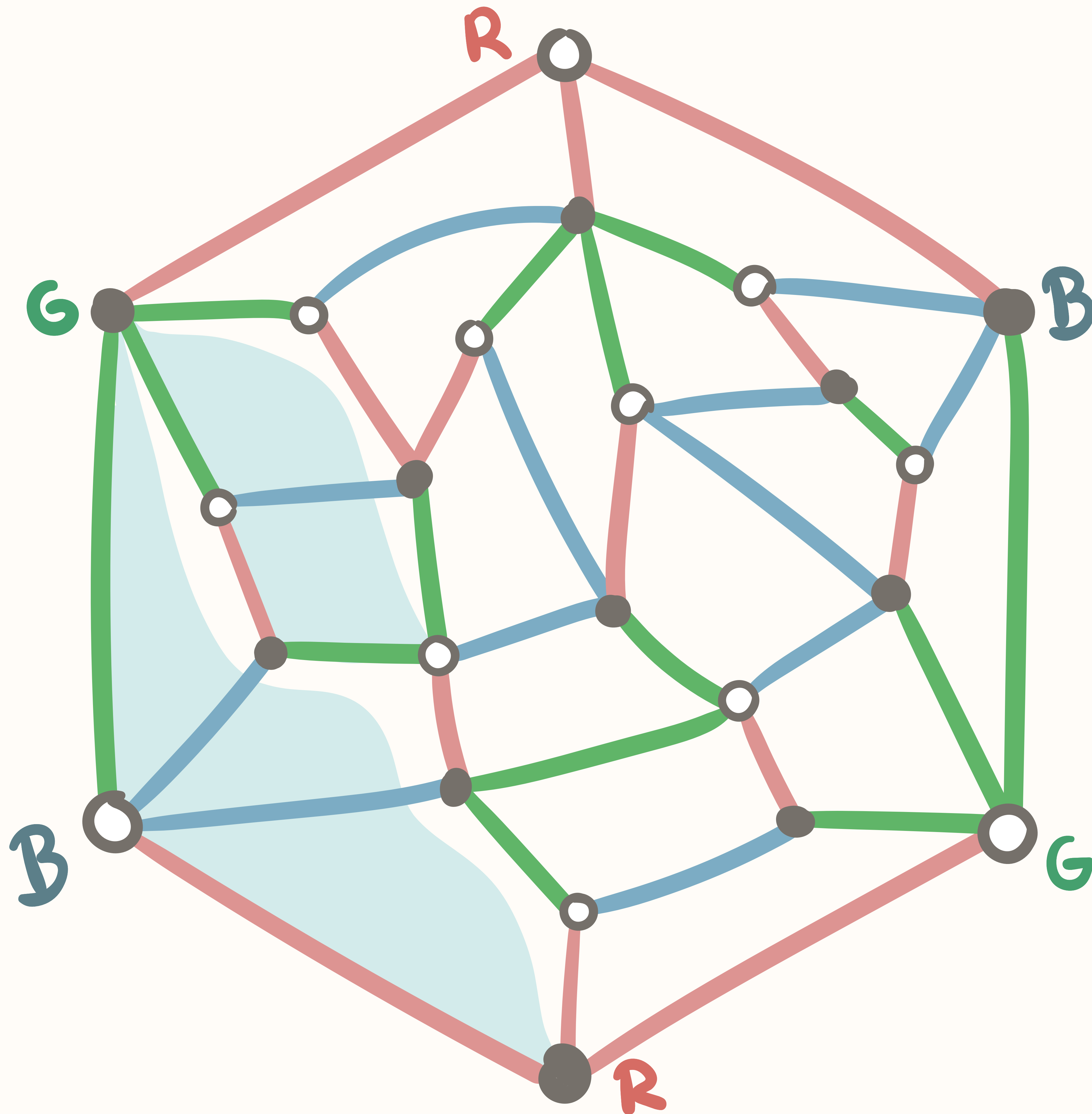
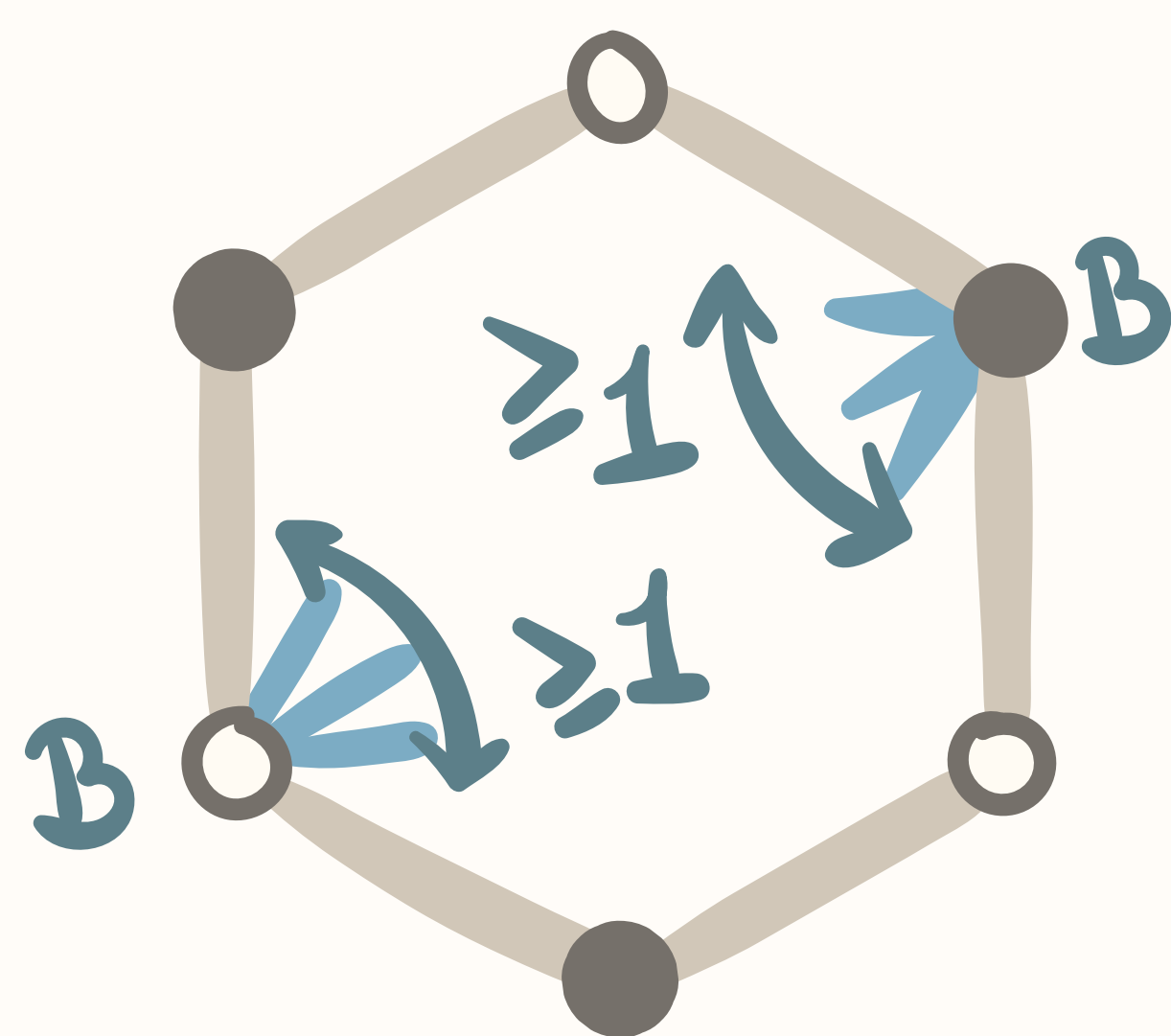
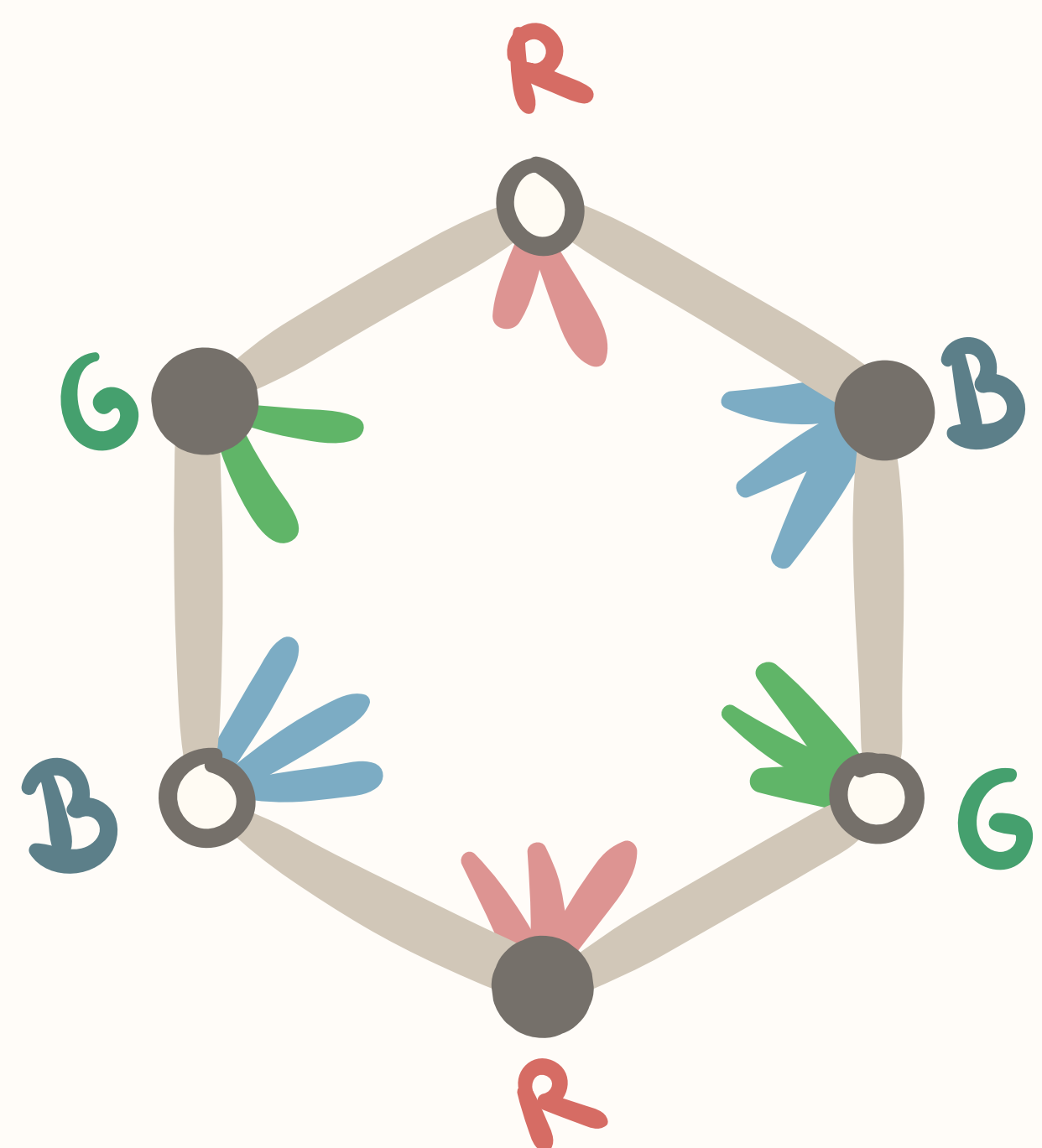
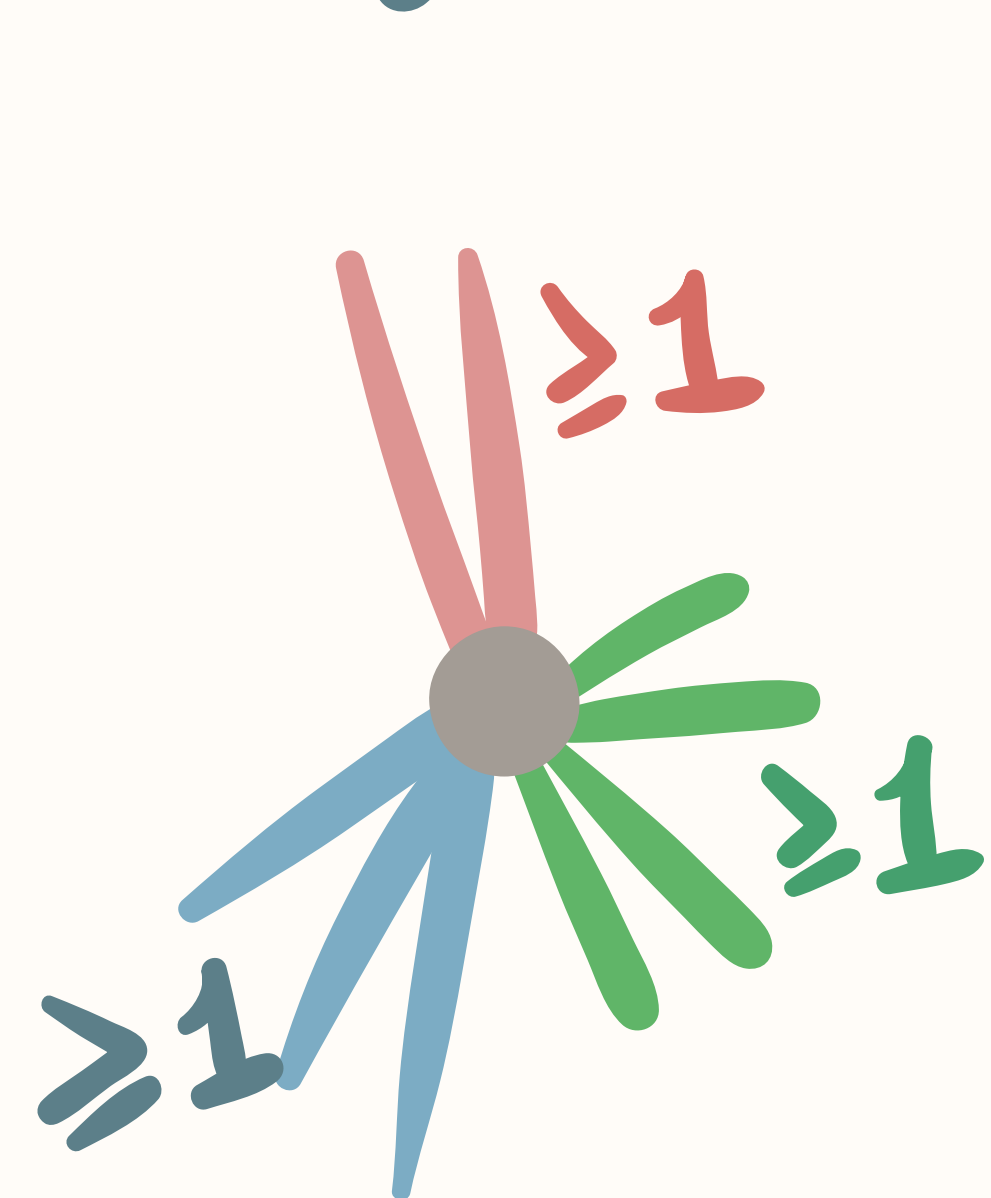
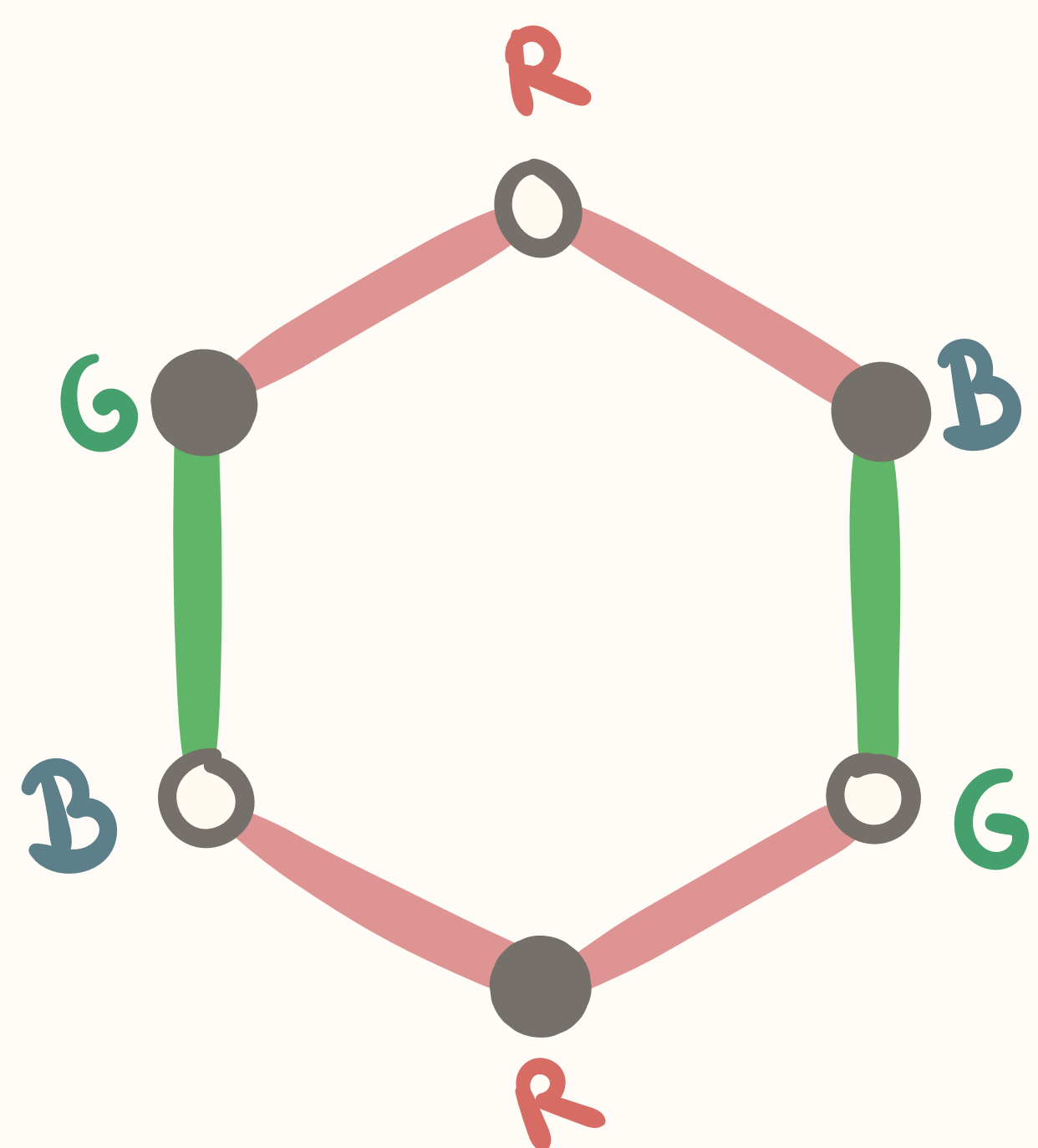
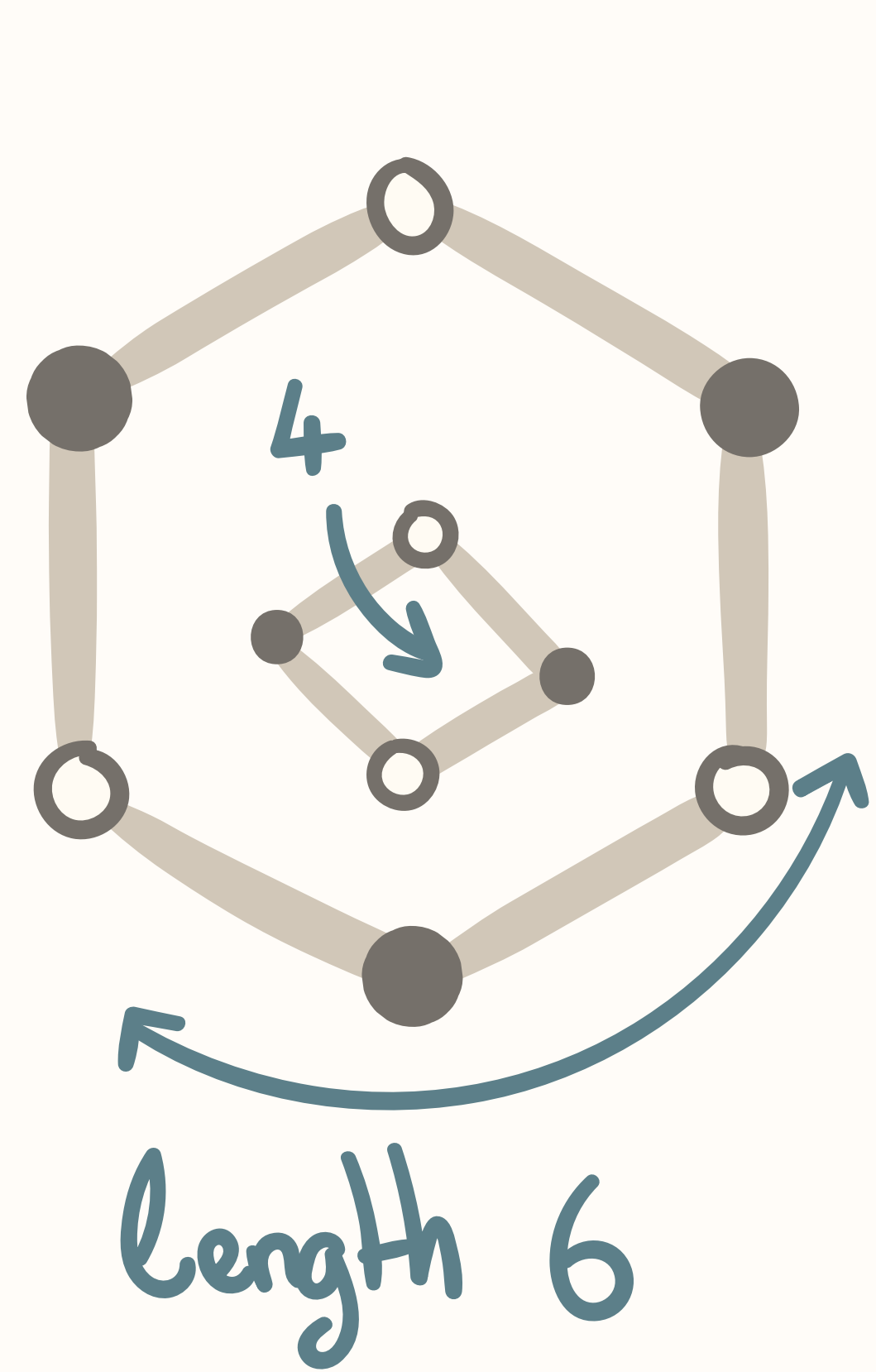
Schnyder labellings



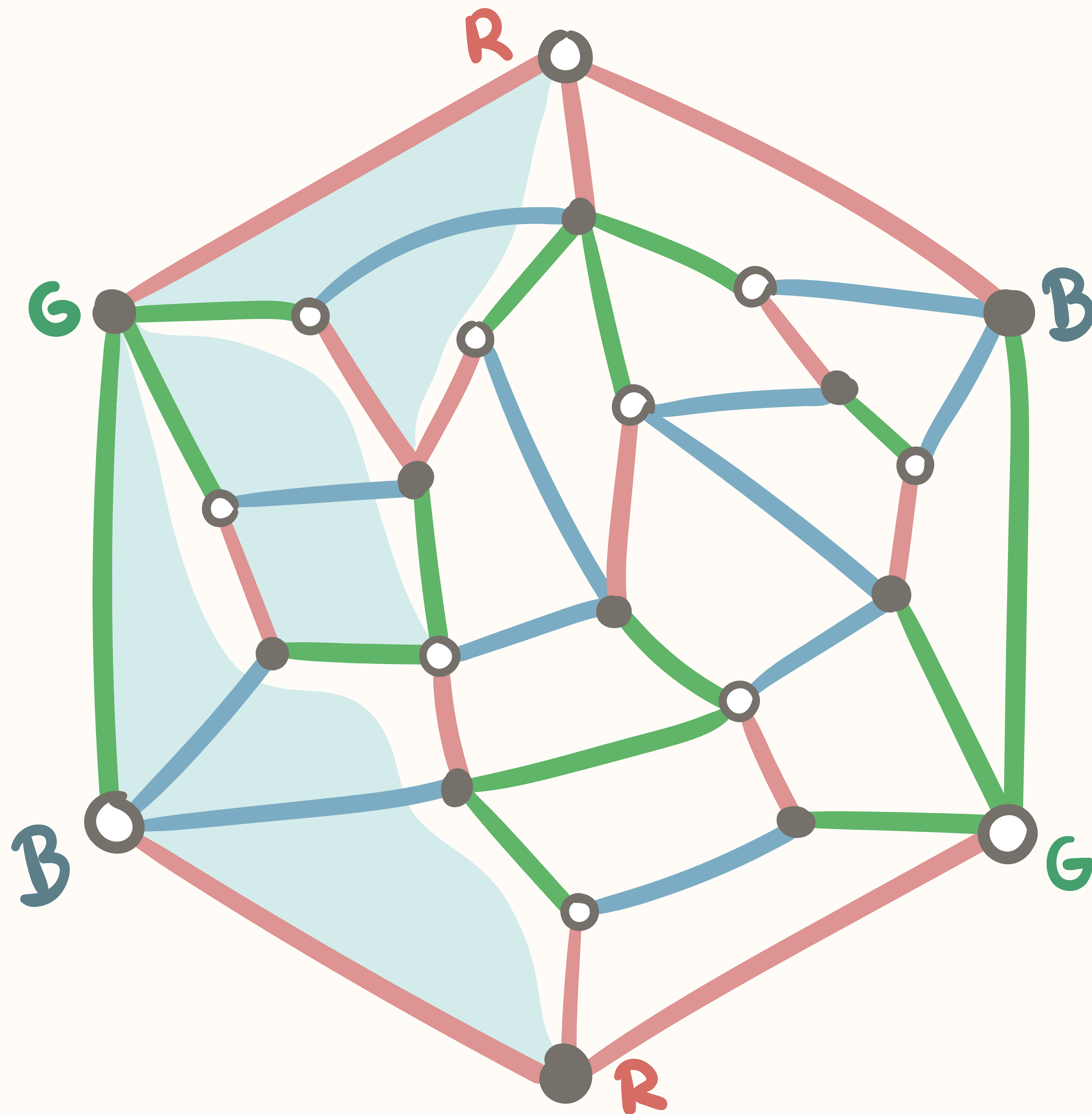
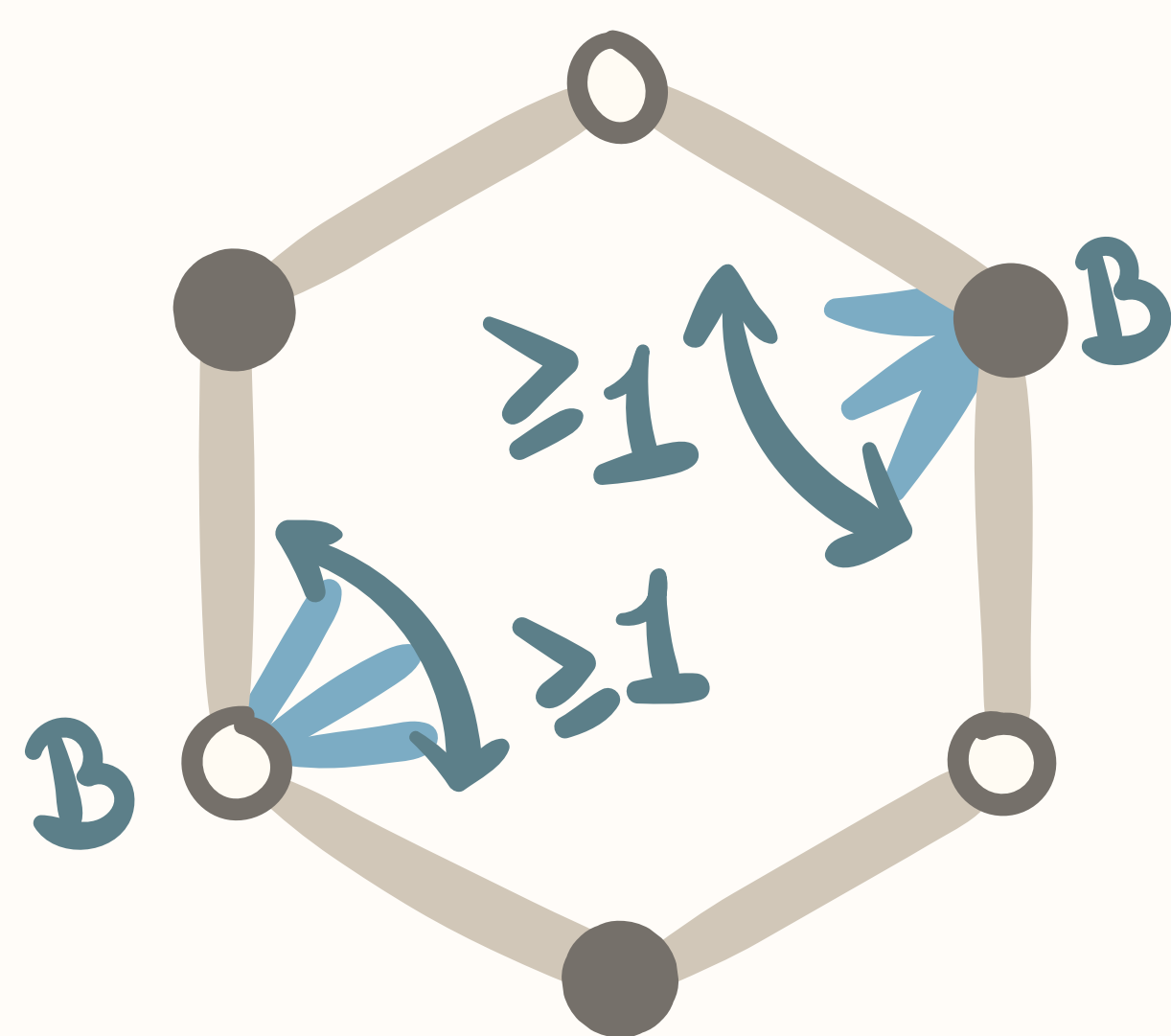
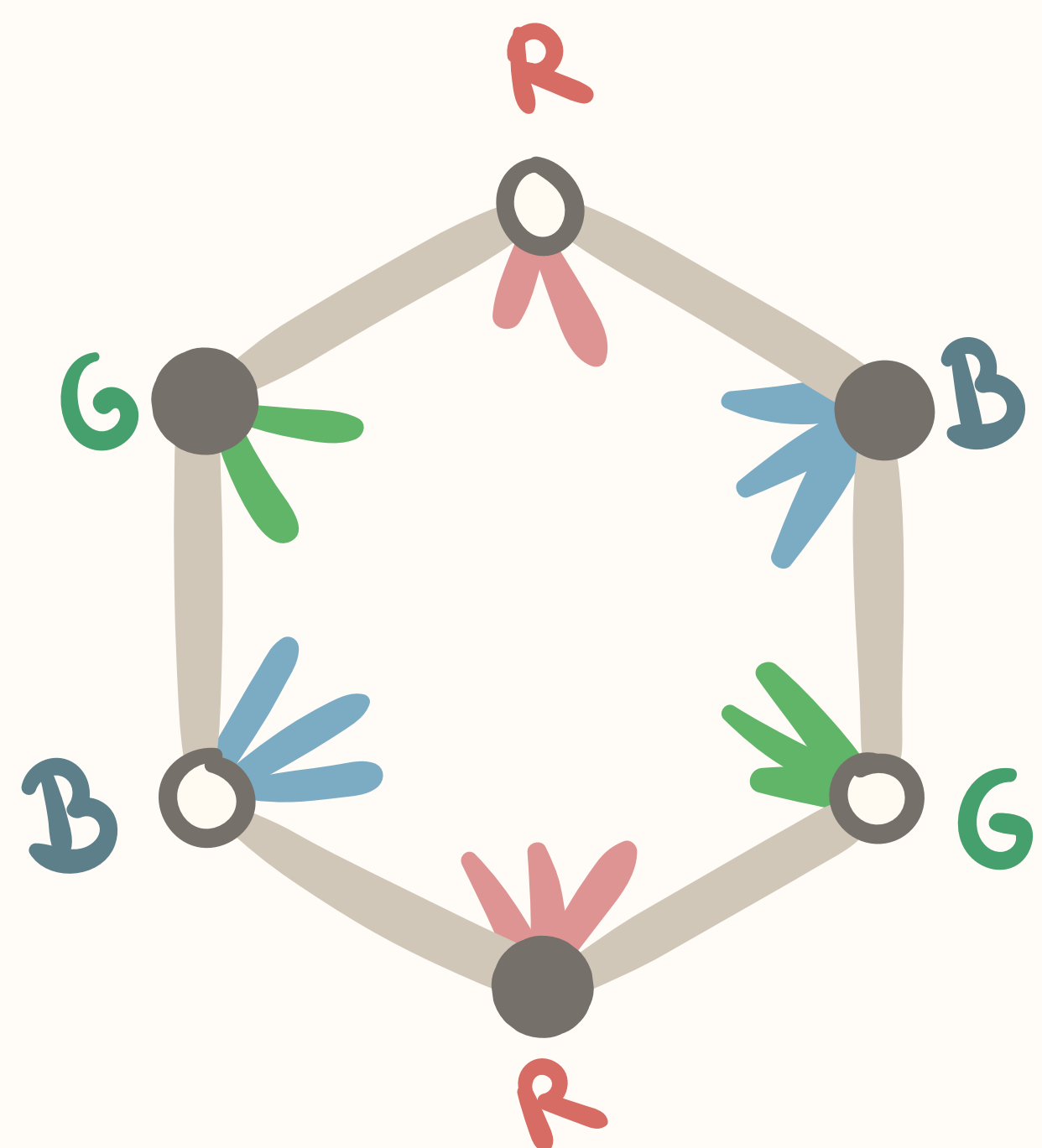
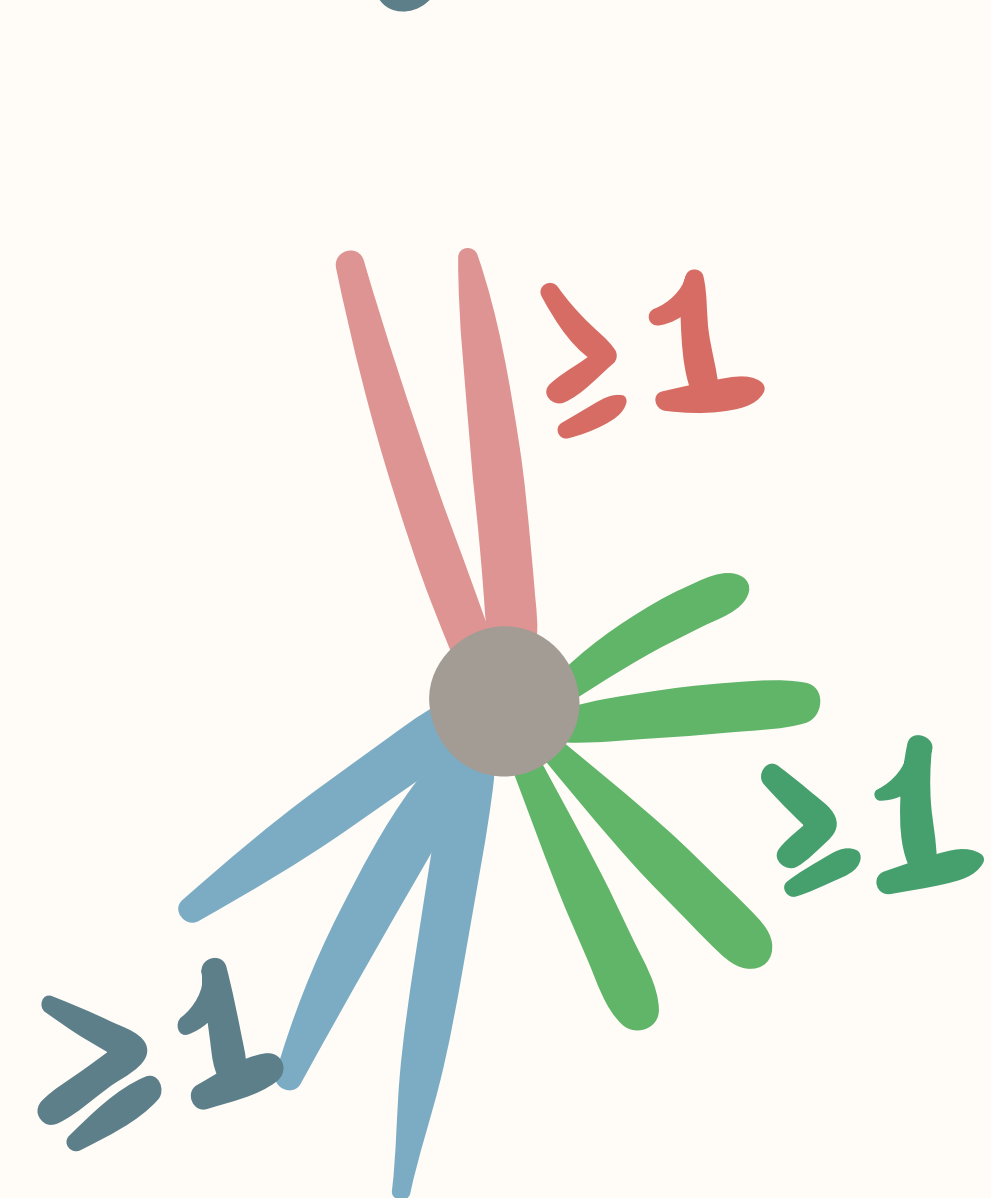
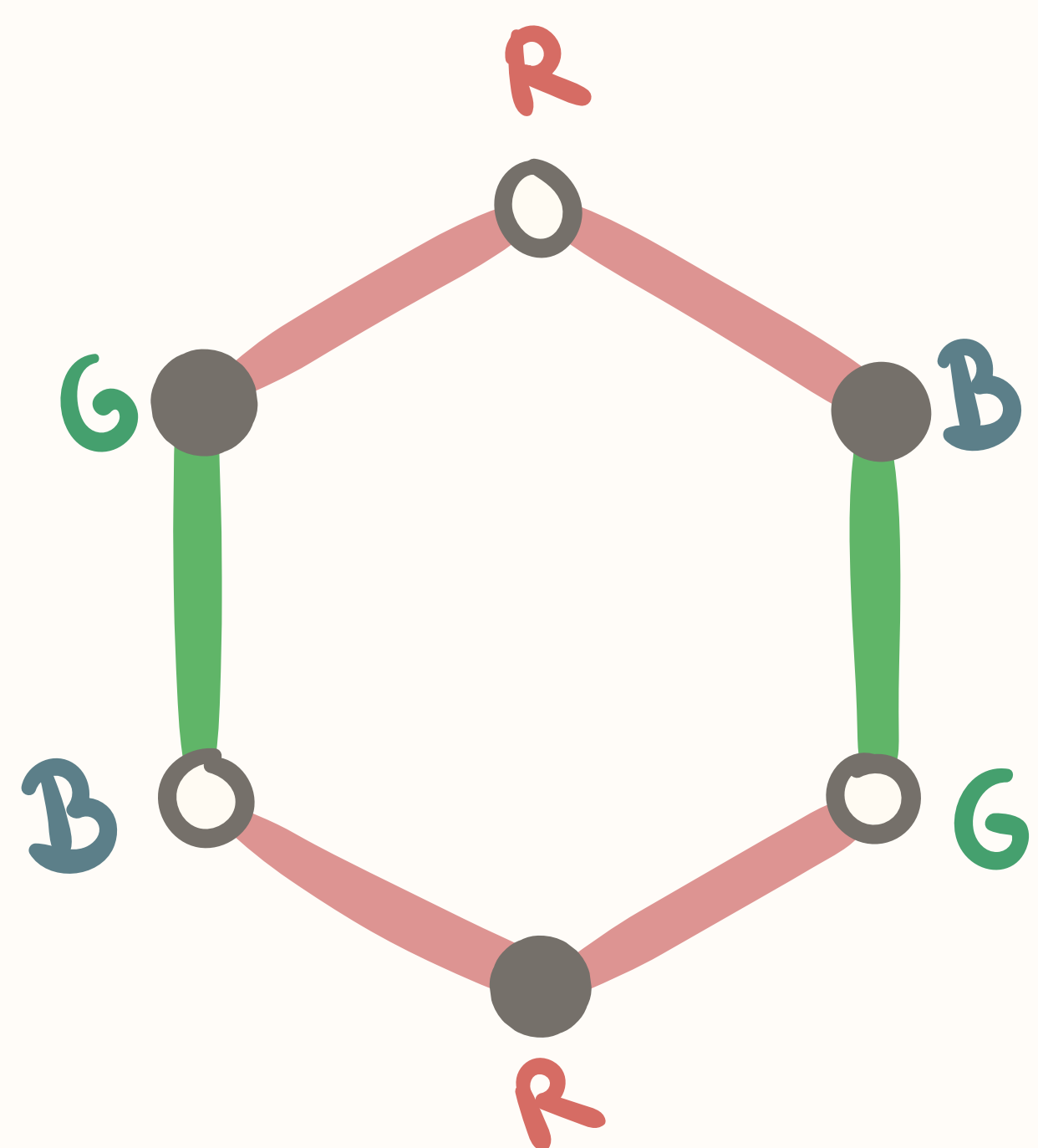
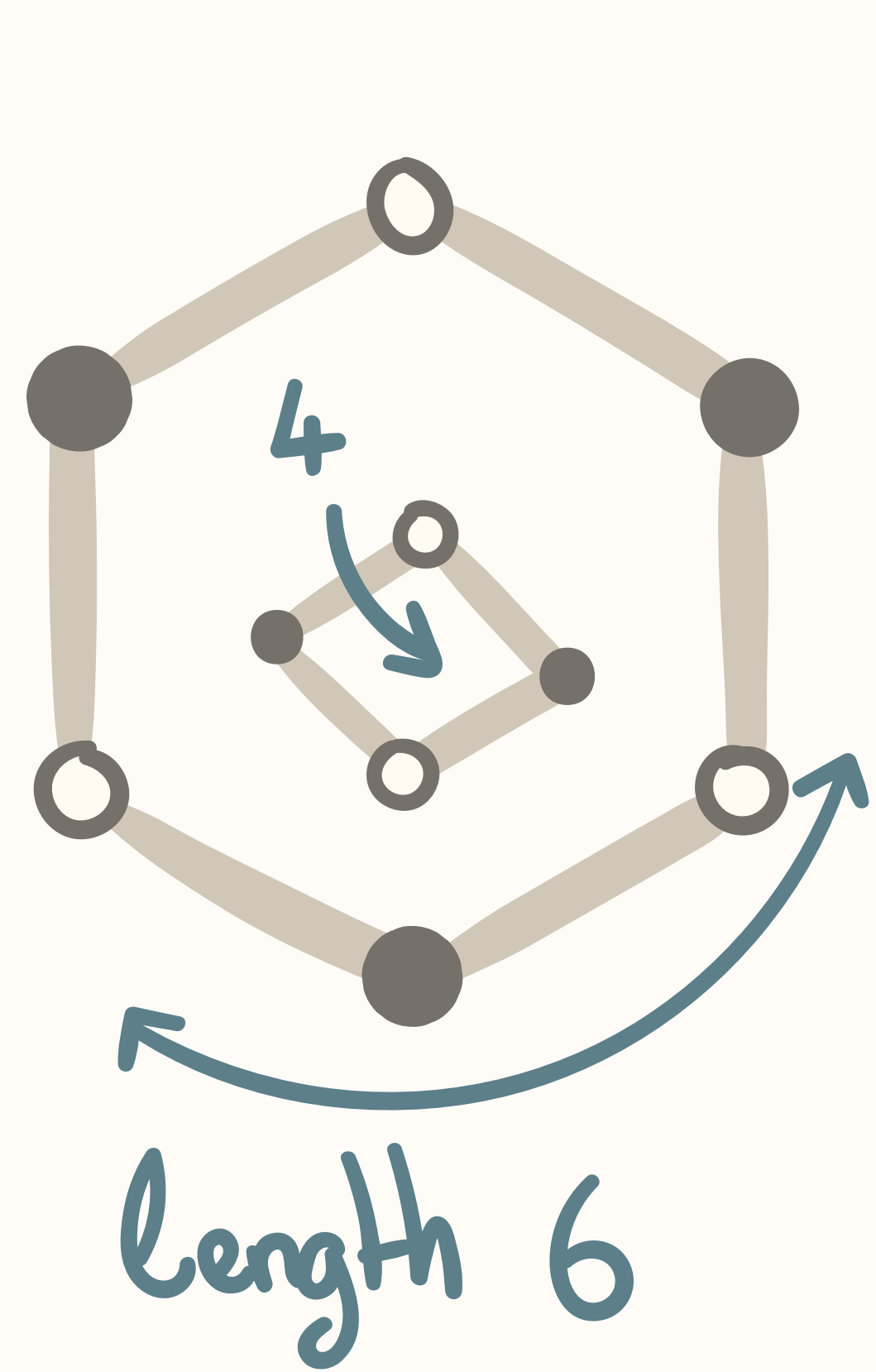
Schnyder labellings



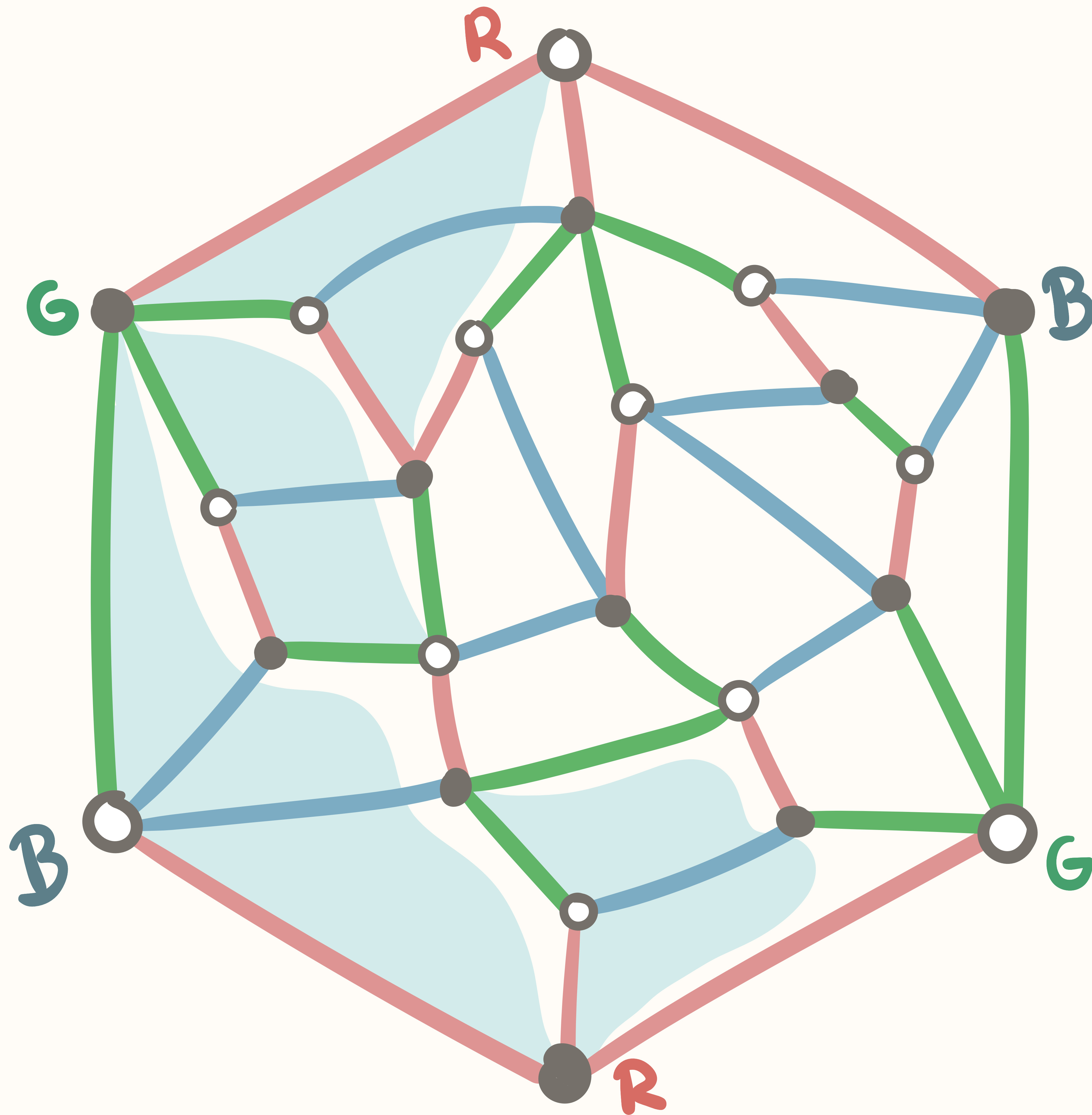
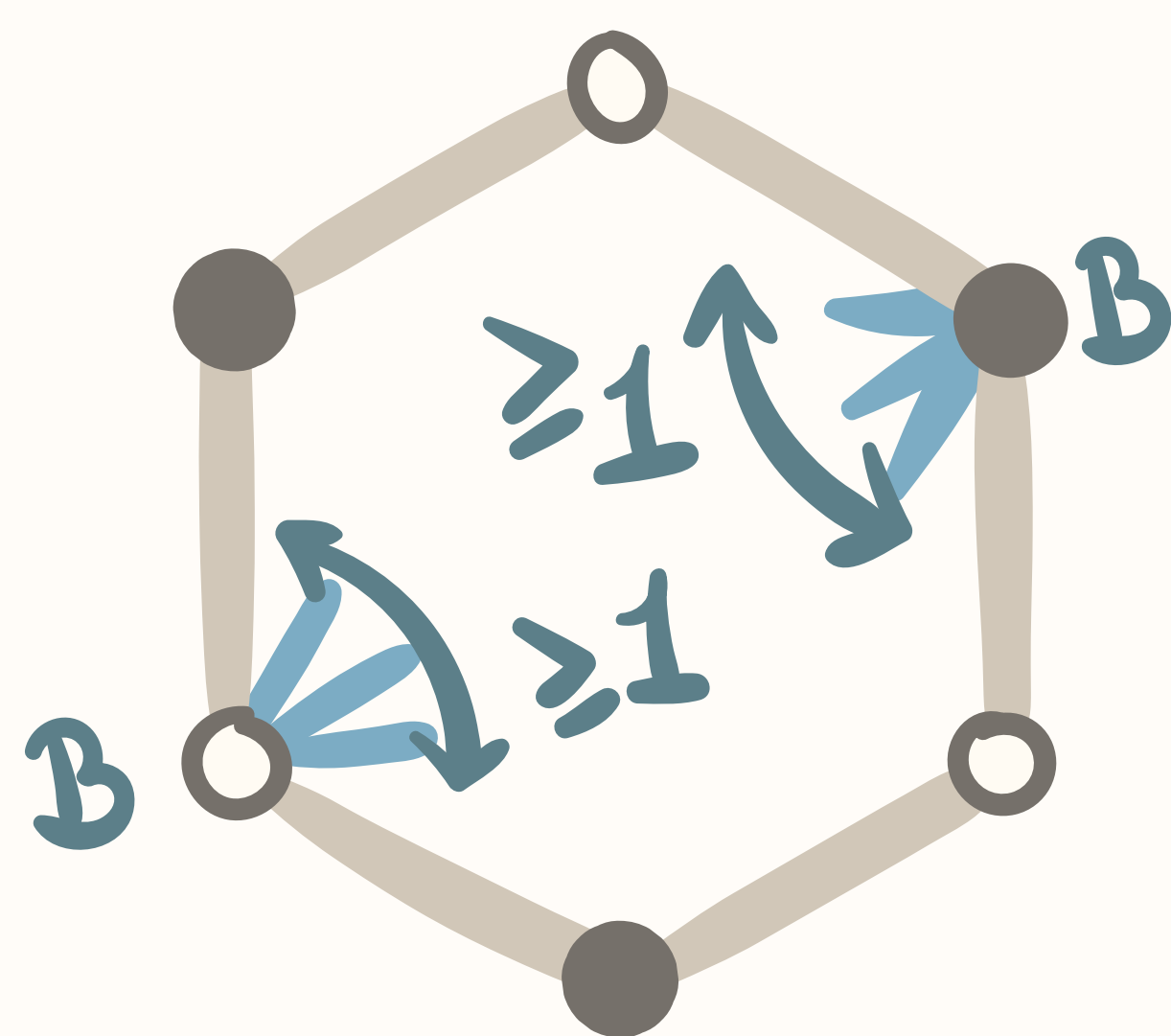
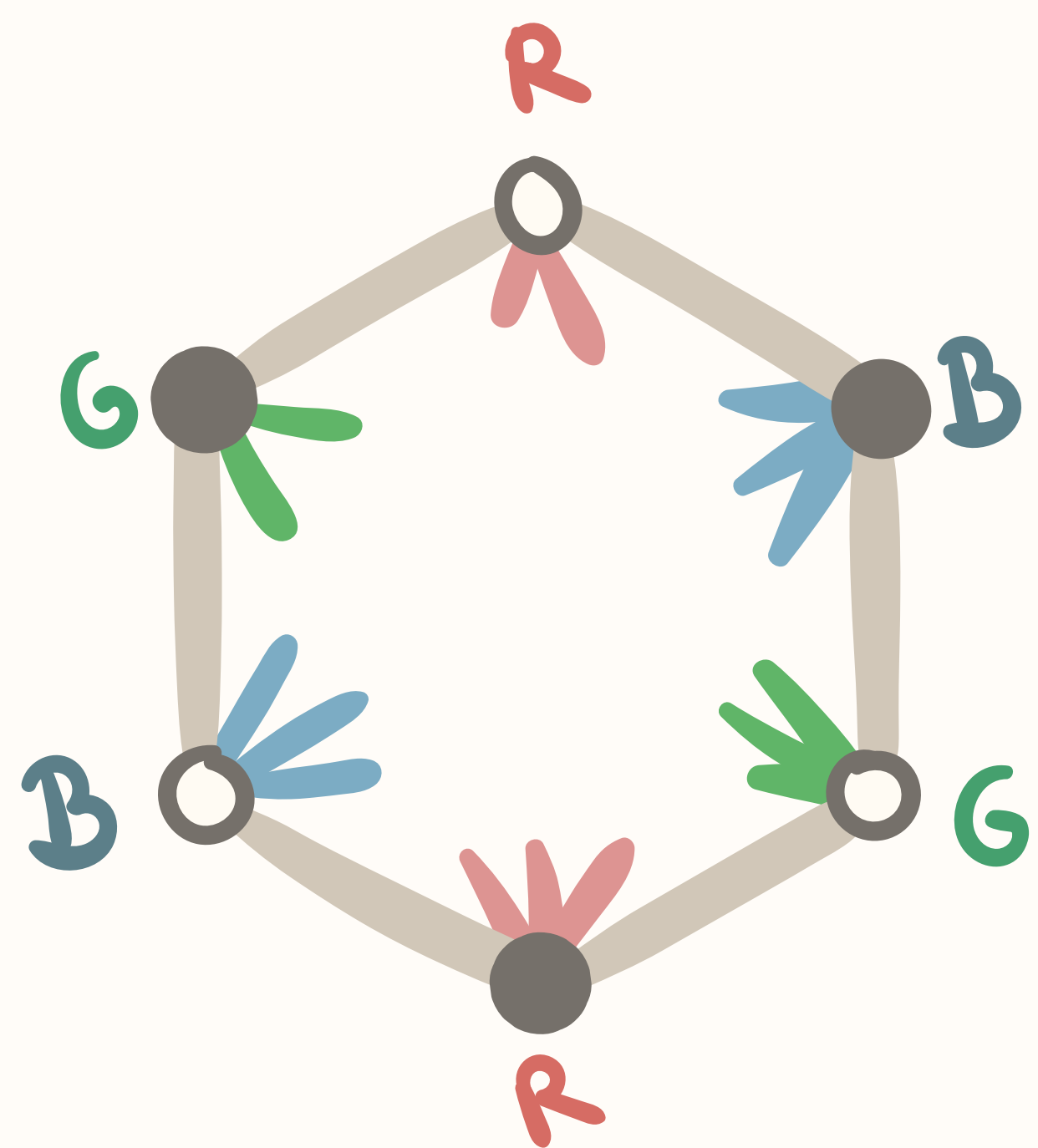
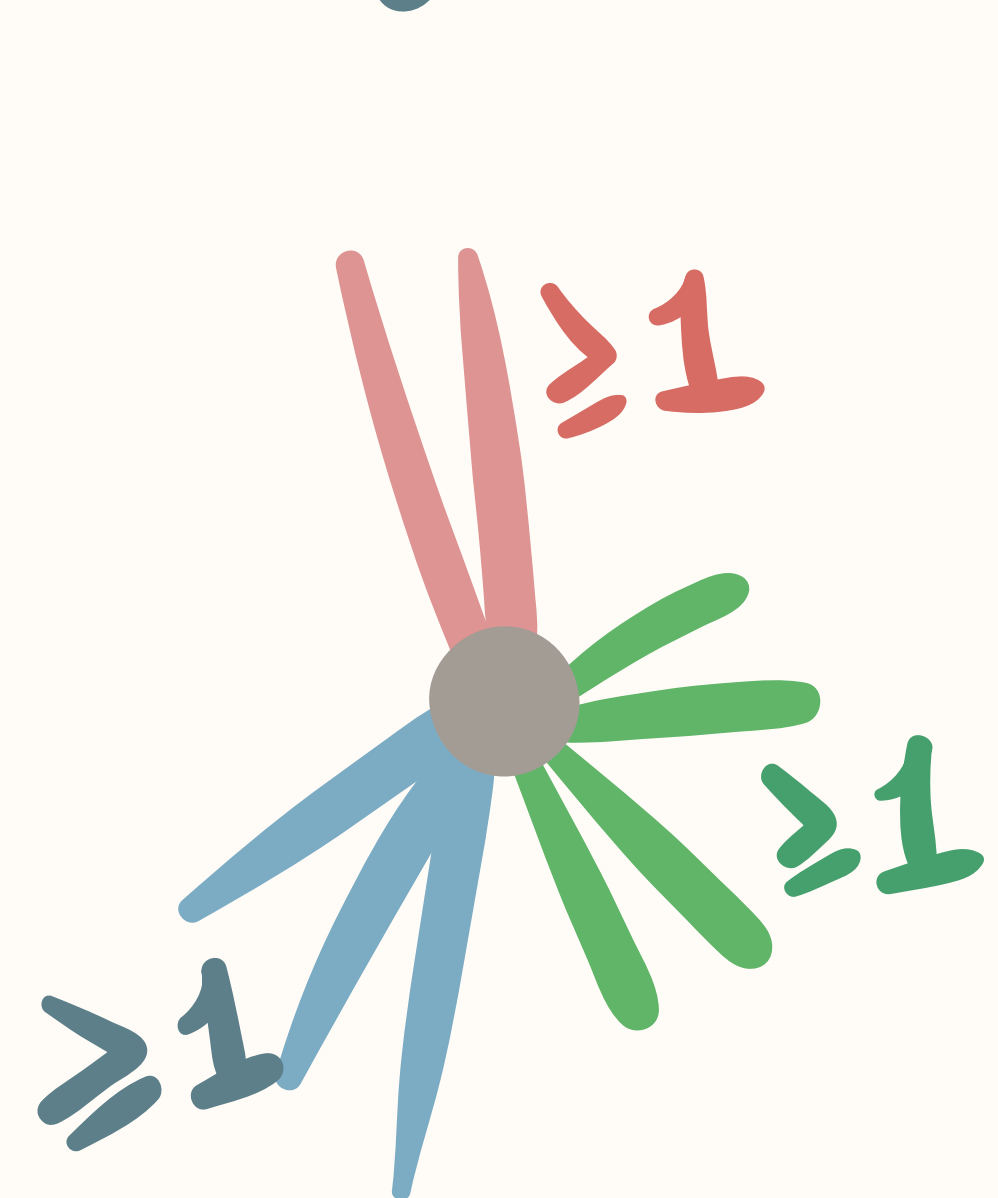
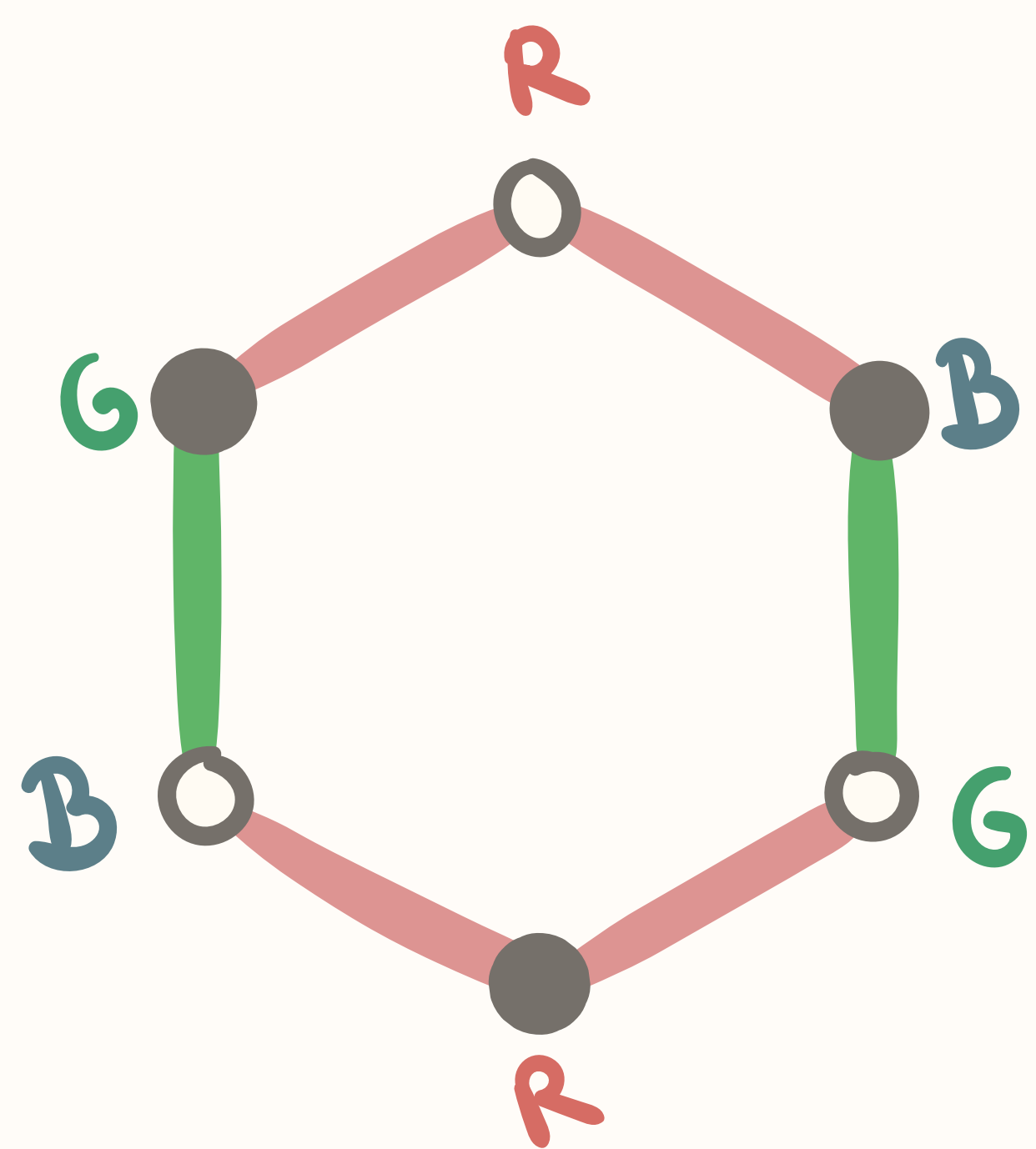
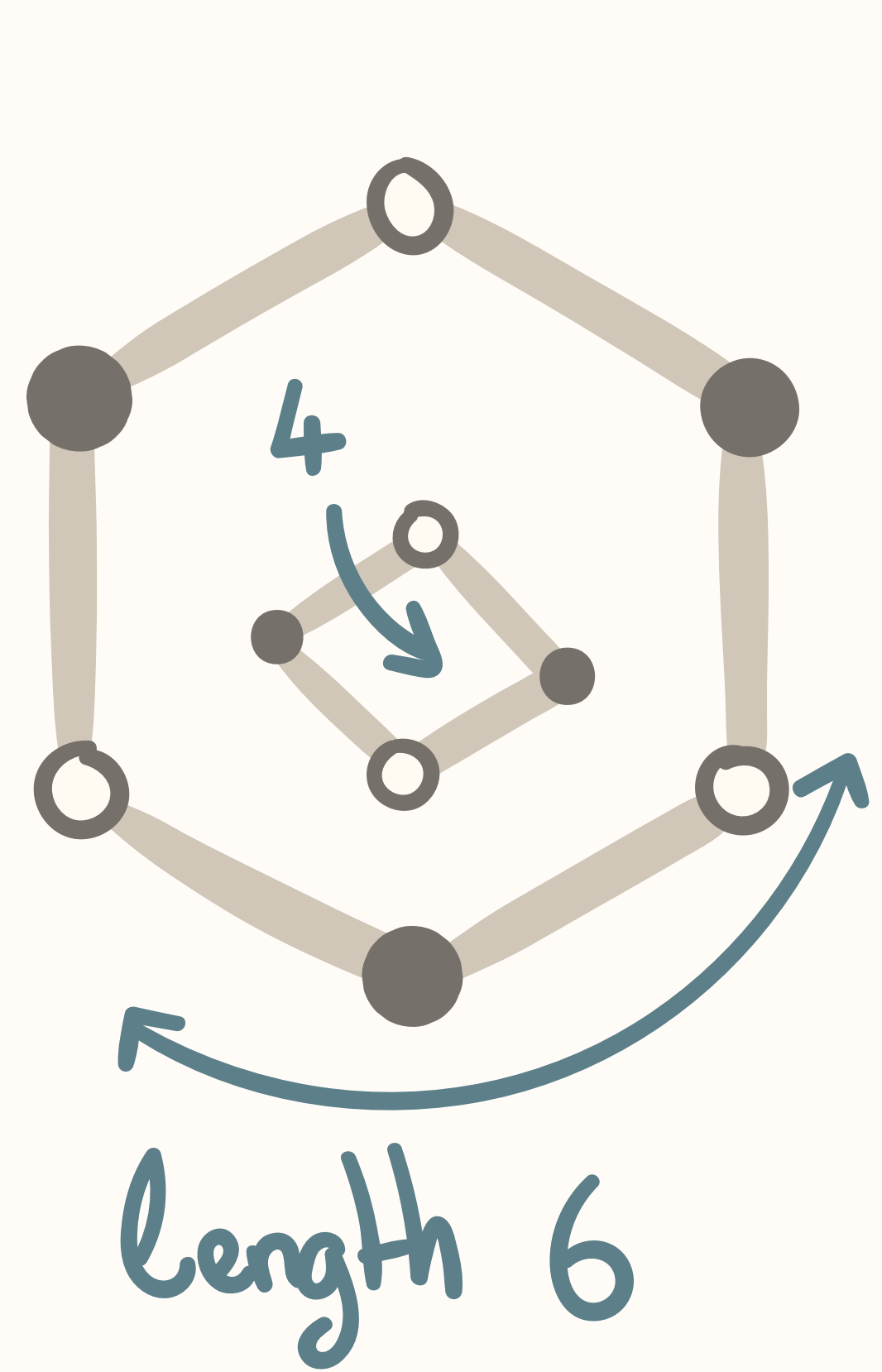
Schnyder labellings



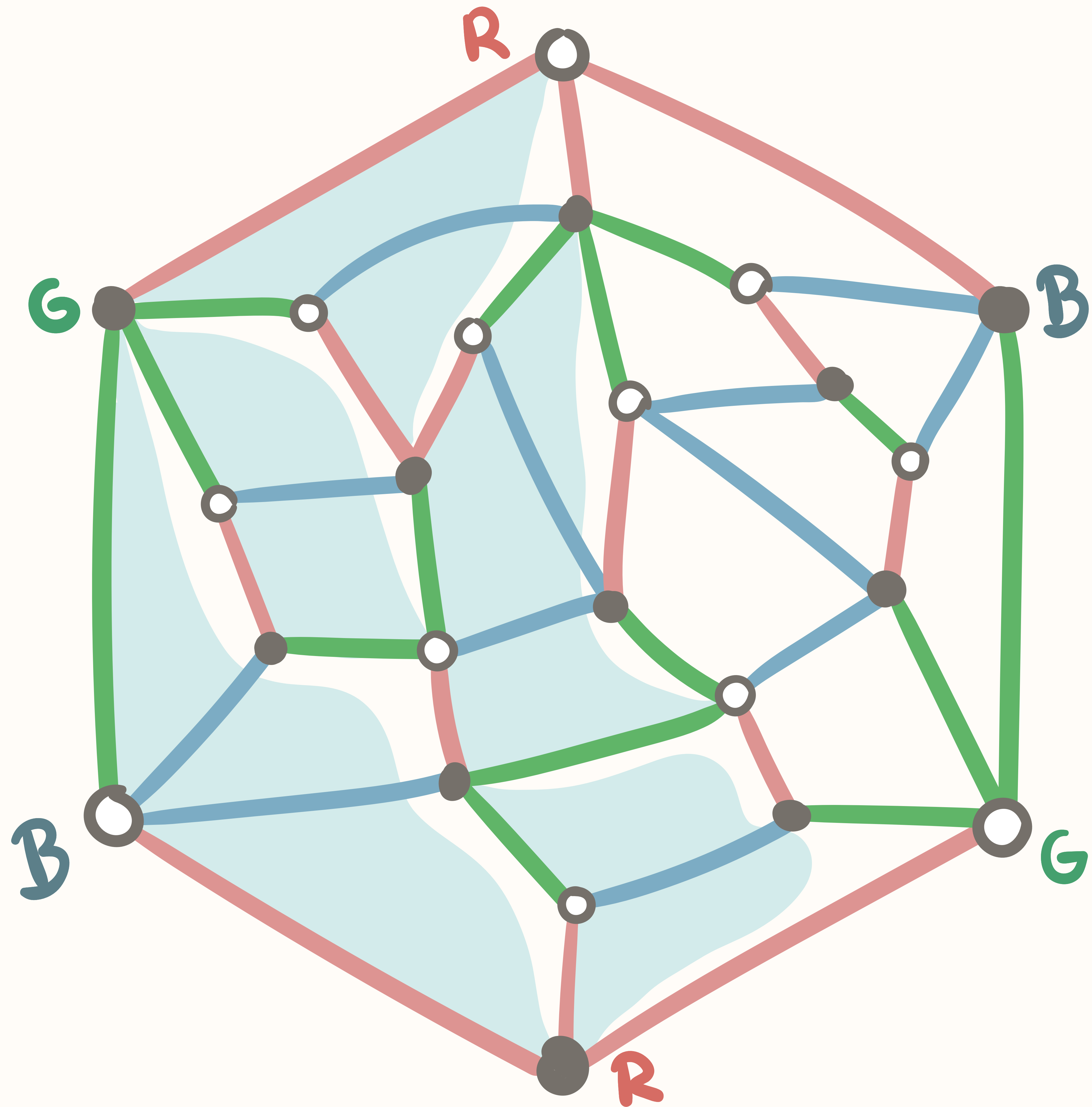
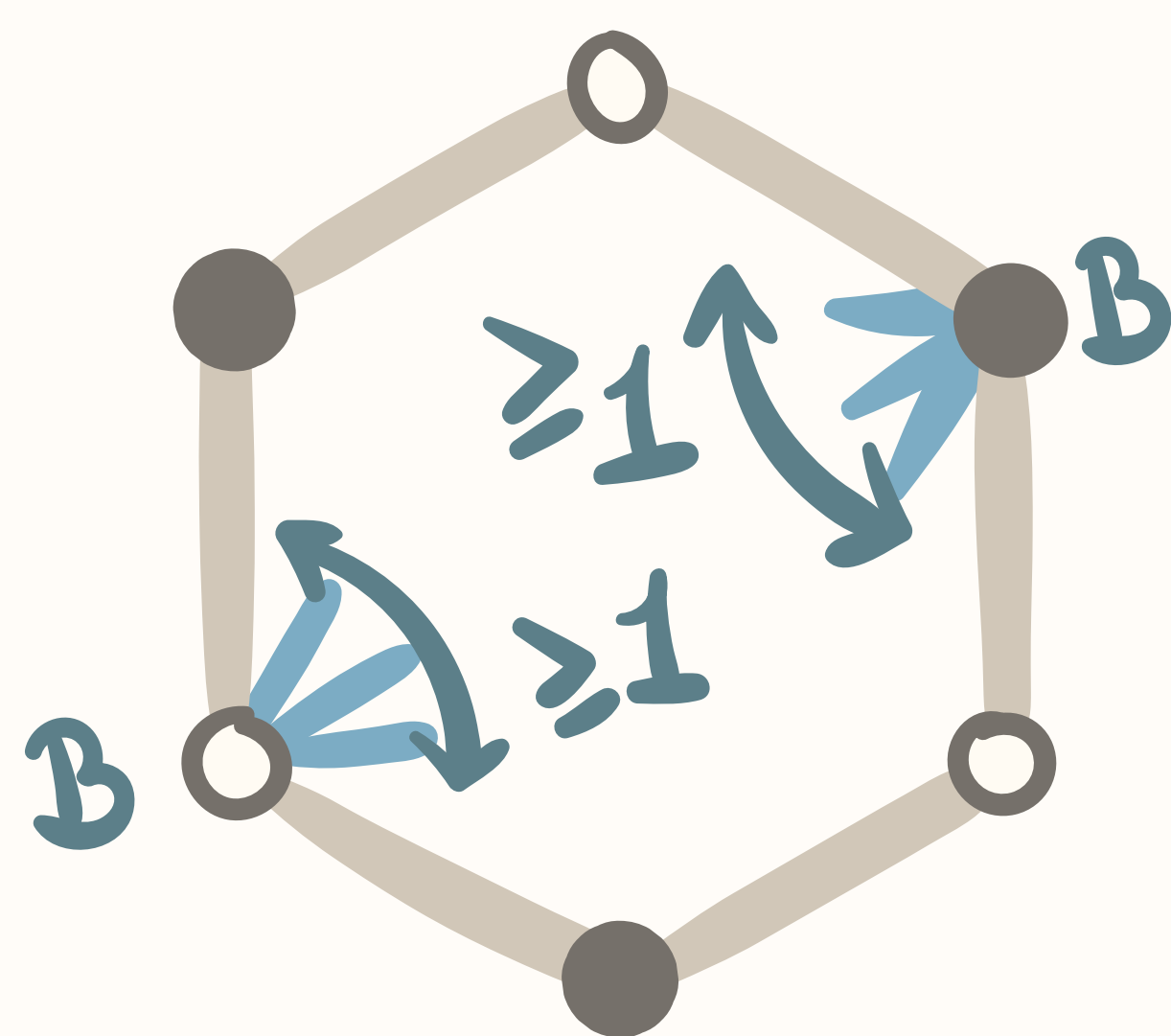
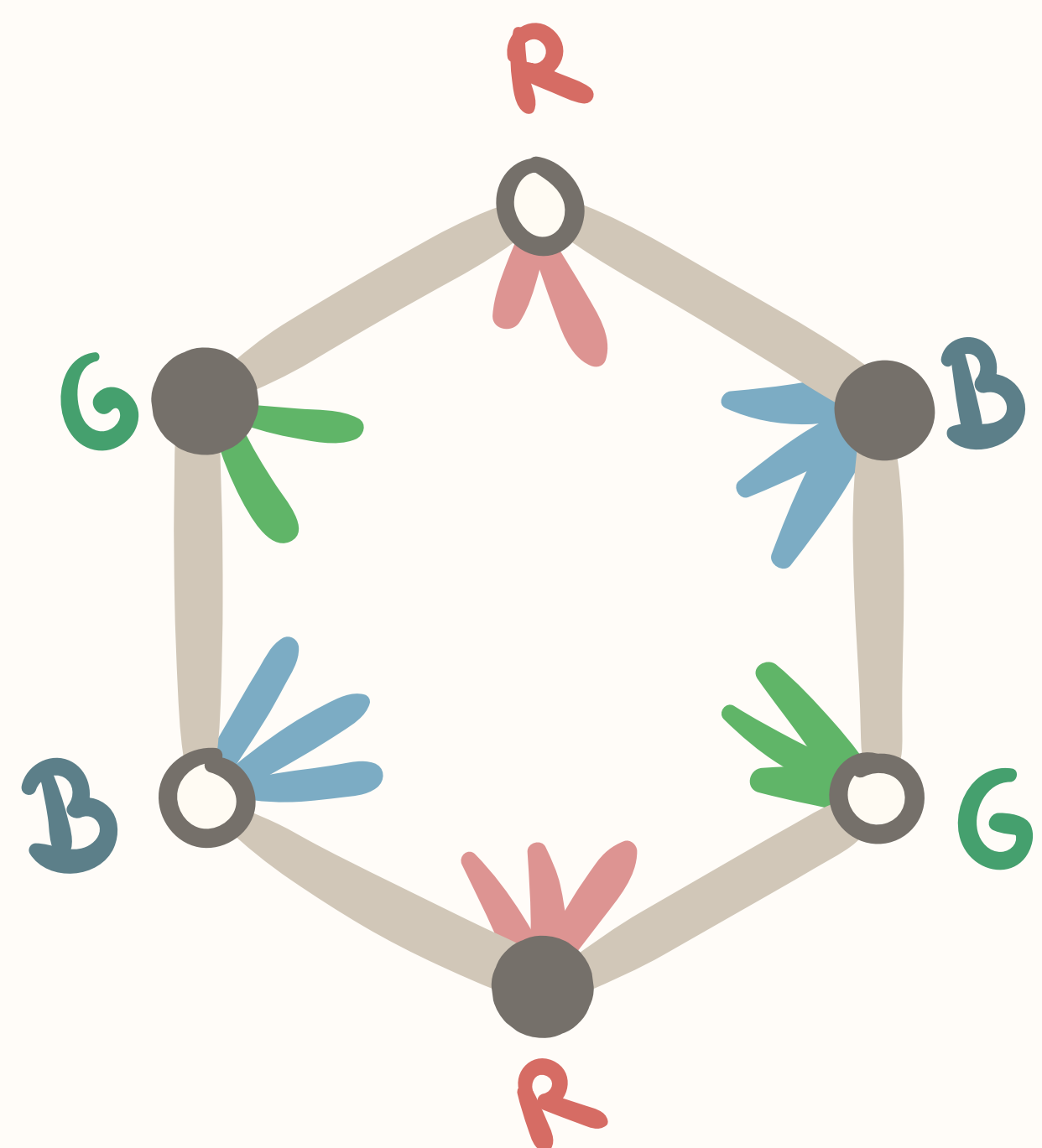
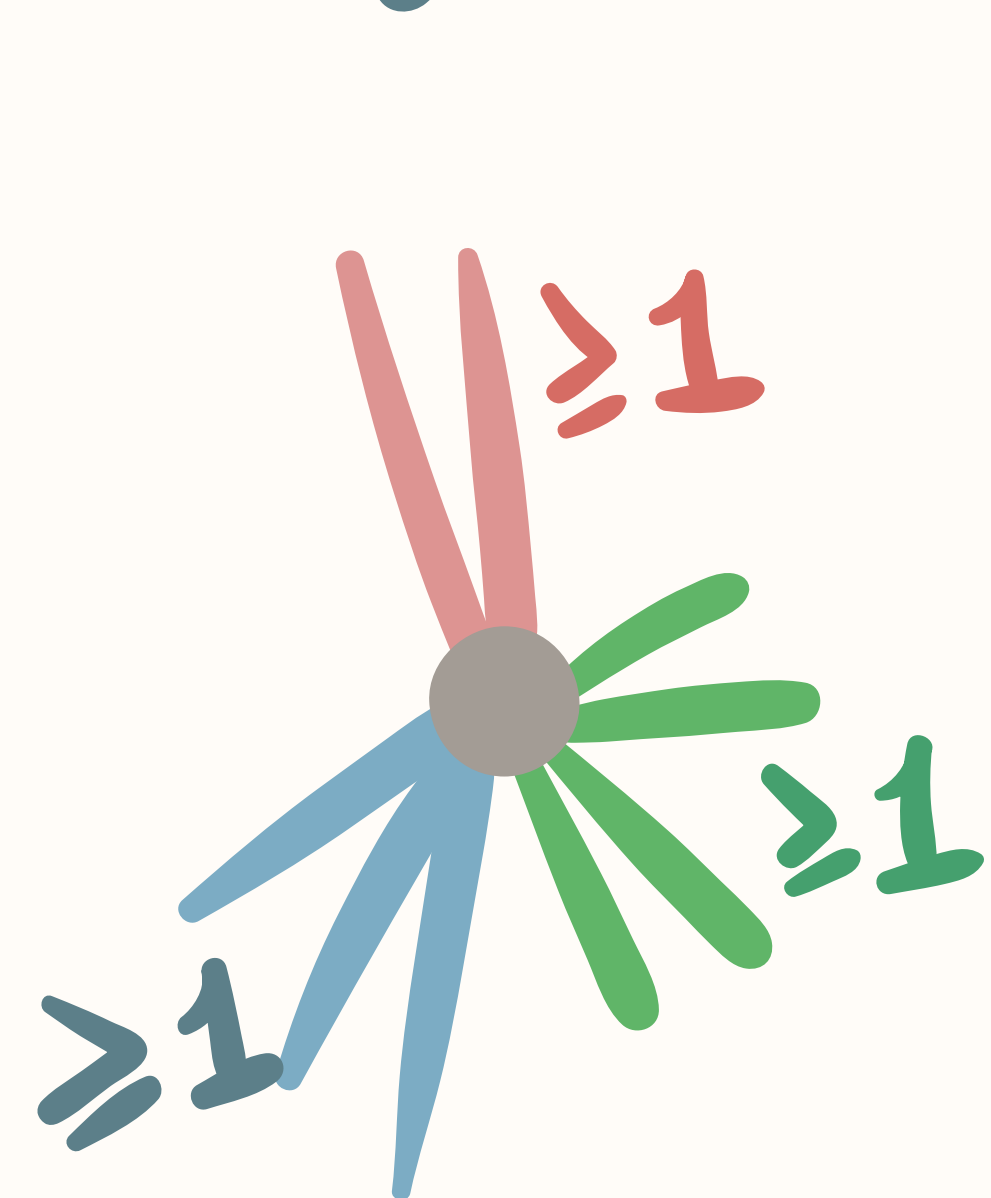
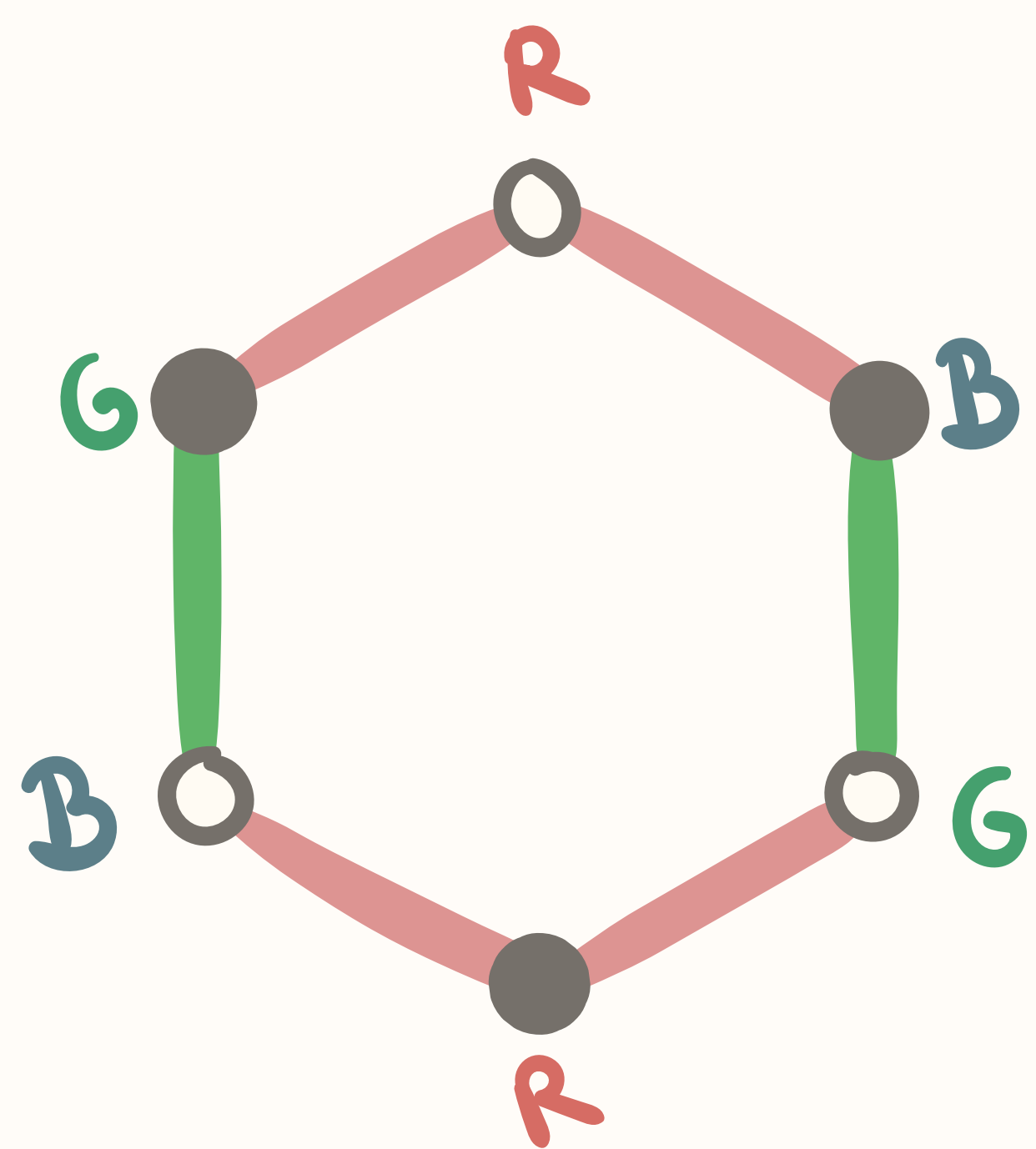
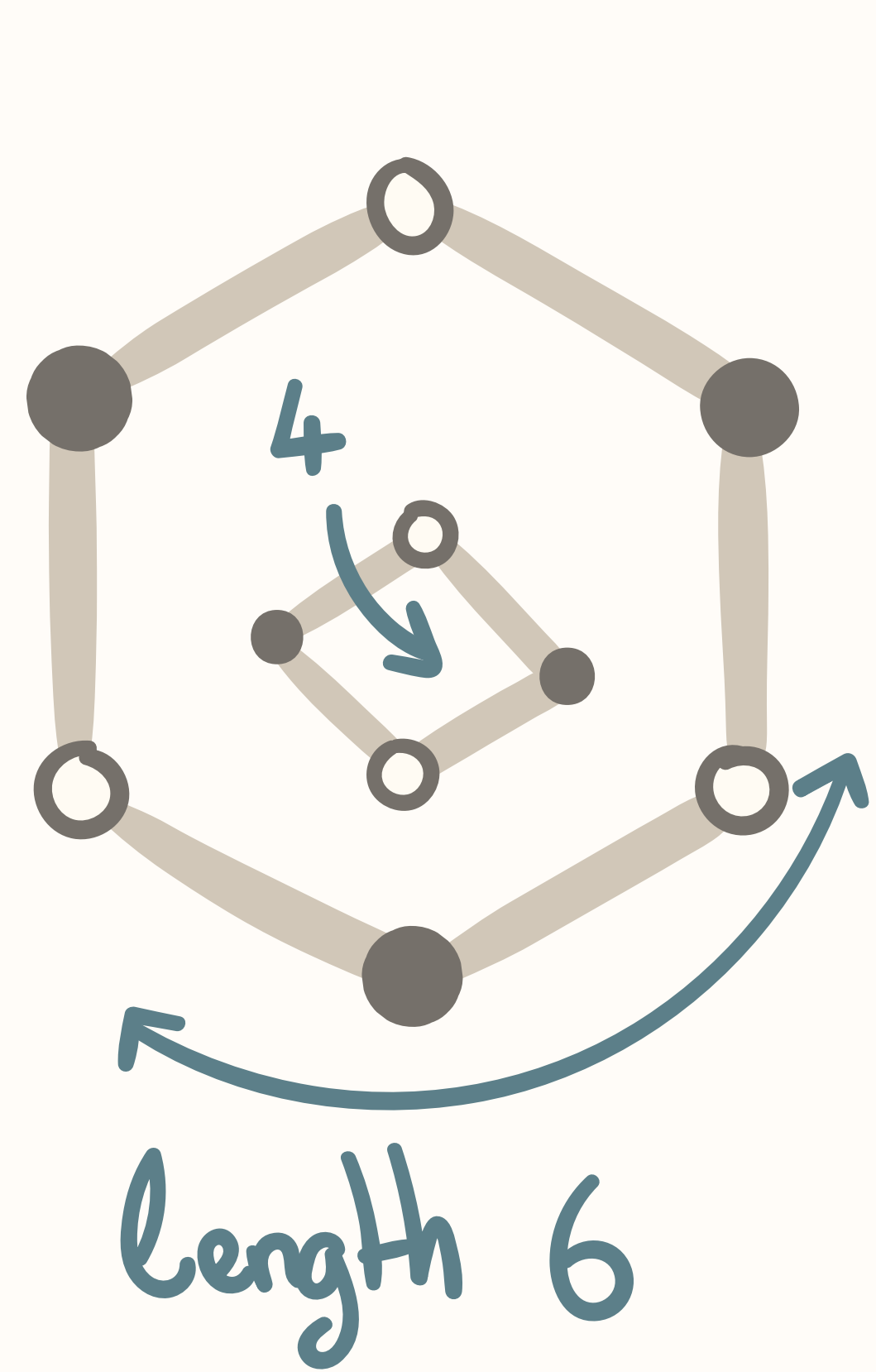
Schnyder labellings



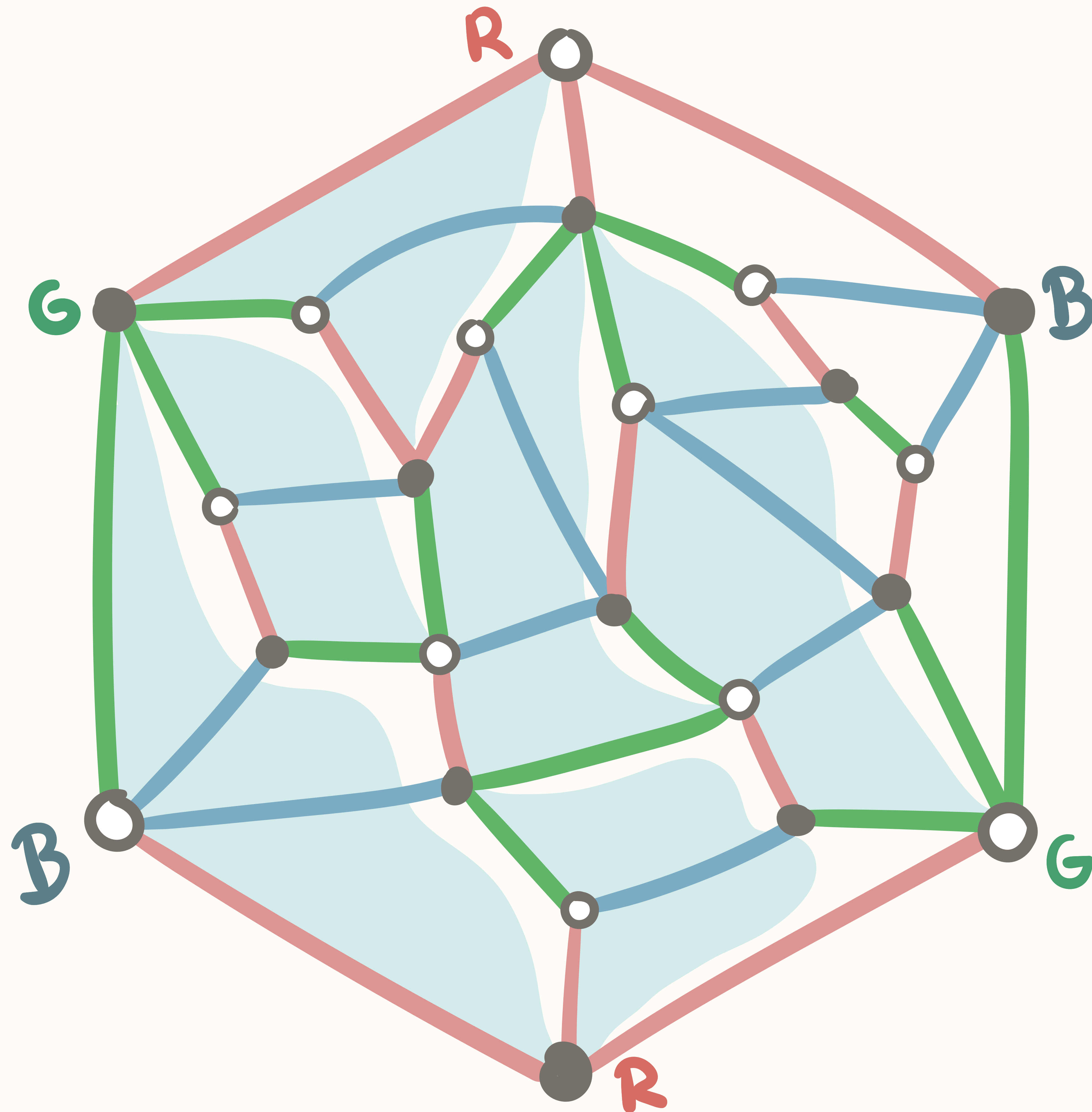
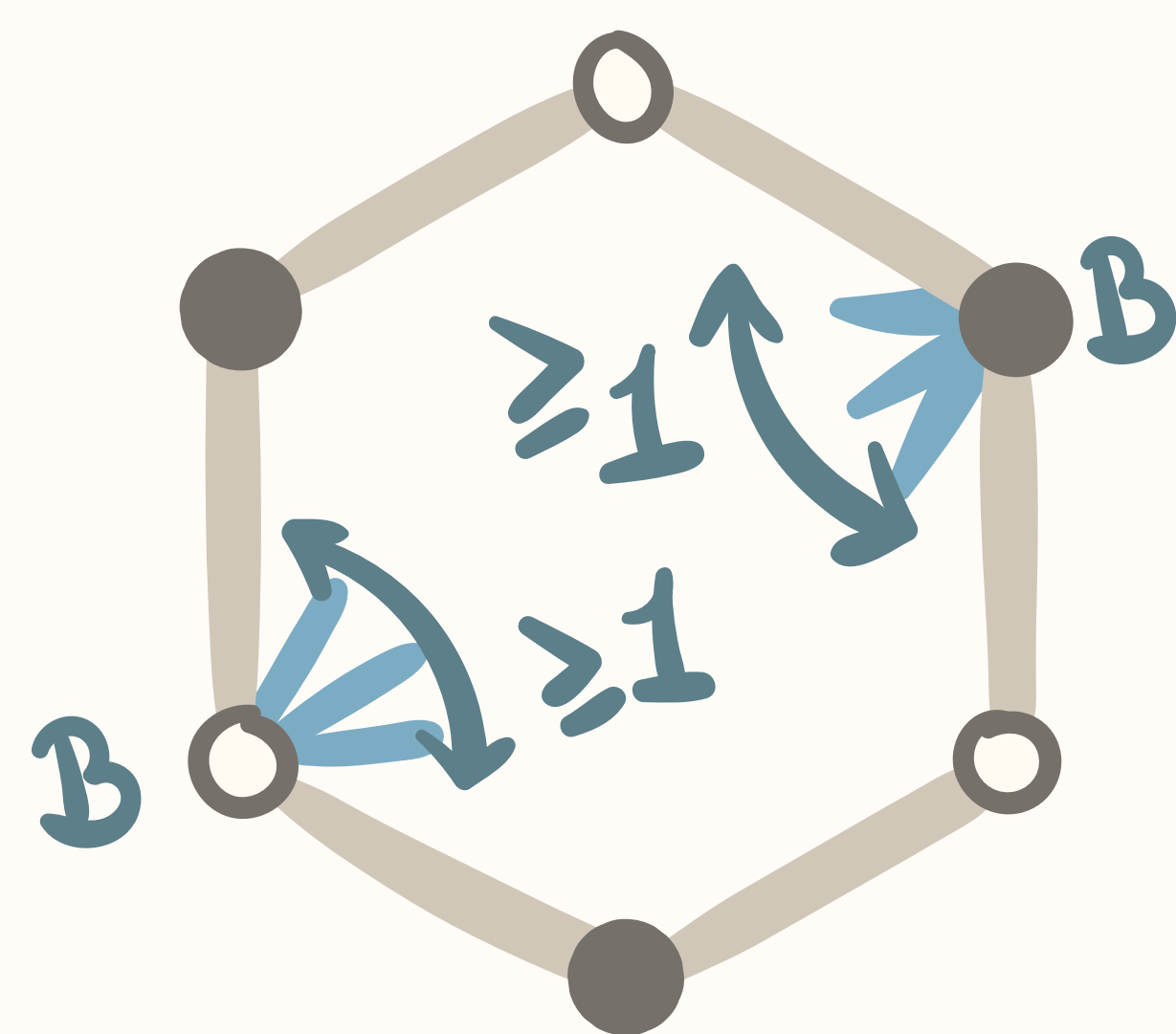
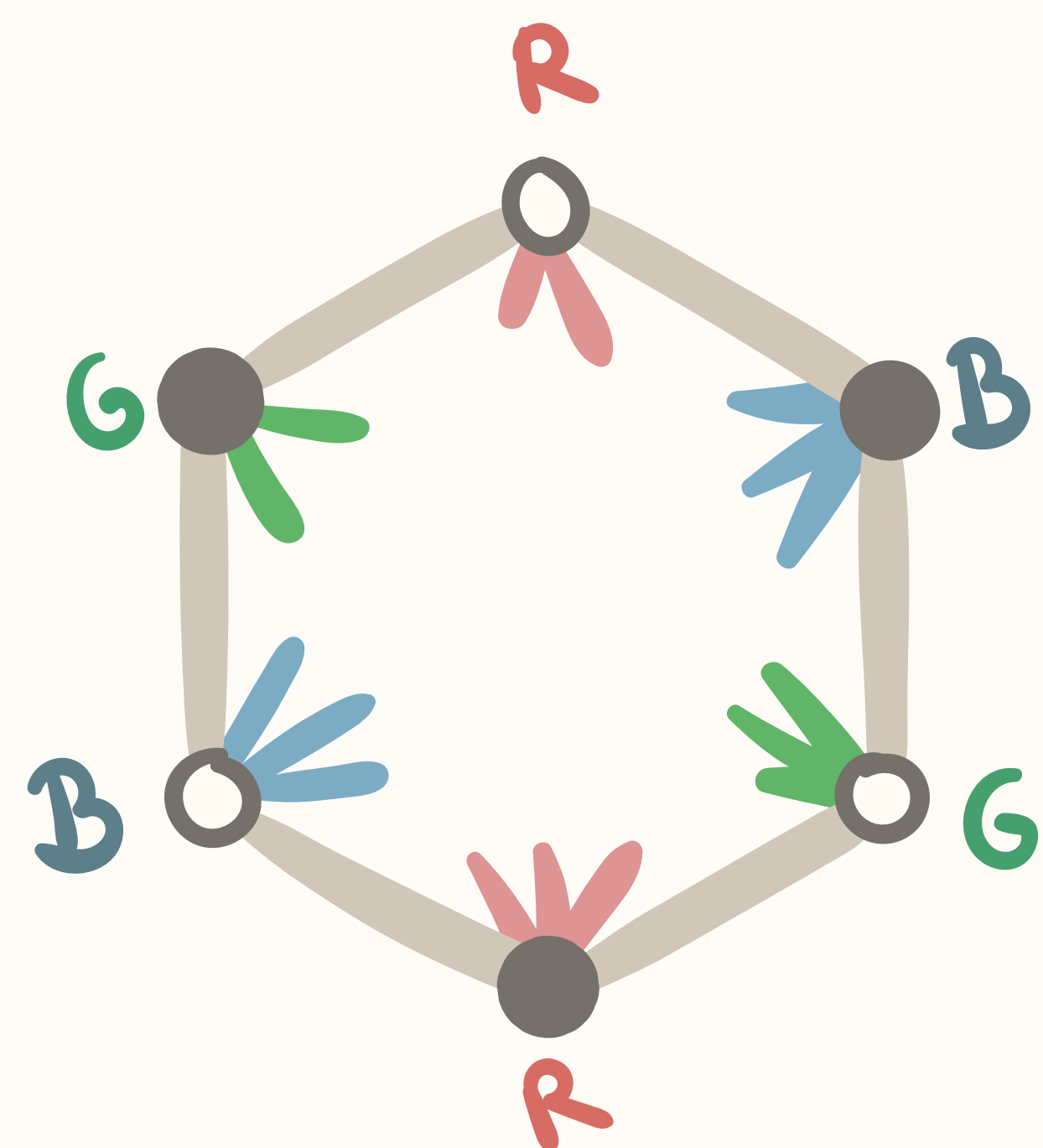
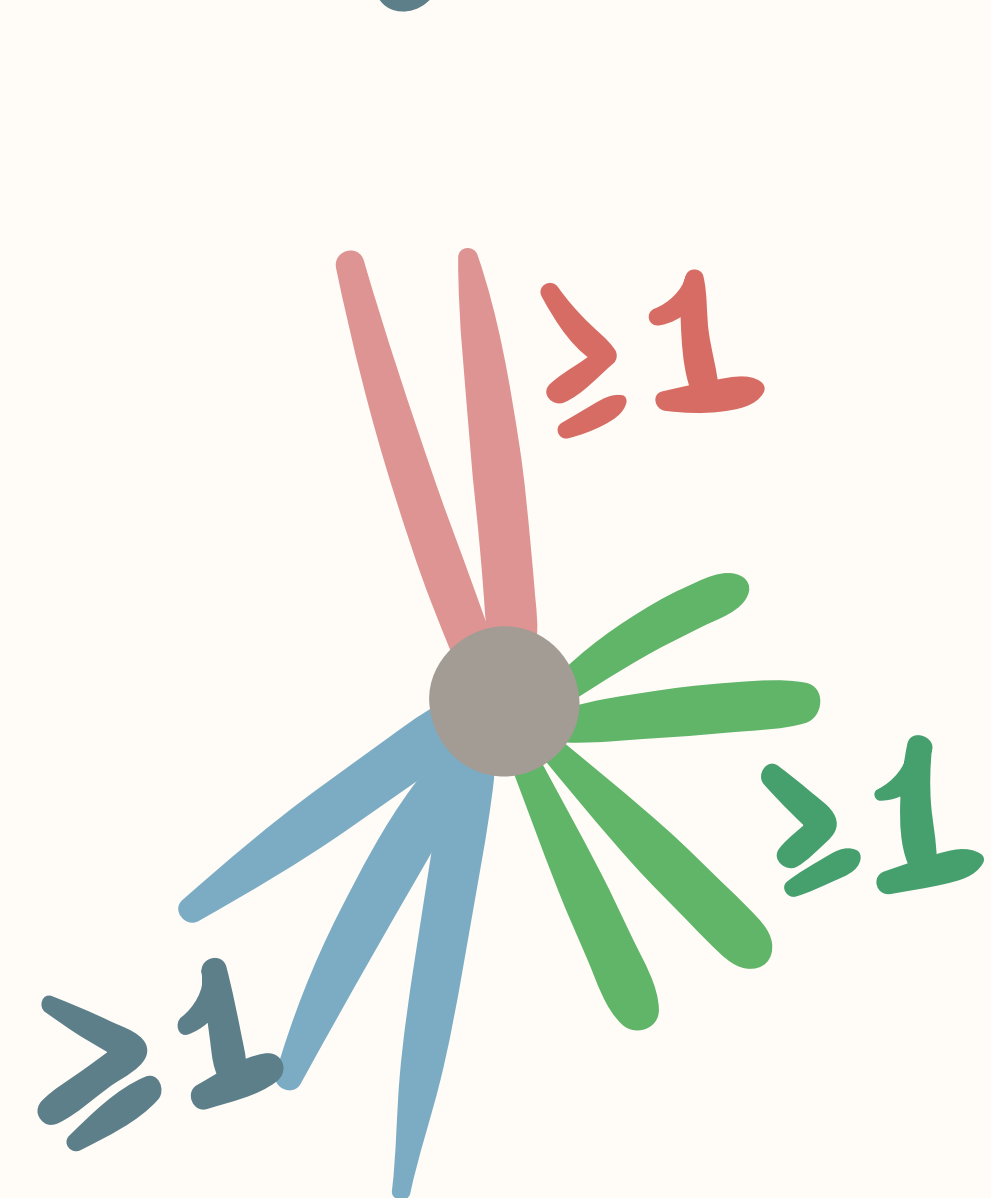
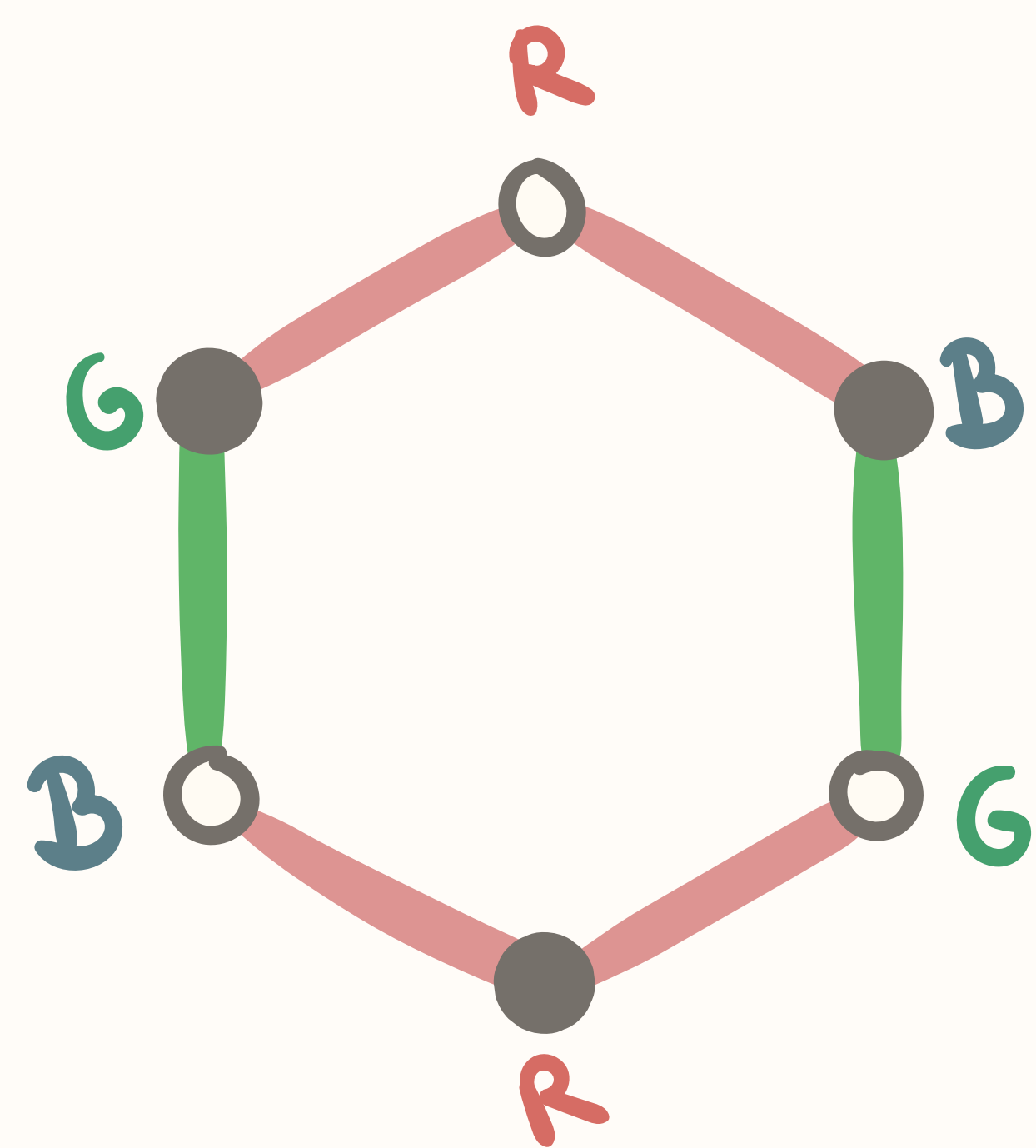
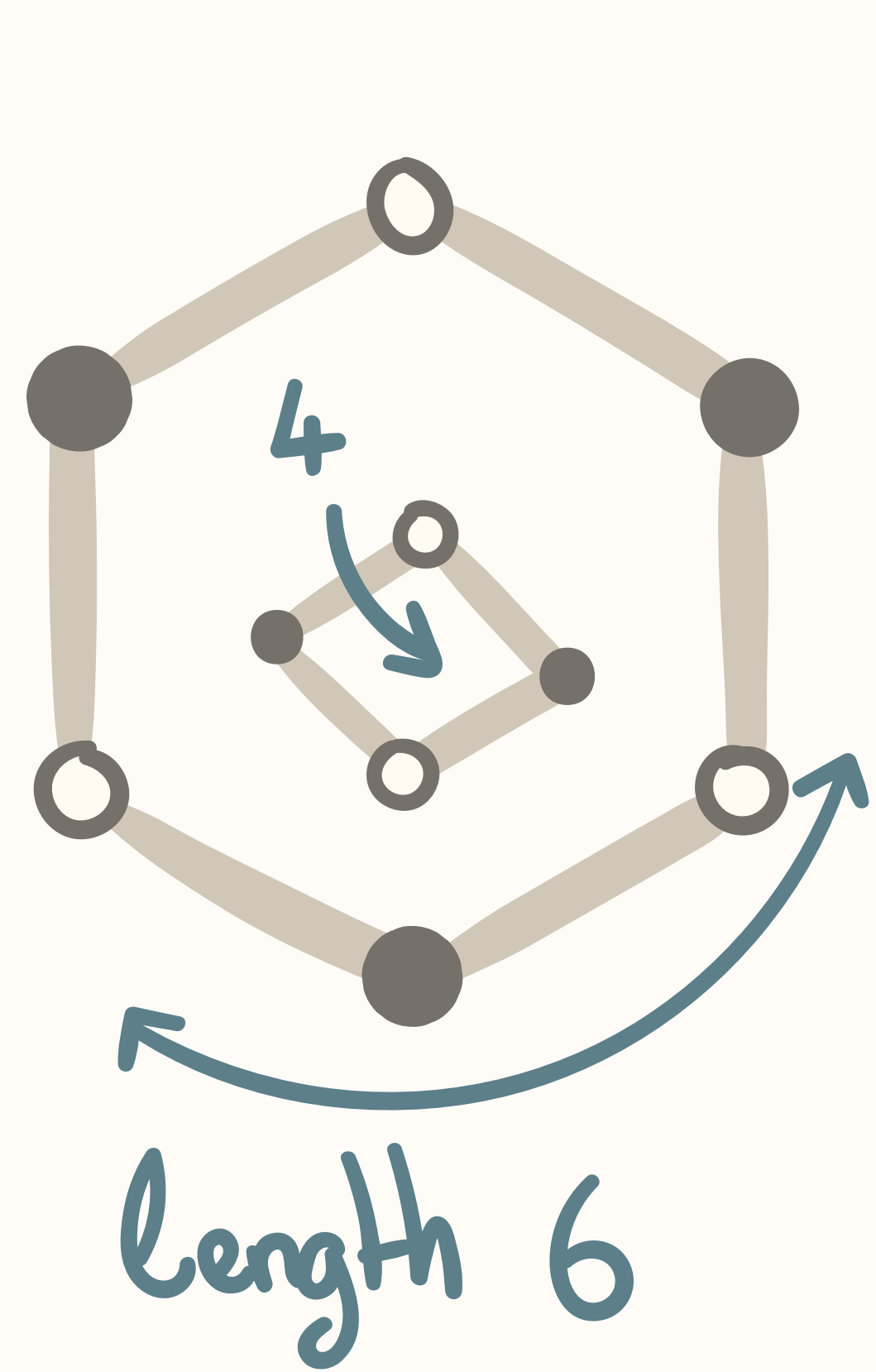
Schnyder labellings



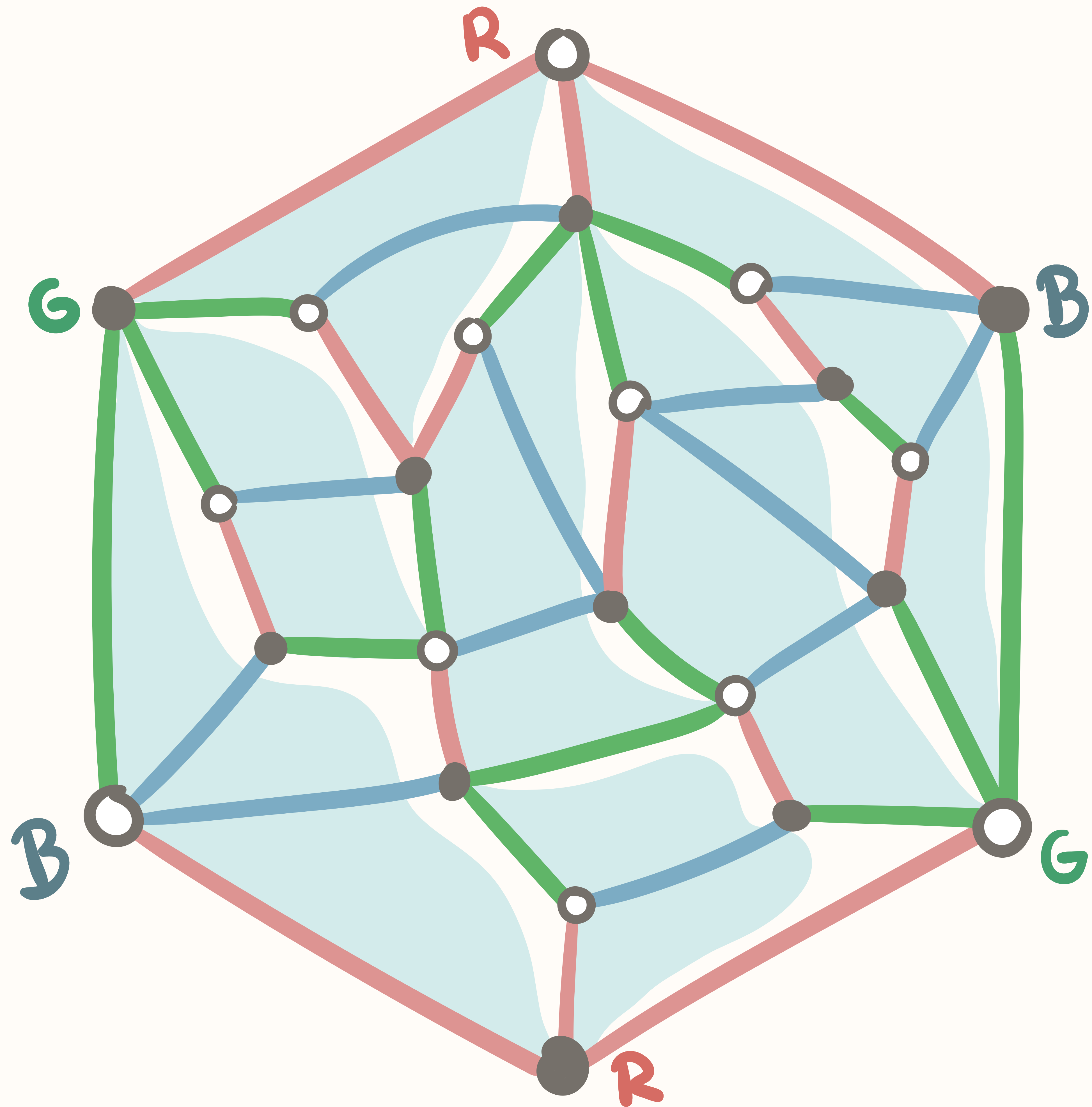
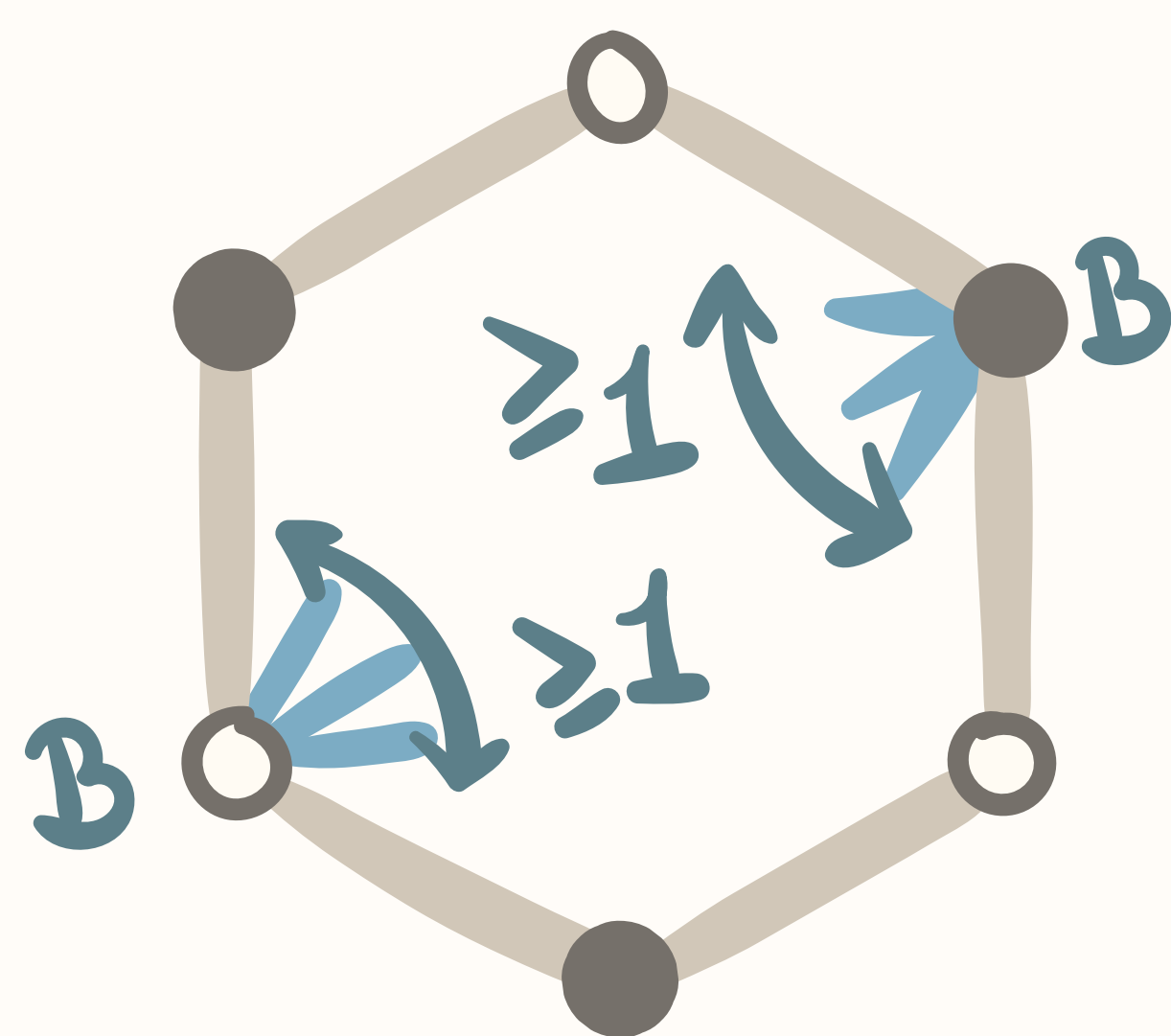
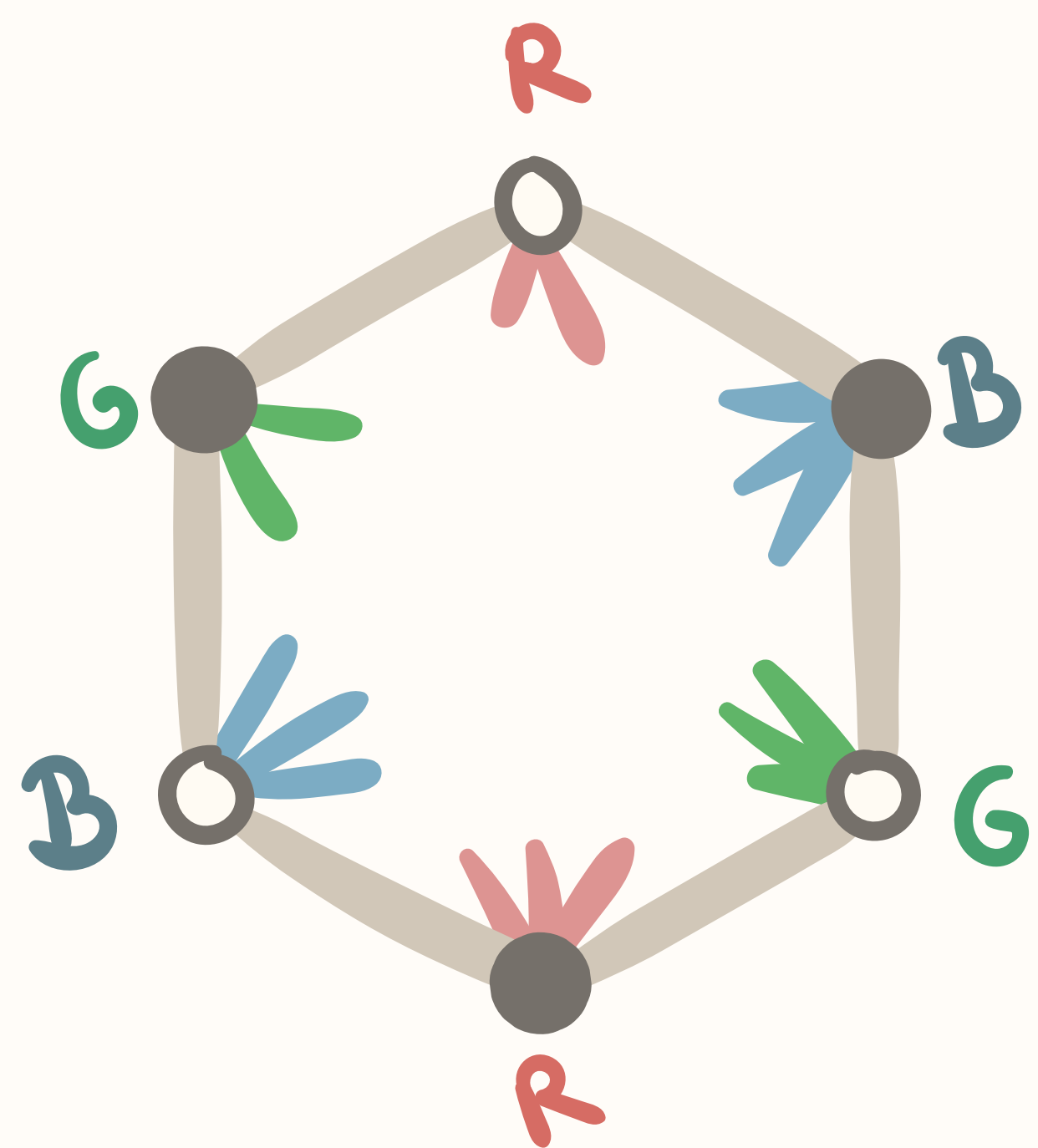
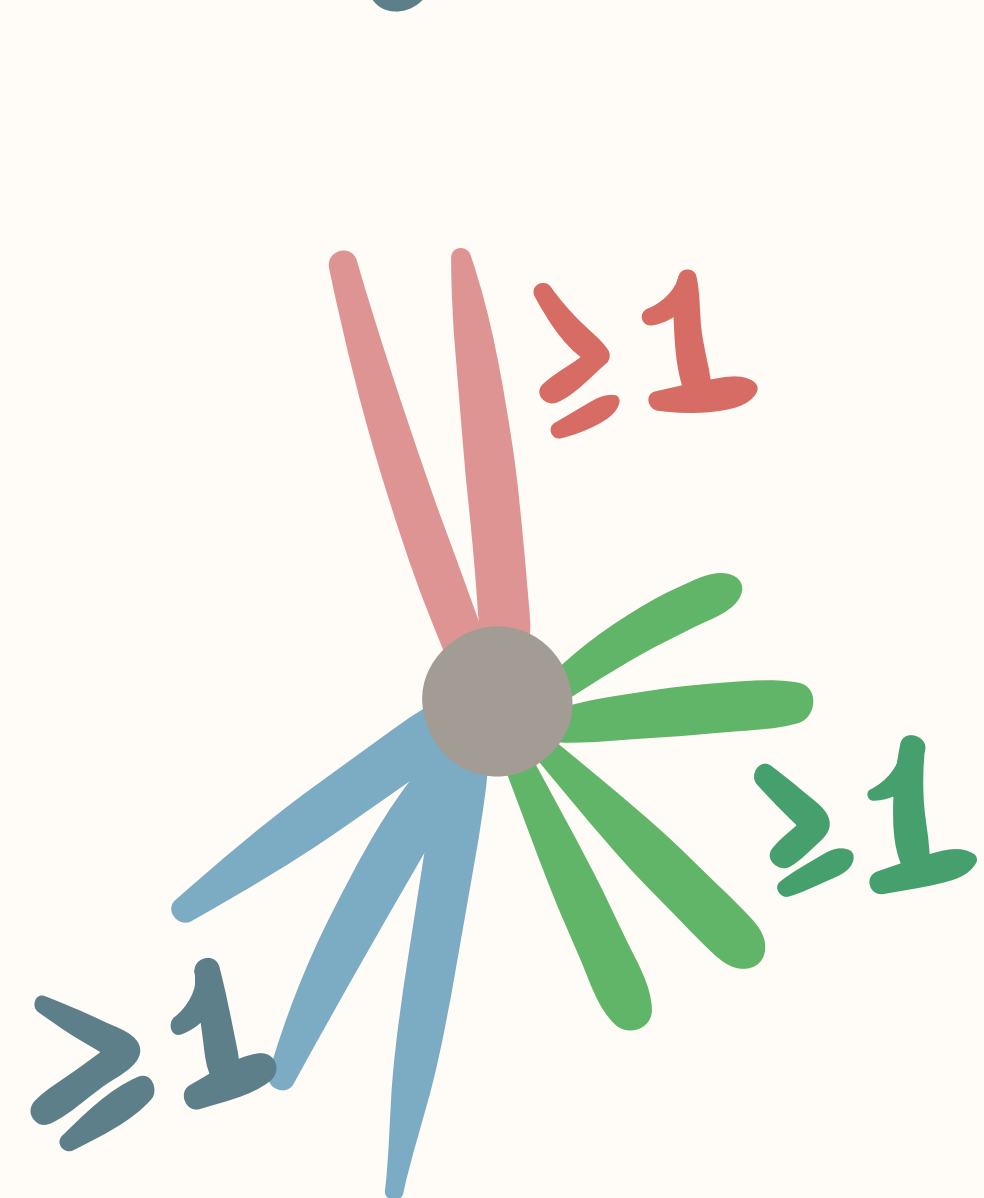
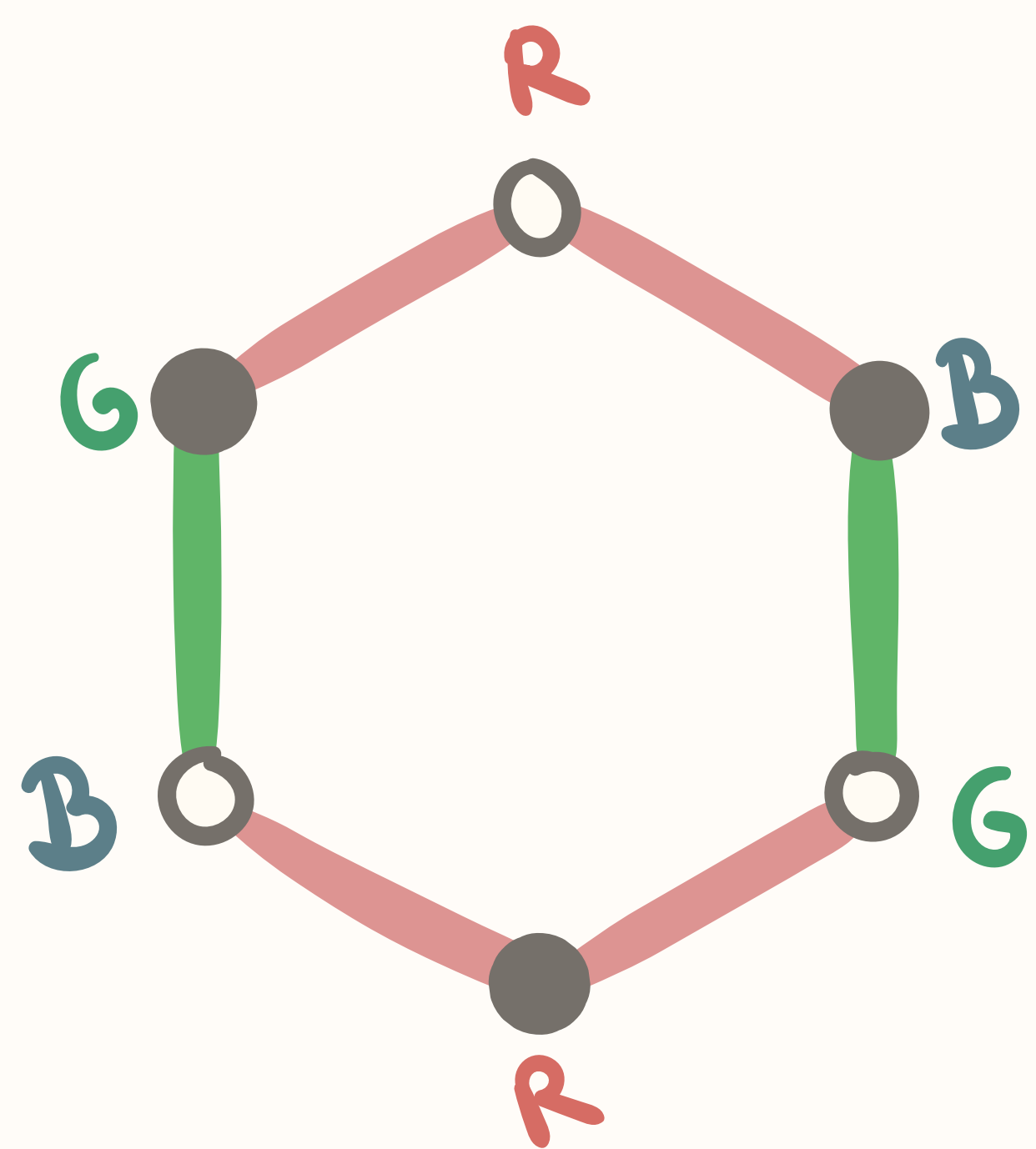
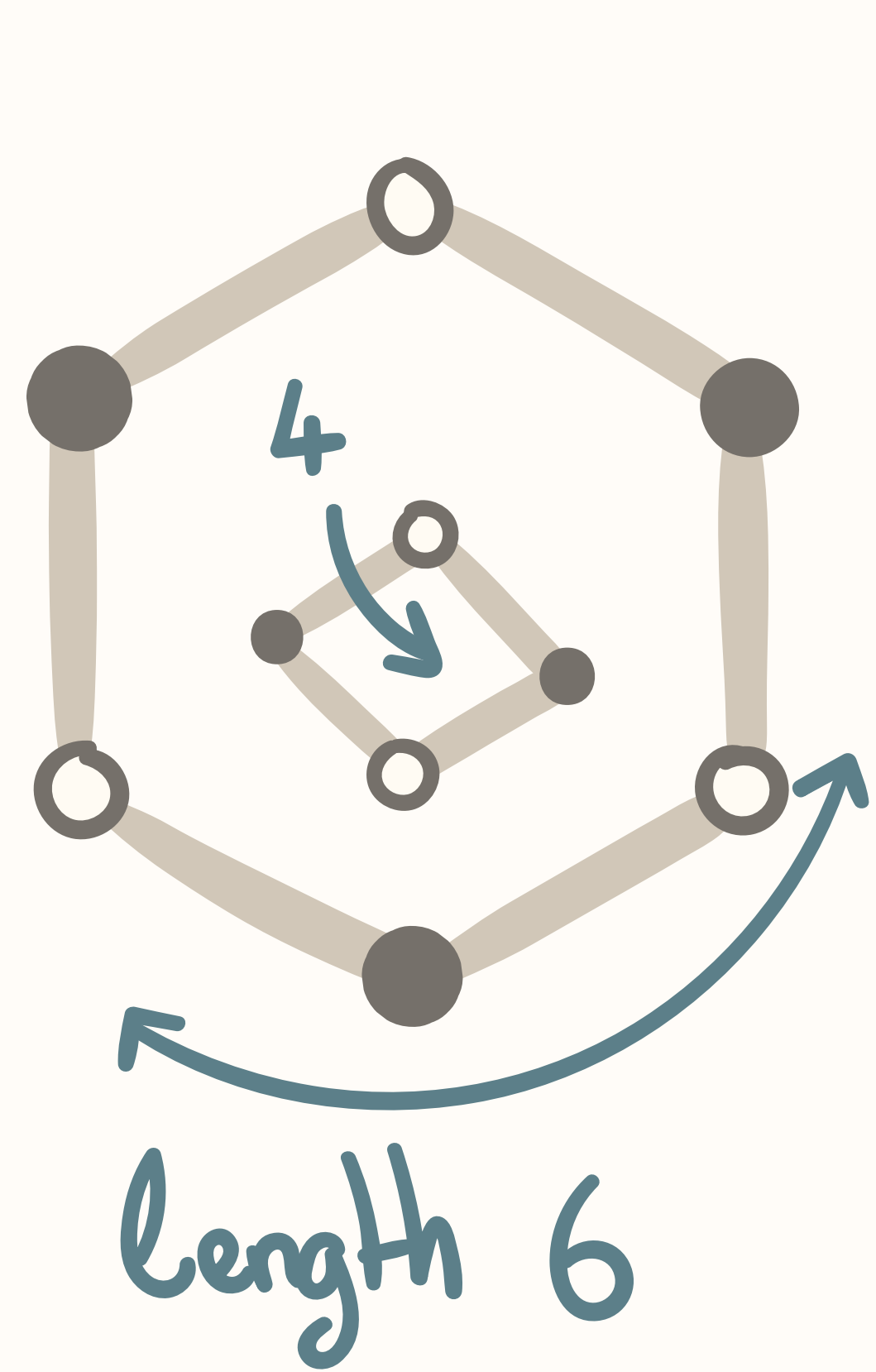
Schnyder labellings



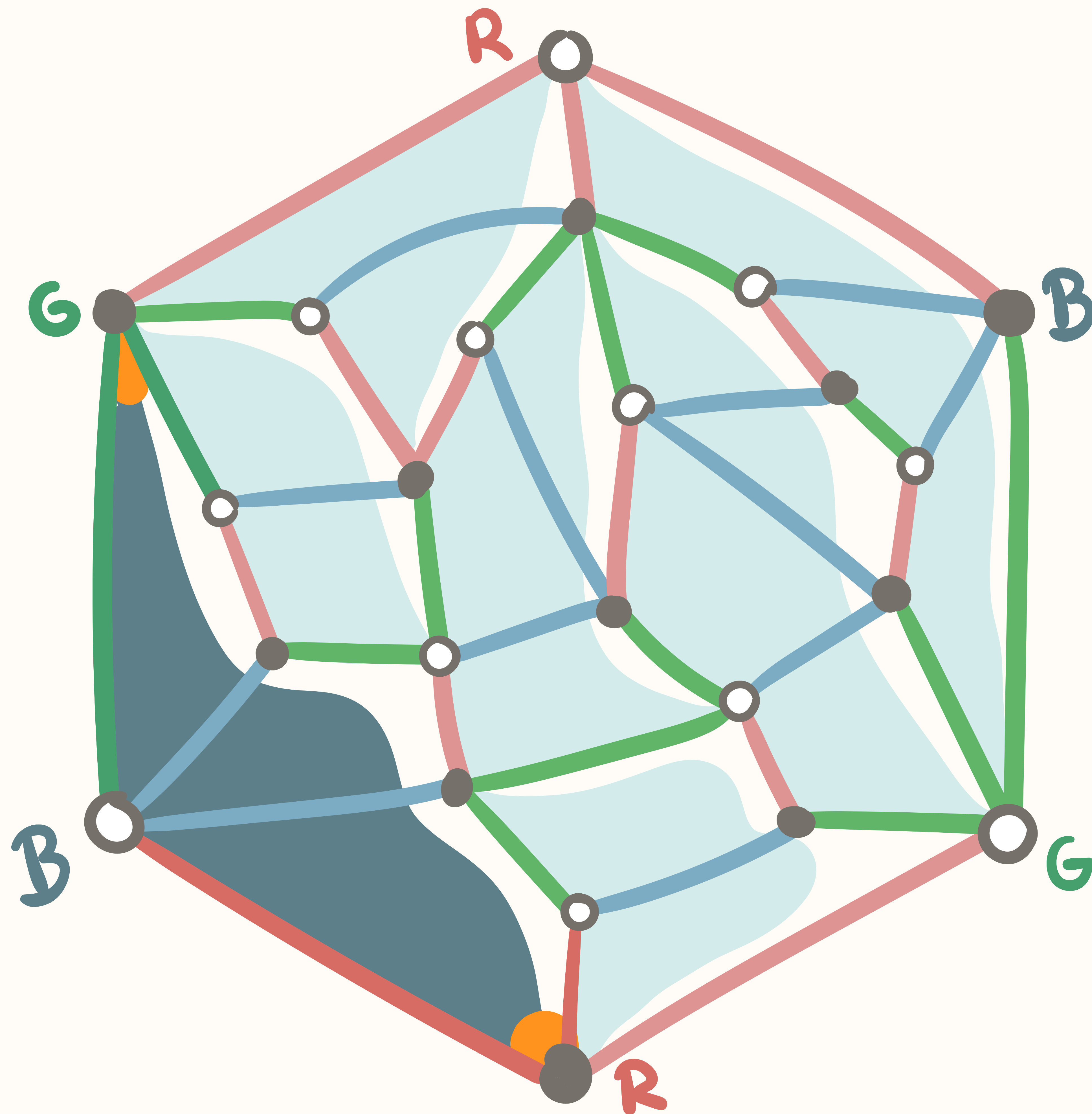
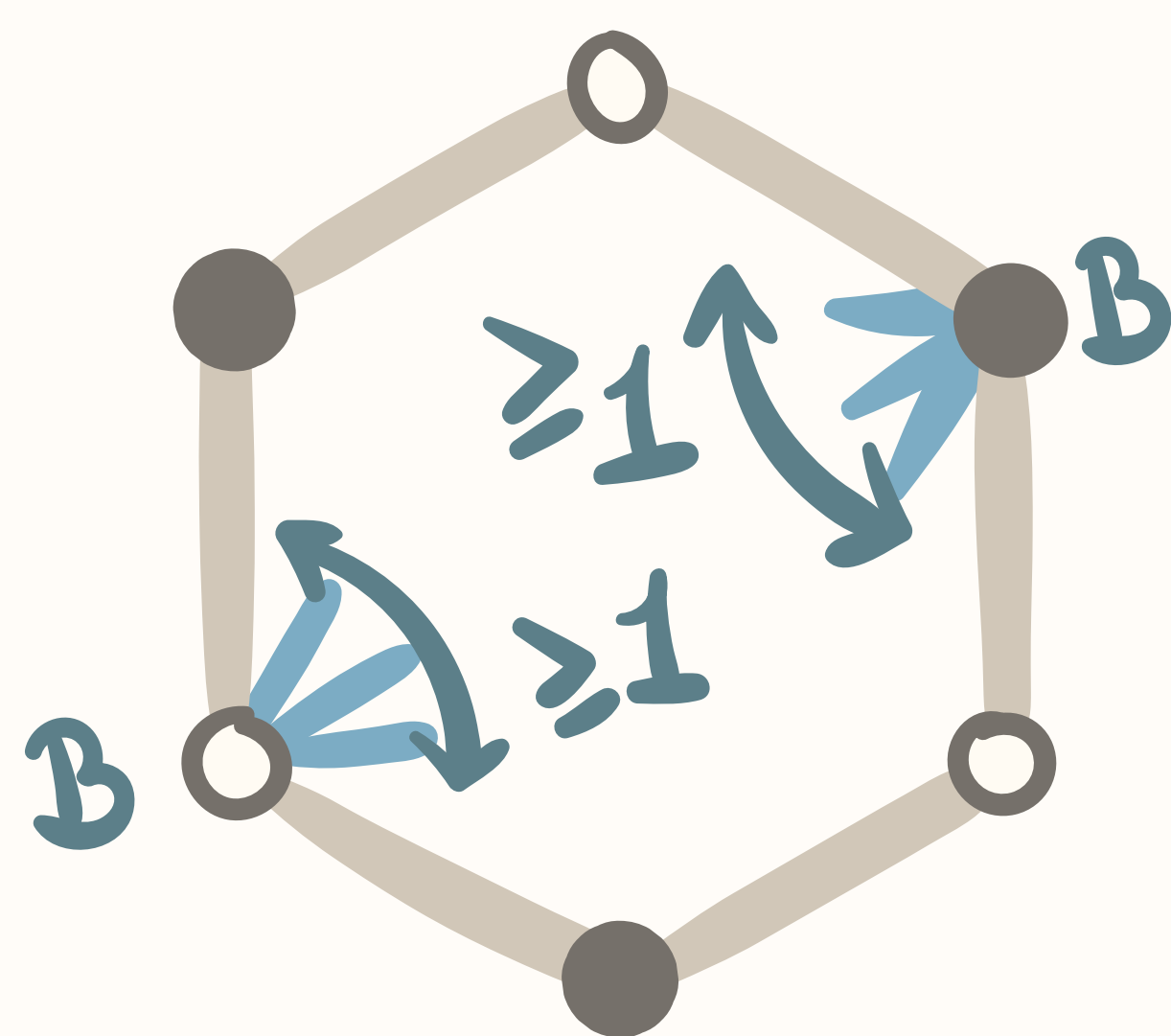
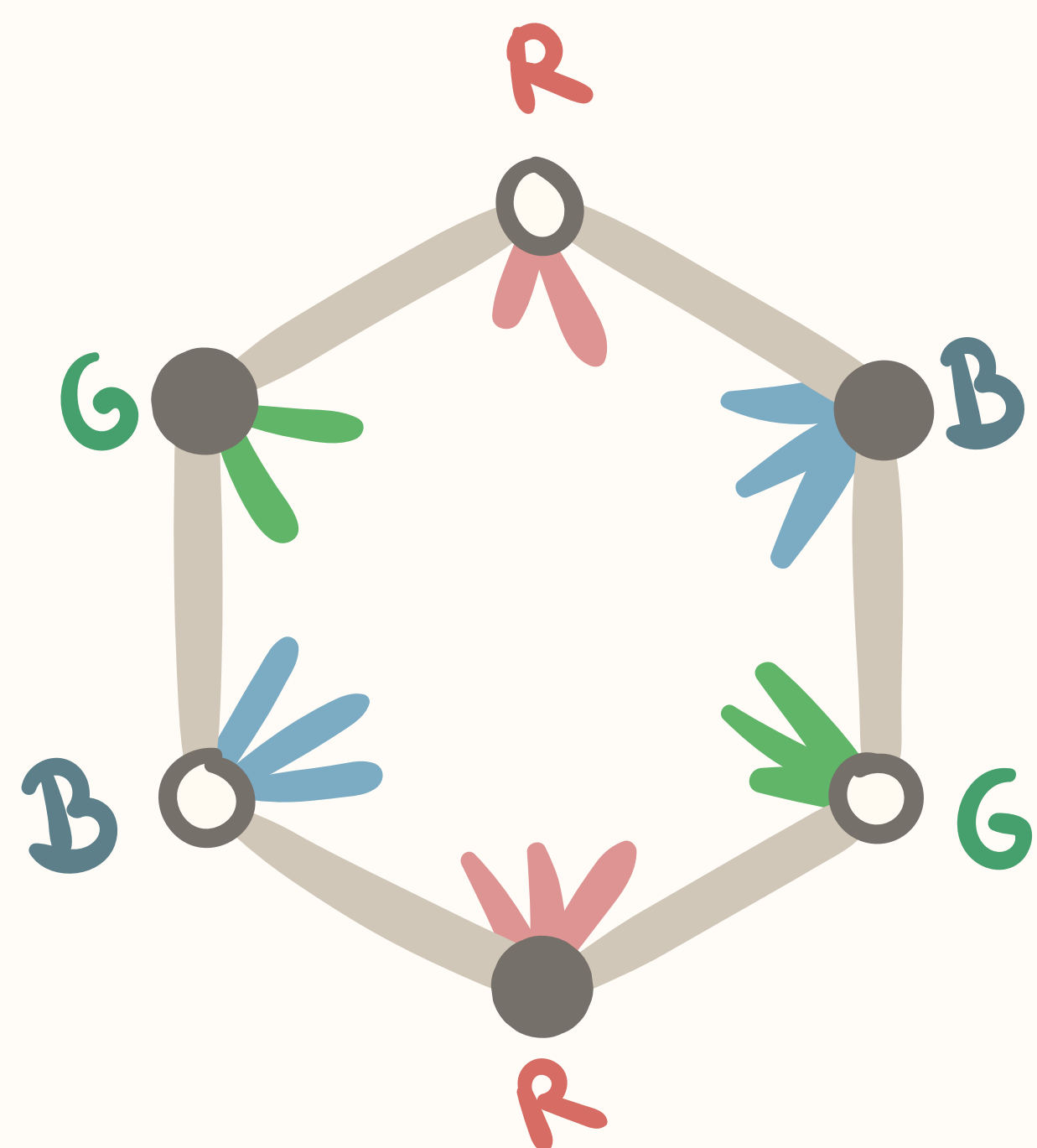
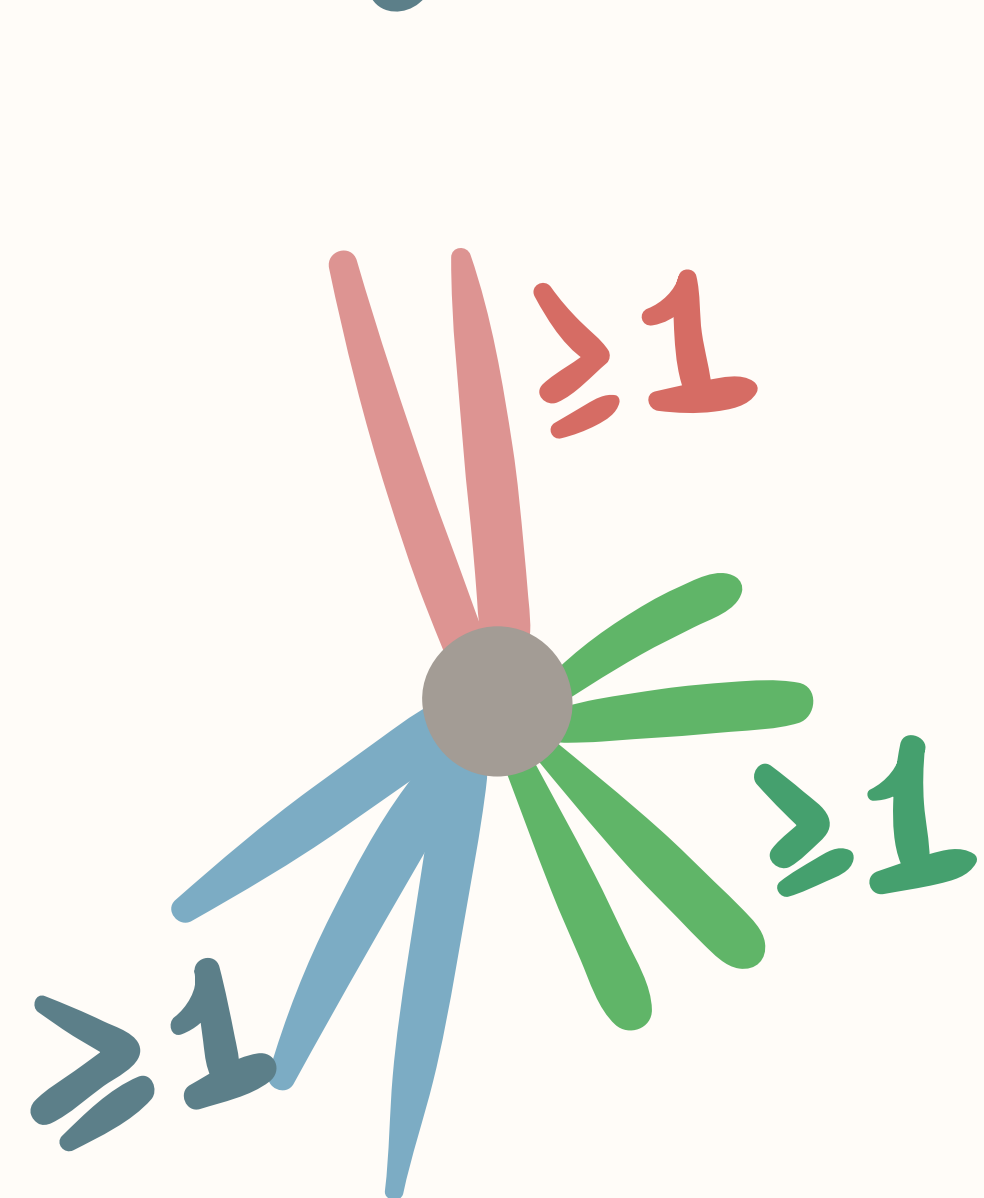
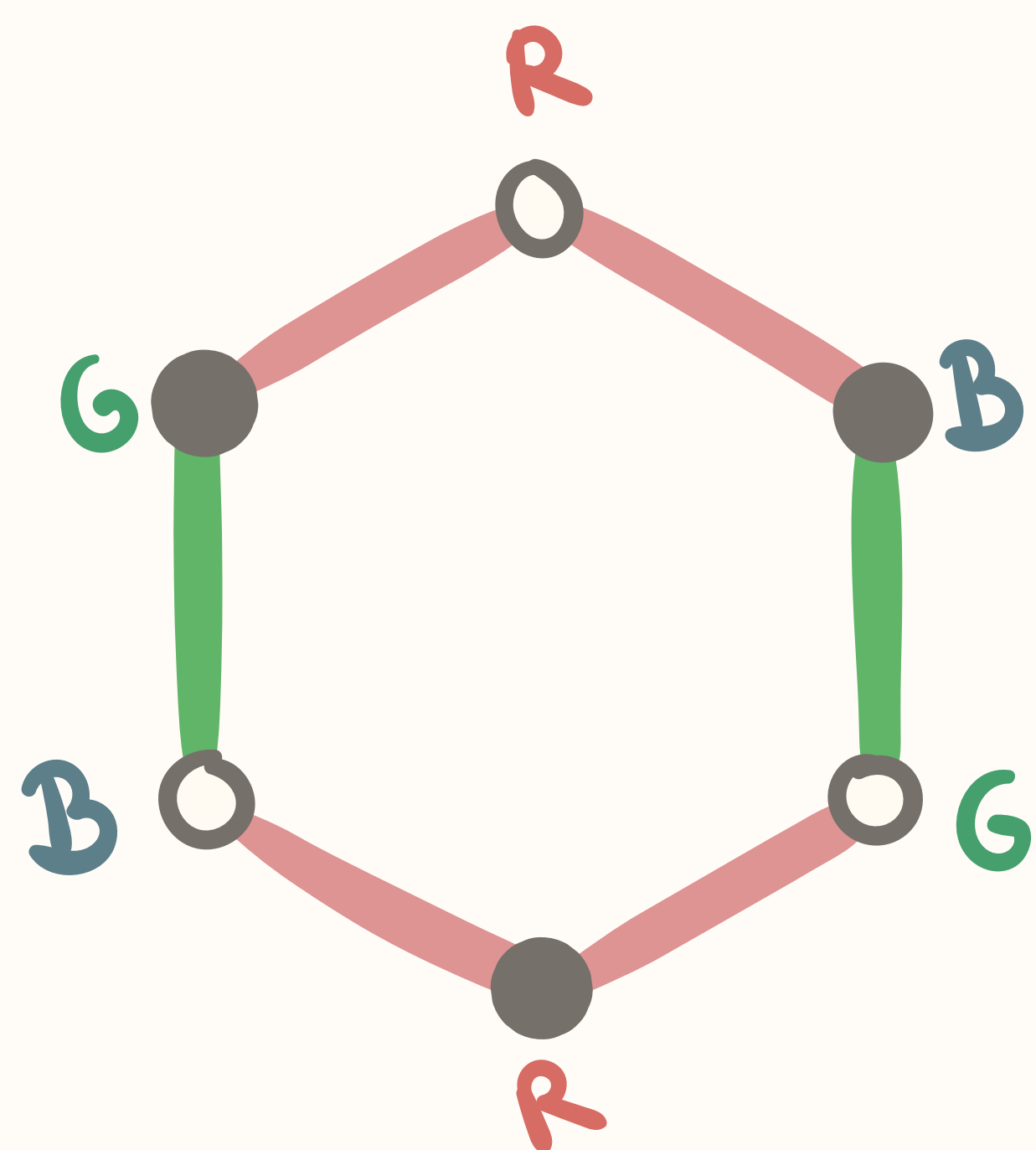
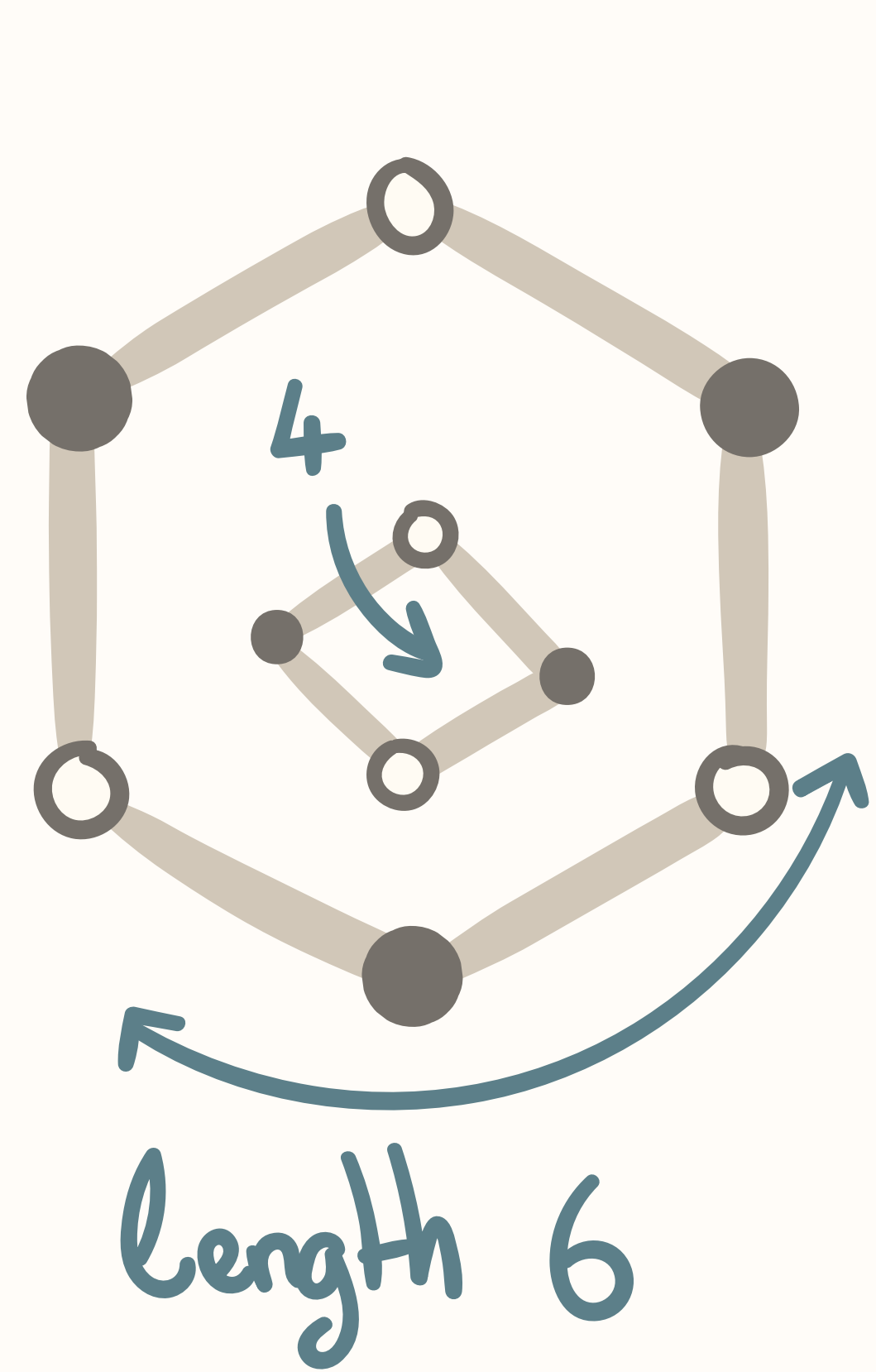
Schnyder labellings



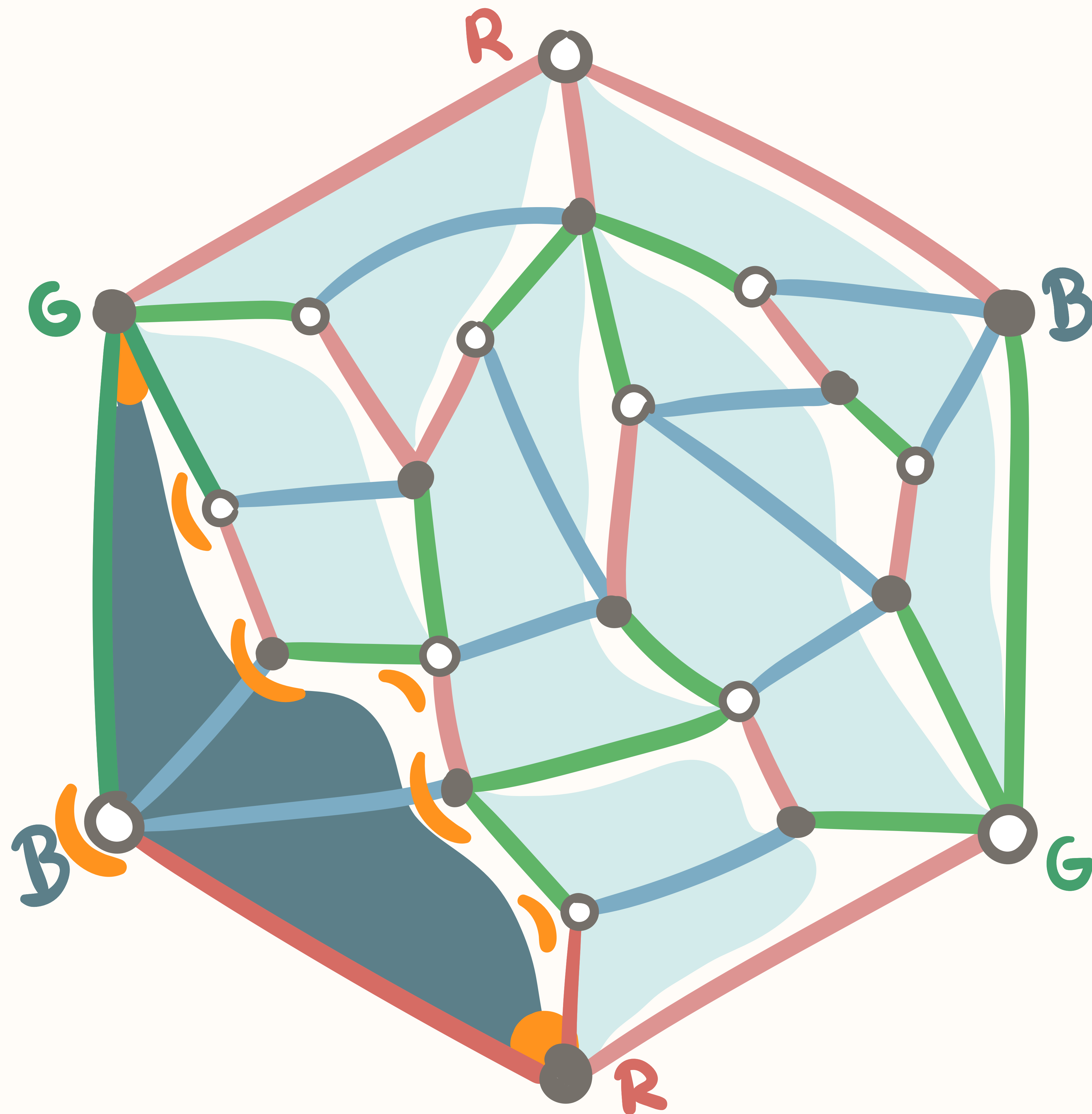
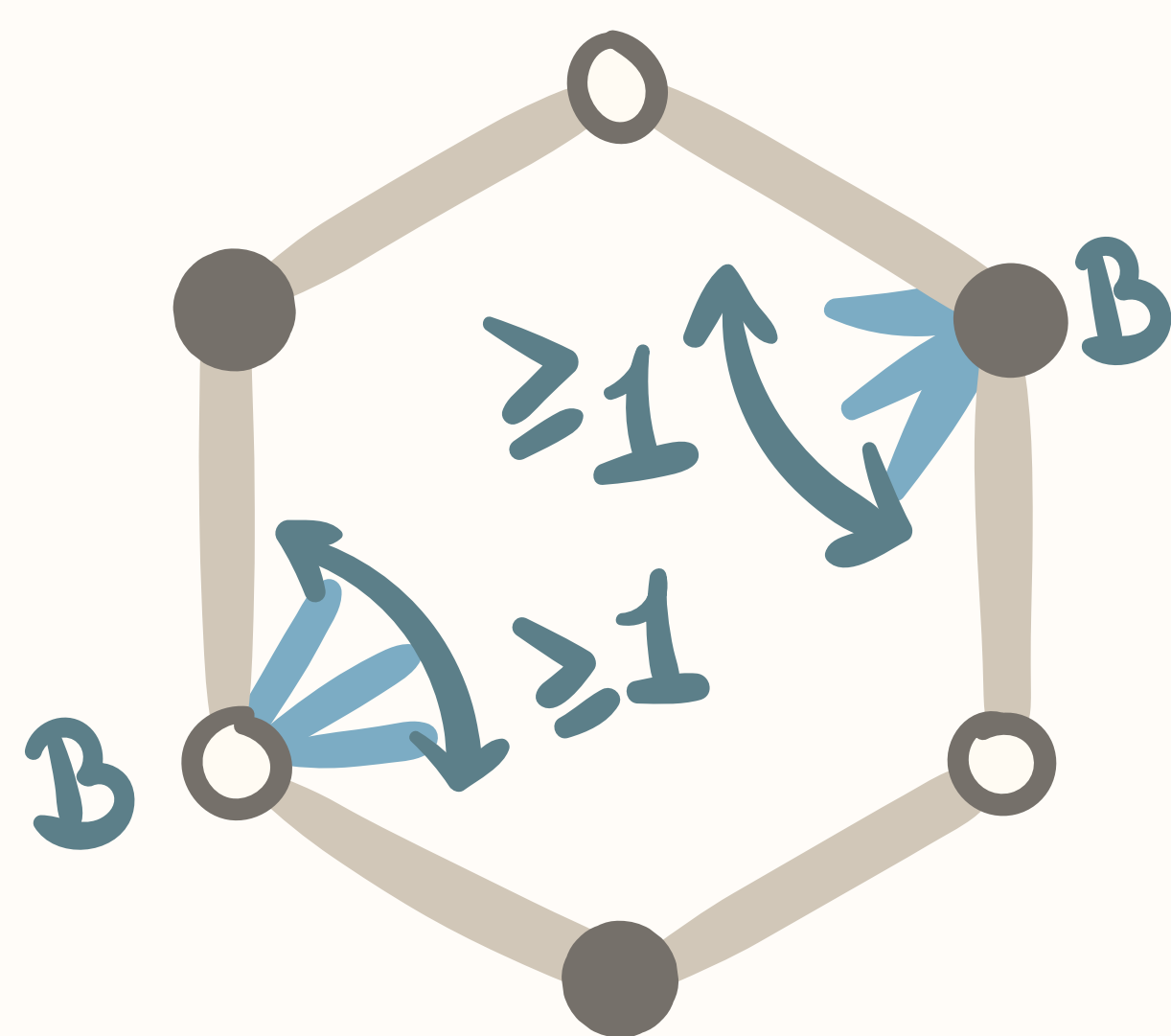
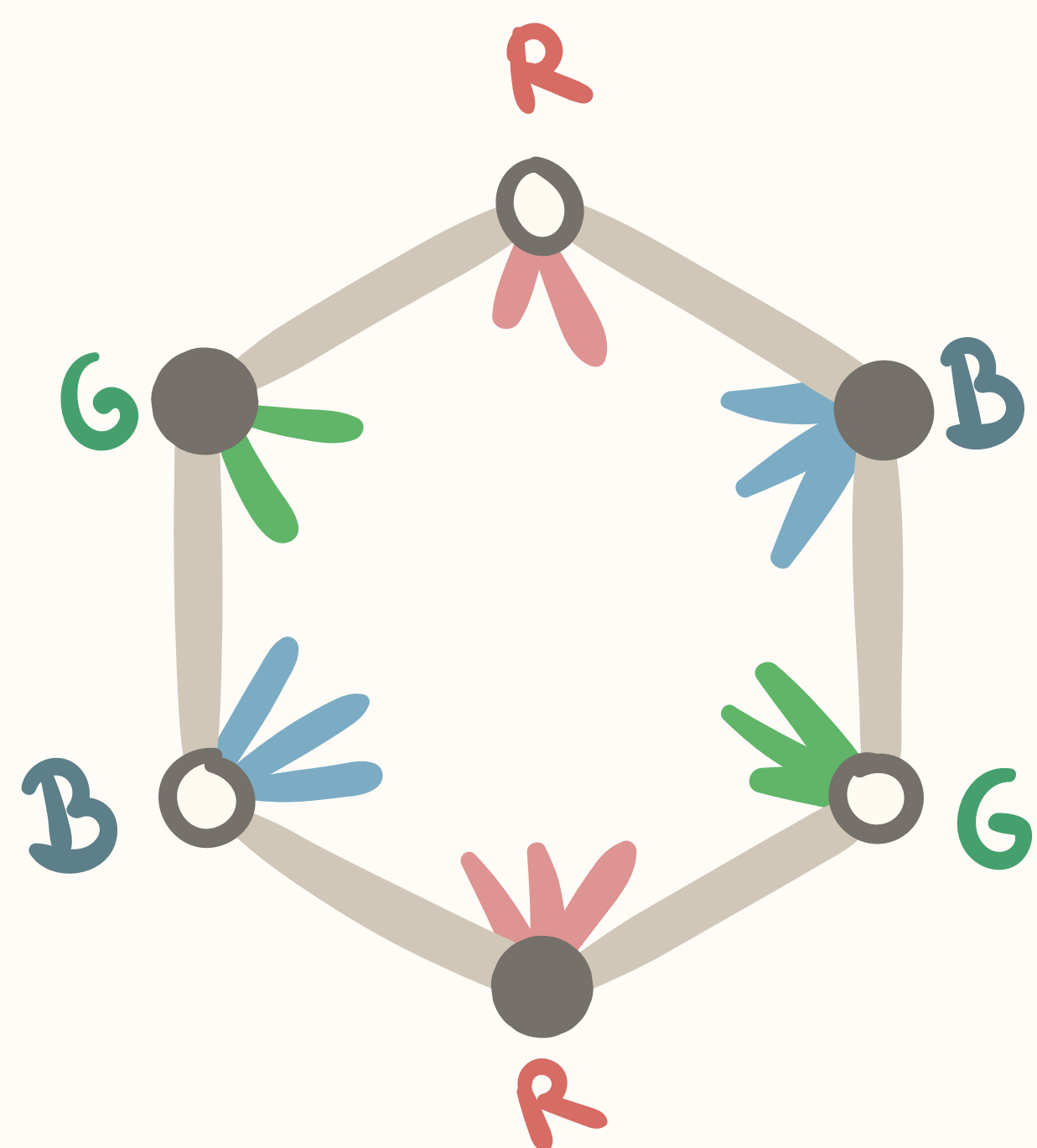
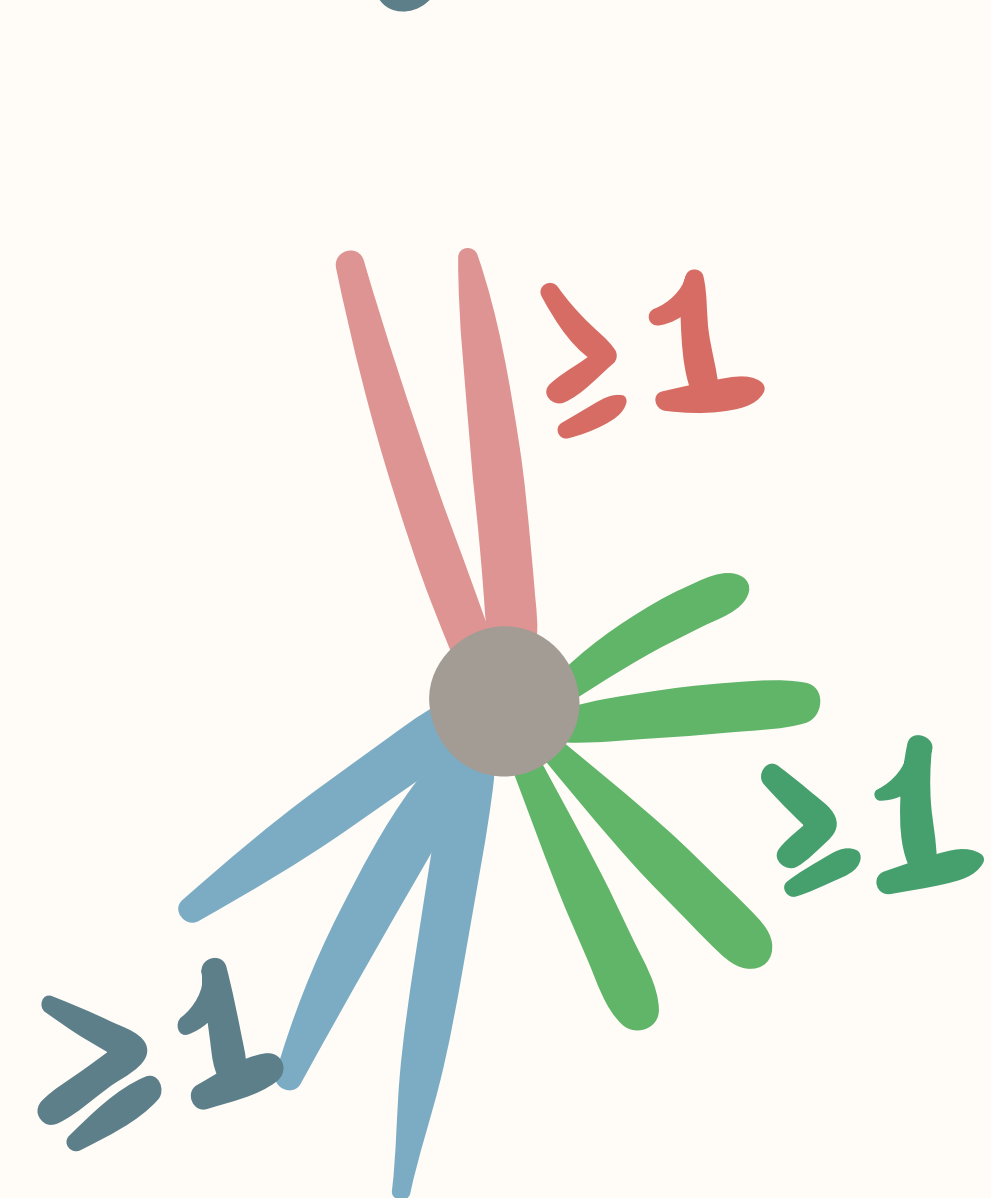
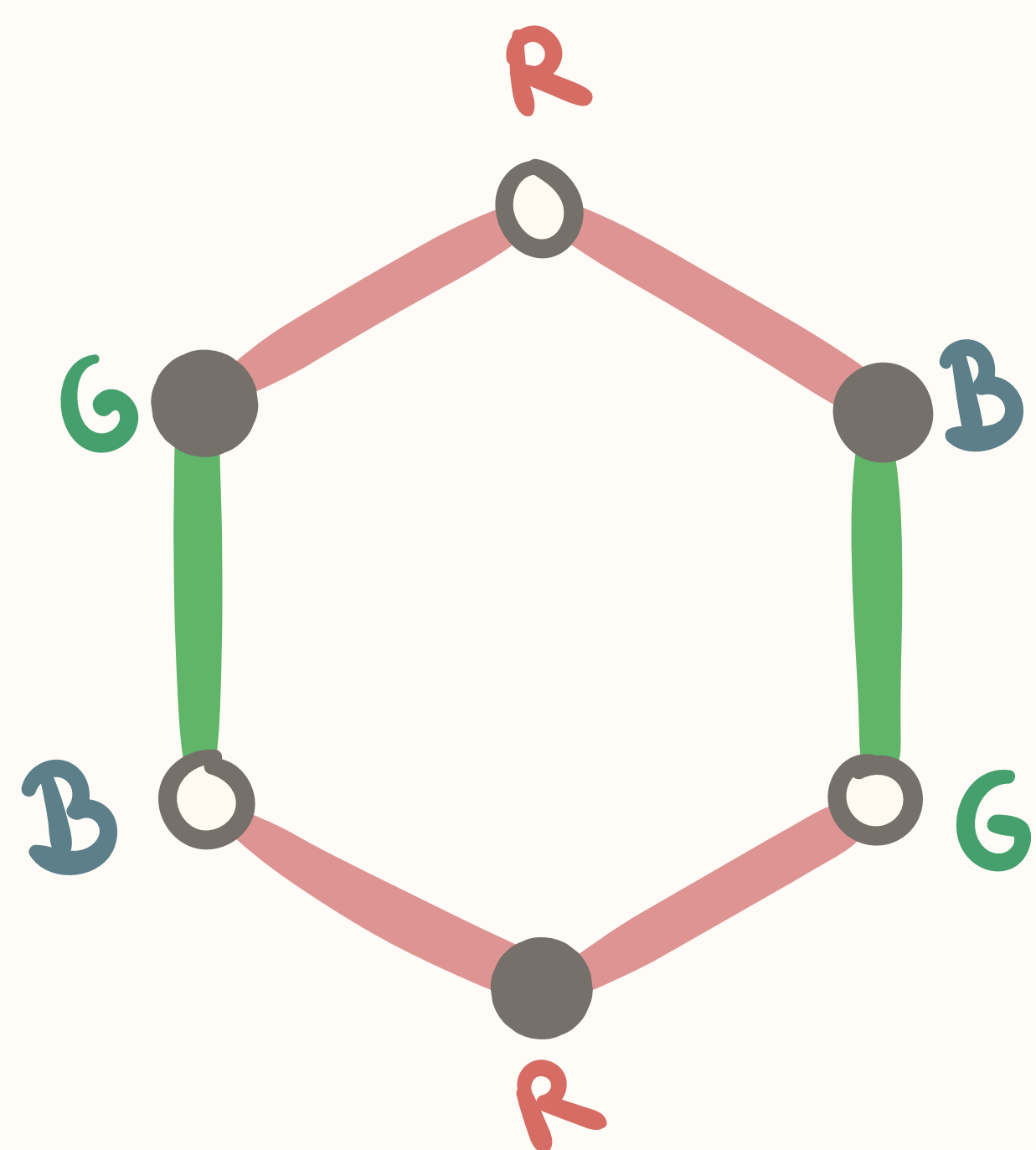
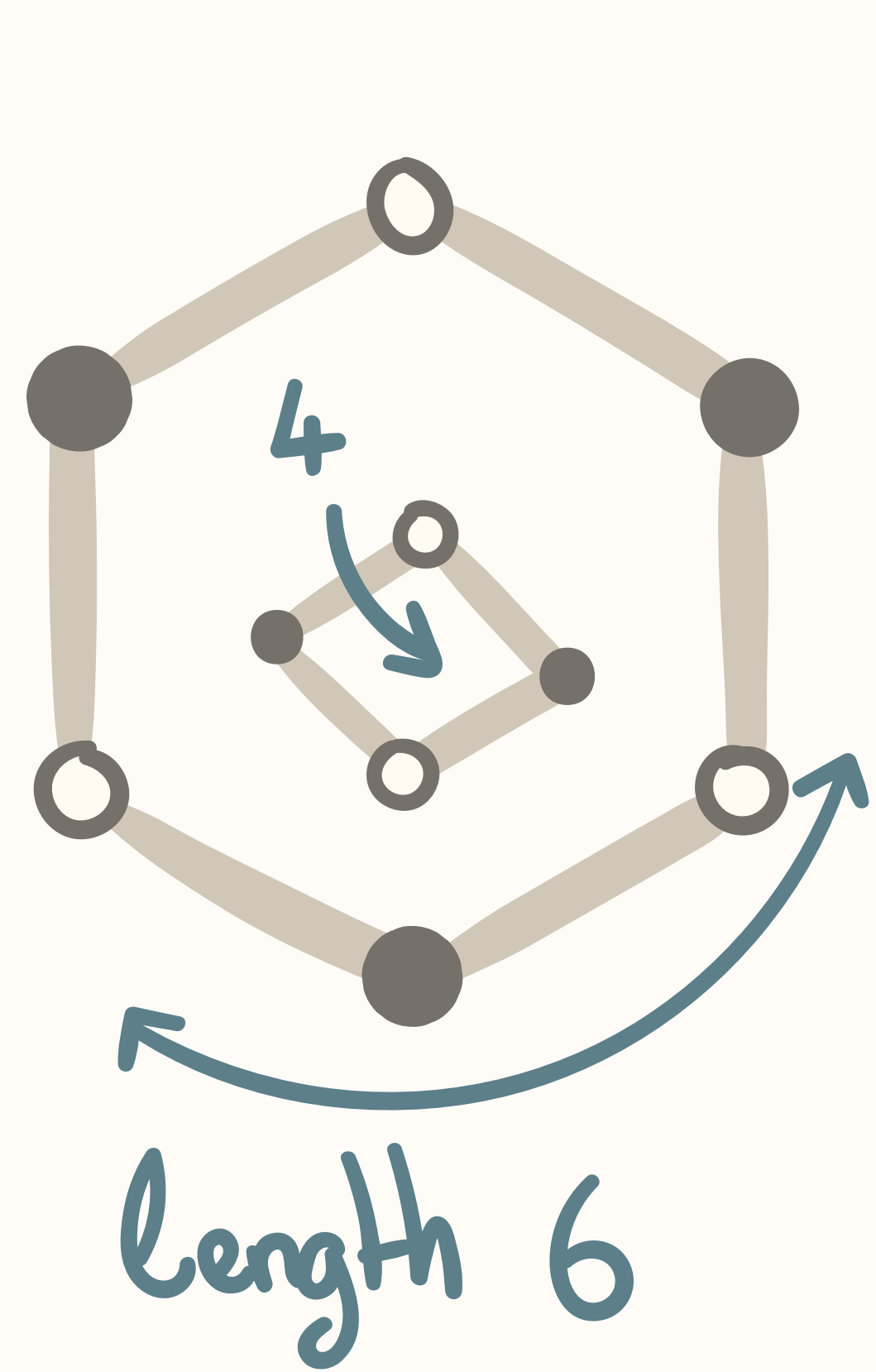
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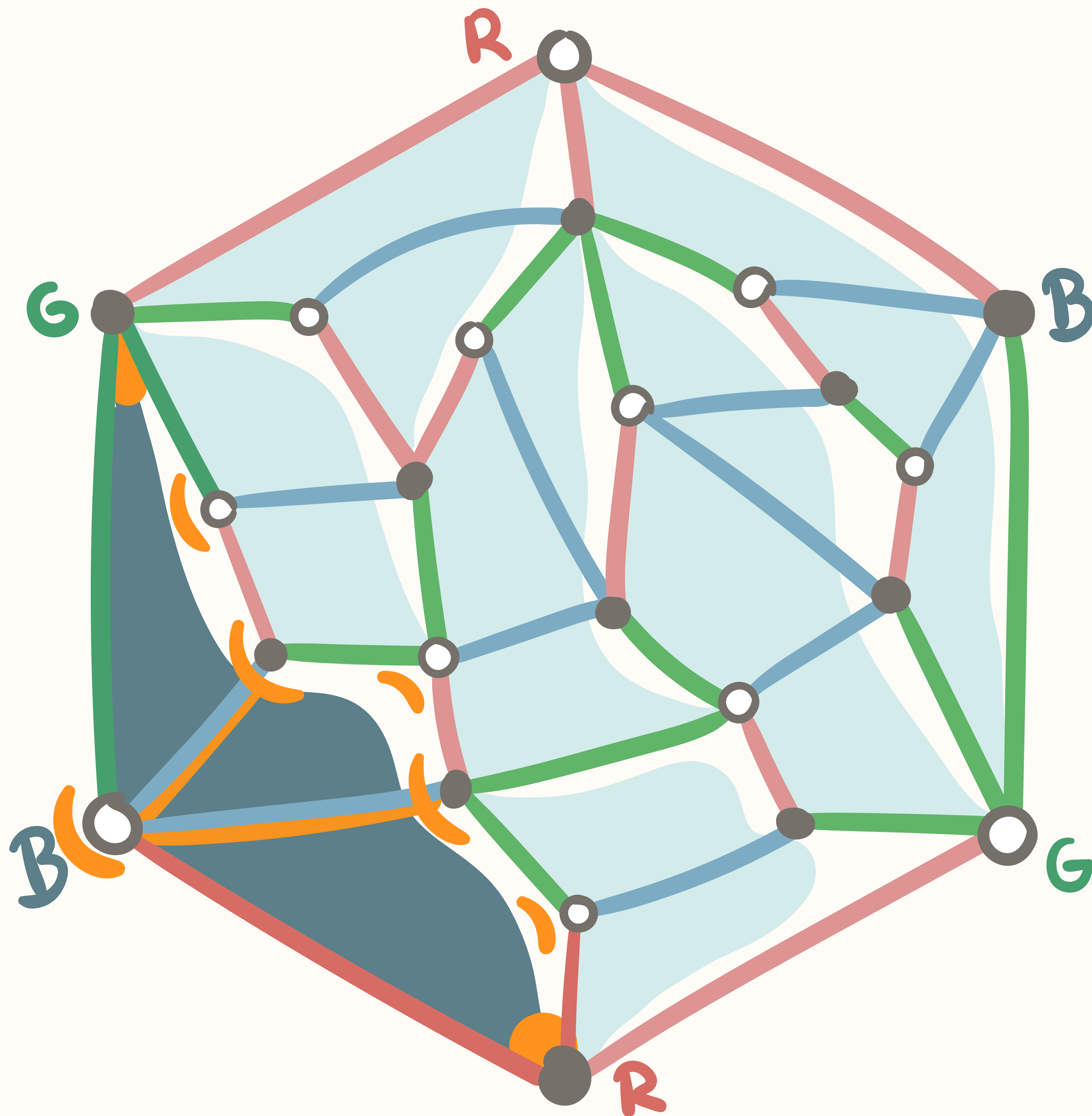
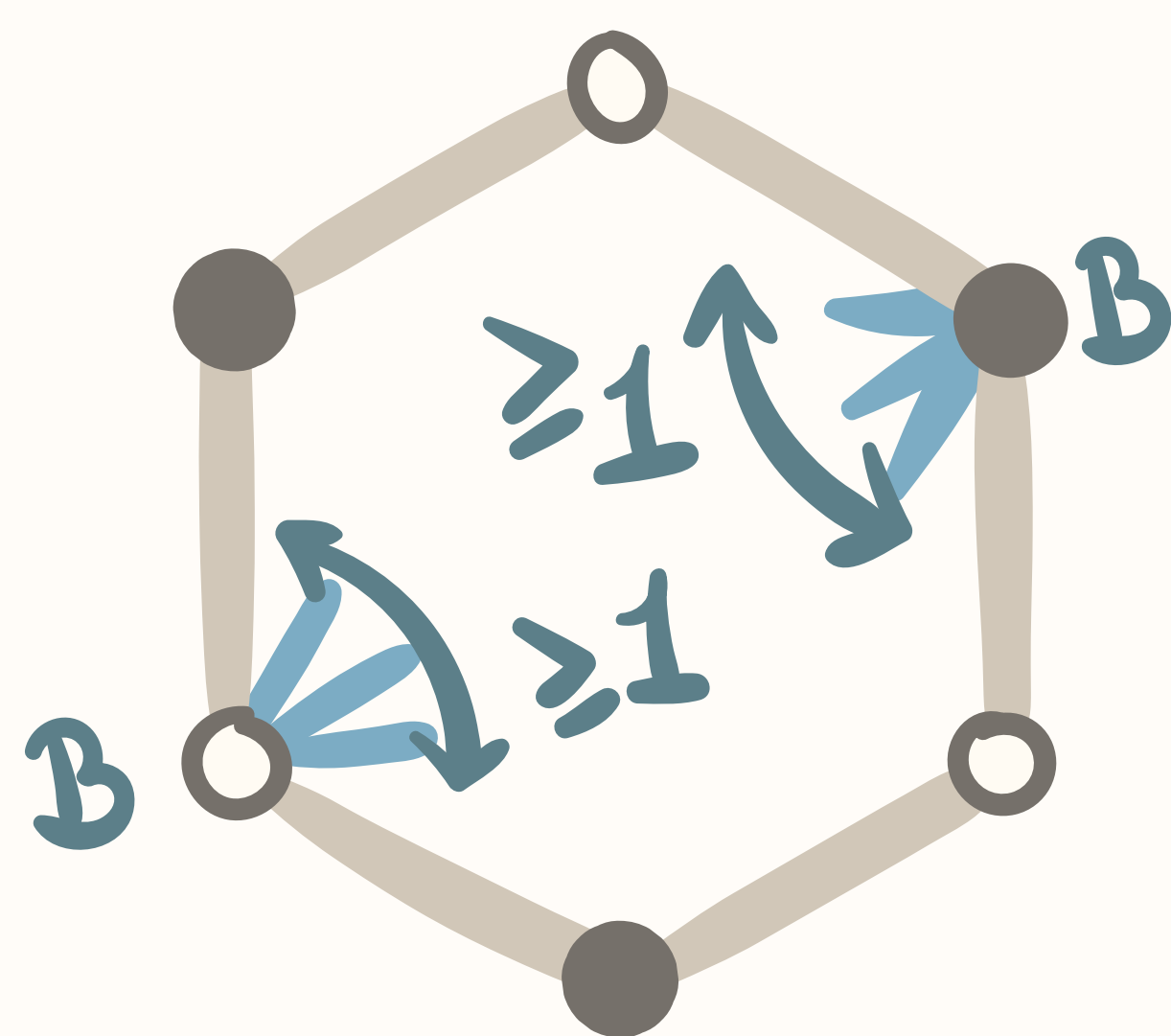
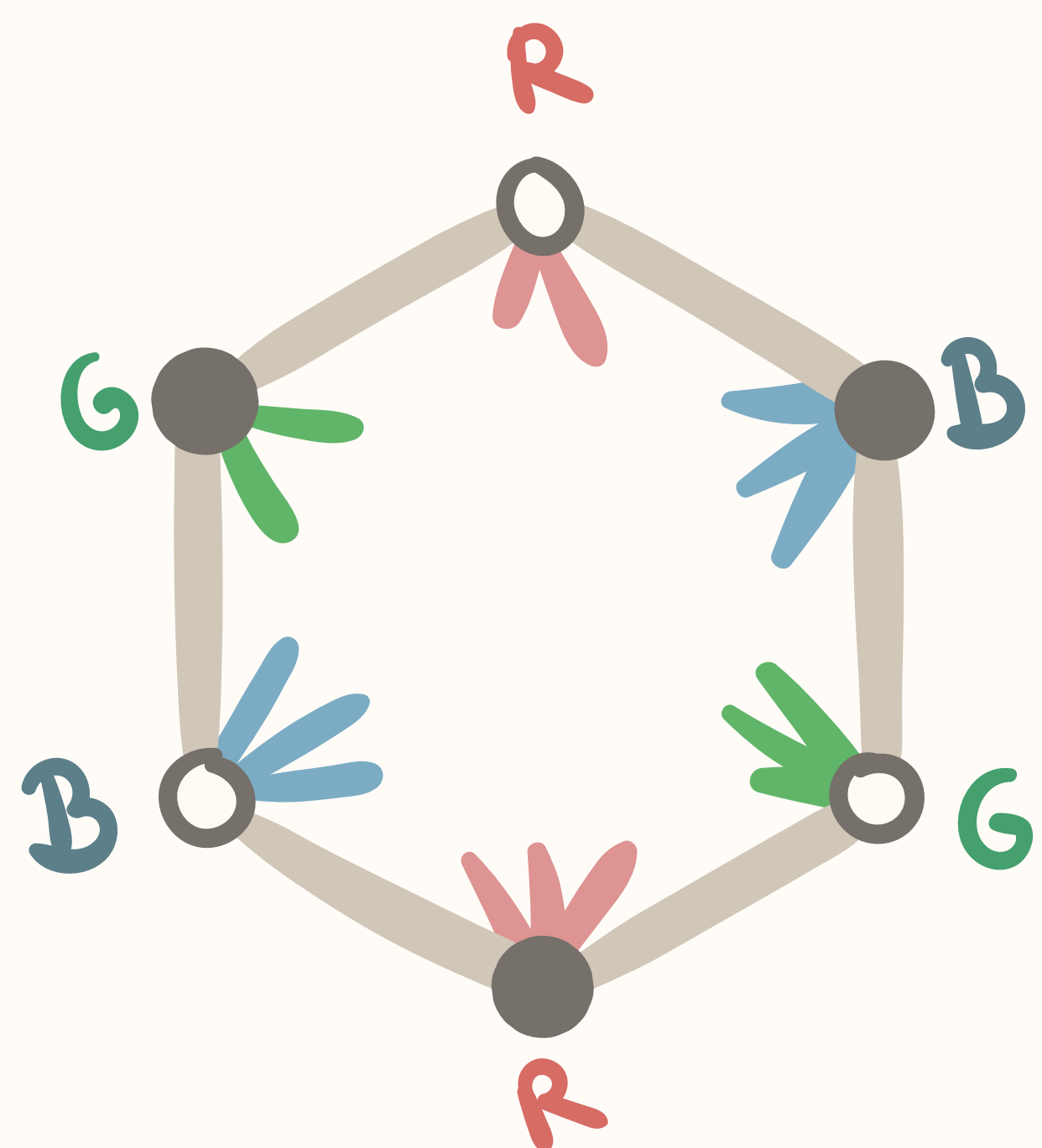
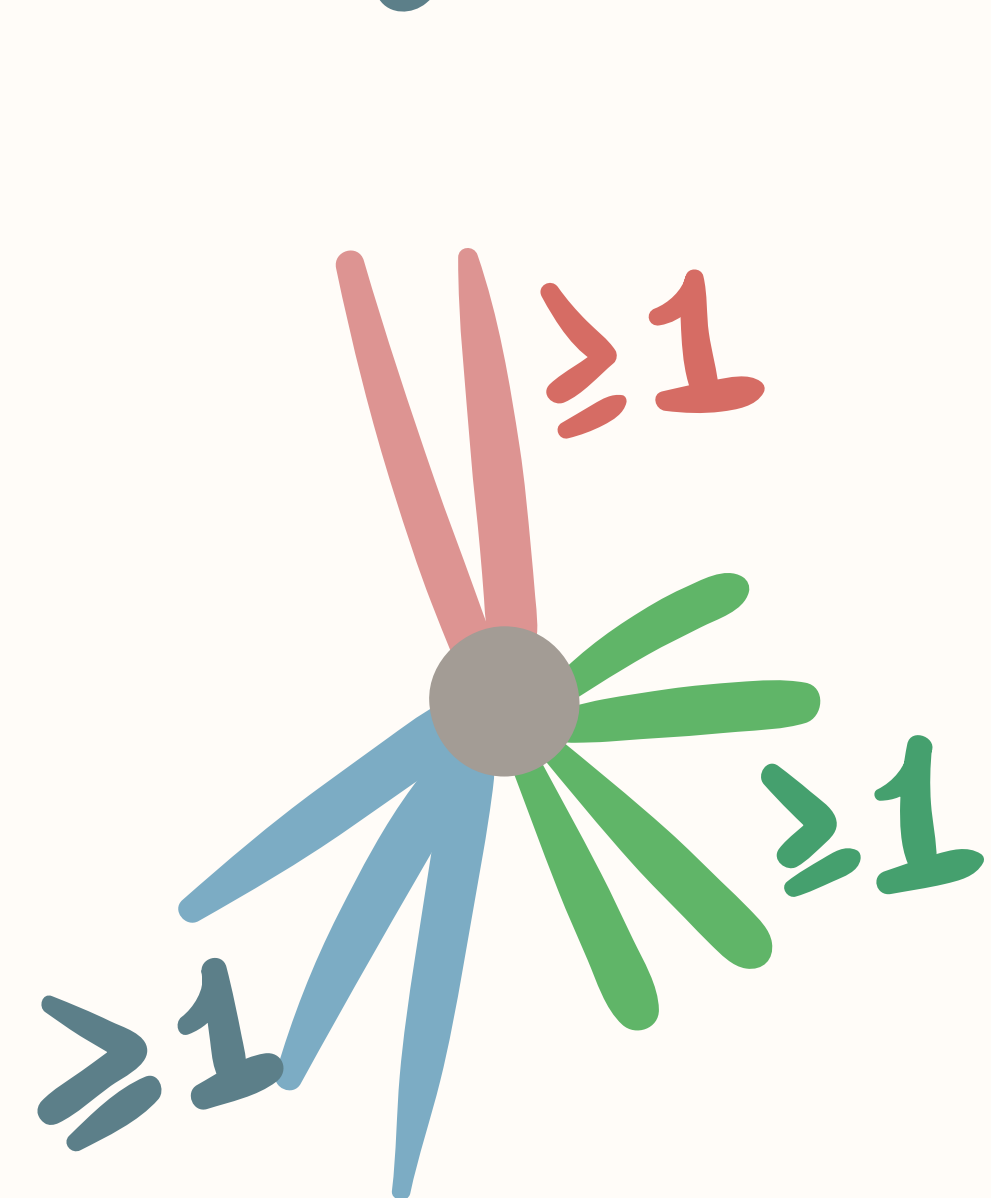
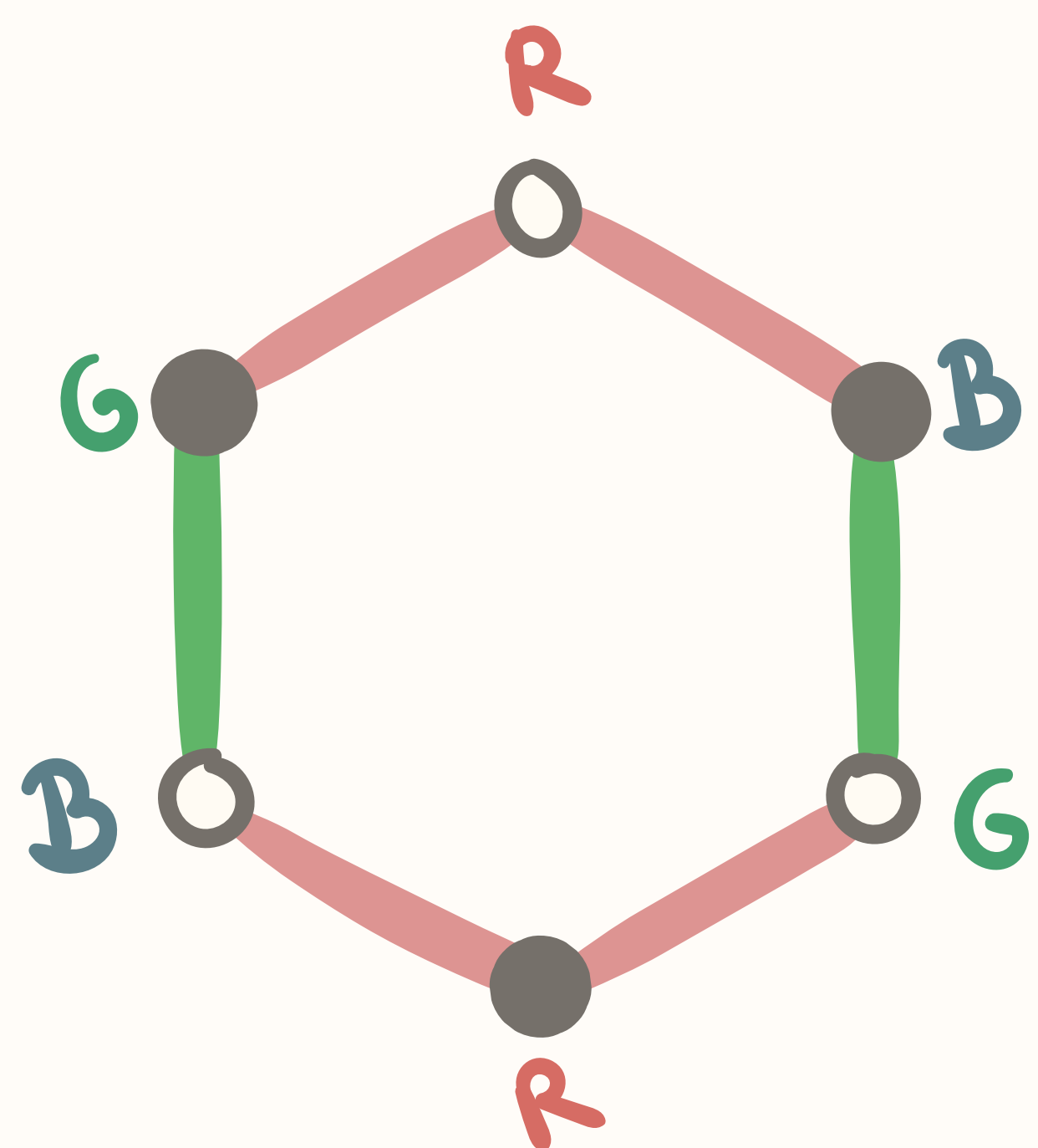
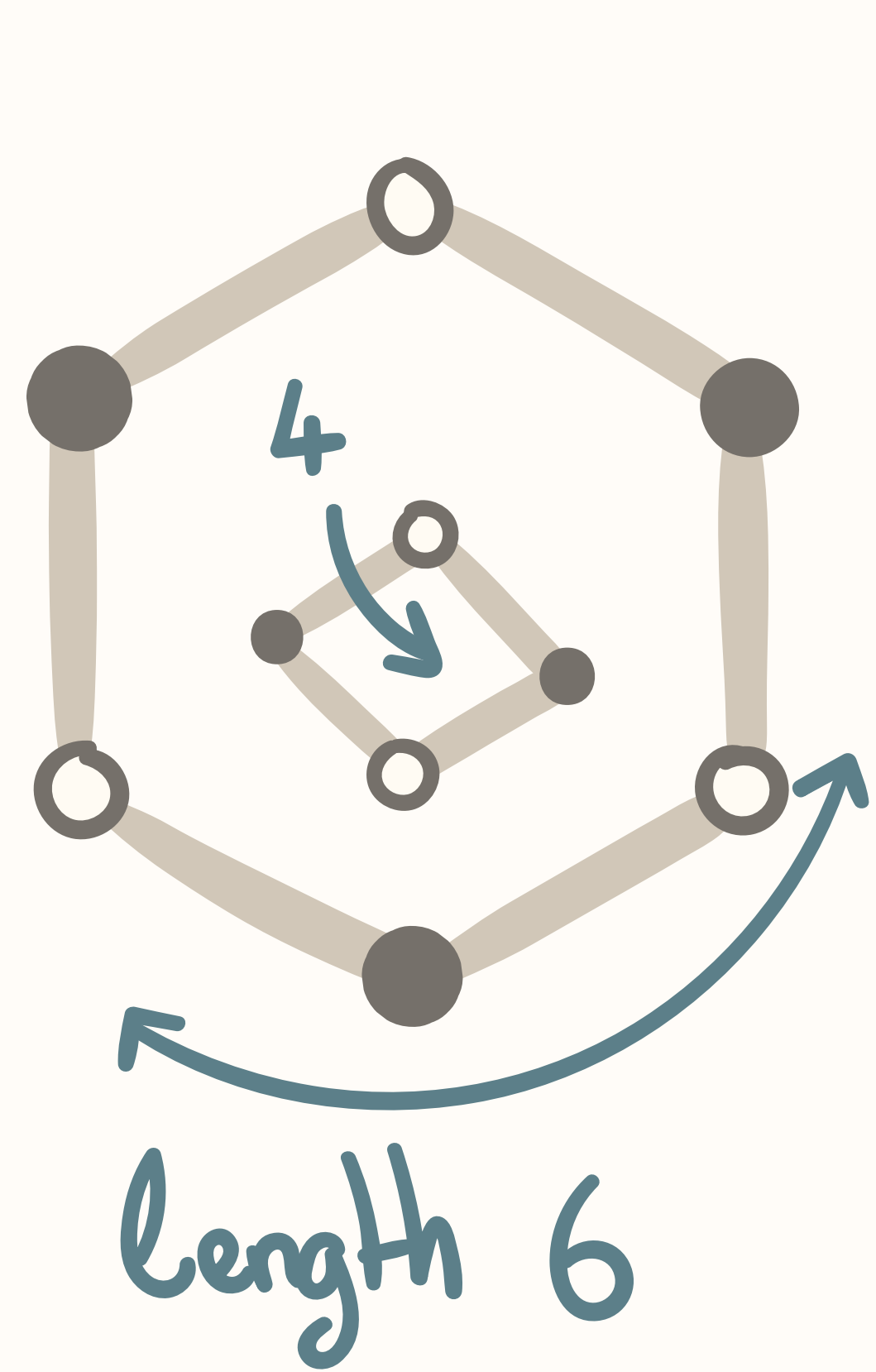
Schnyder labellings



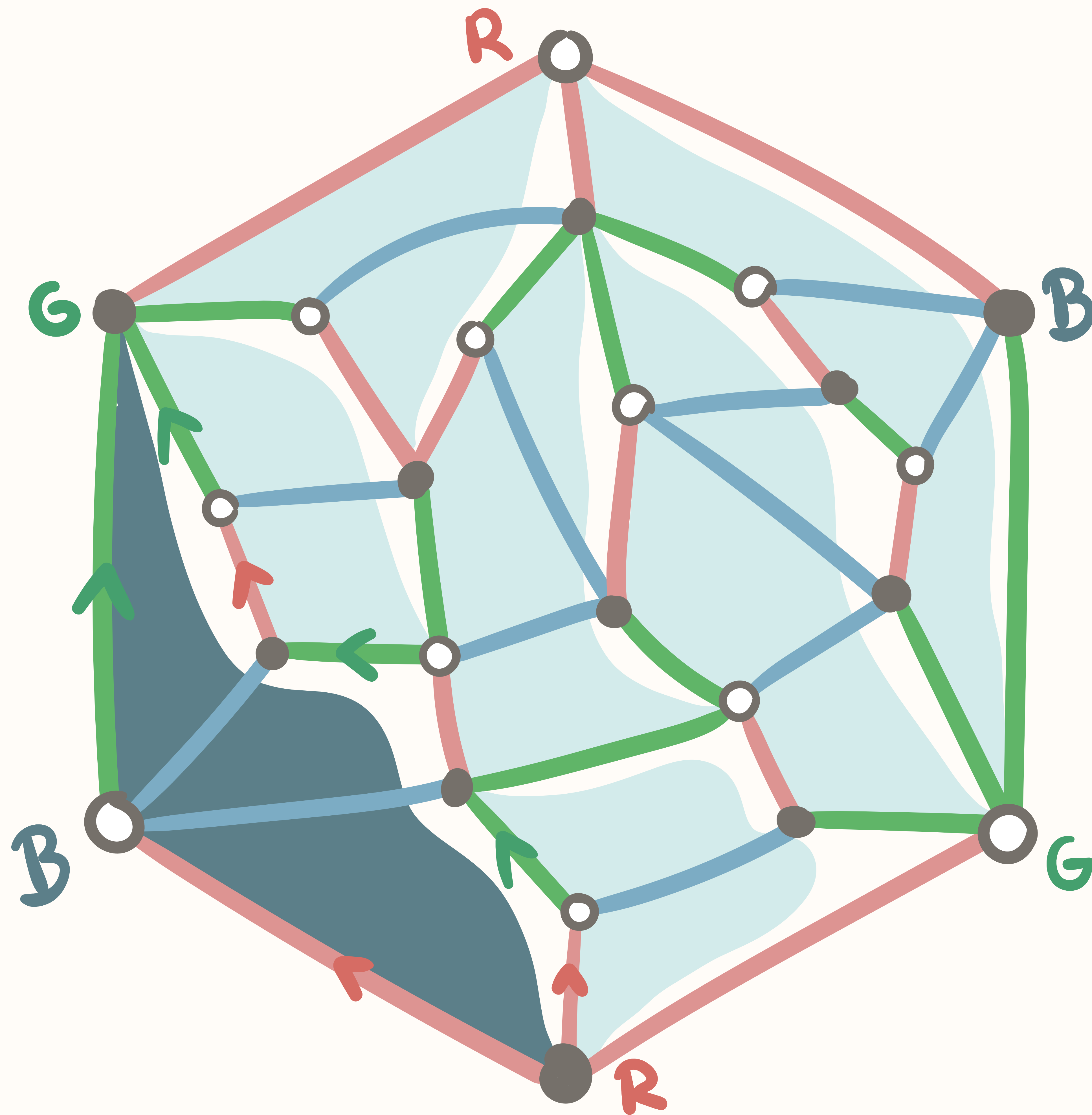
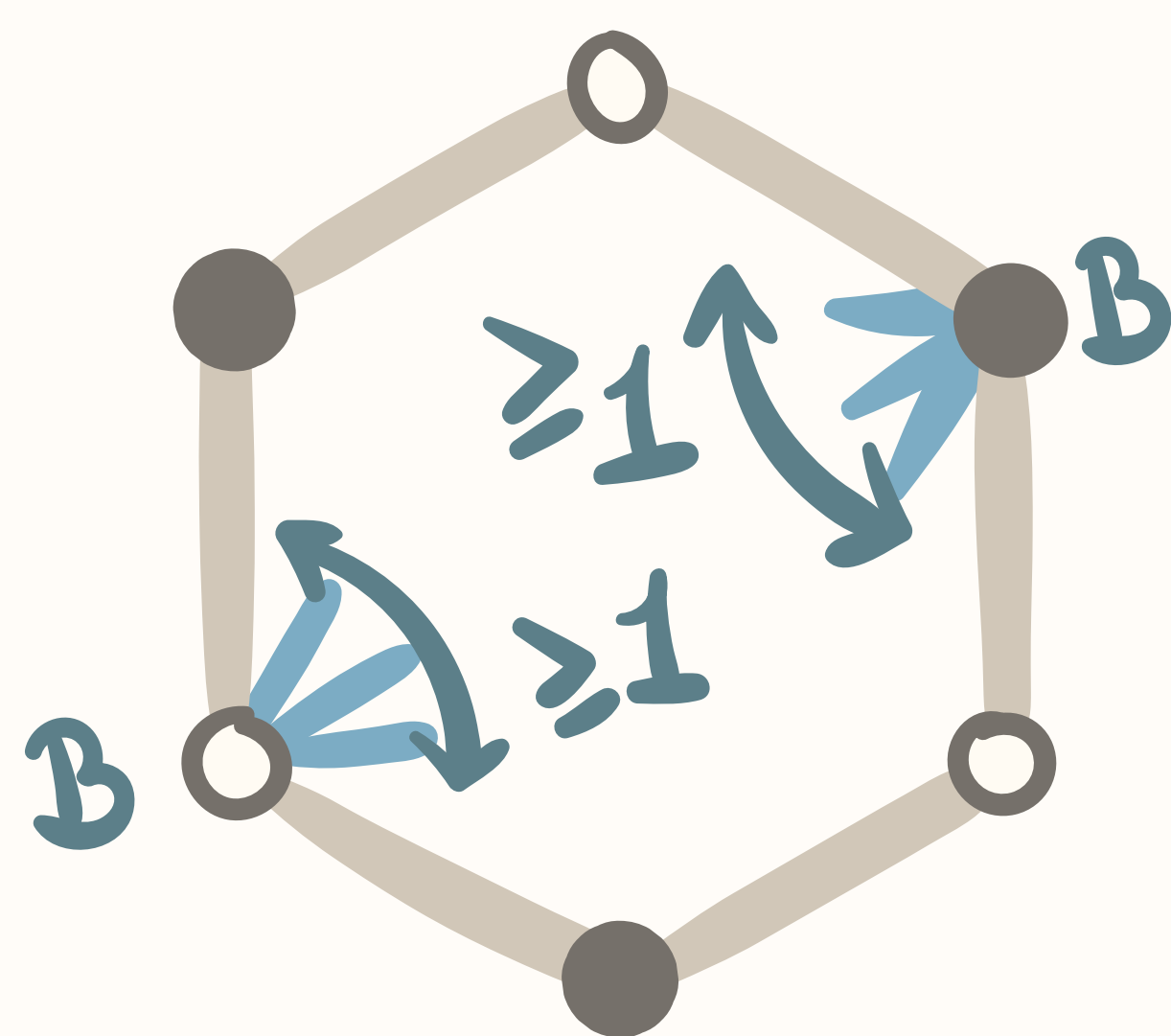
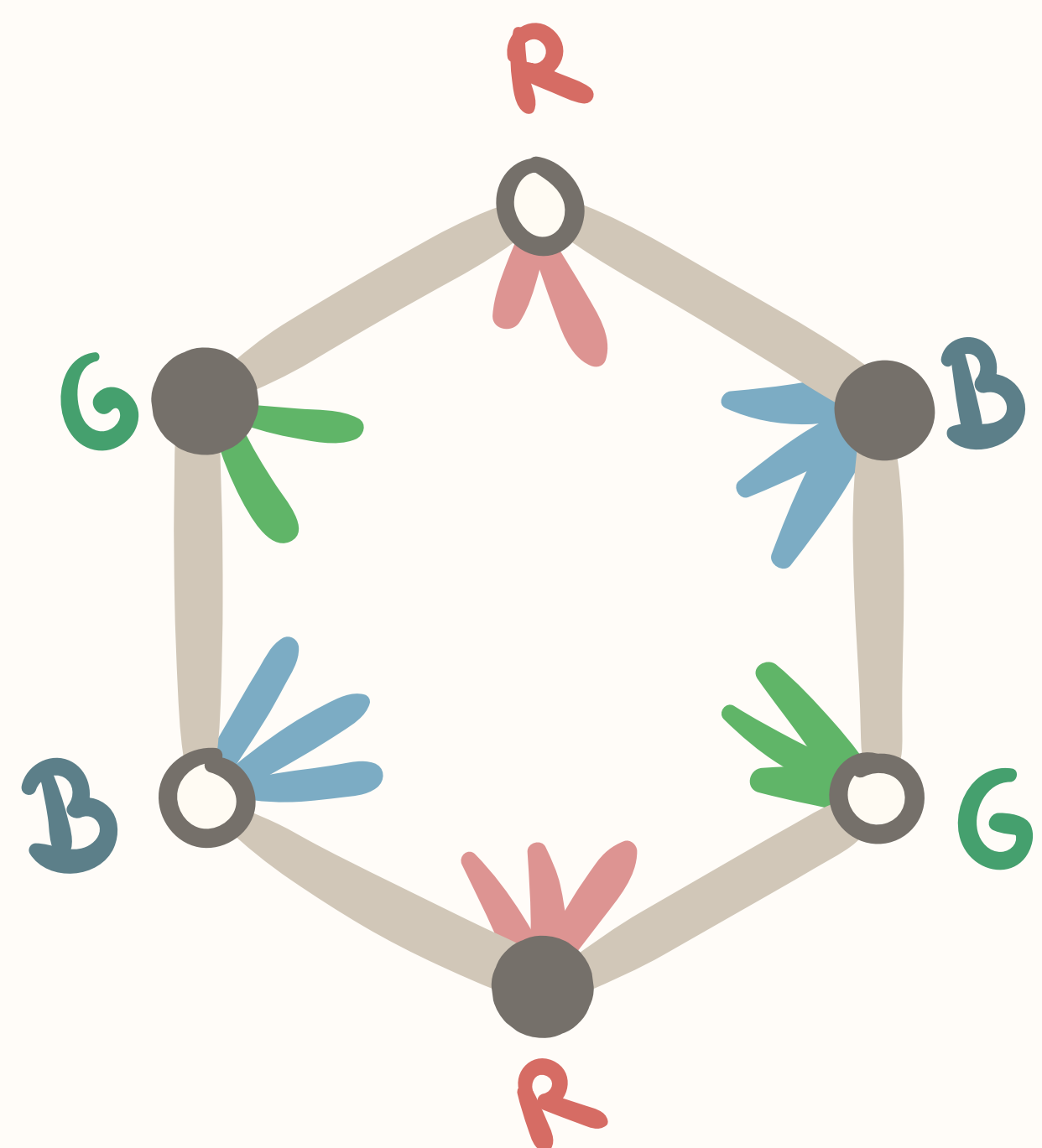
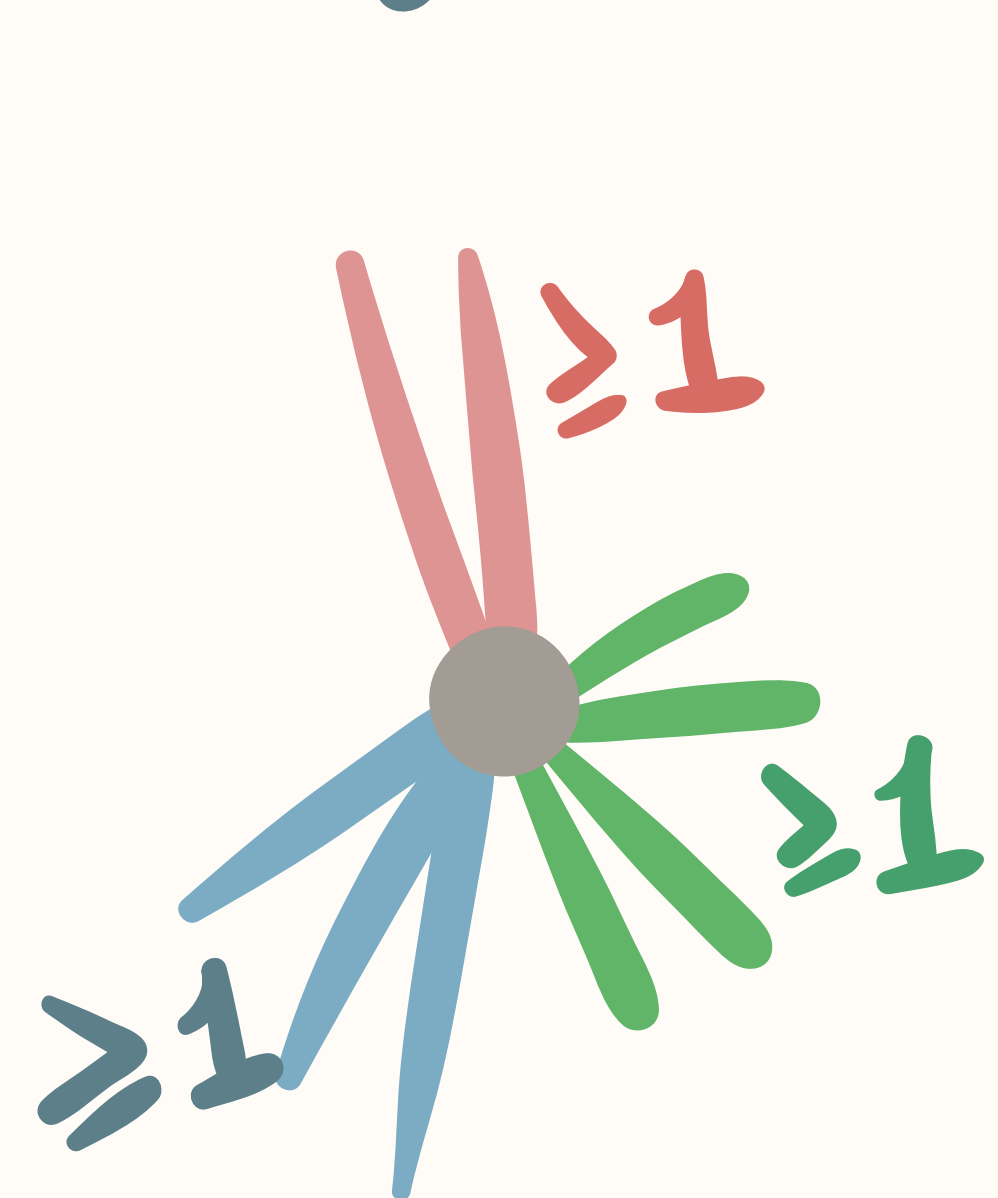
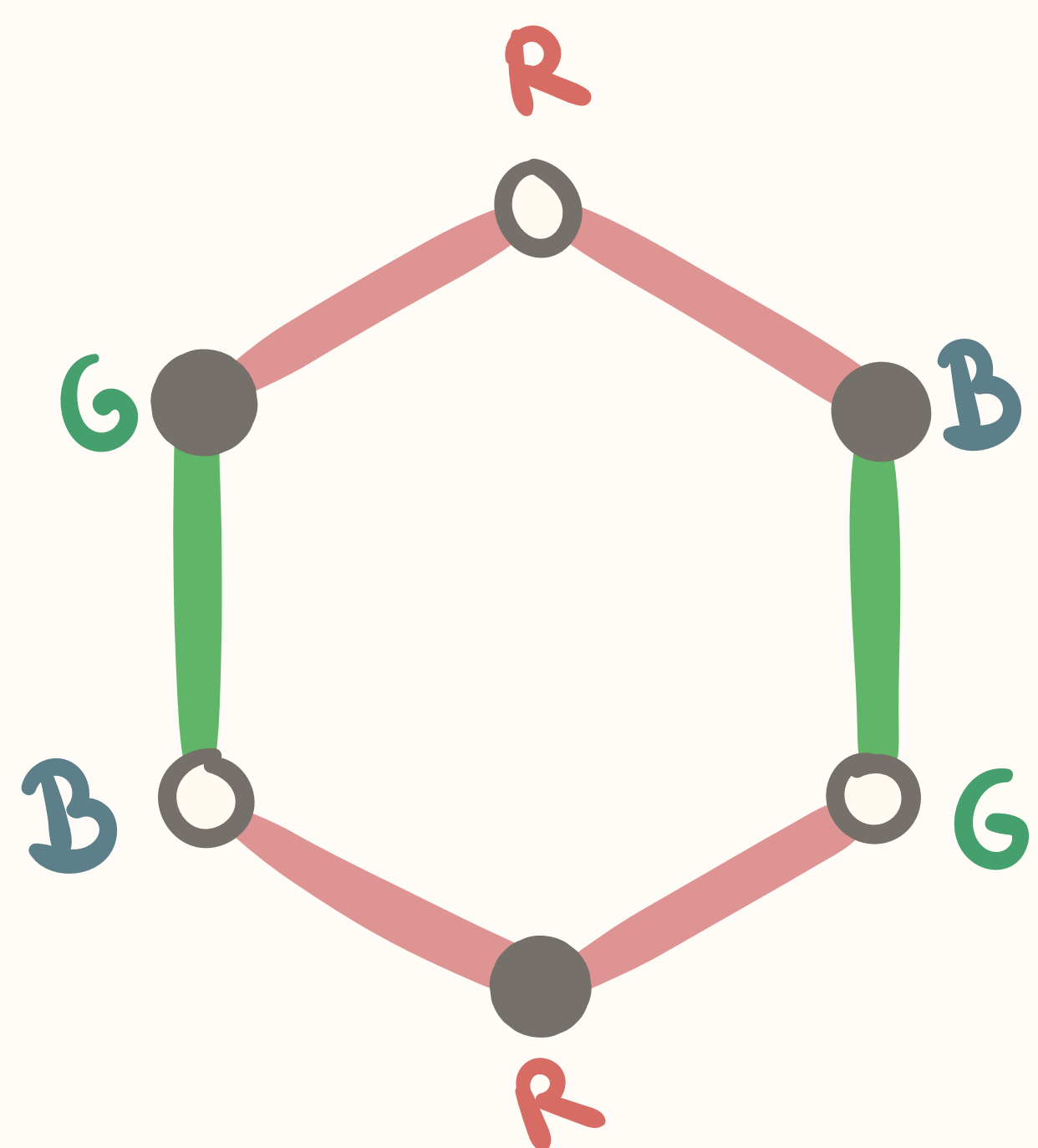
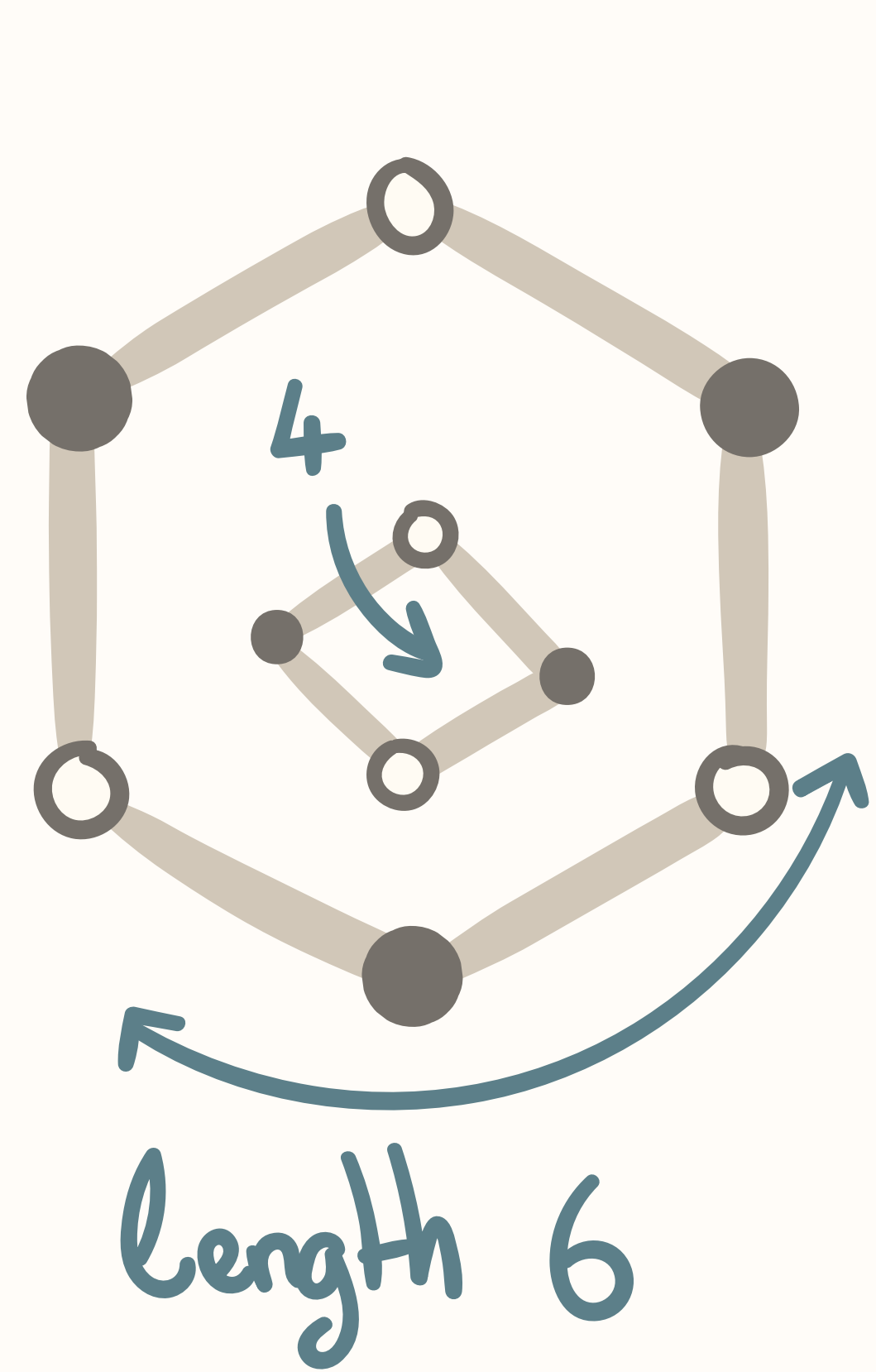
Schnyder labellings



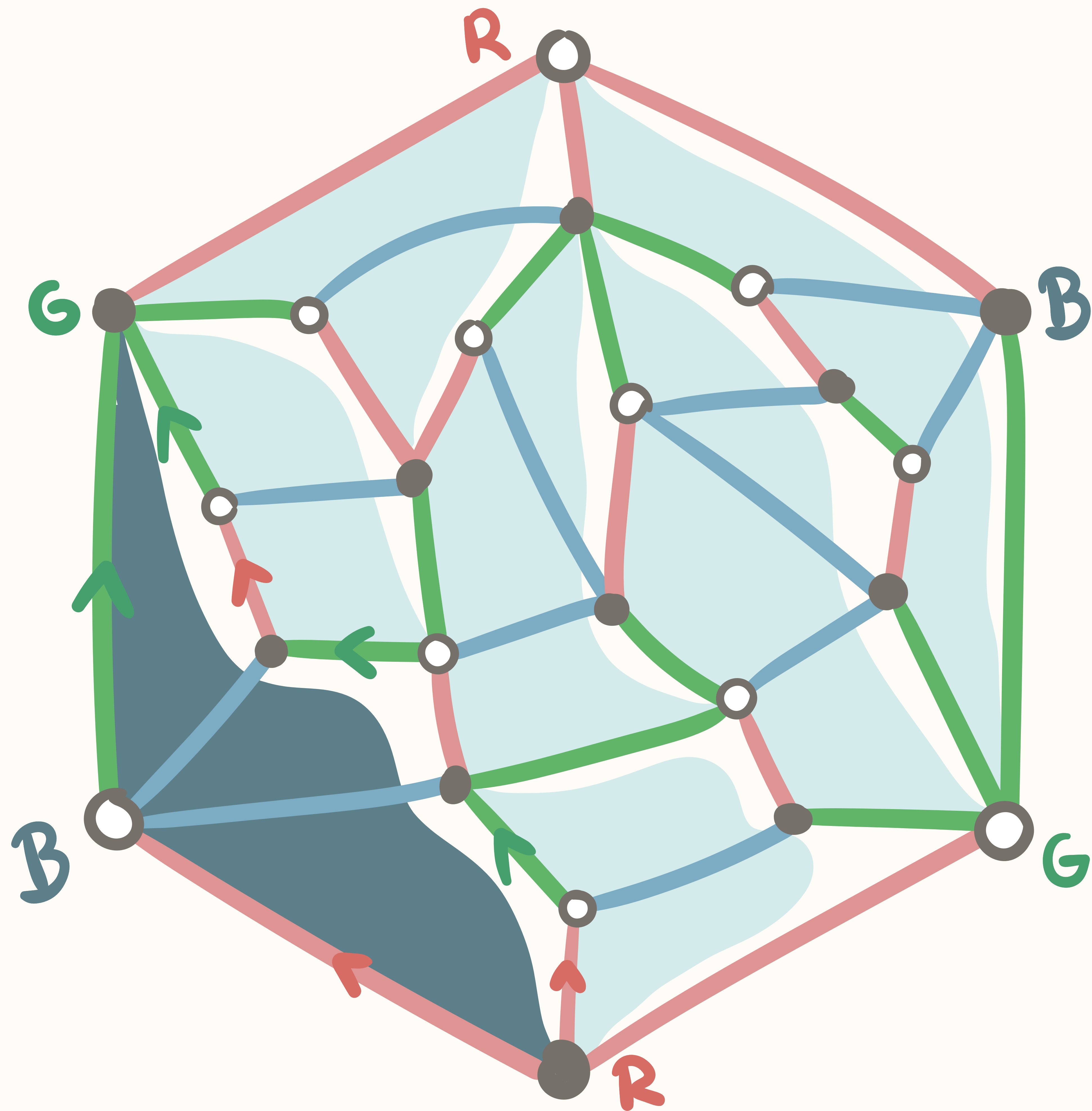
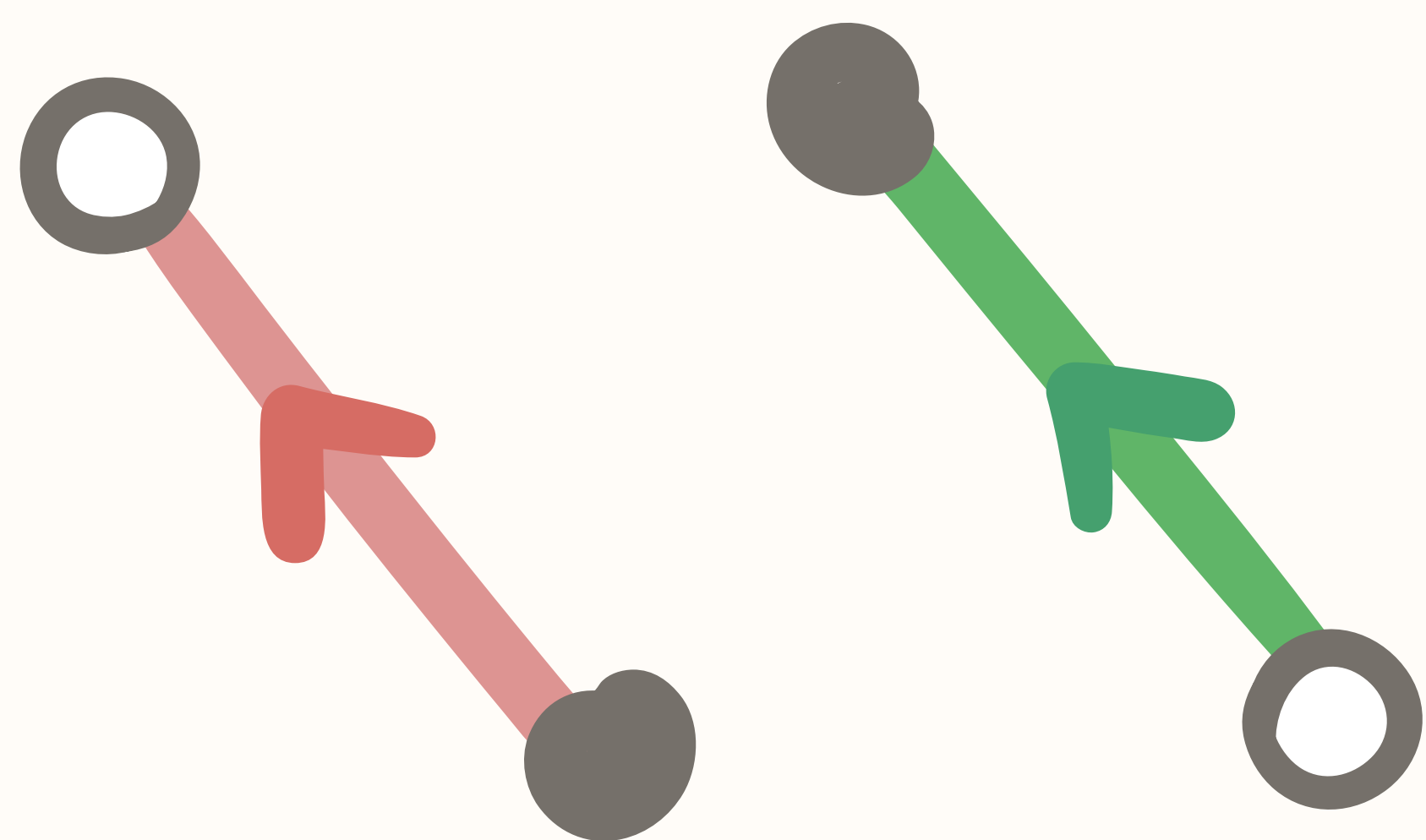
Schnyder labellings



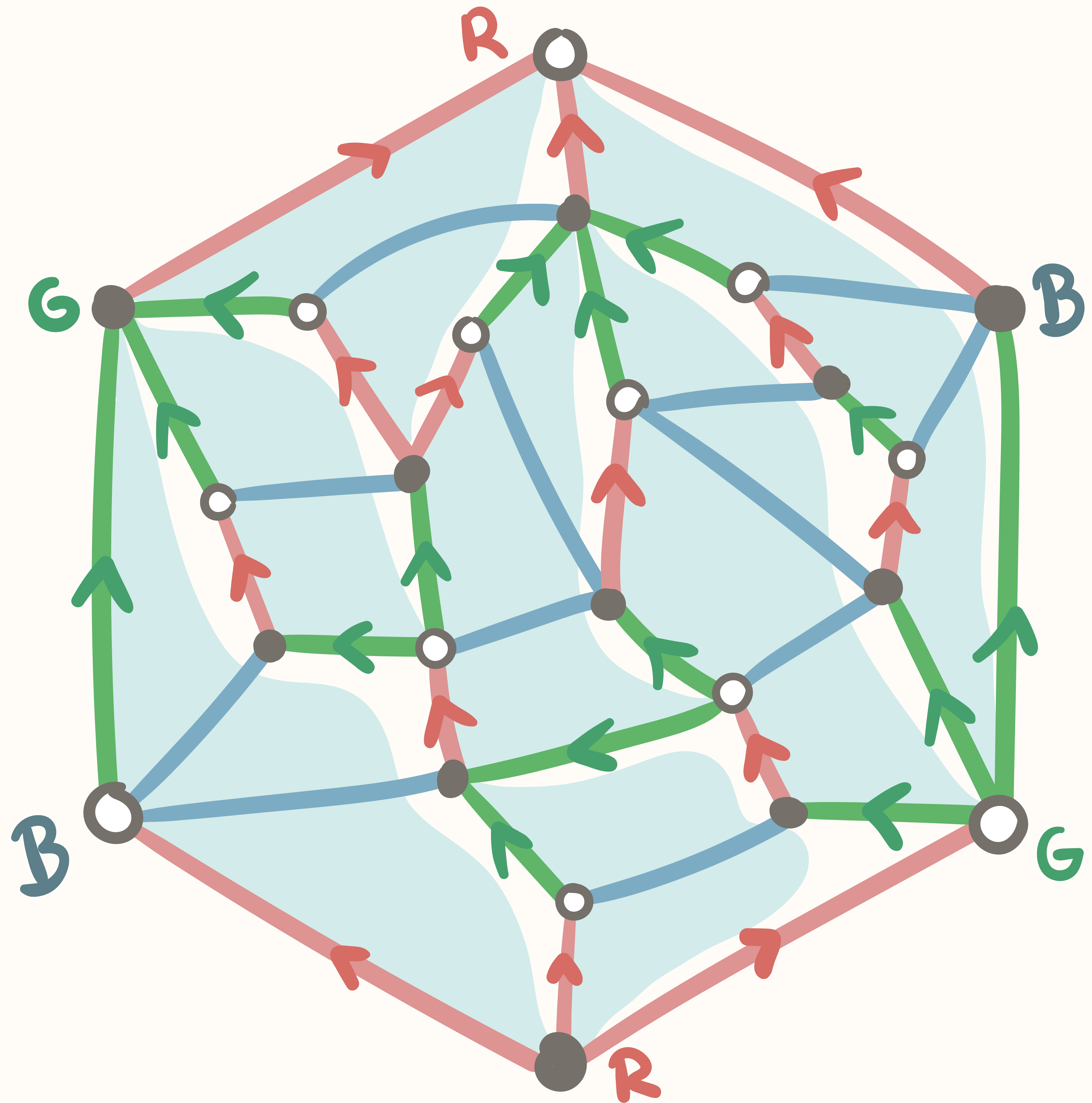
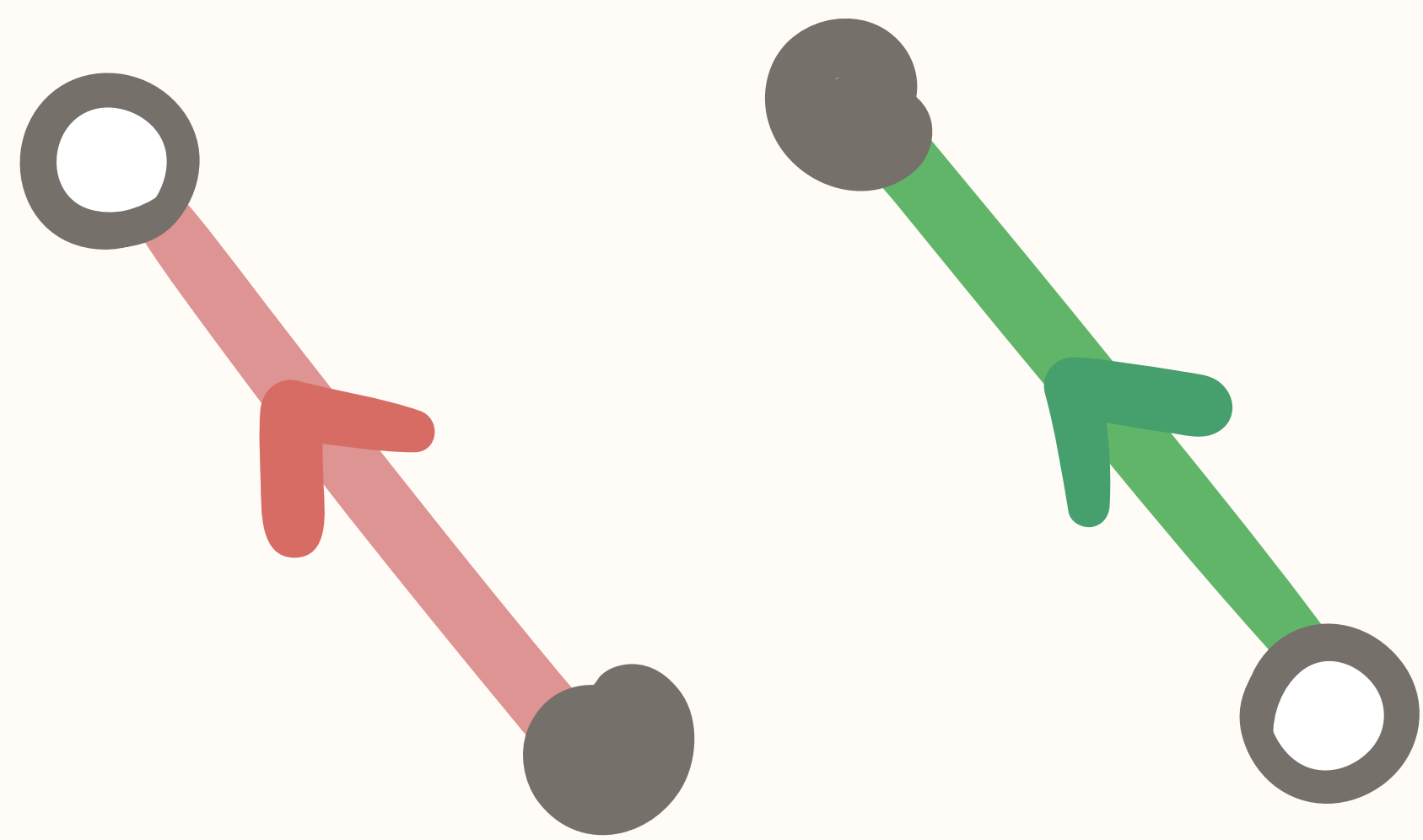
Schnyder labellings



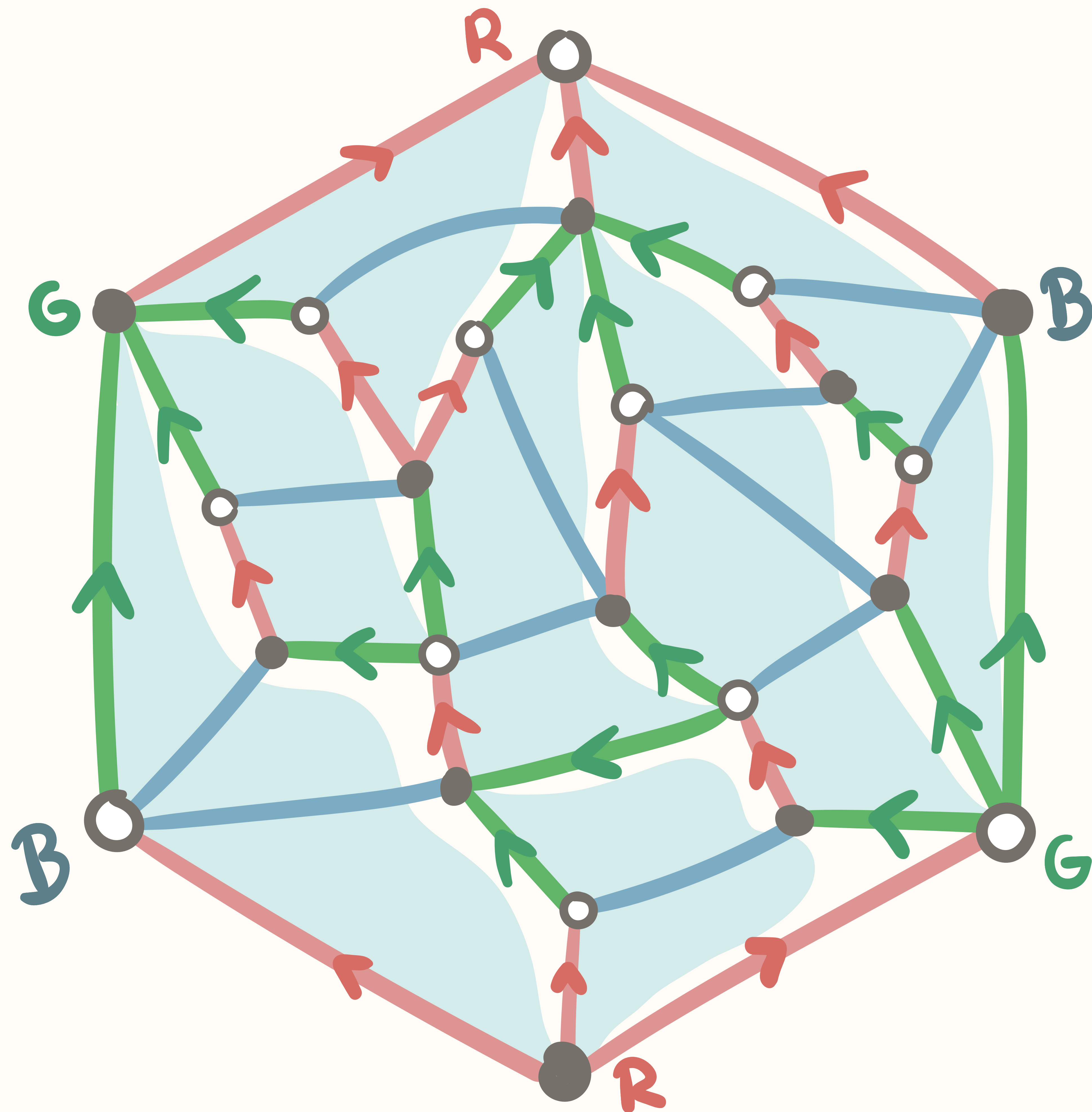
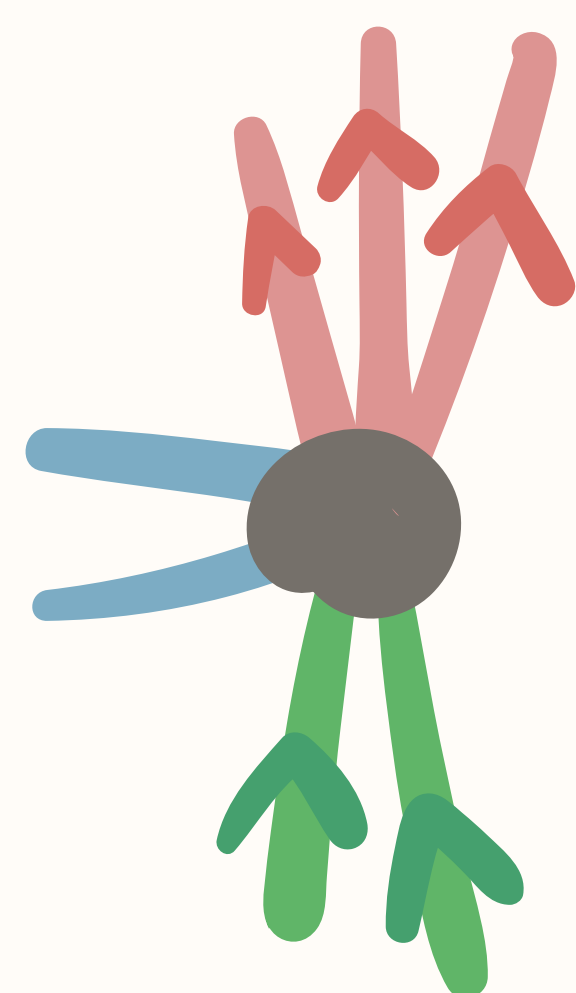
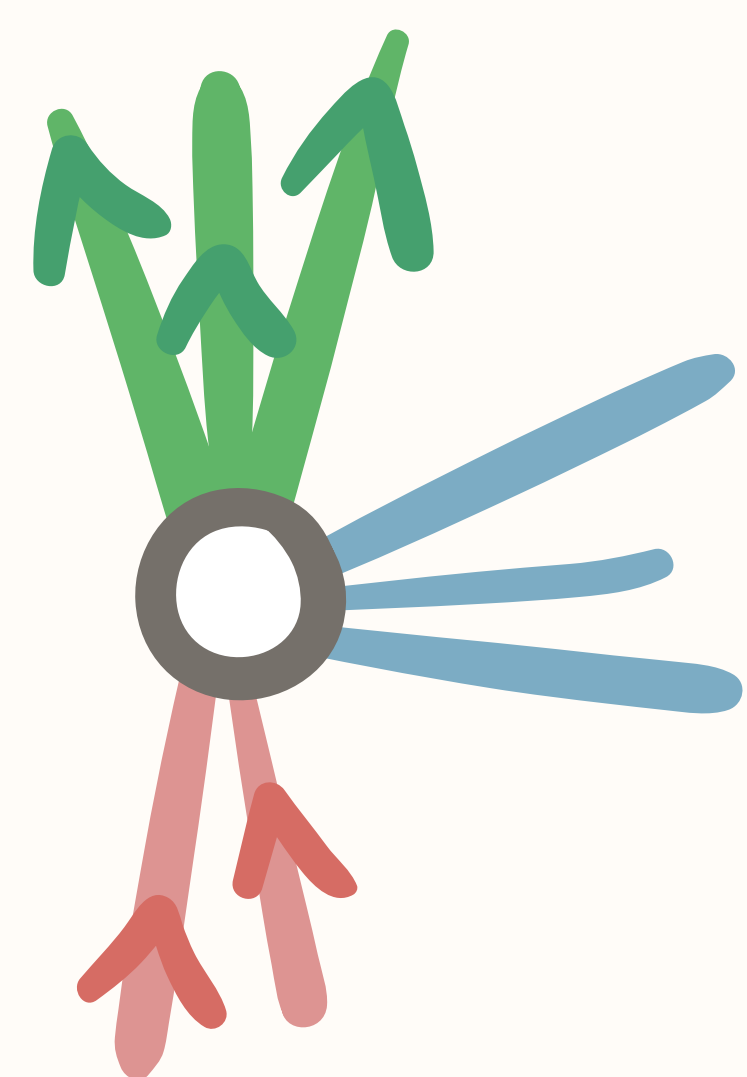
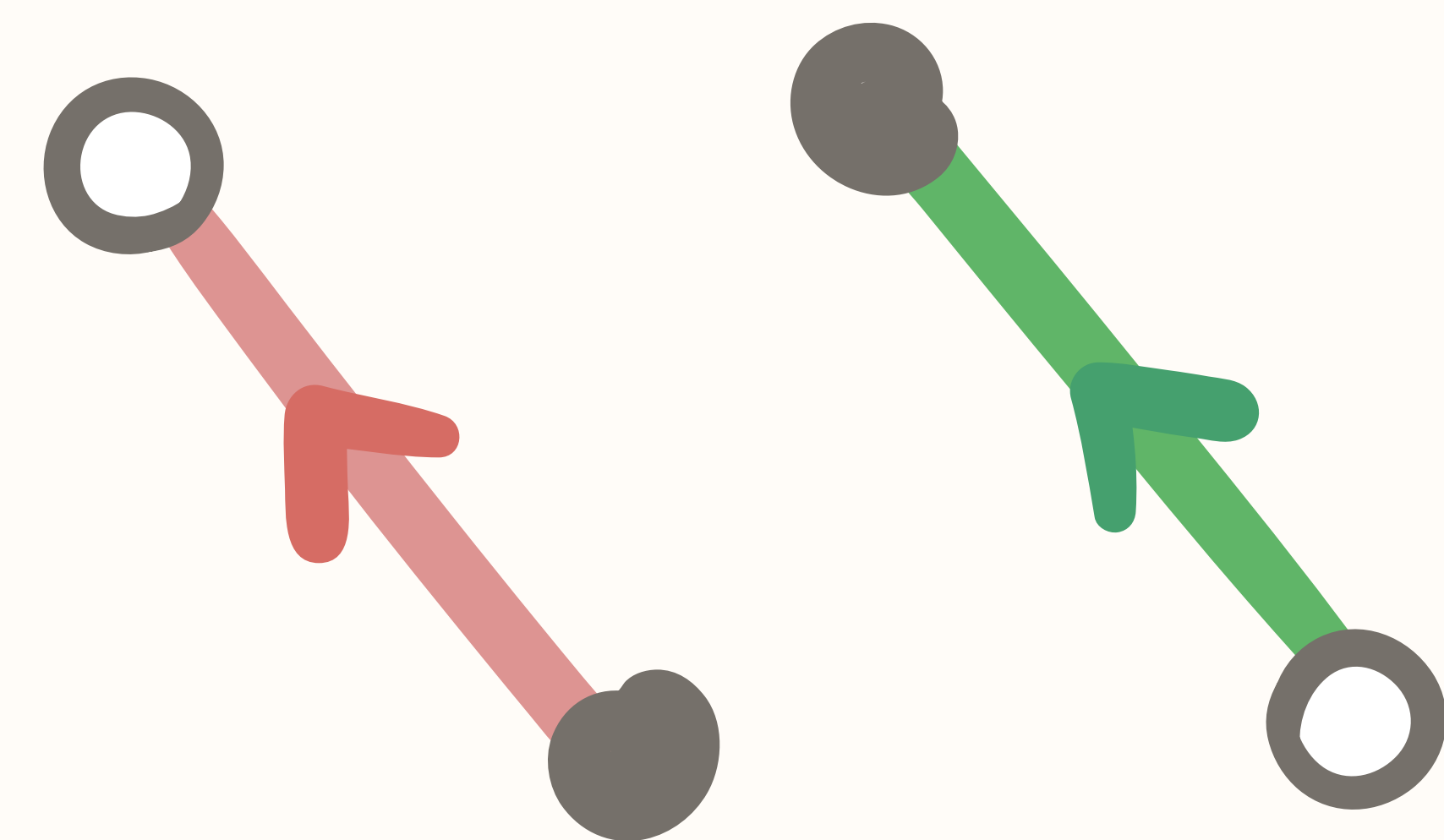
Schnyder labellings



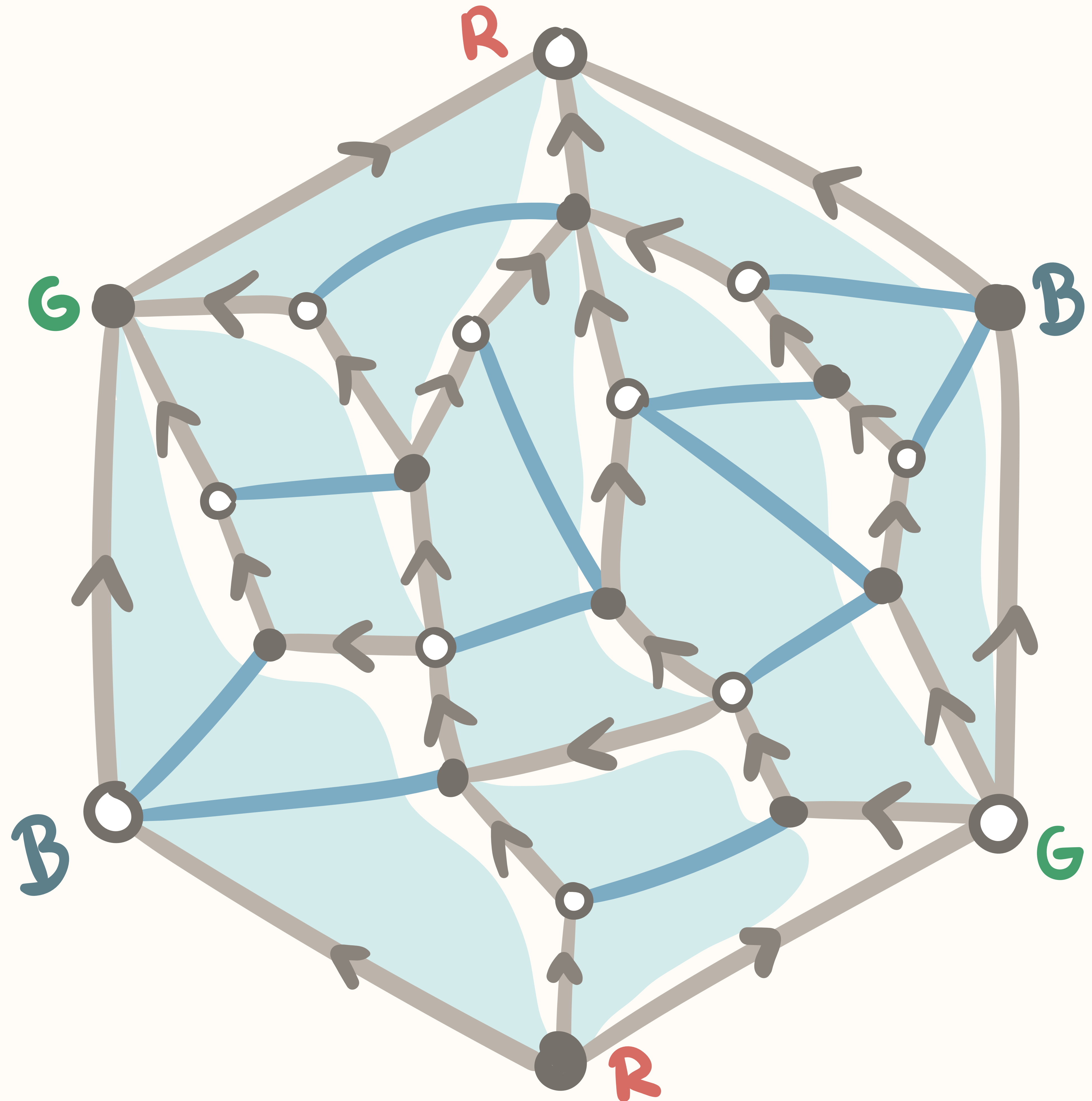
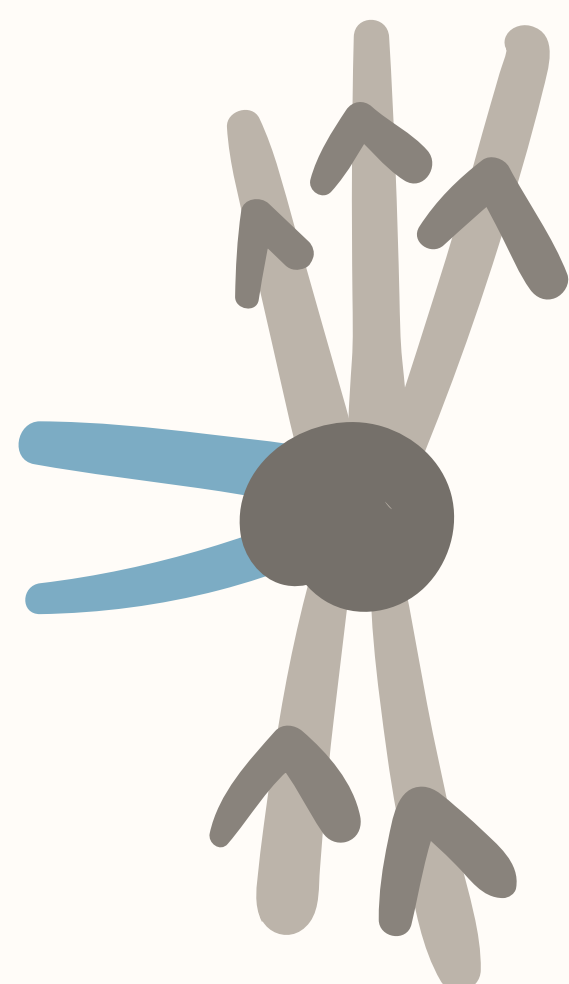
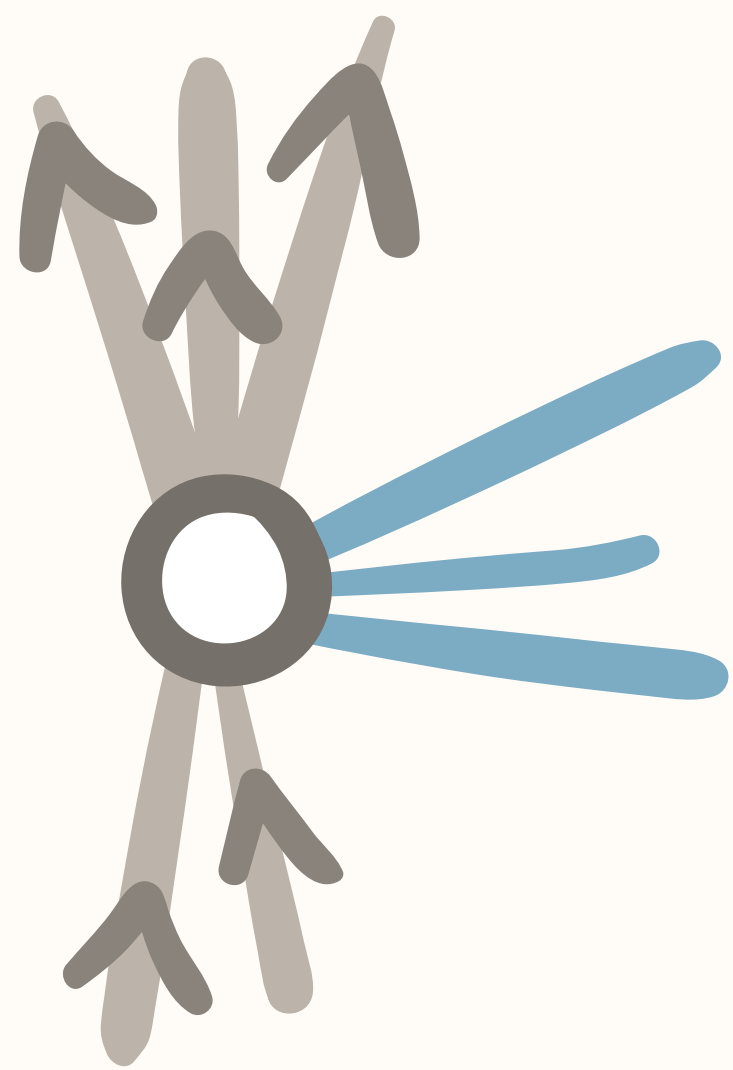
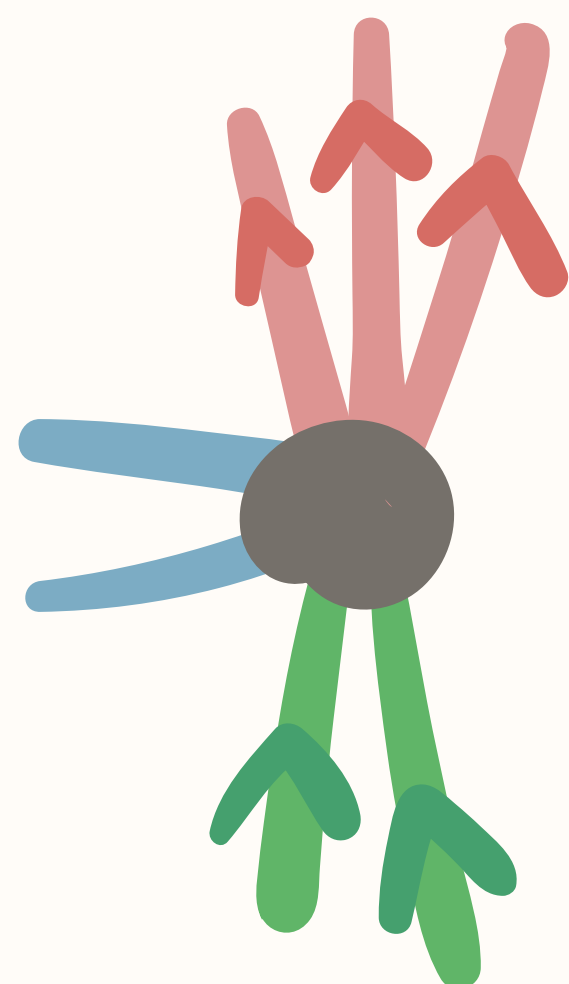
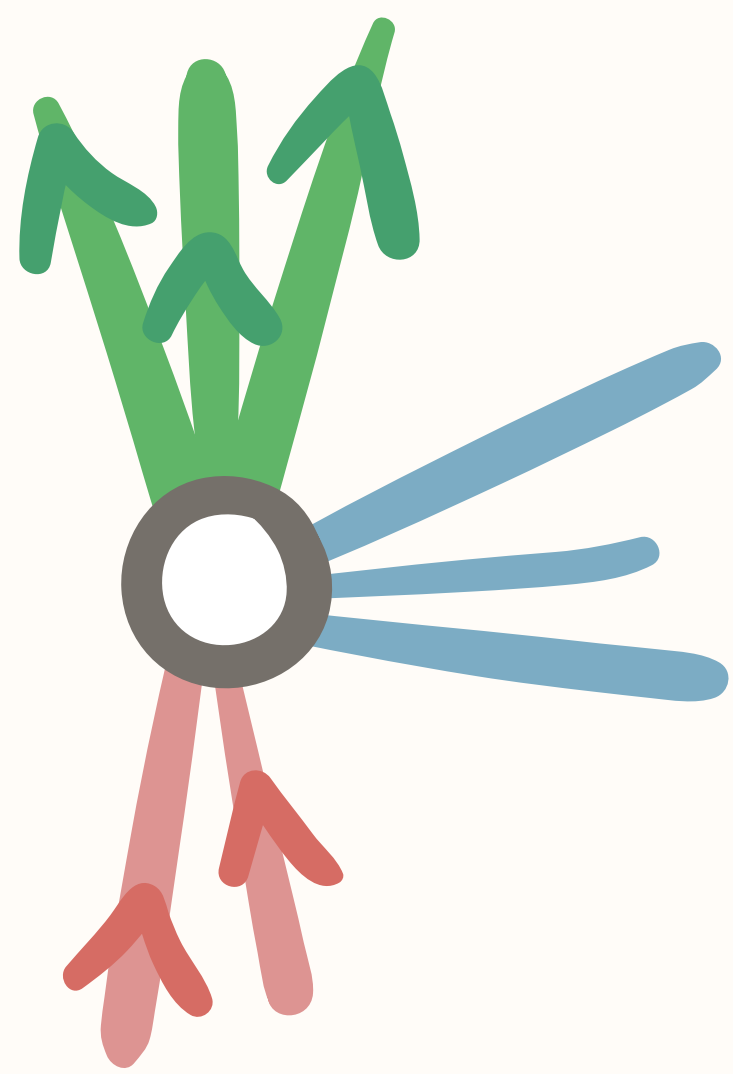
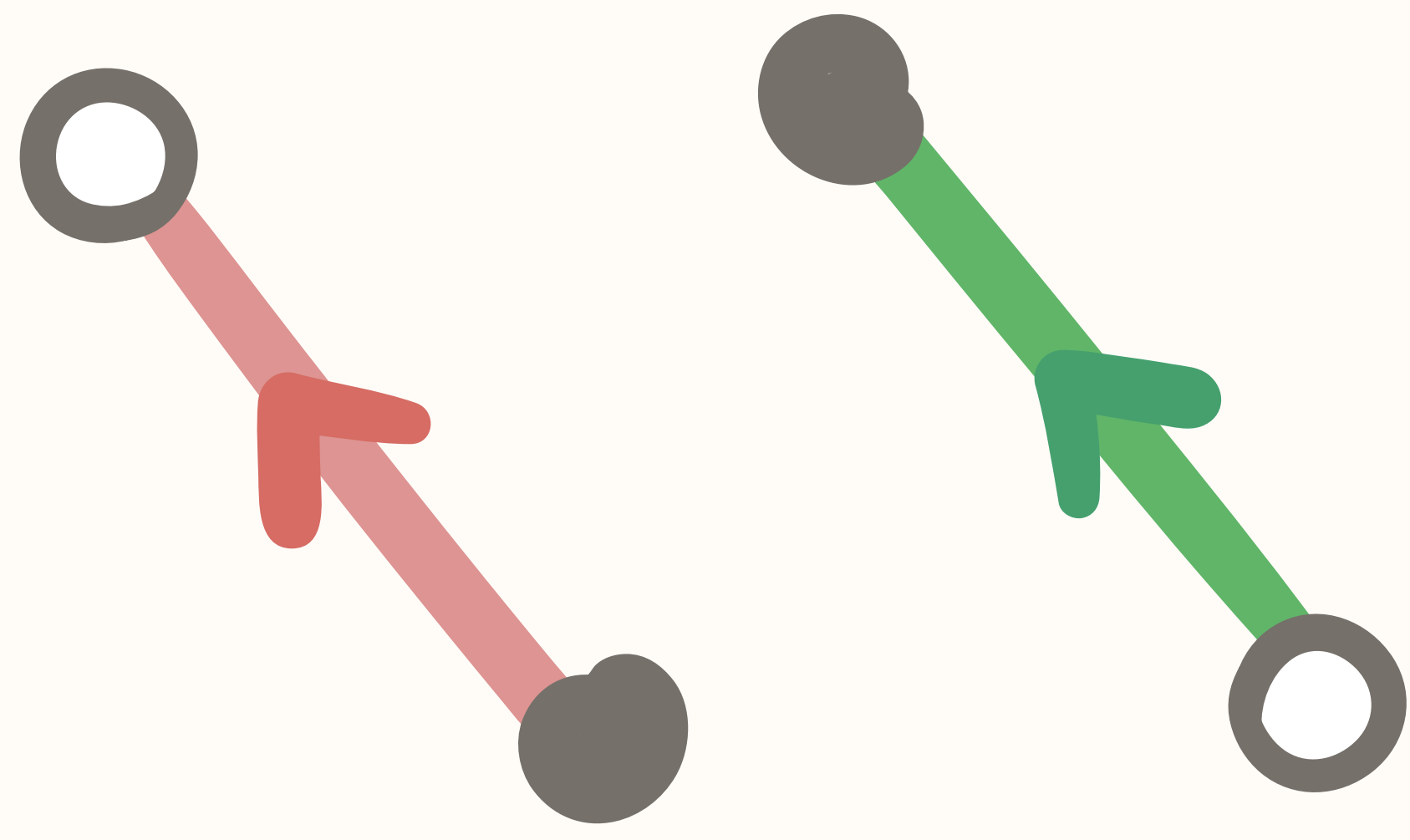
Schnyder labellings



Schnyder labellings



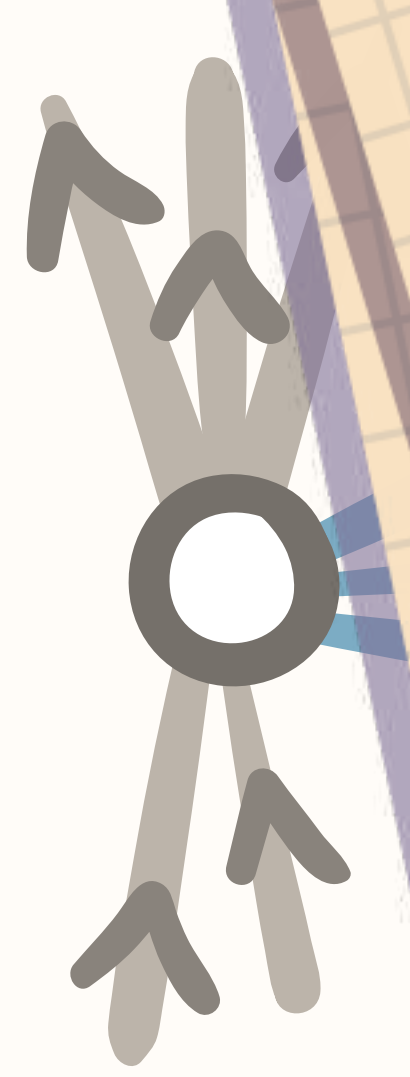
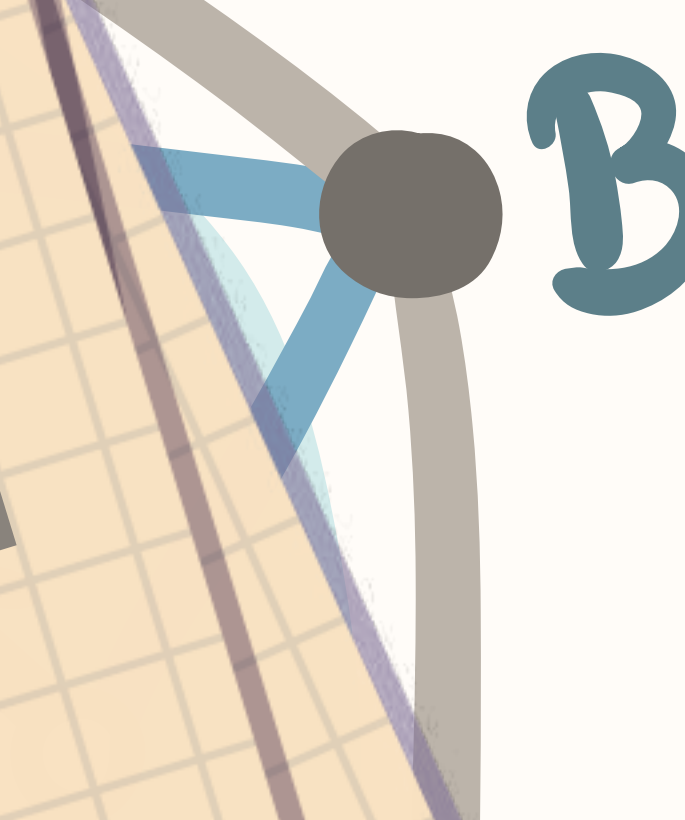
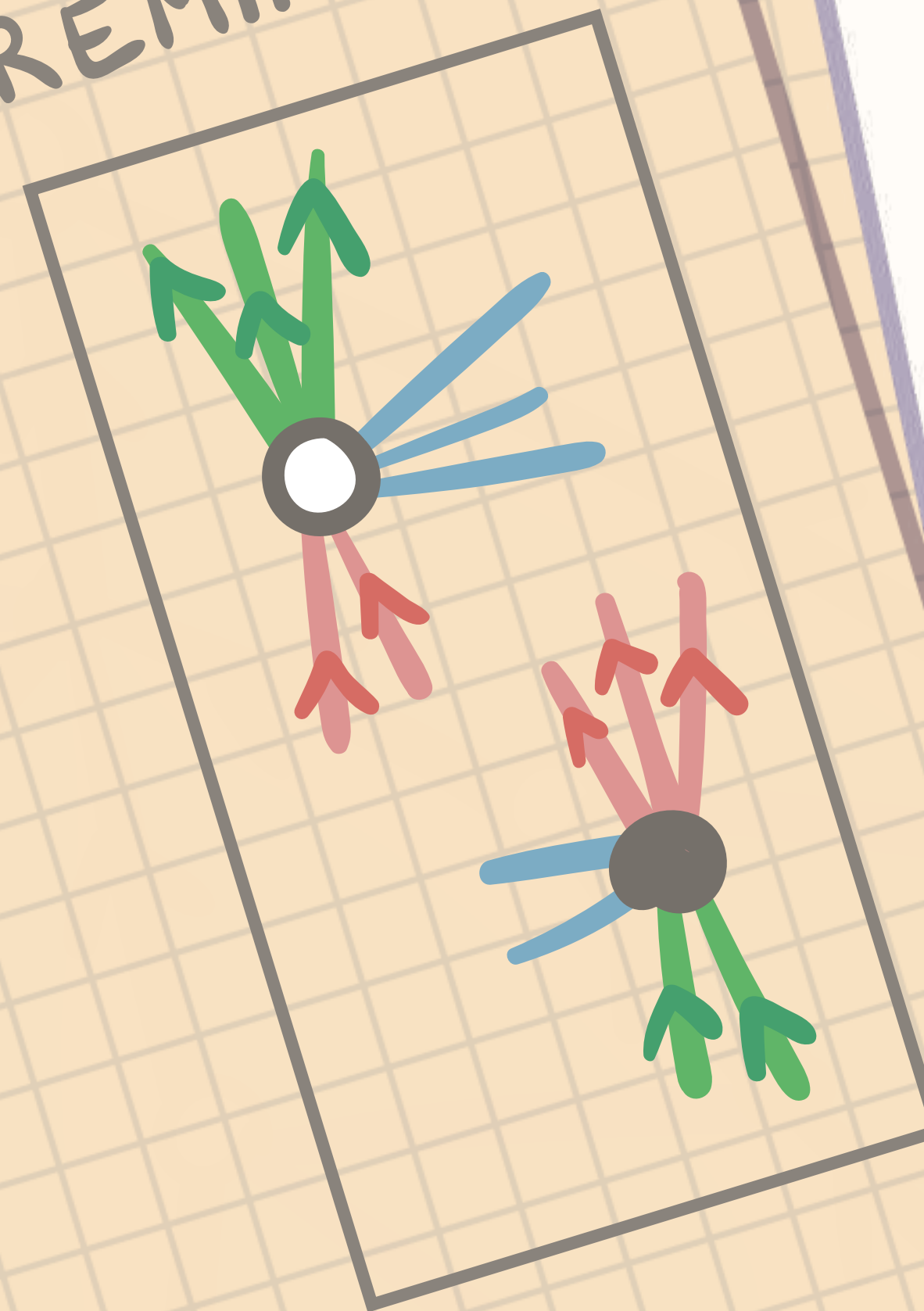
Schnyder labellings



Schnyder labellings

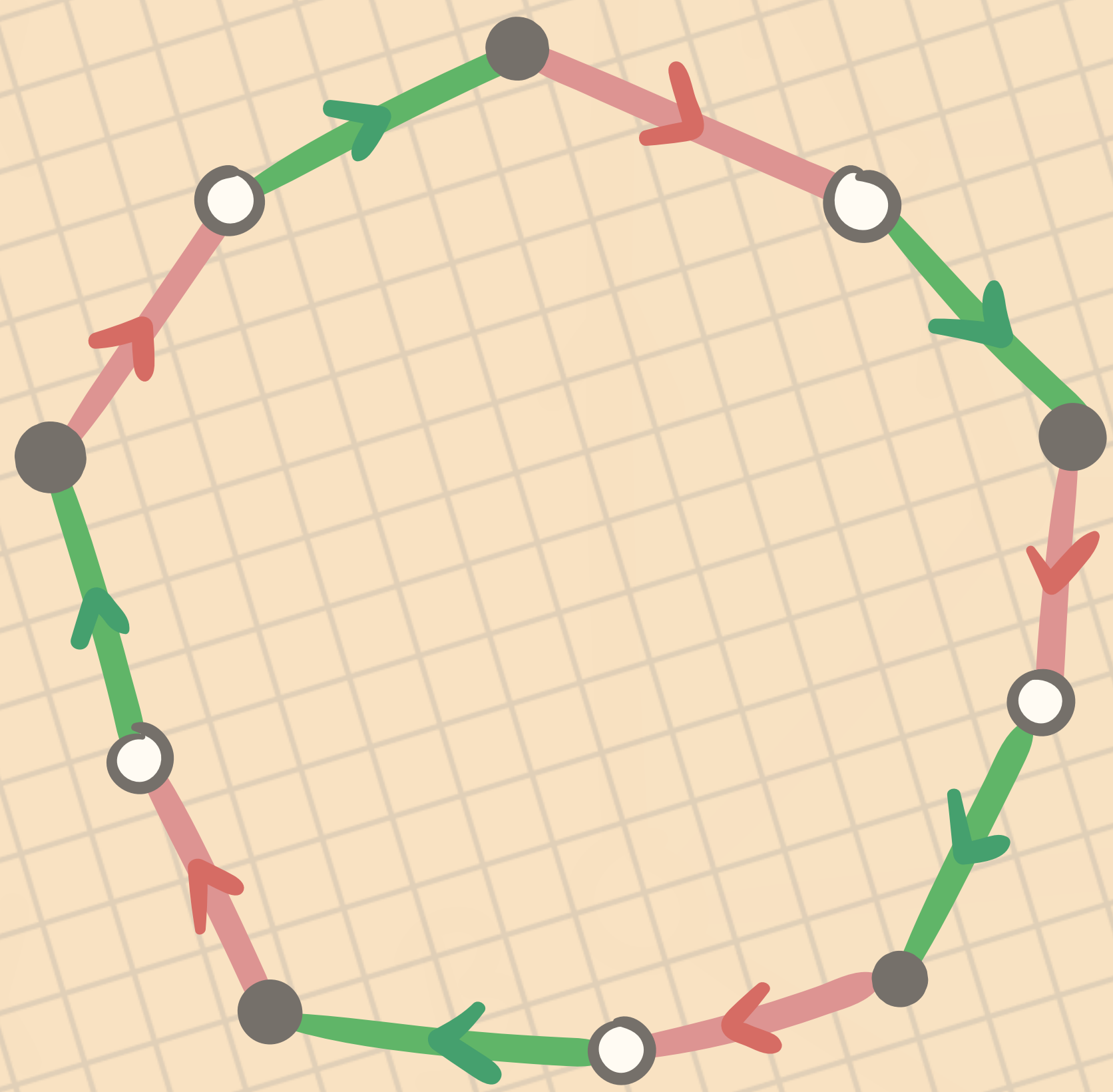
IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC

REMINDER

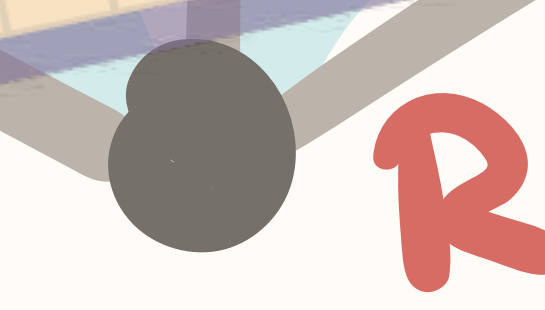
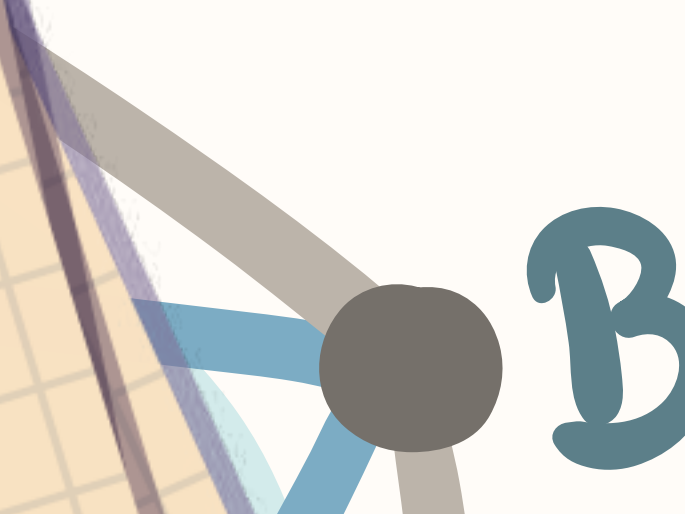
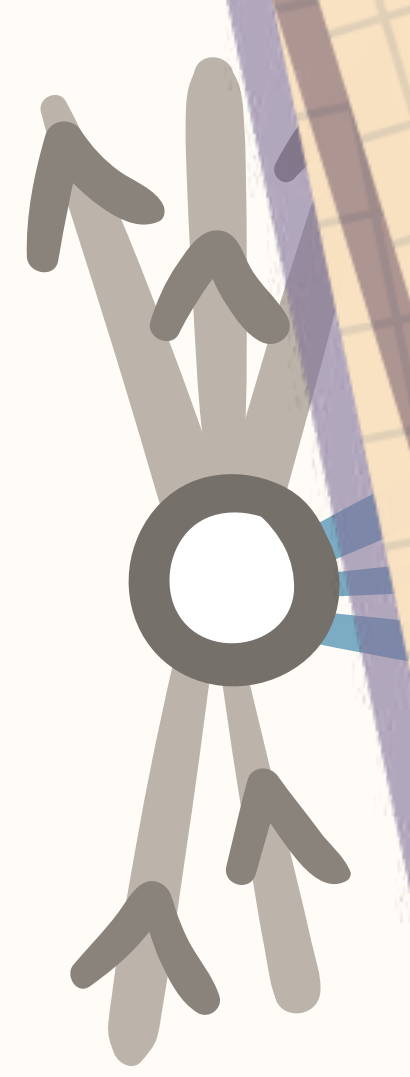
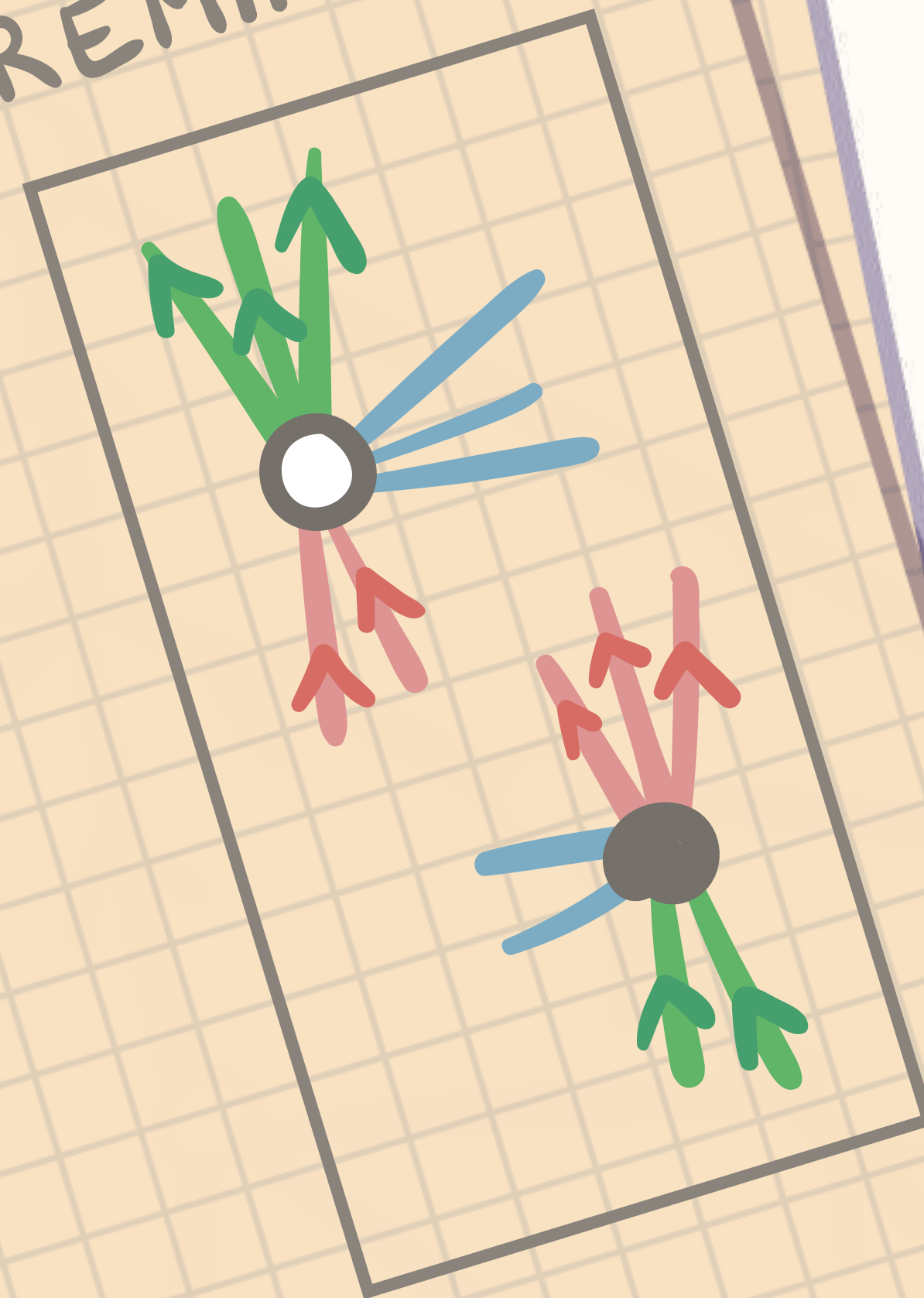


Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



REMINDER

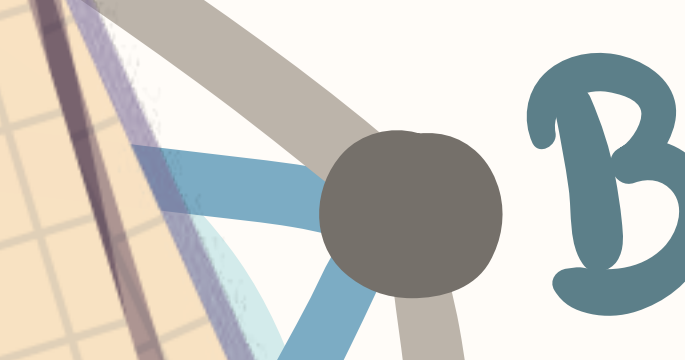
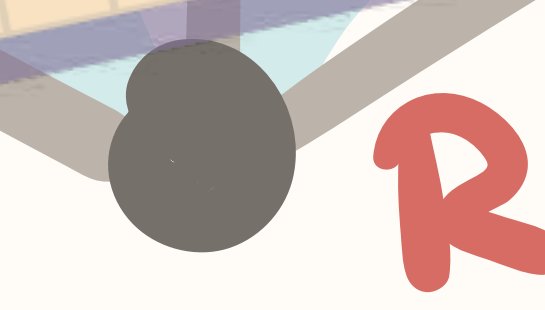
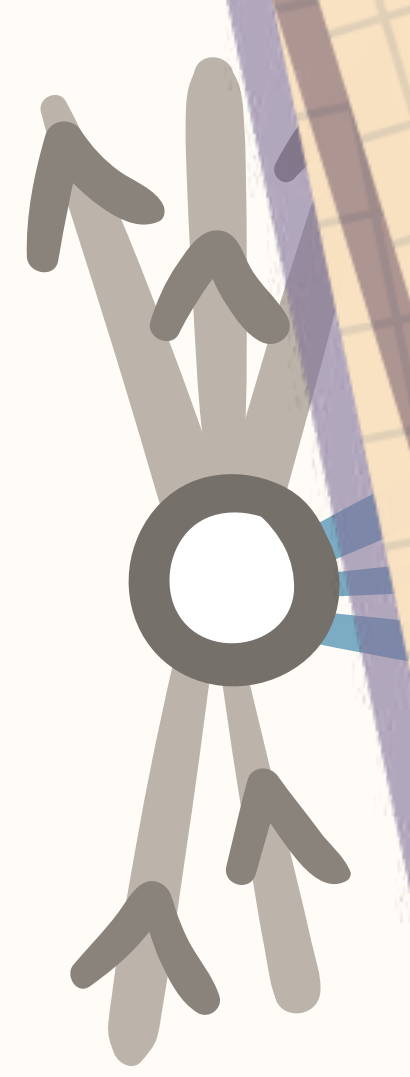
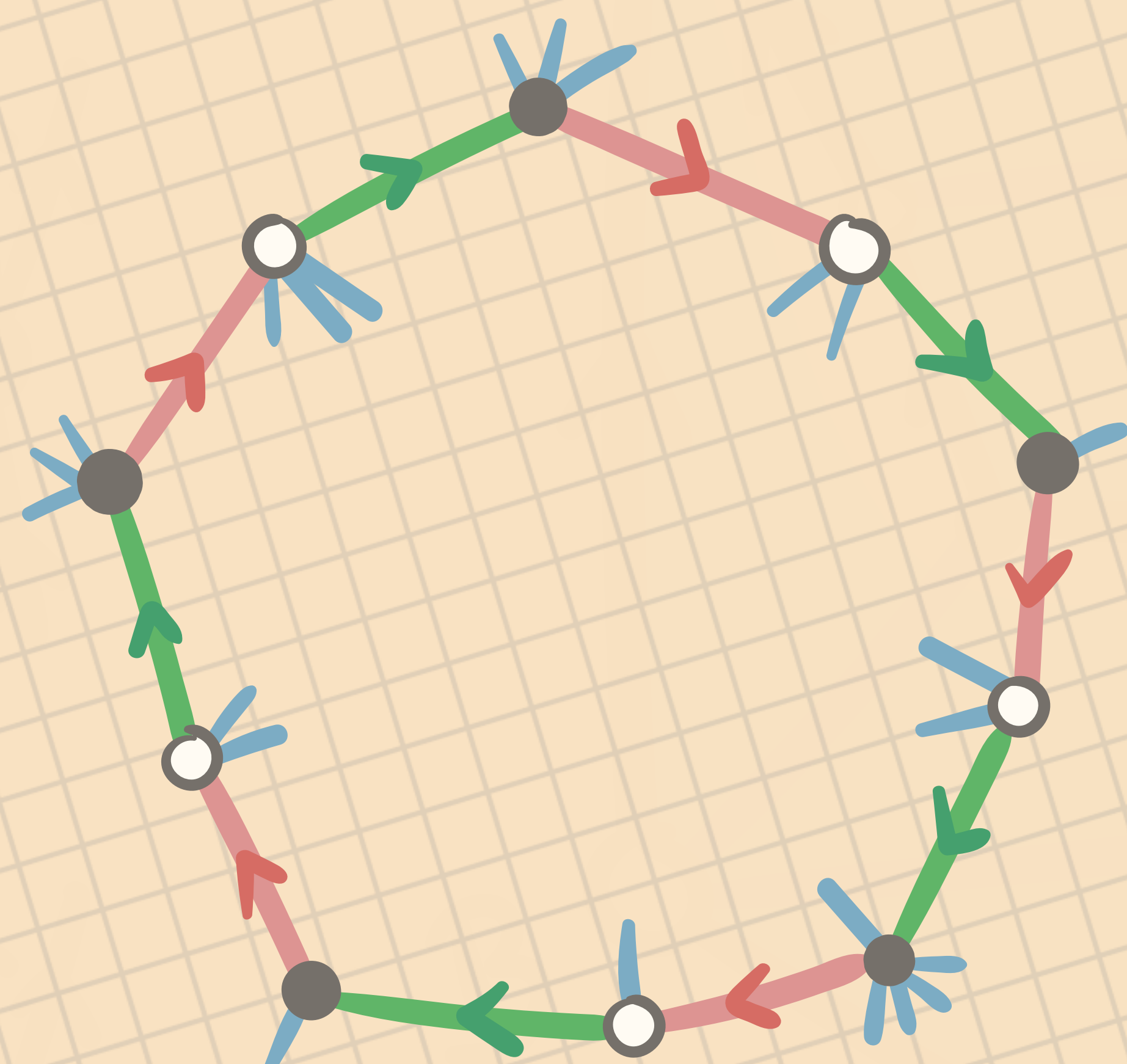
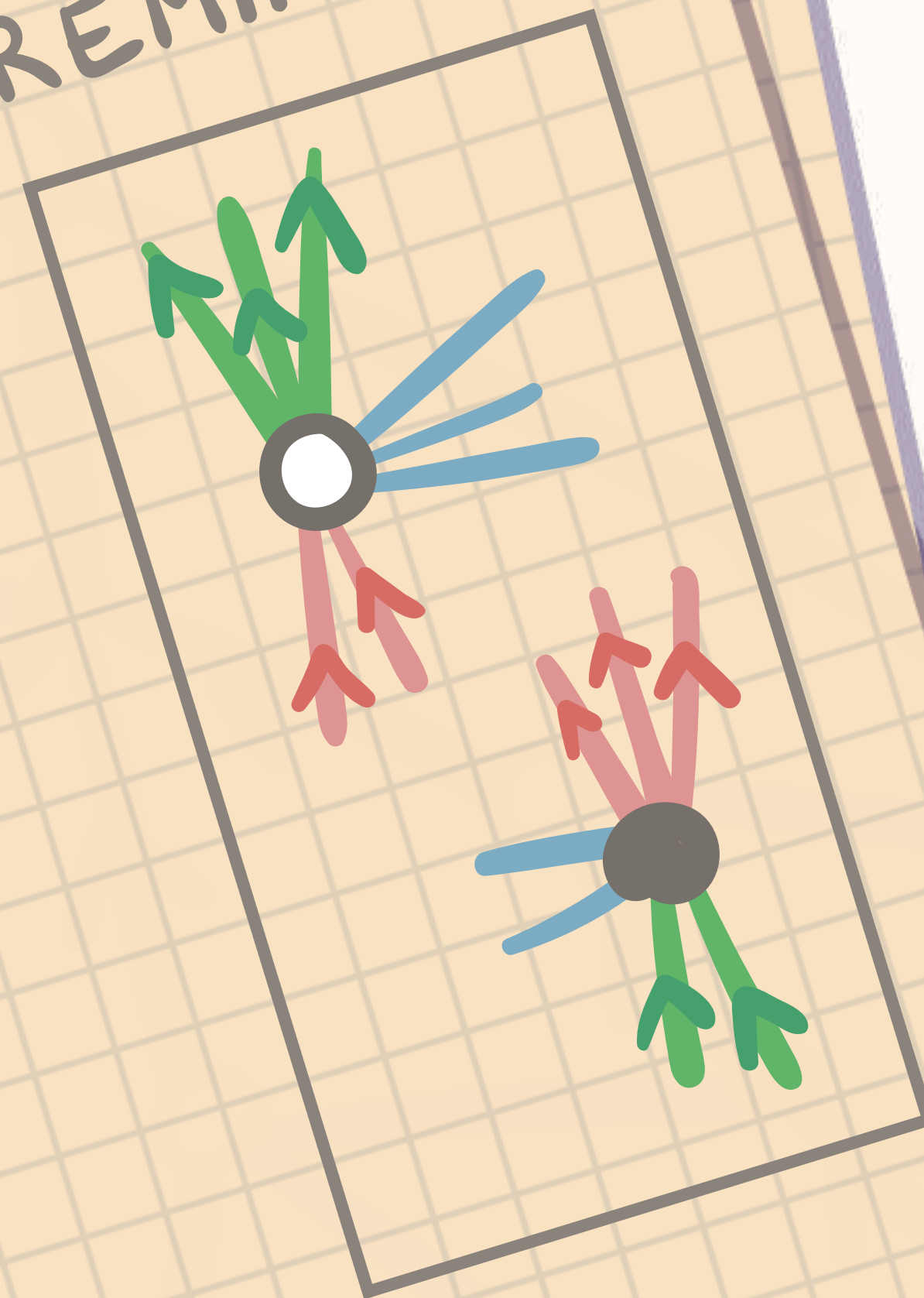


G

Schnyder labellings

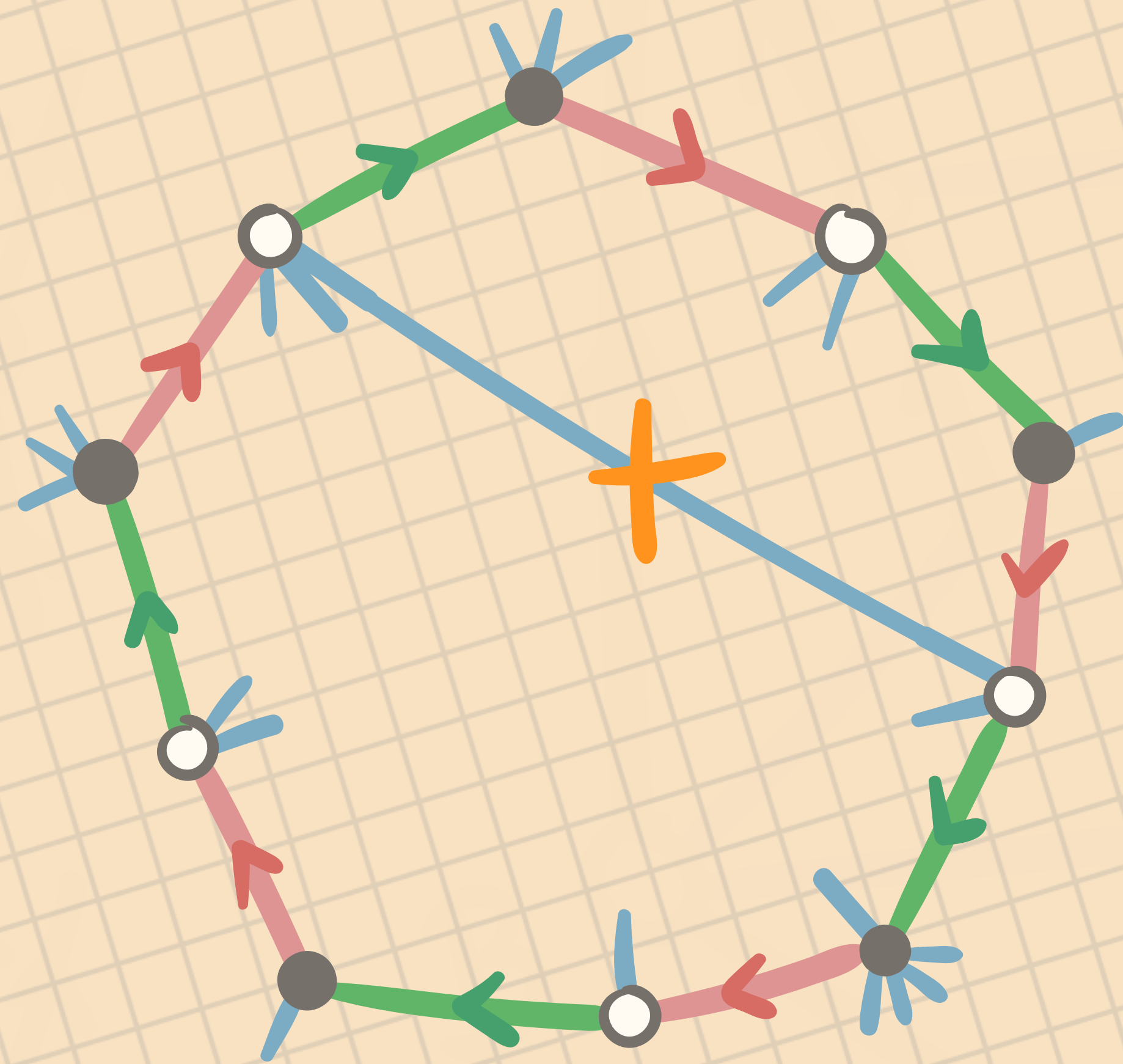
IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL

REMINDER

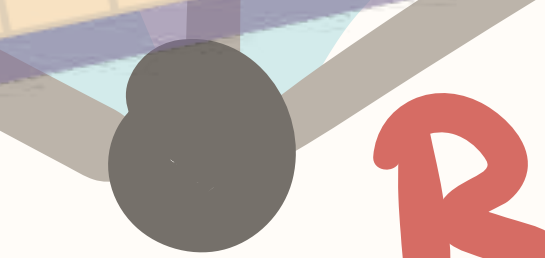
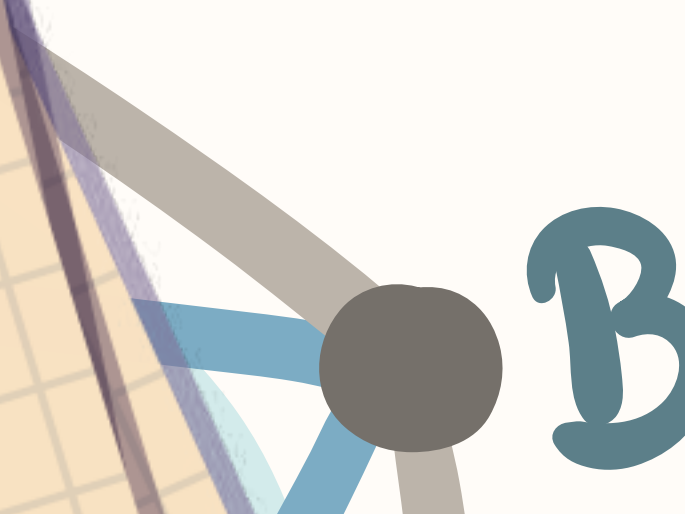
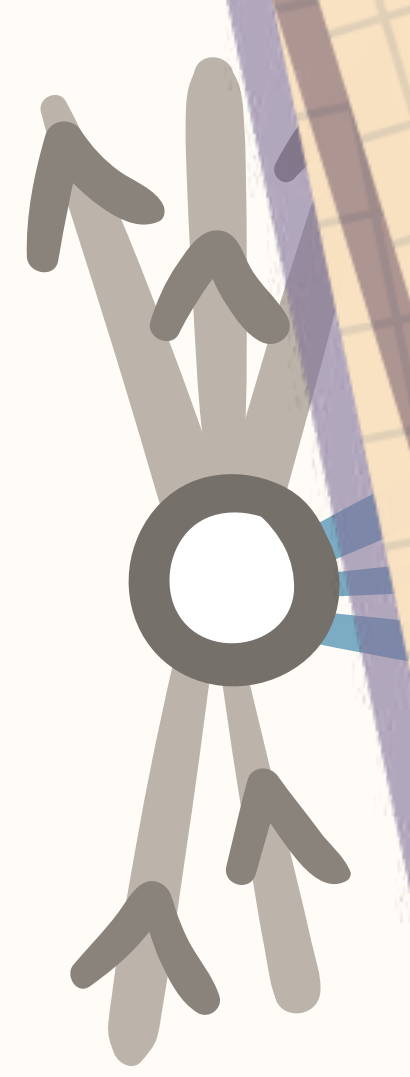
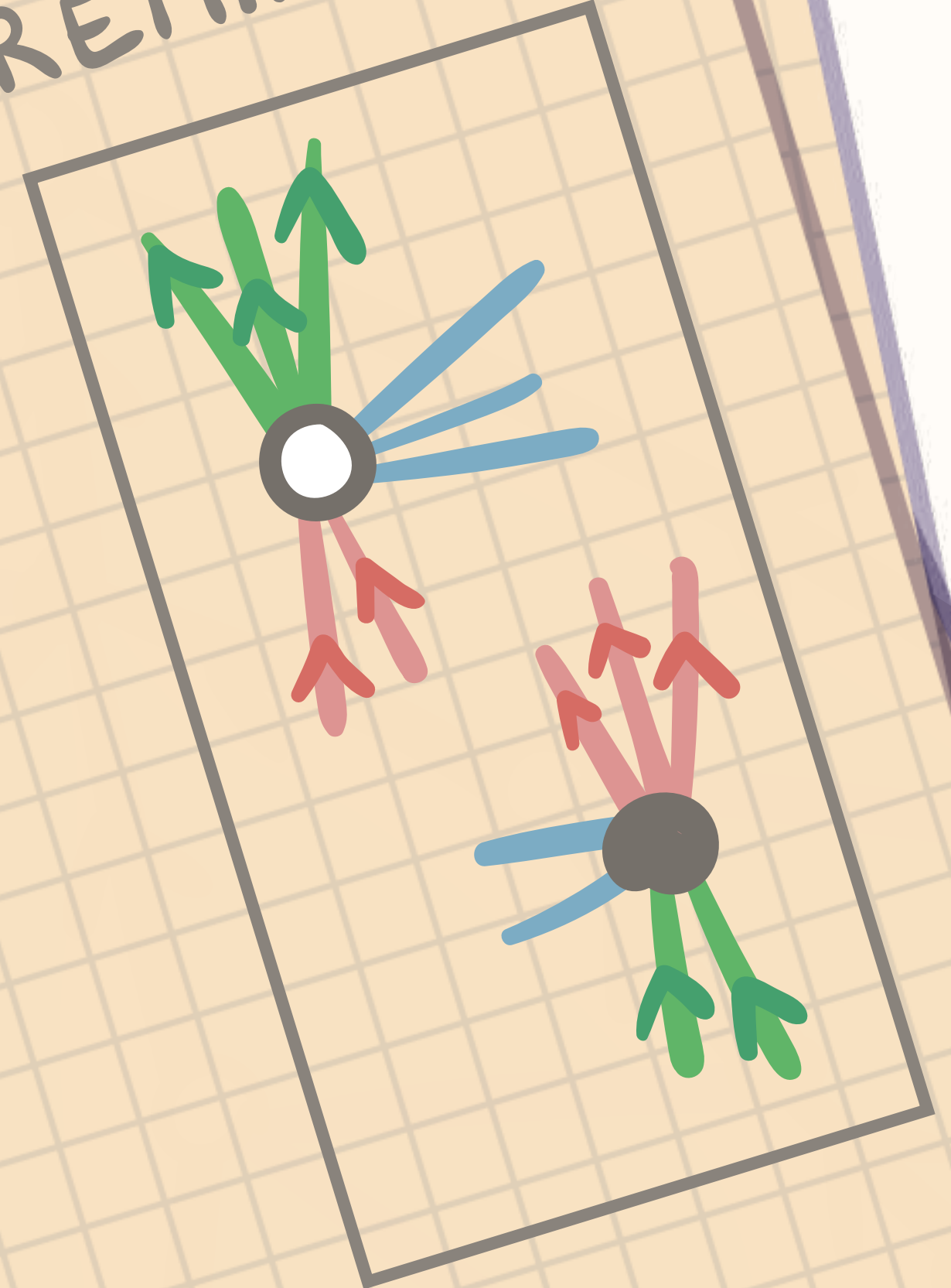


Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



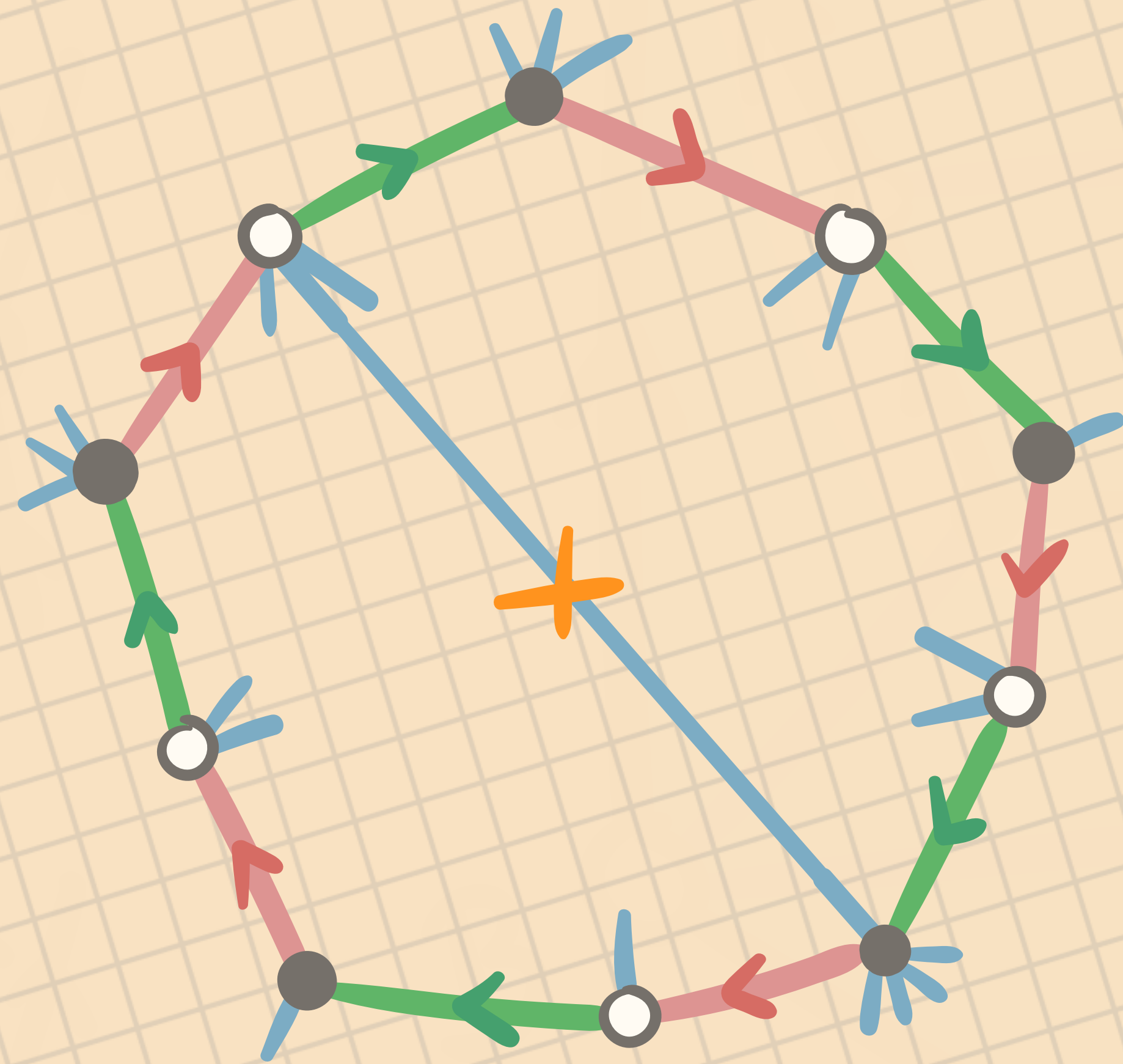
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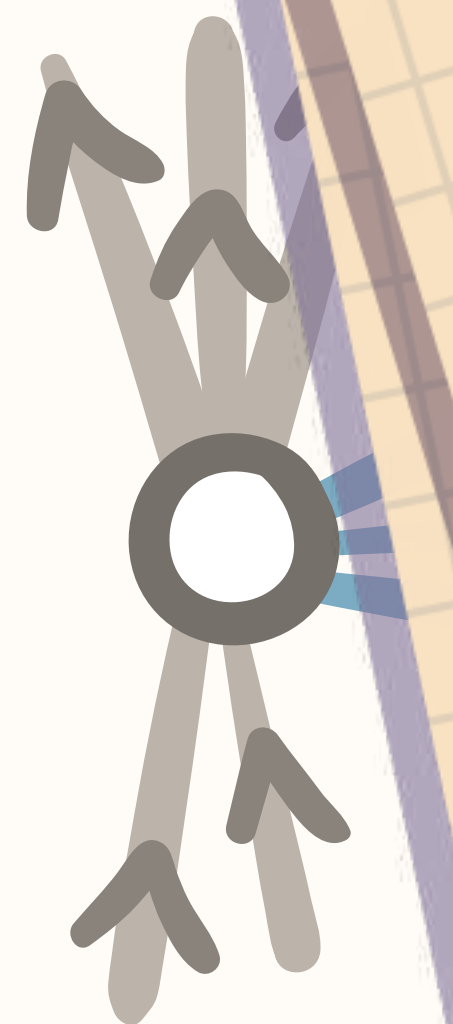
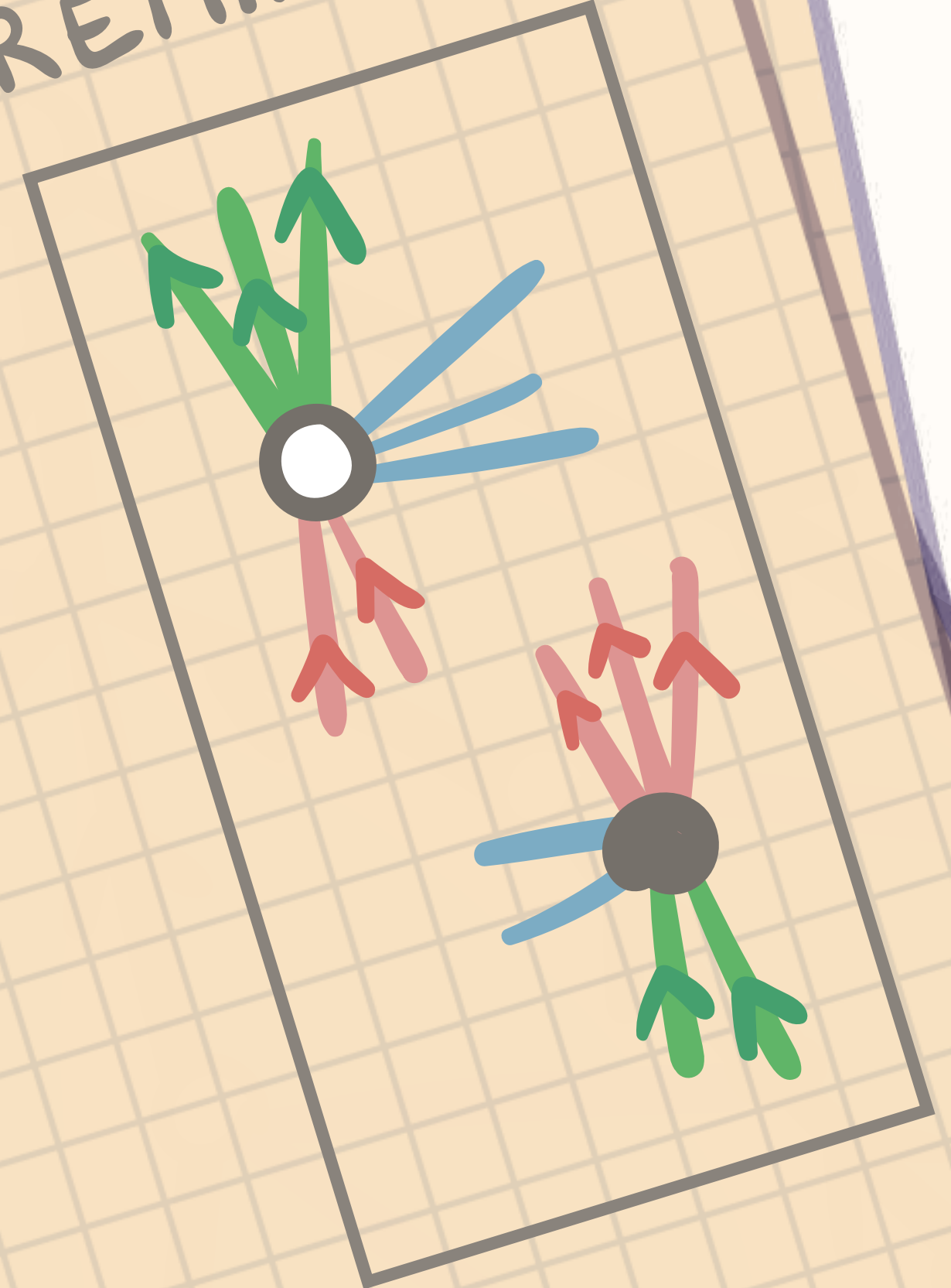
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Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL

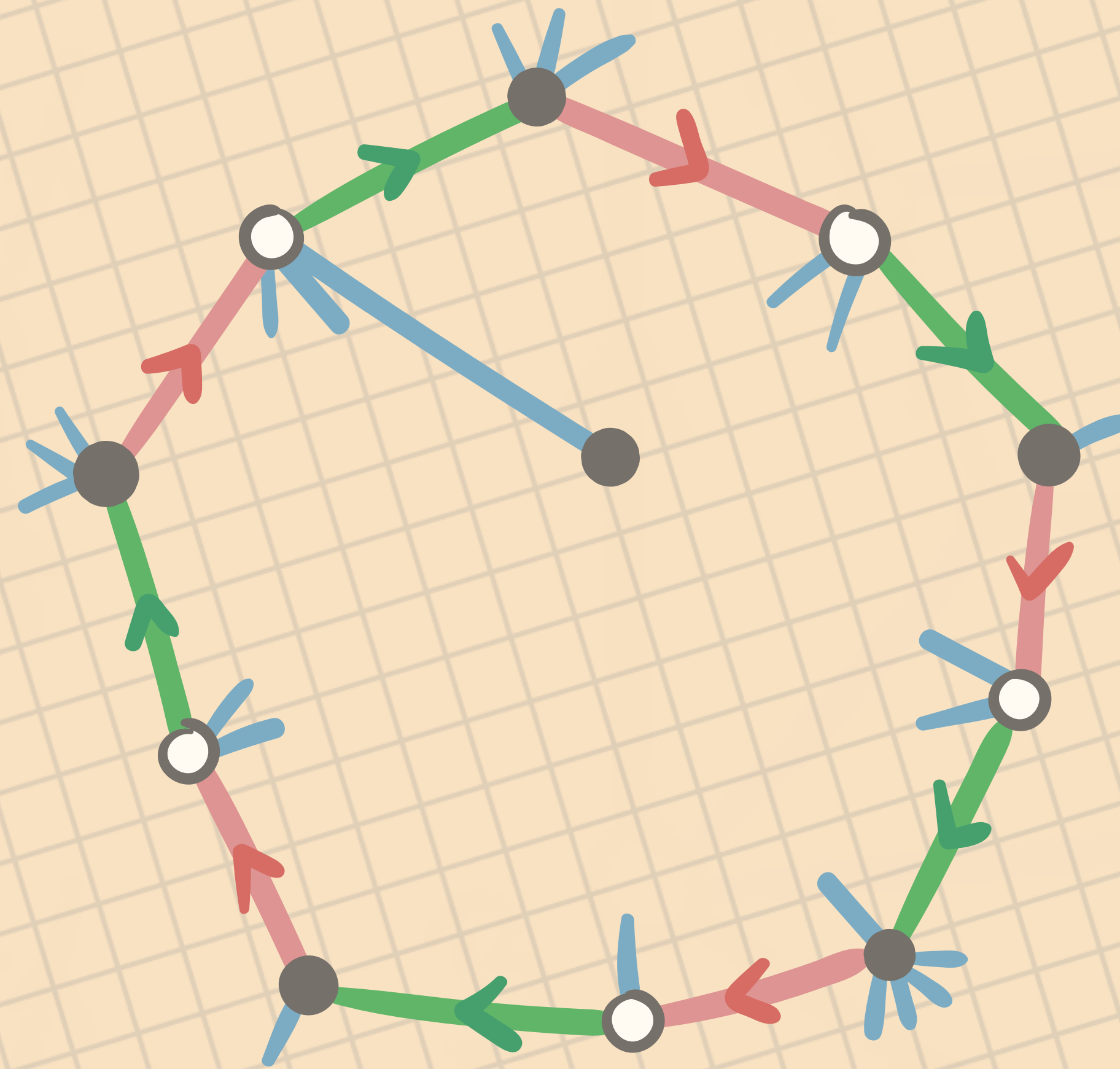


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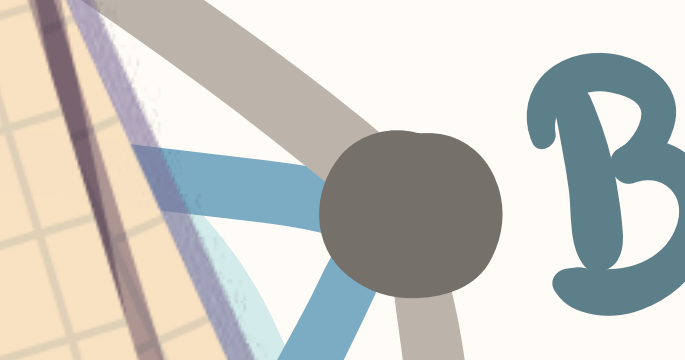
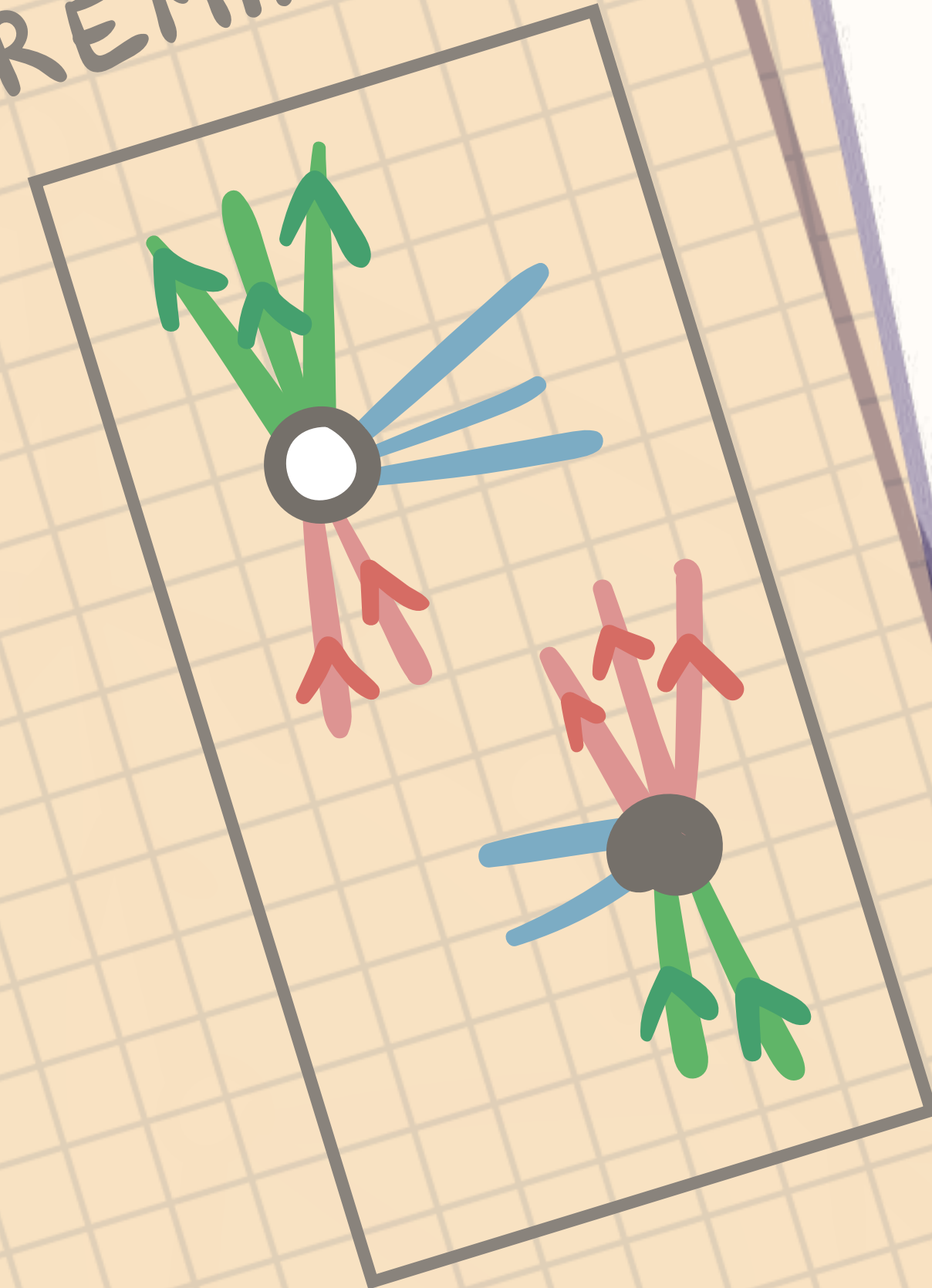


Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL

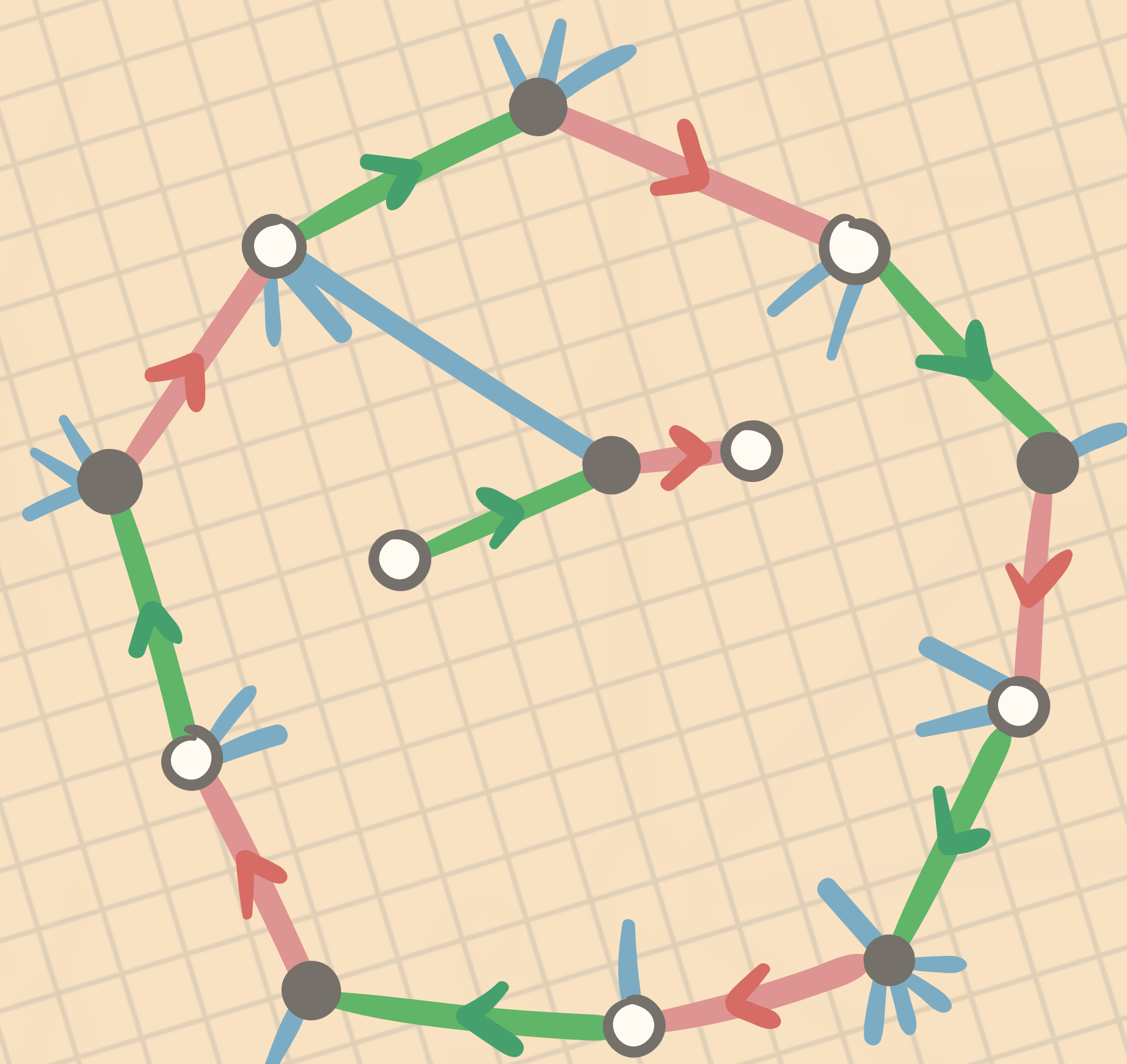


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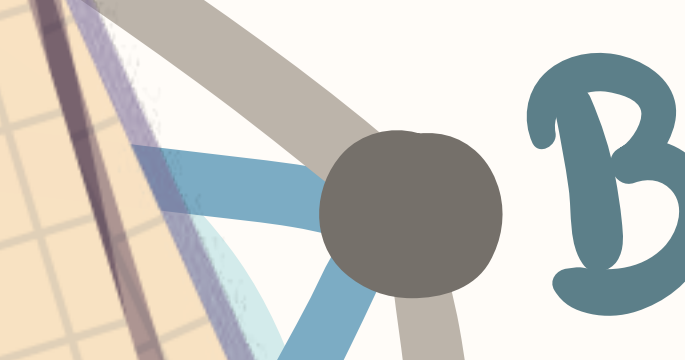
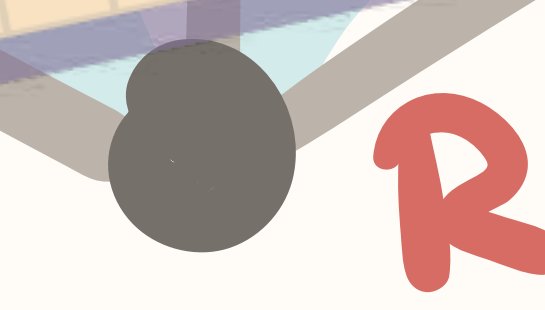
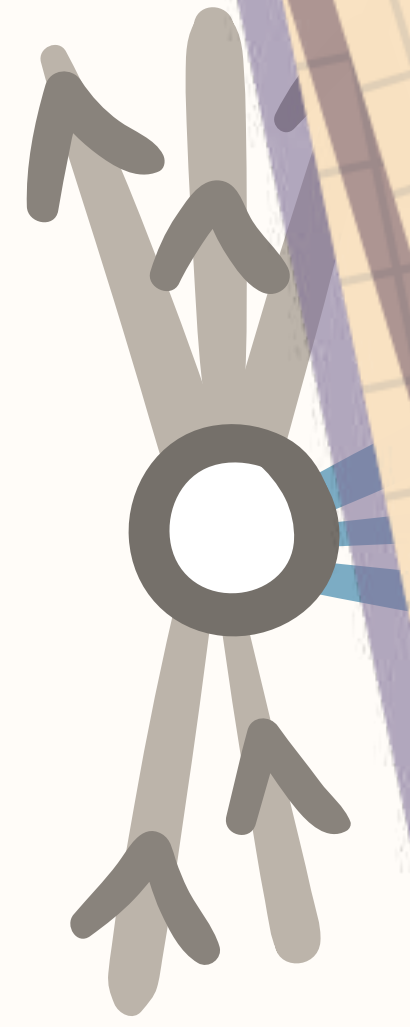
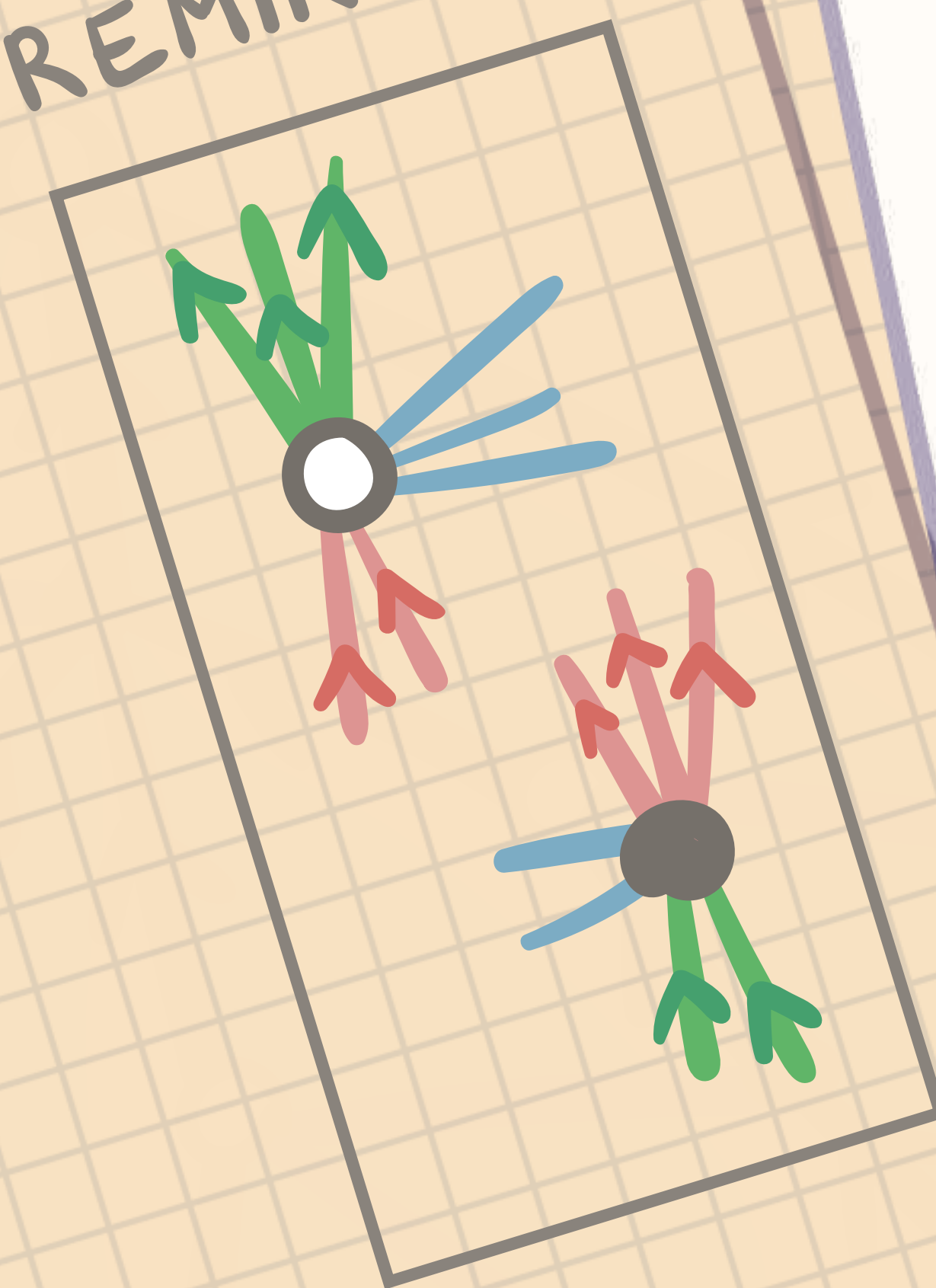


Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL

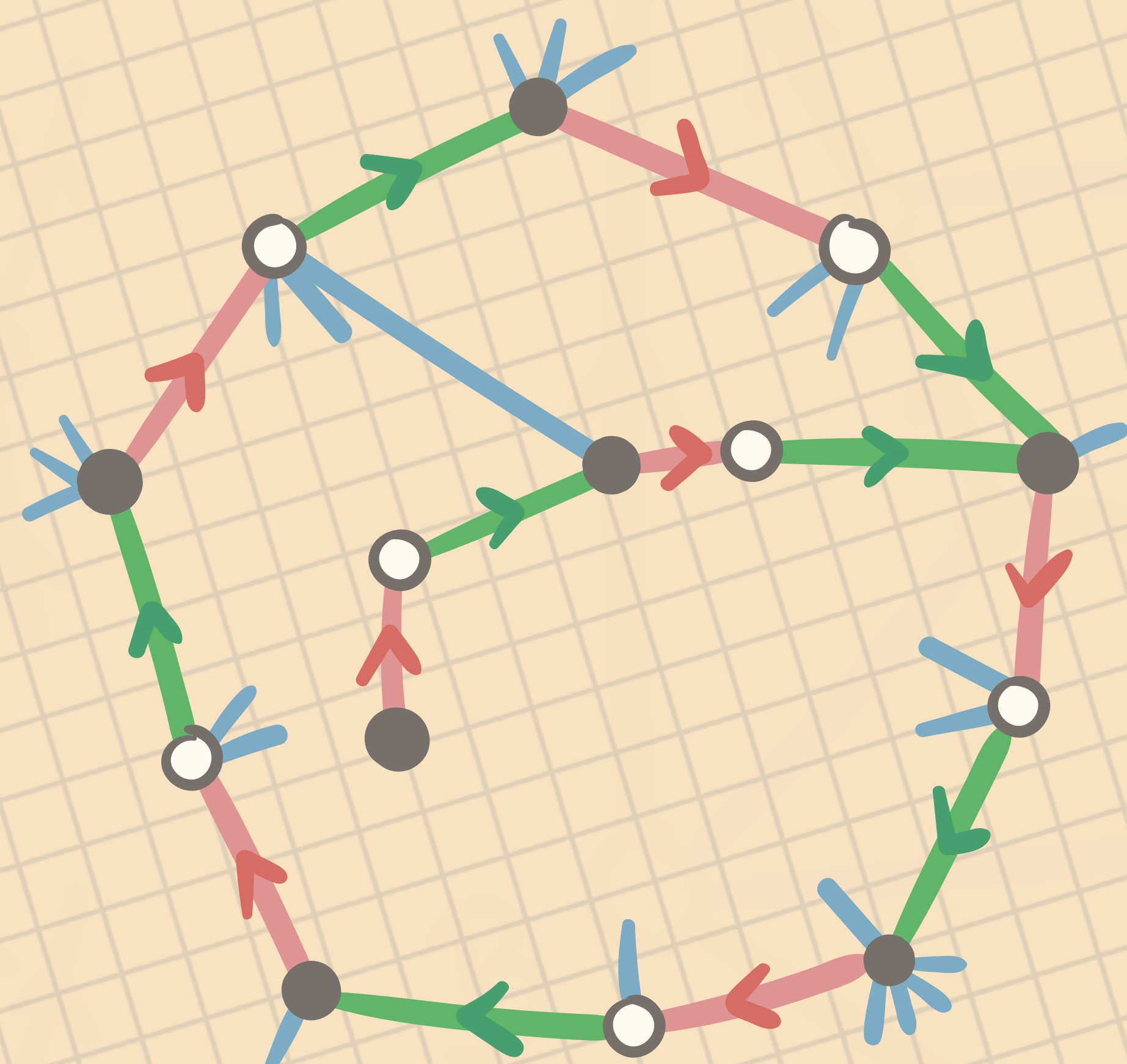


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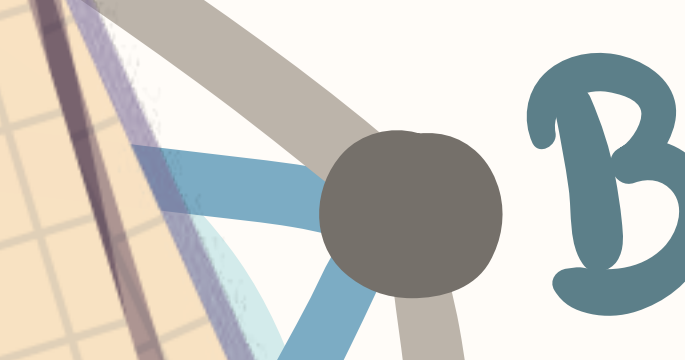
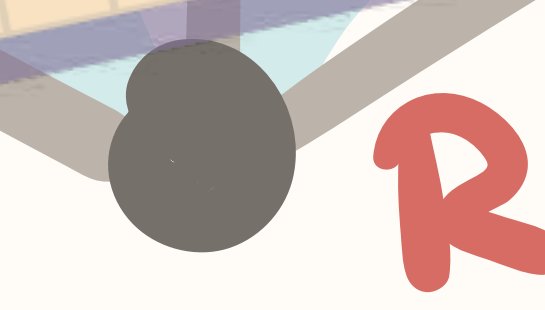
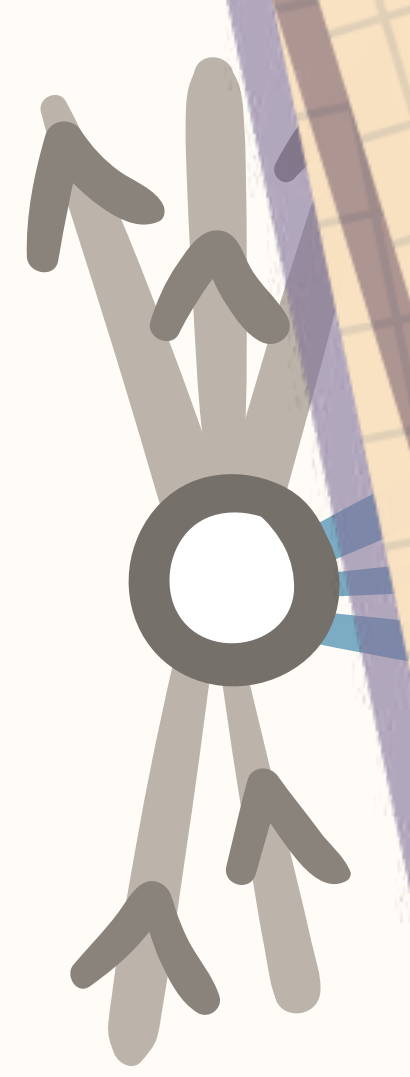
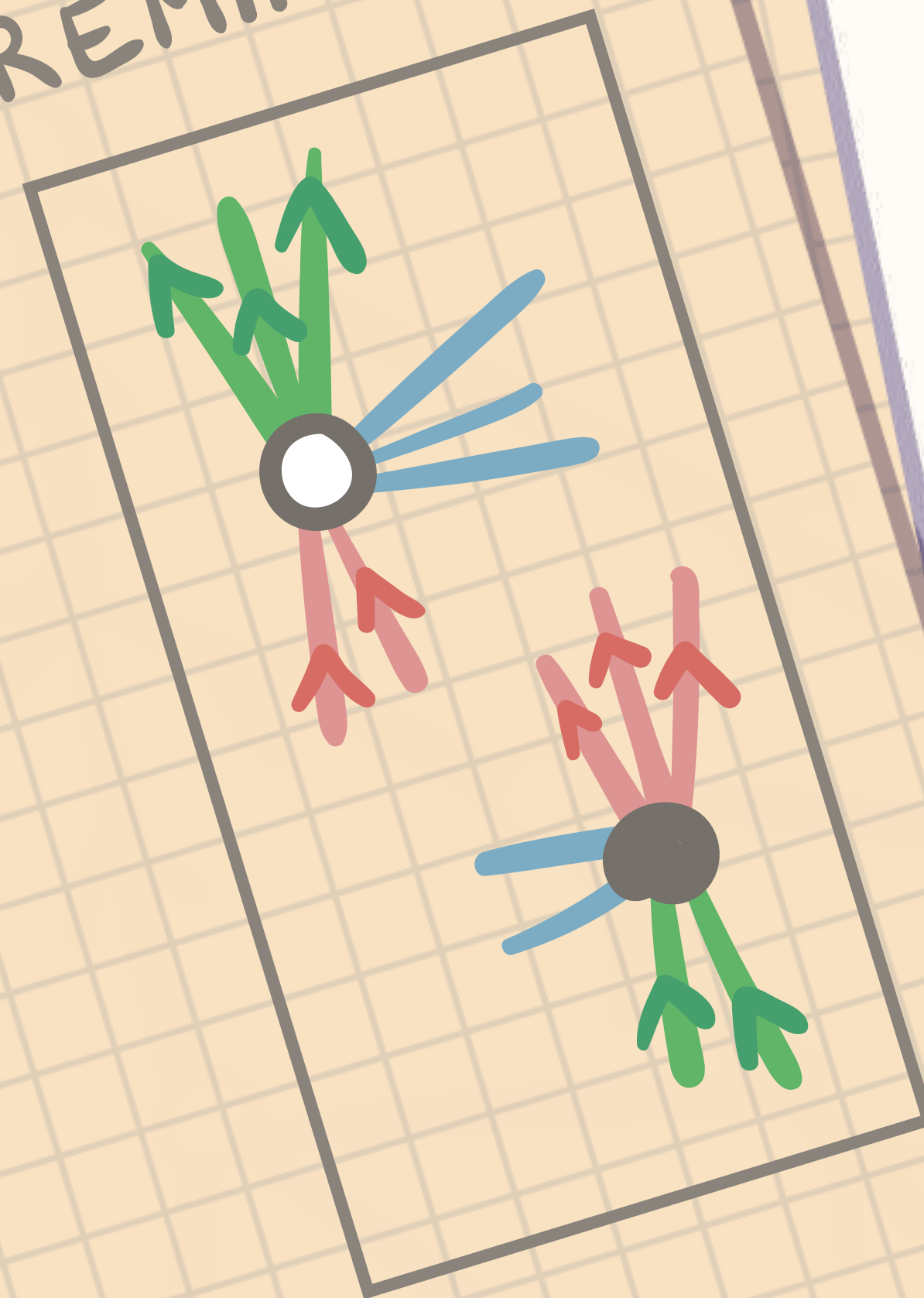


Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



REMINDER



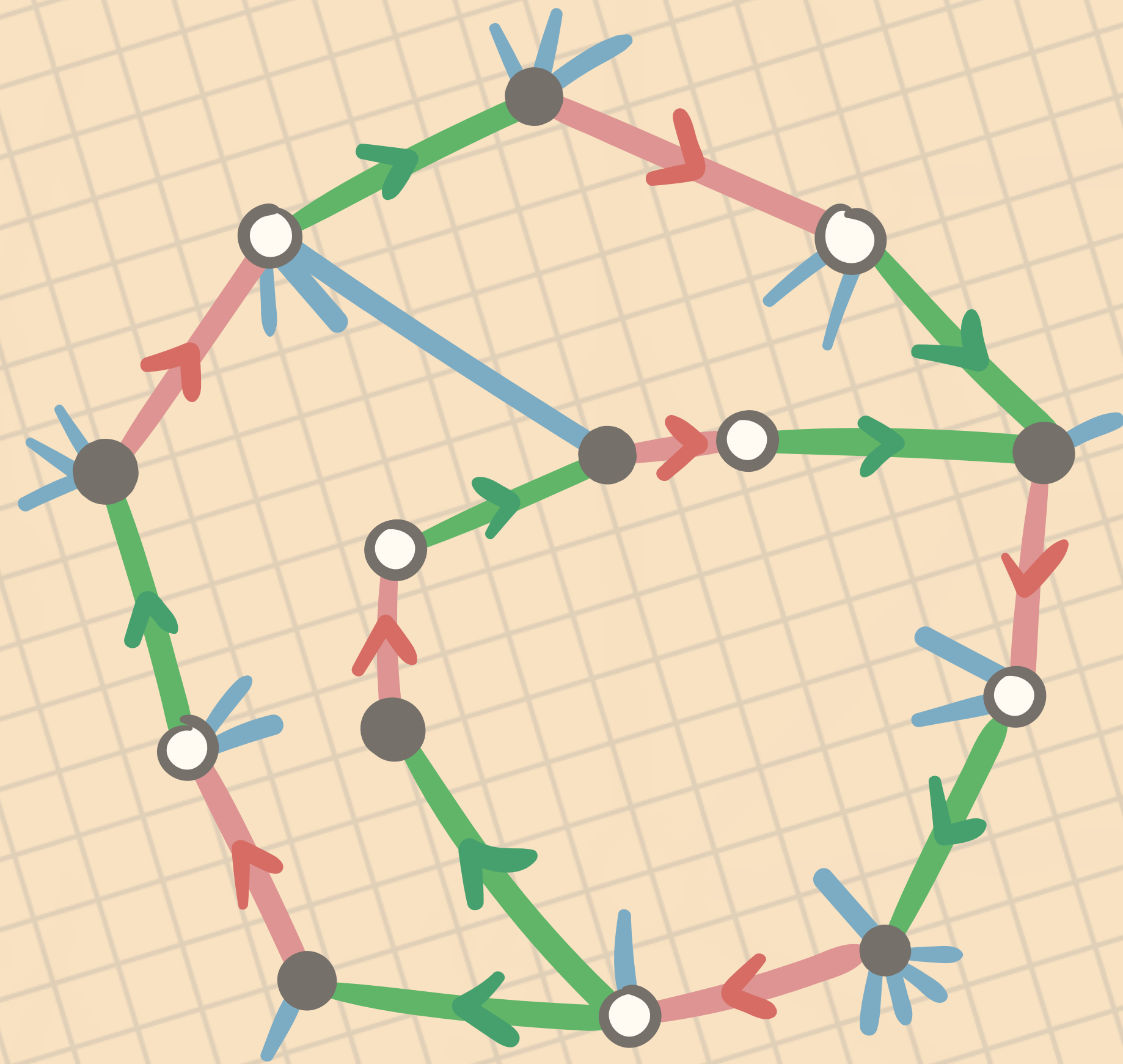
G

B

R

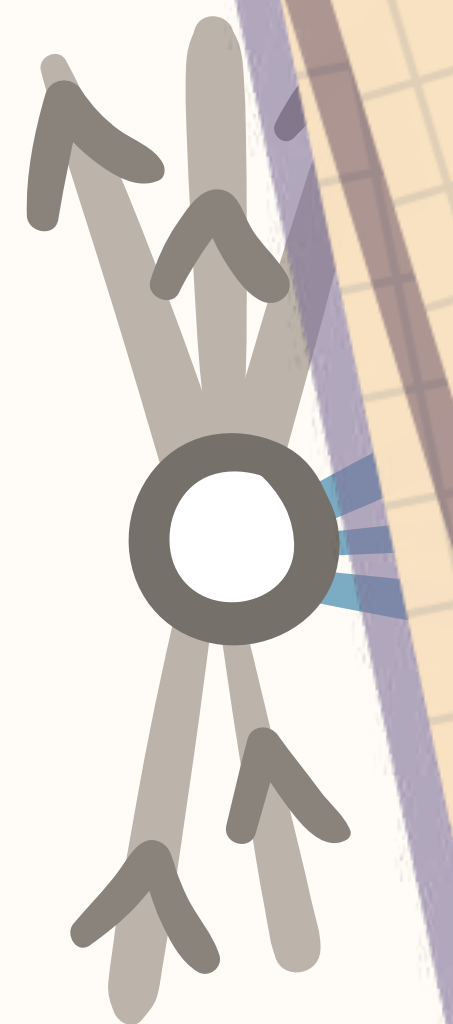
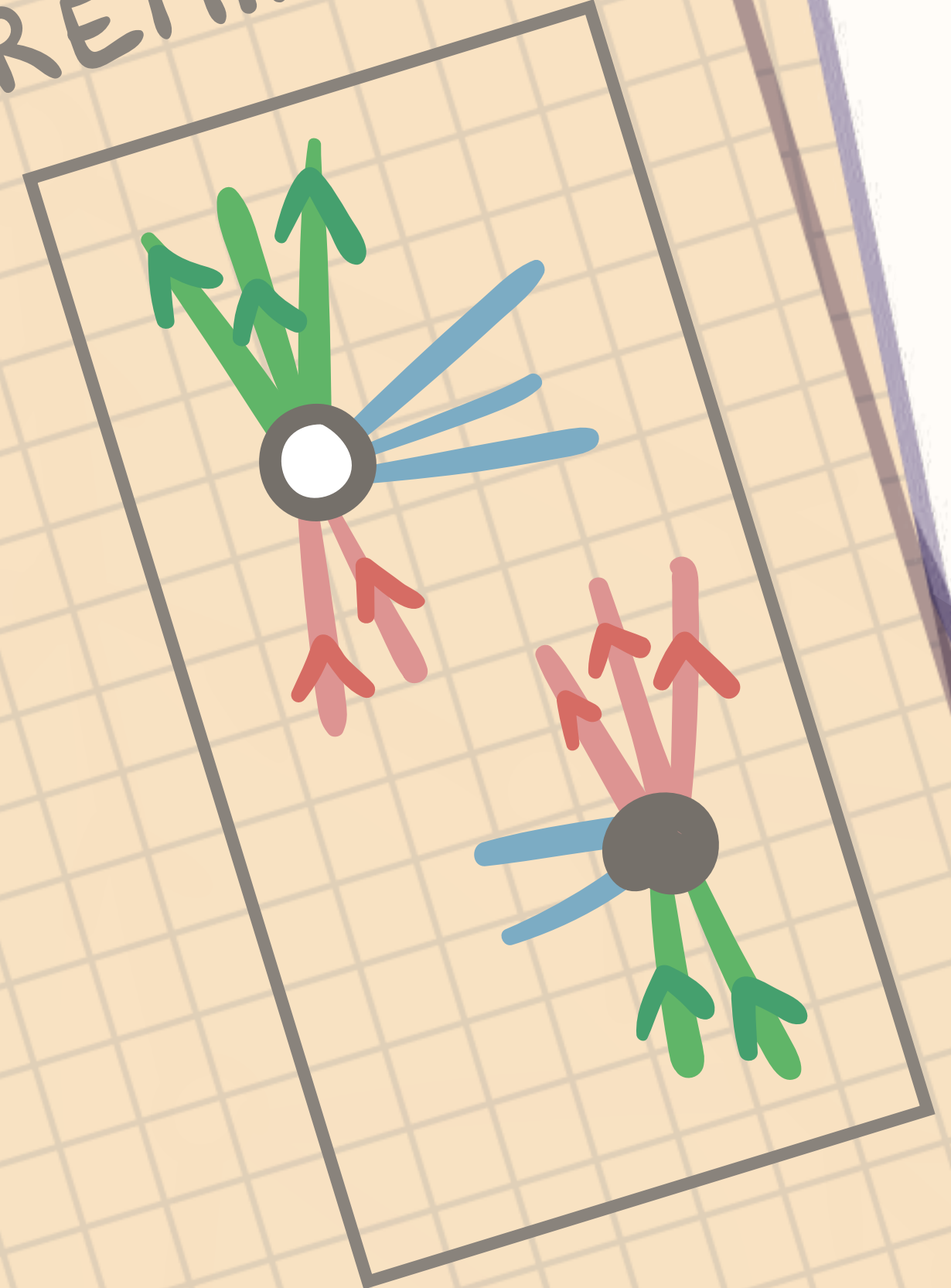
Schnyder labellings

IT IS A BIPOLAR ORIENTATION :
1) ACYCLIC
ASSUME \exists CYCLE, TAKEN MINIMAL



NOT MINIMAL

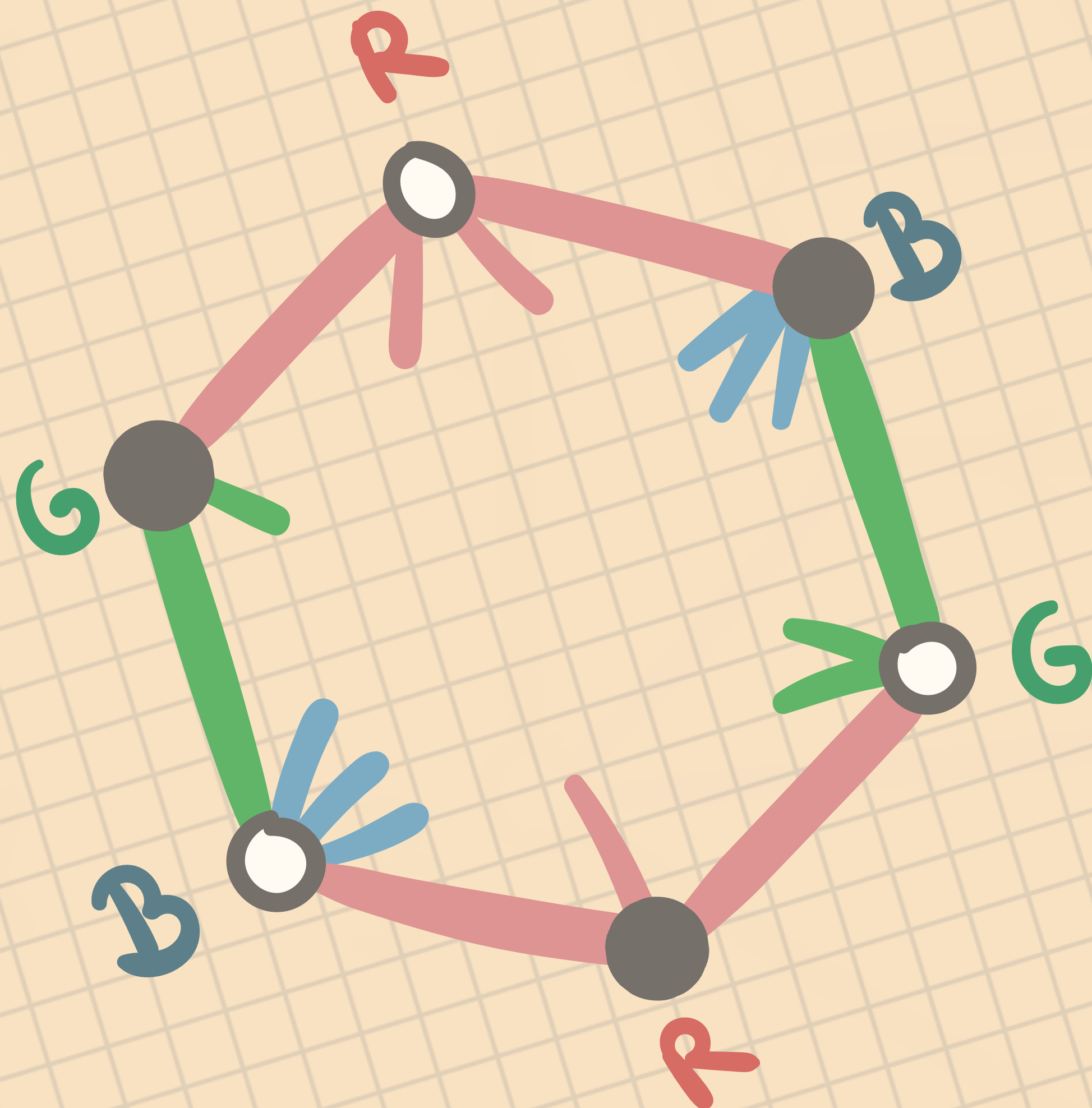
REMINDER



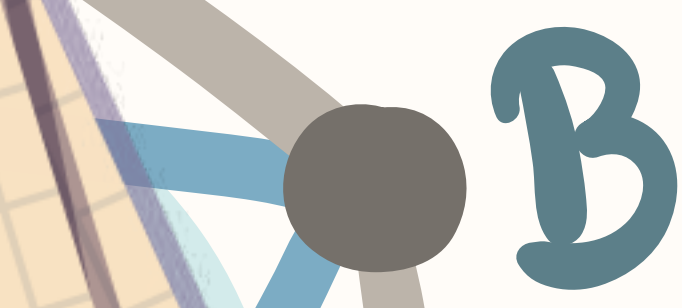
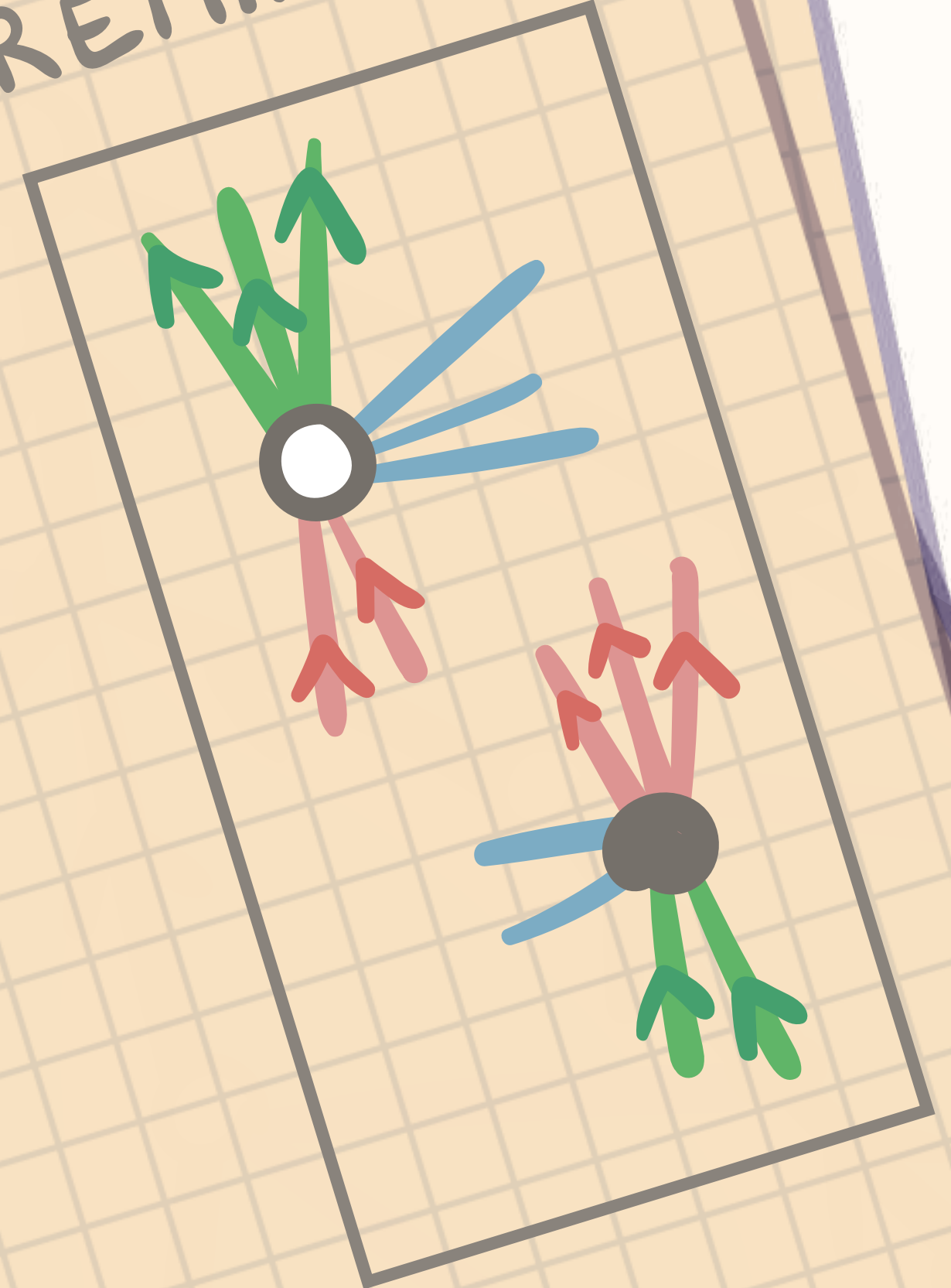
G

Schnyder labellings

- IT IS A BIPOLAR ORIENTATION :
- 1) ACYCLIC
 - 2) SINGLE SINK / SOURCE



REMINDER

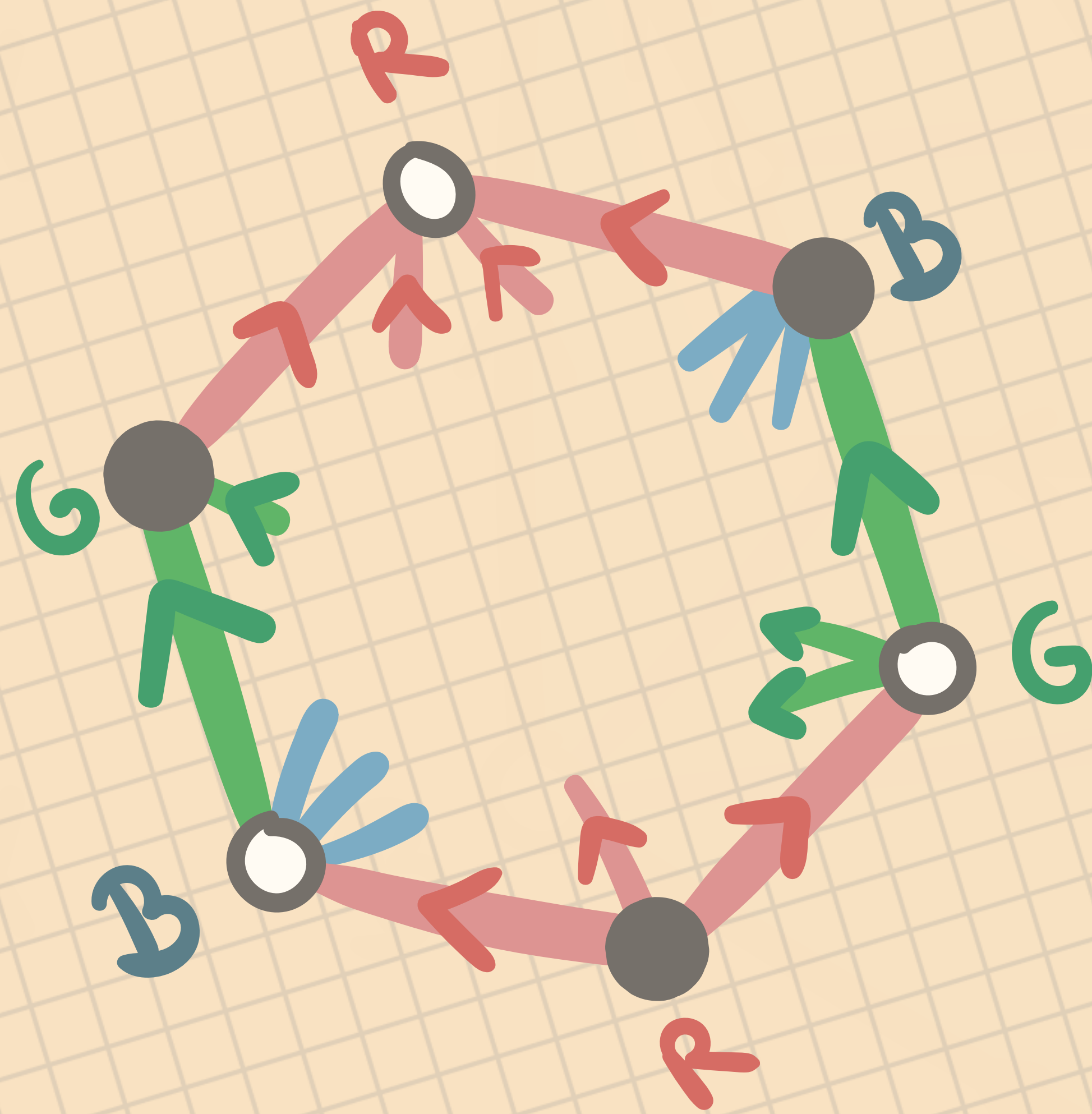


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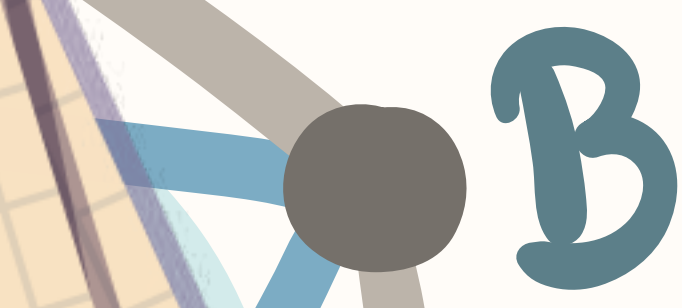
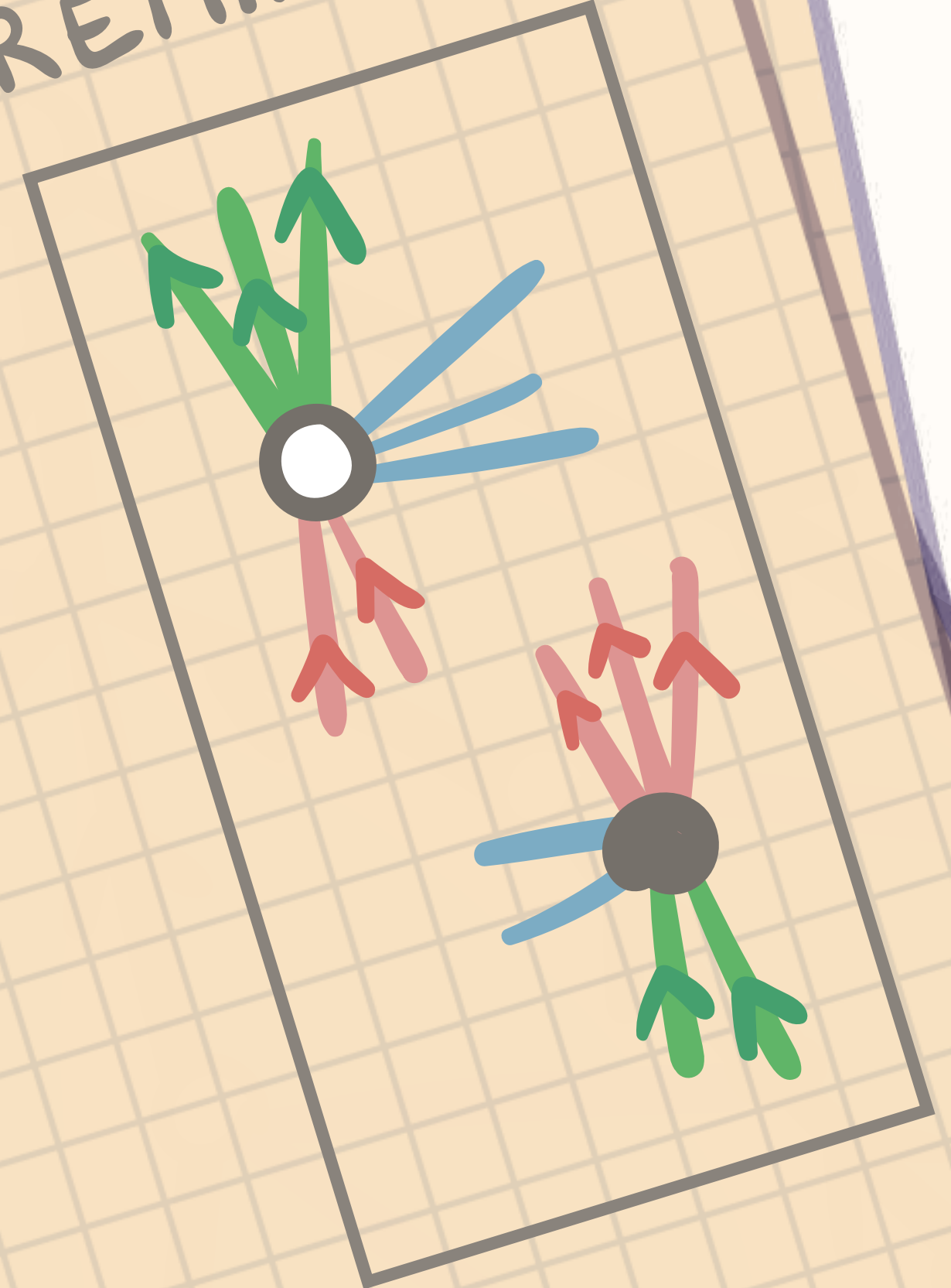
R

Schnyder labellings

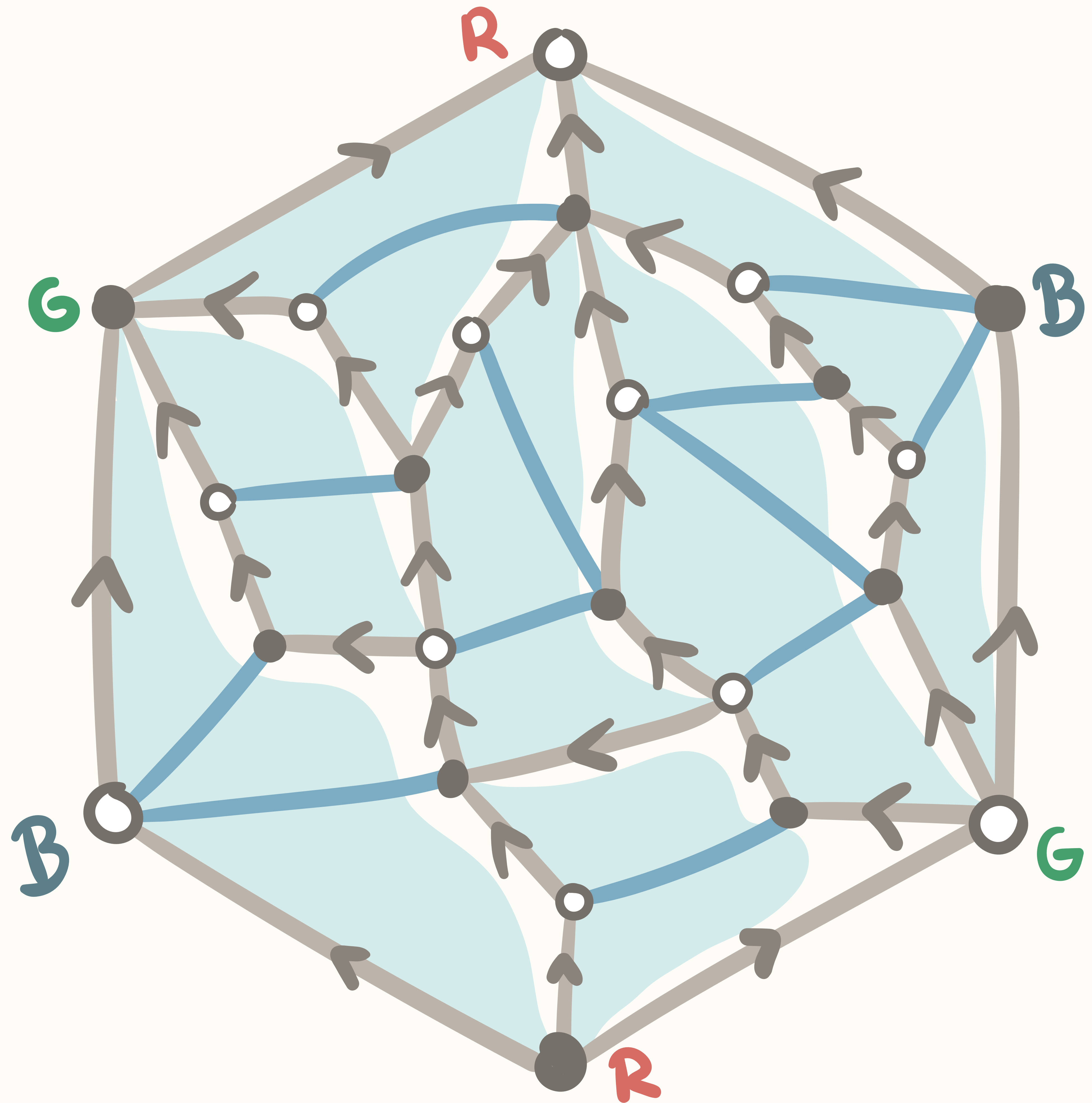
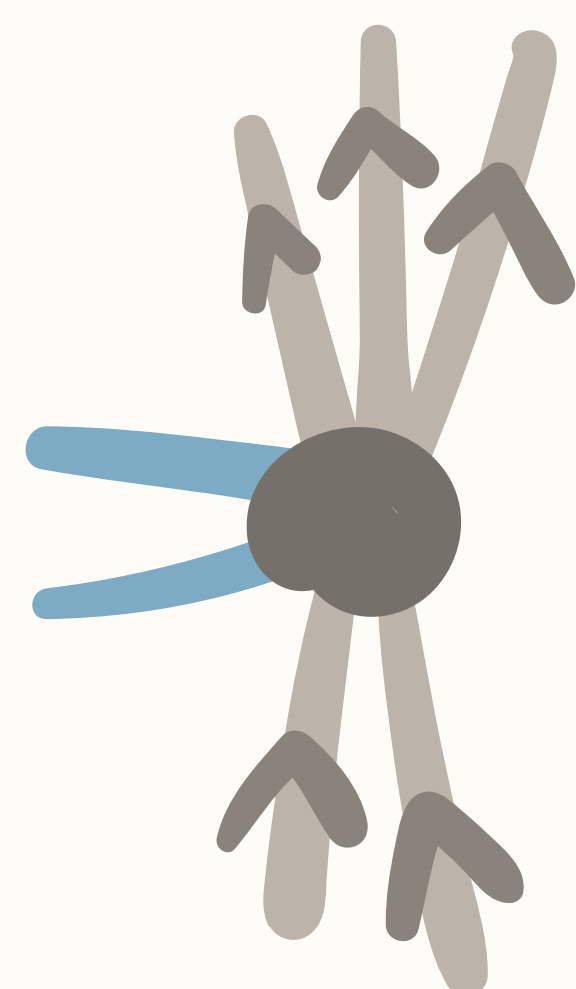
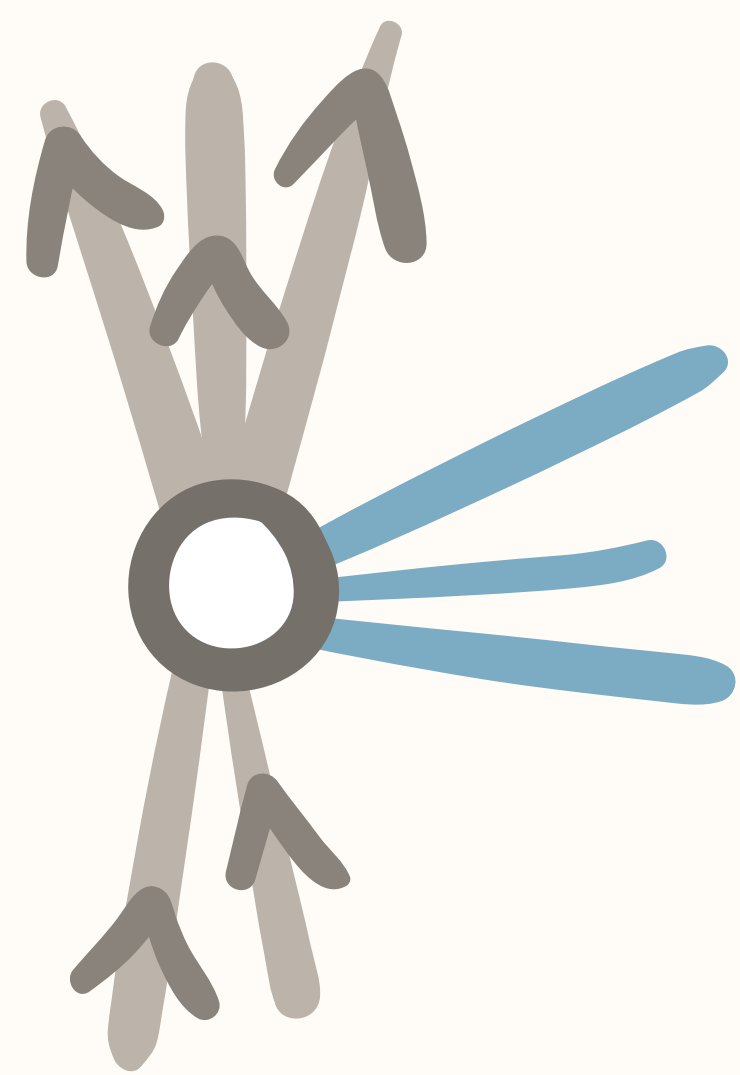
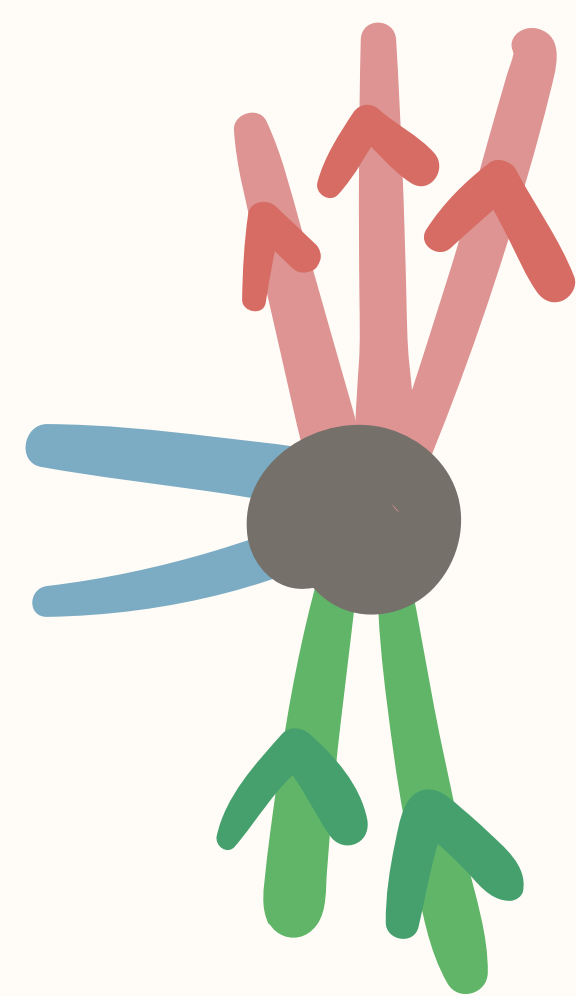
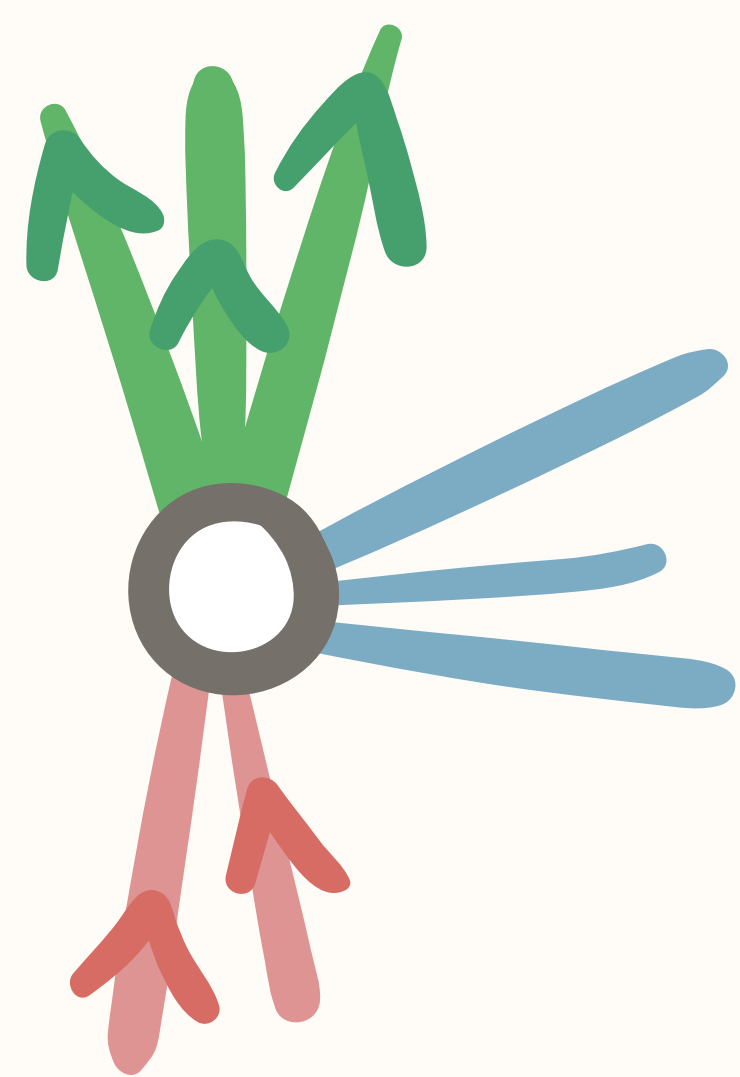
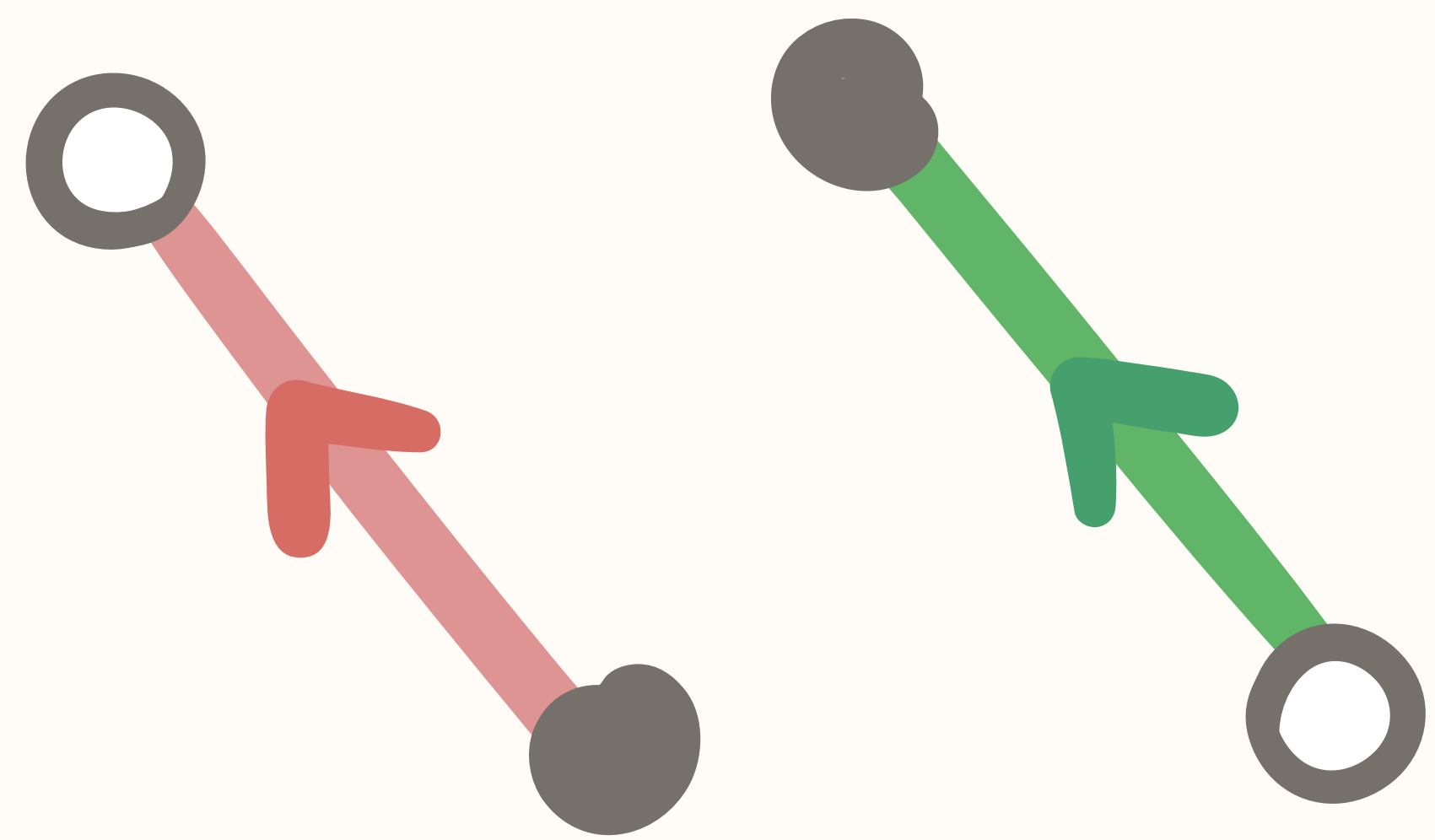
- IT IS A BIPOLAR ORIENTATION :
- 1) ACYCLIC
 - 2) SINGLE SINK / SOURCE



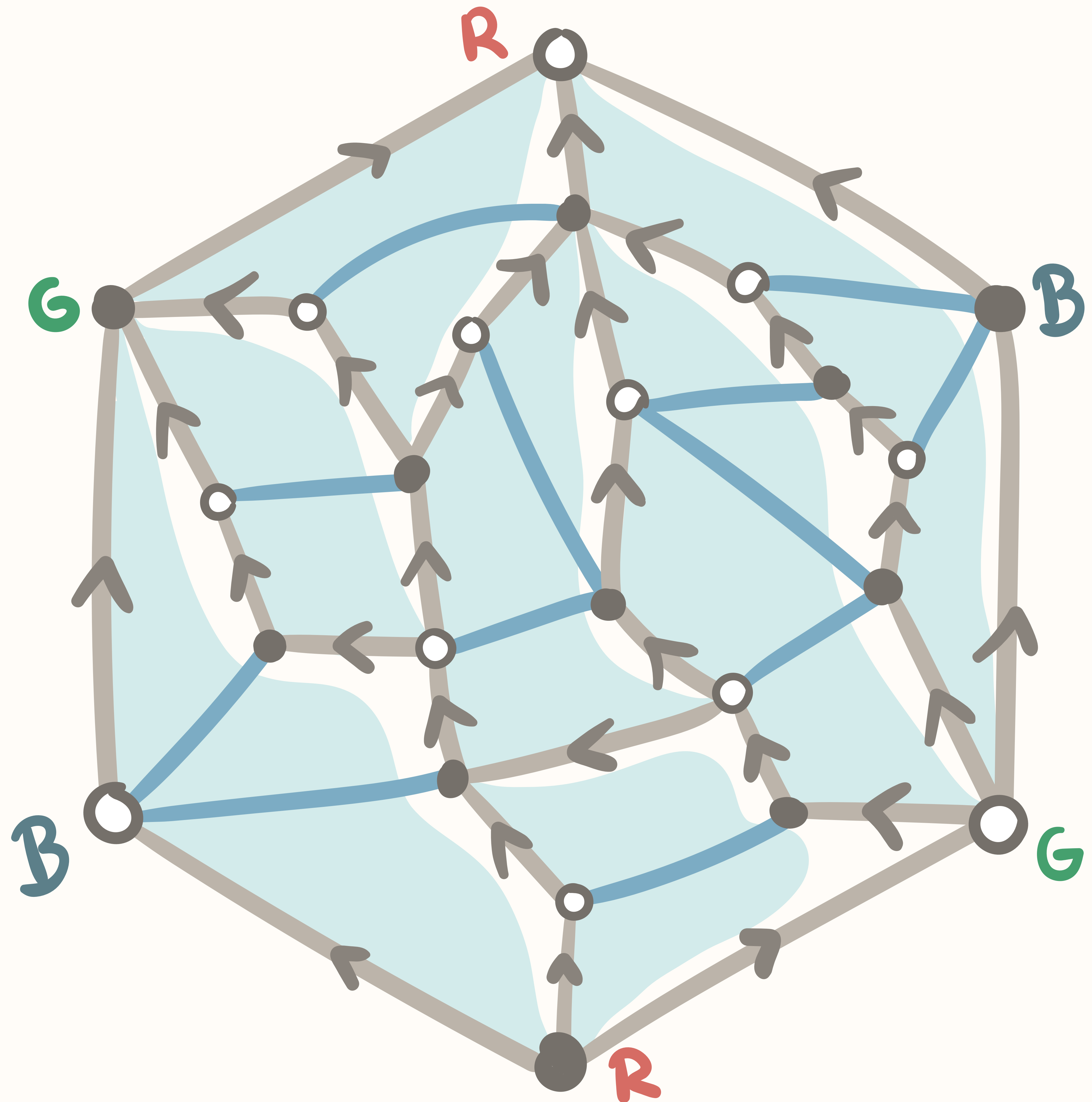
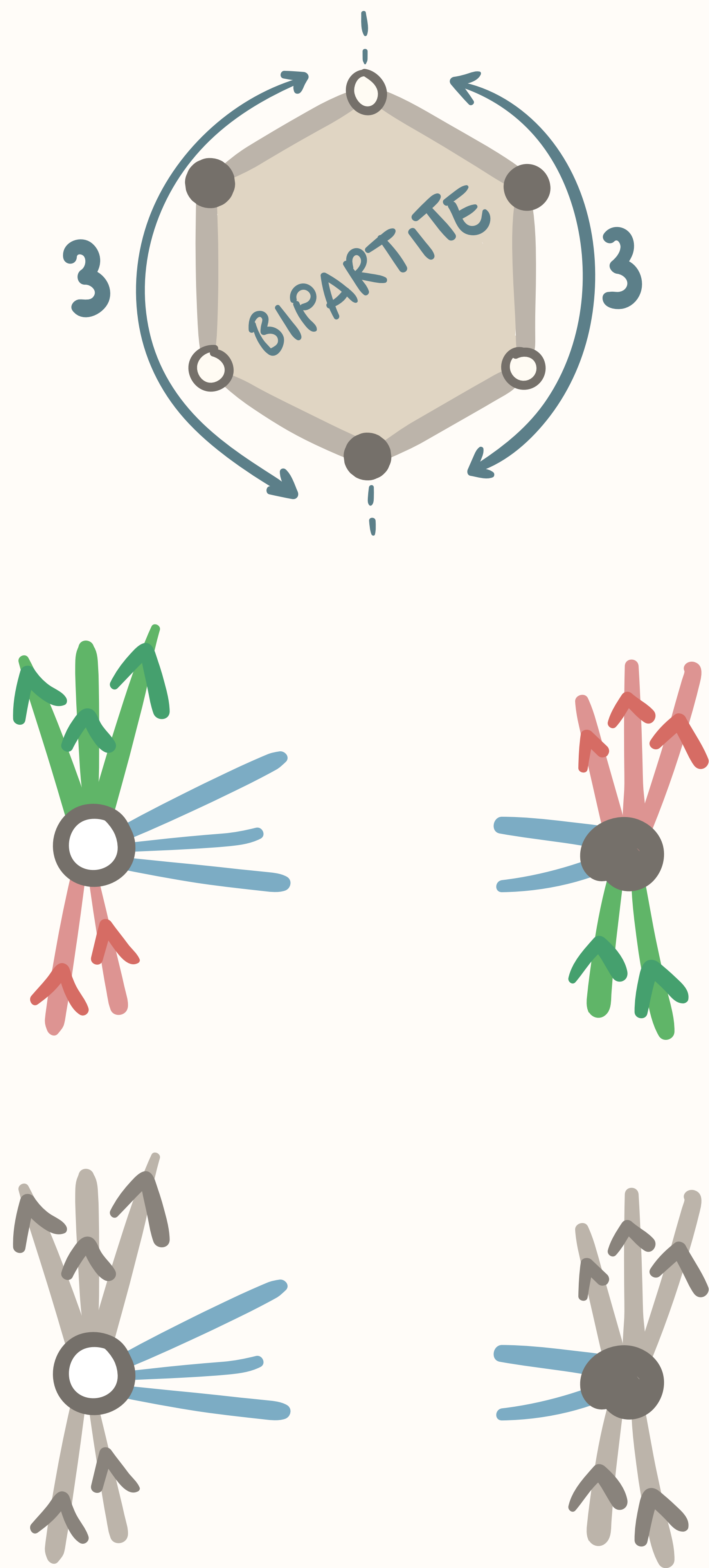
REMINDER



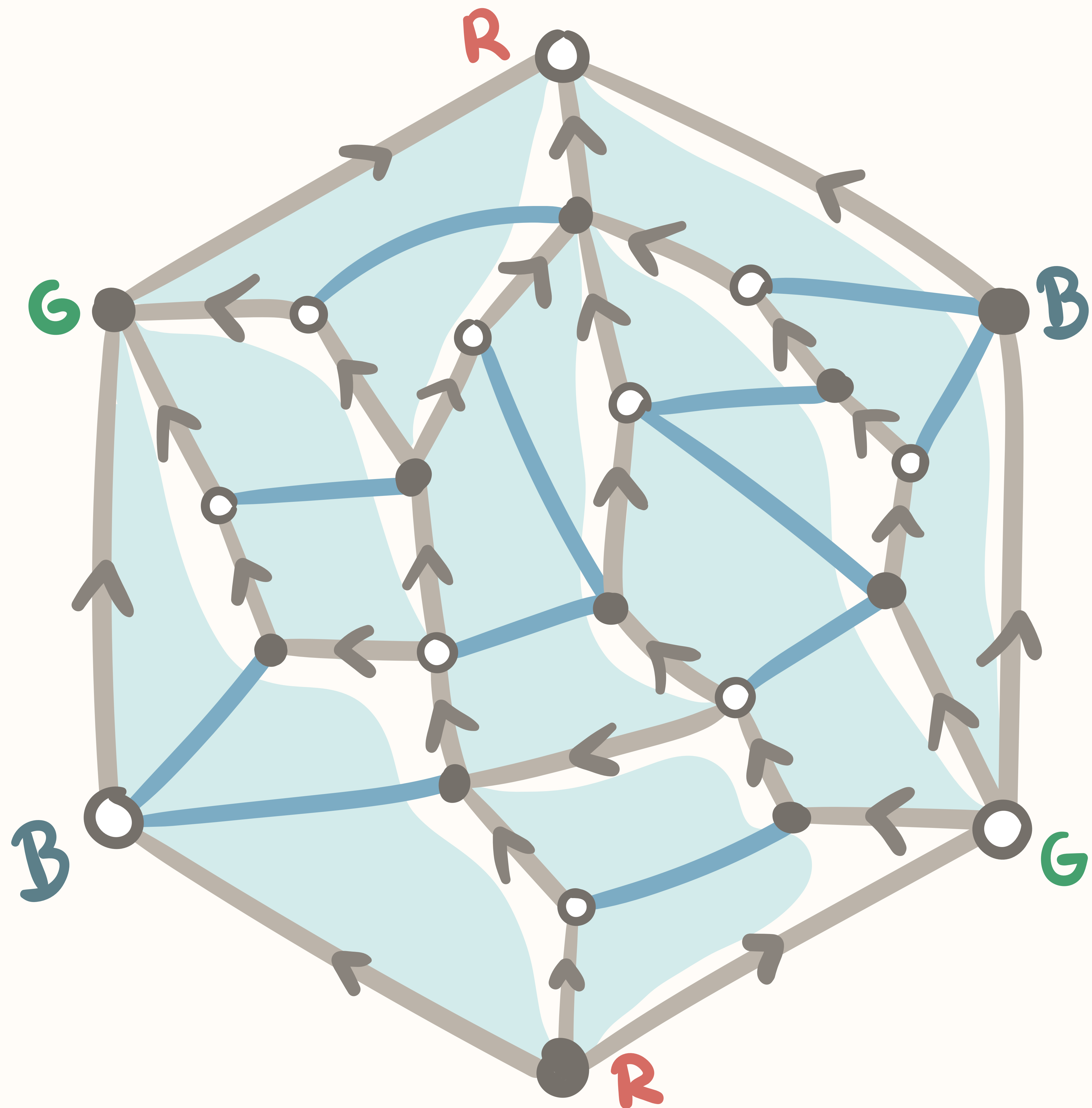
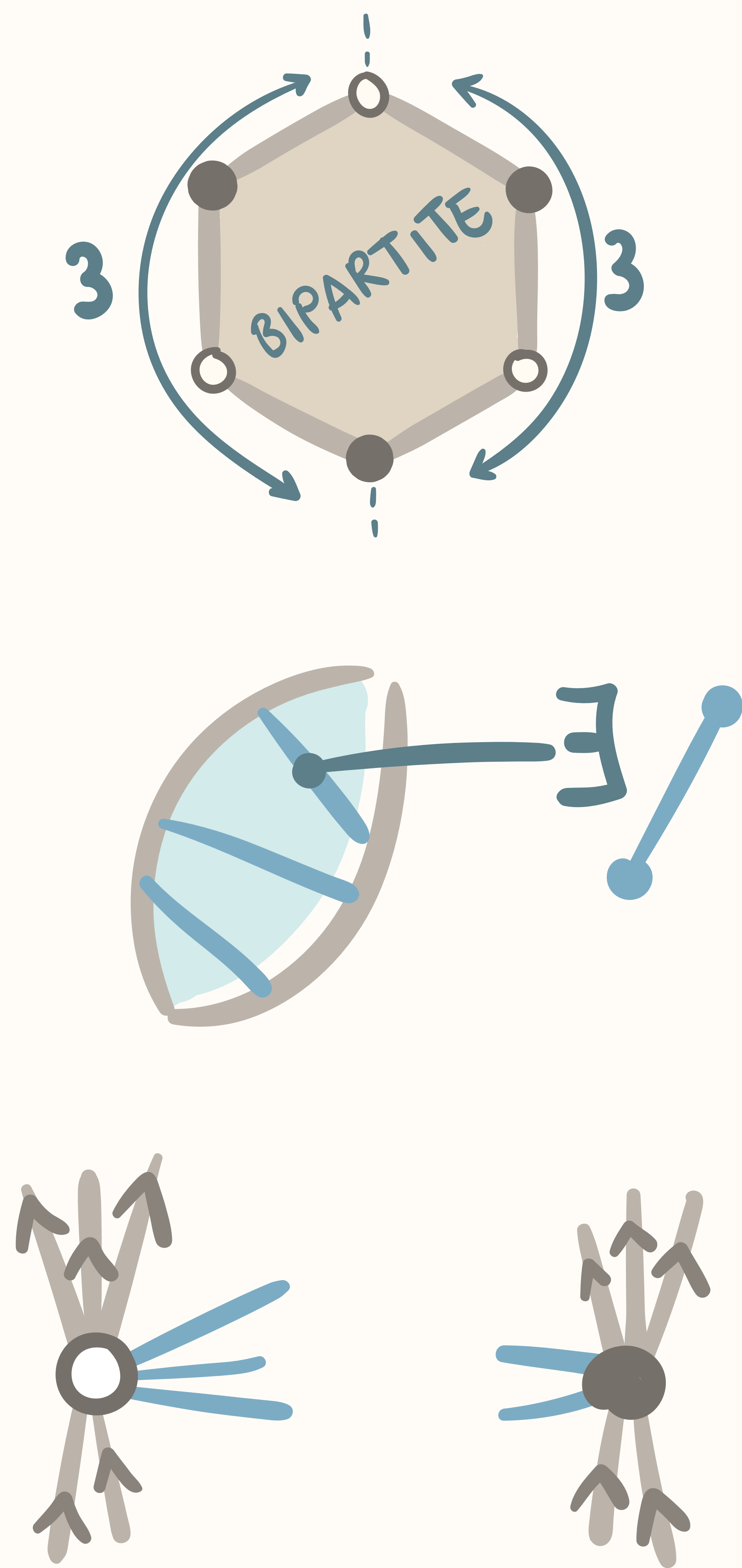
Schnyder labellings



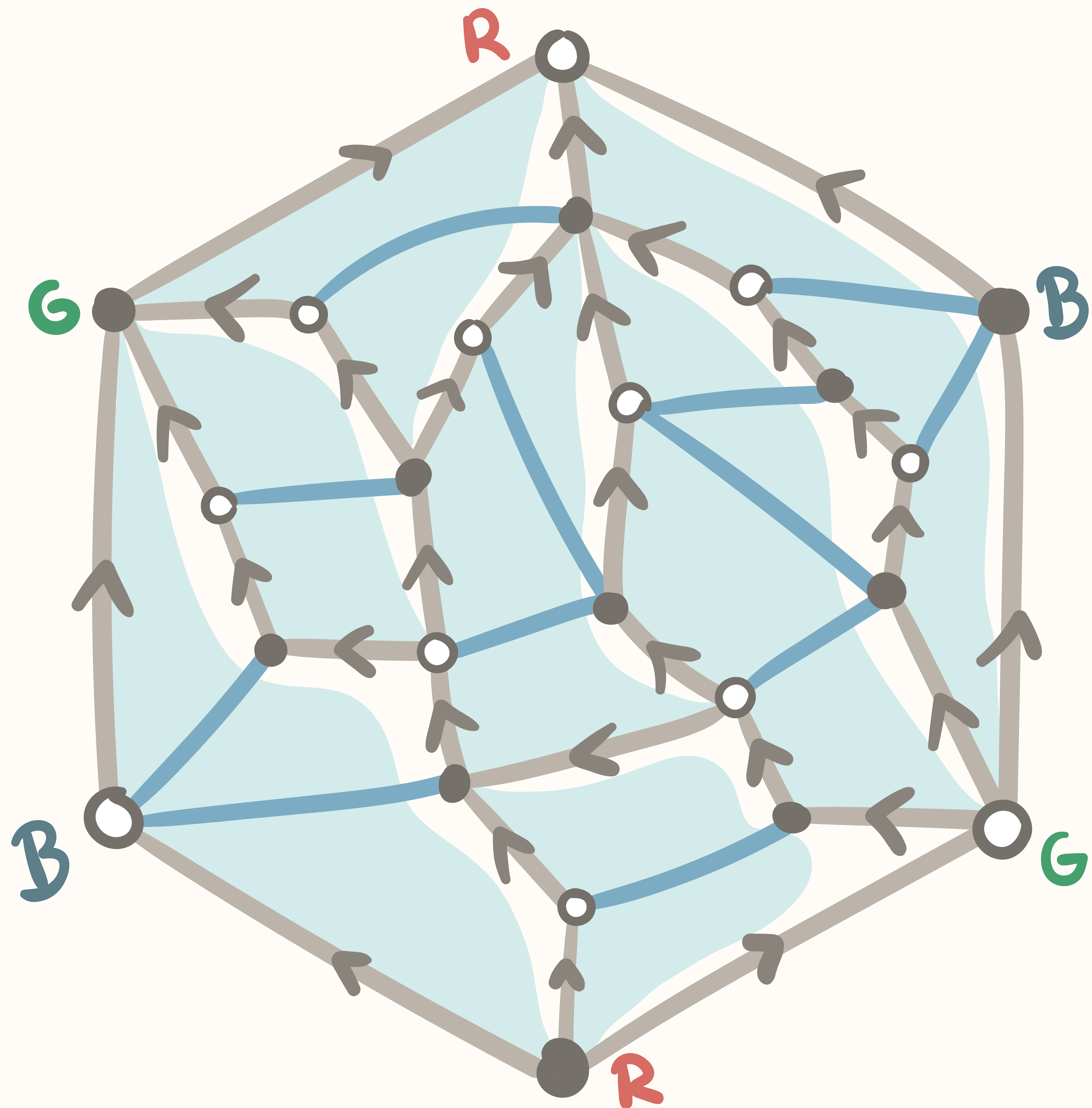
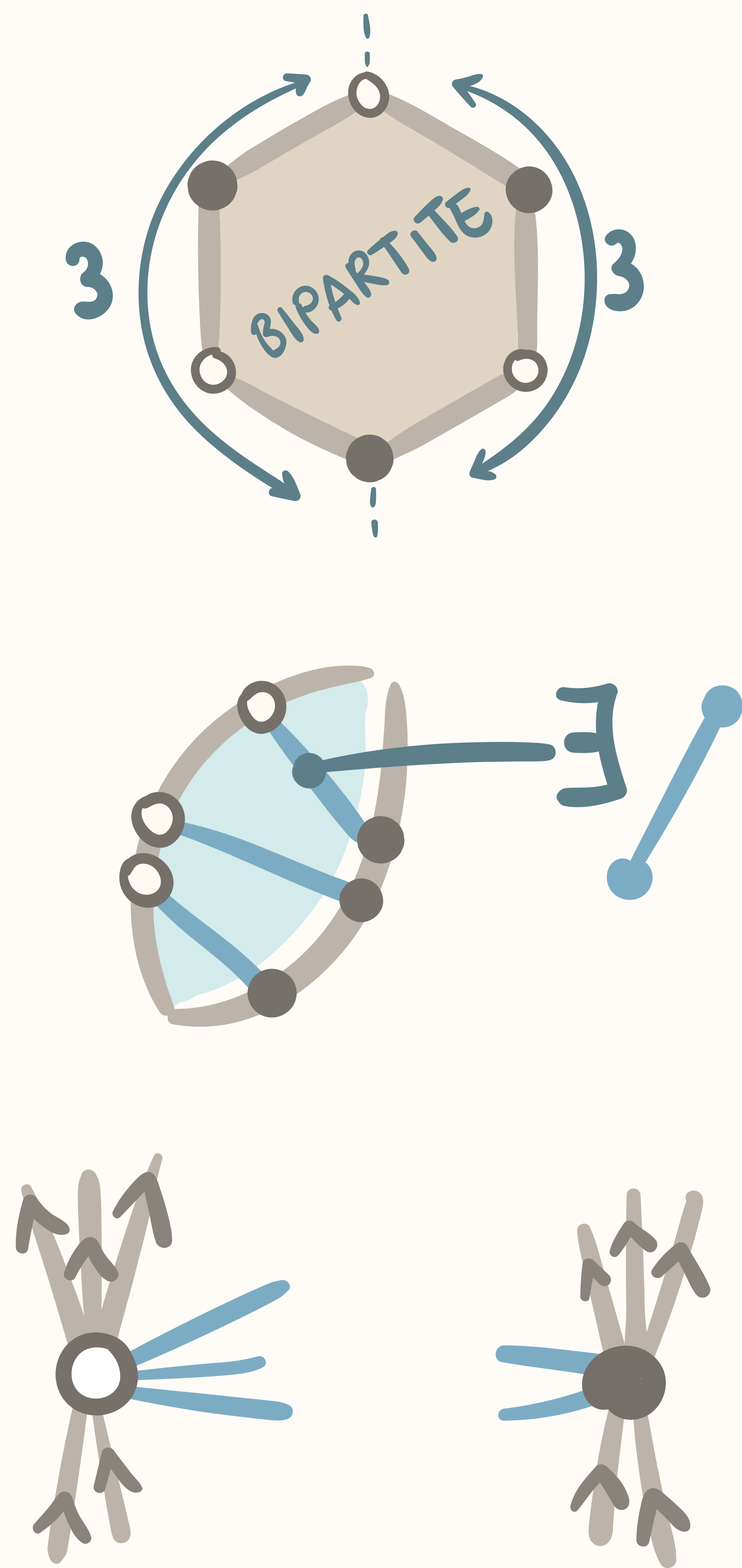
Schnyder labellings



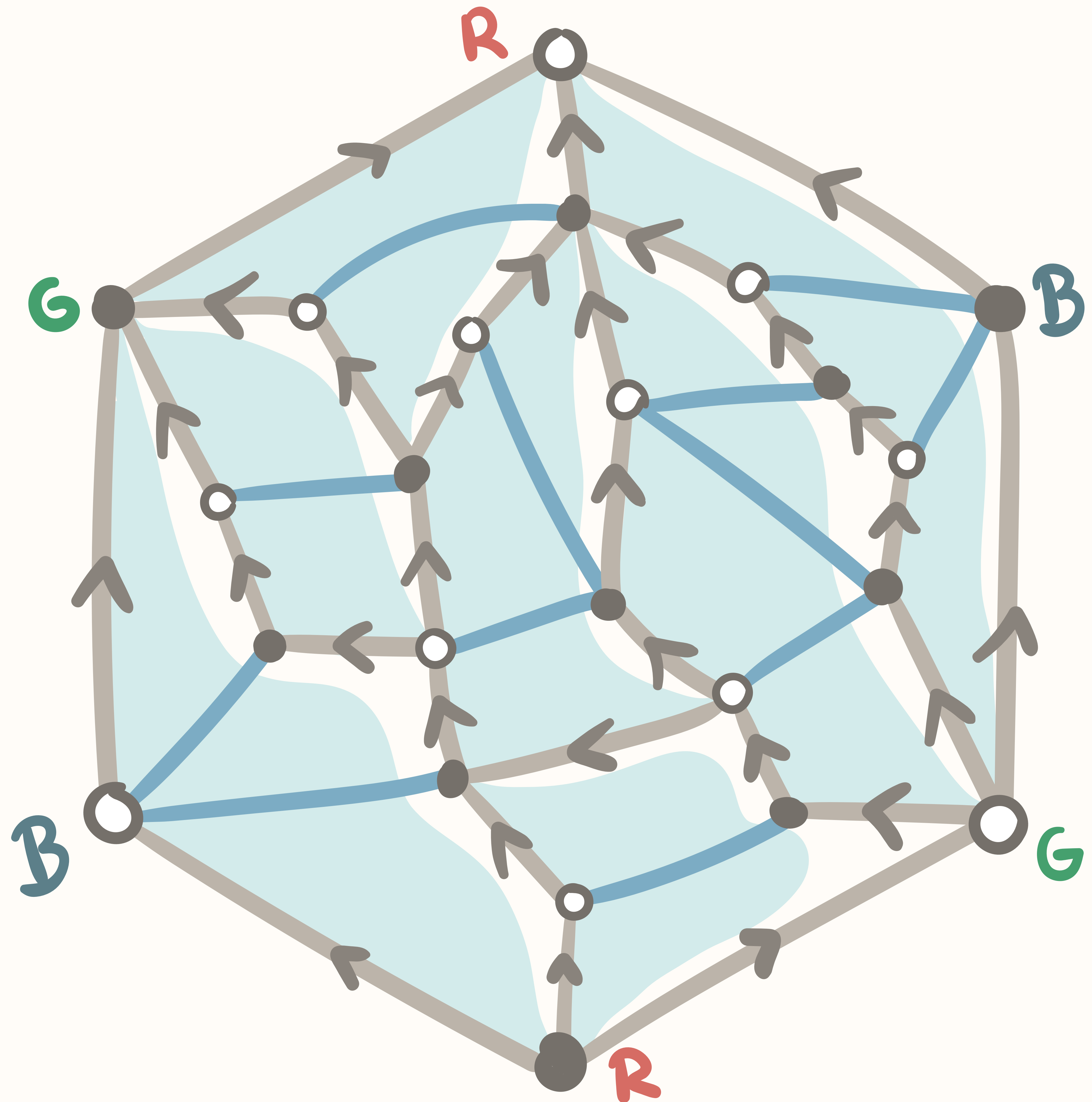
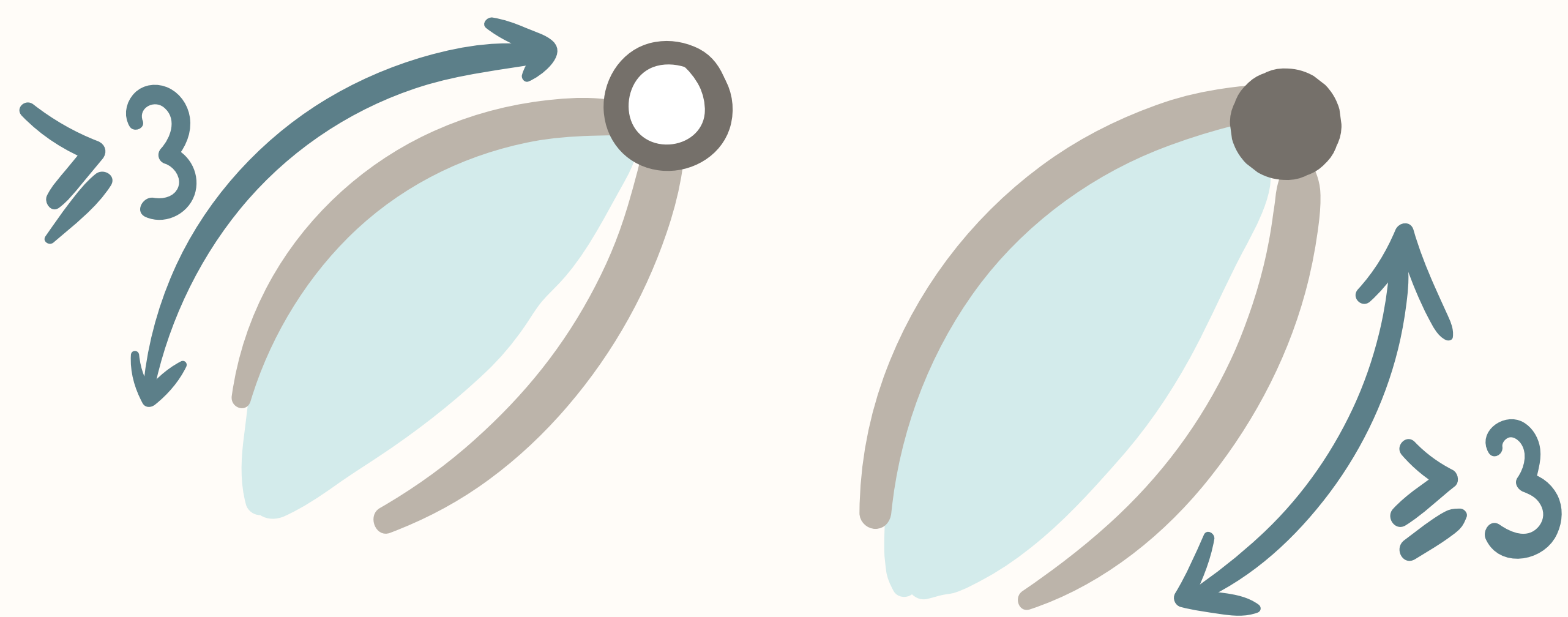
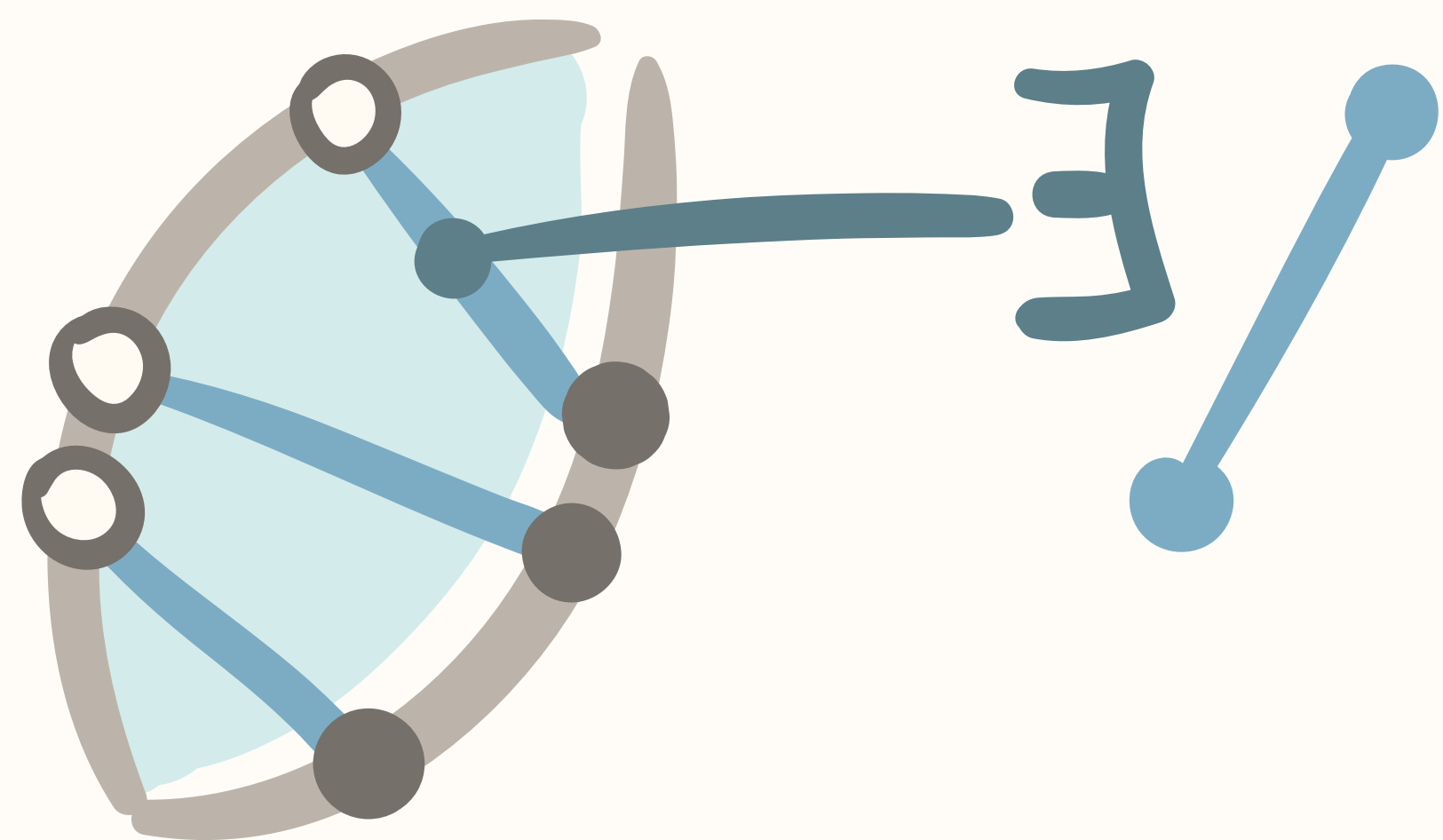
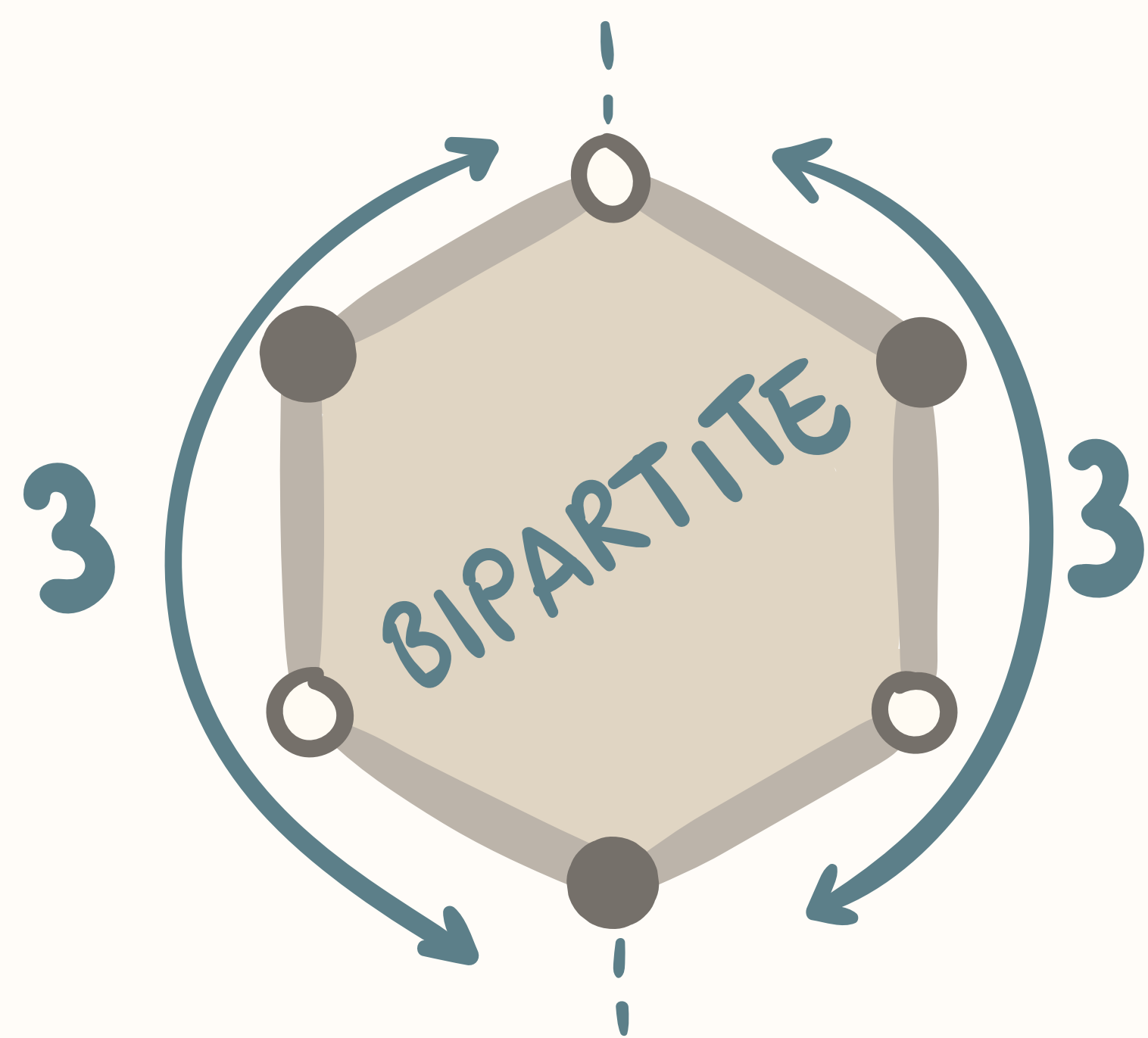
Schnyder labellings



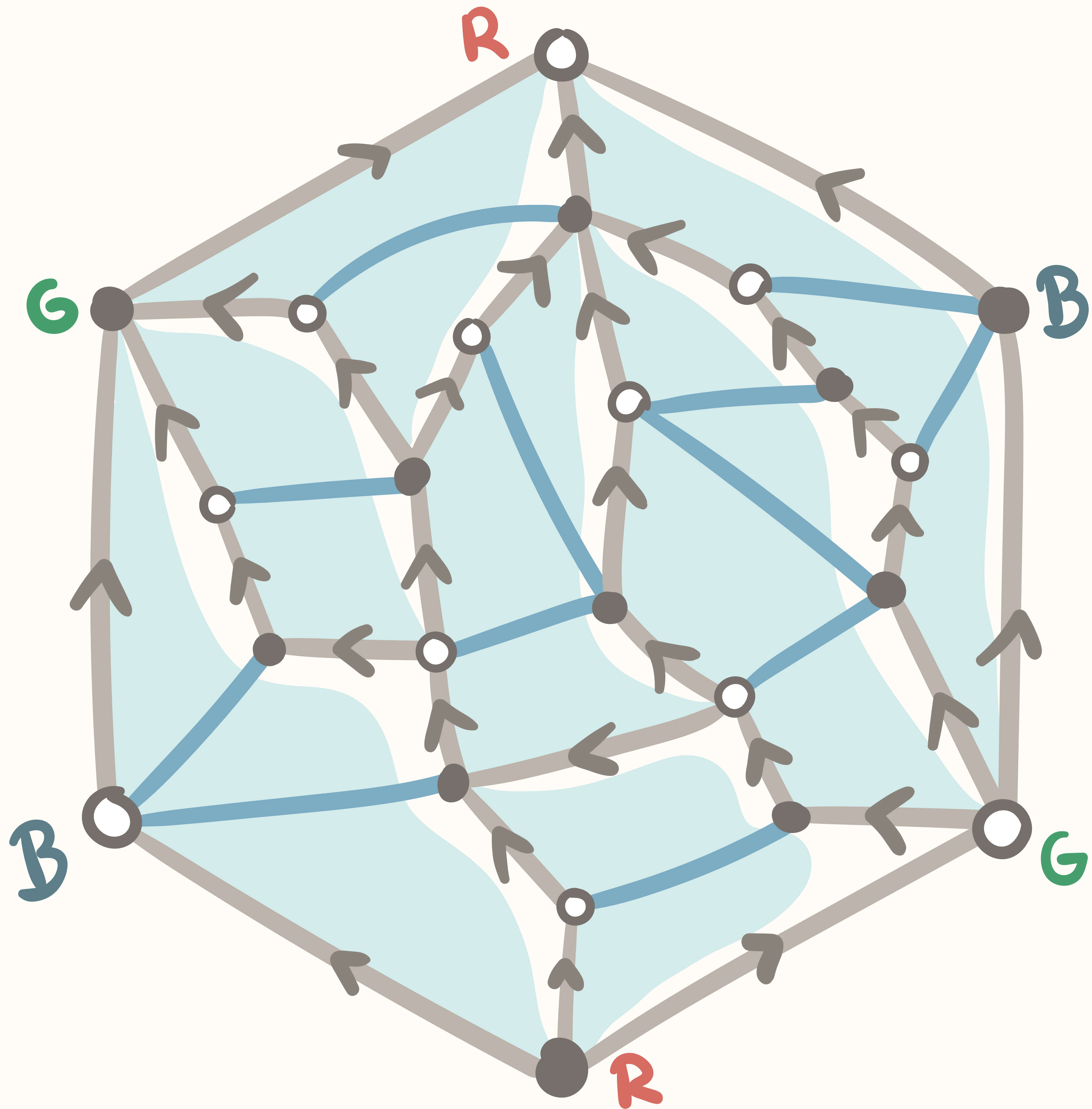
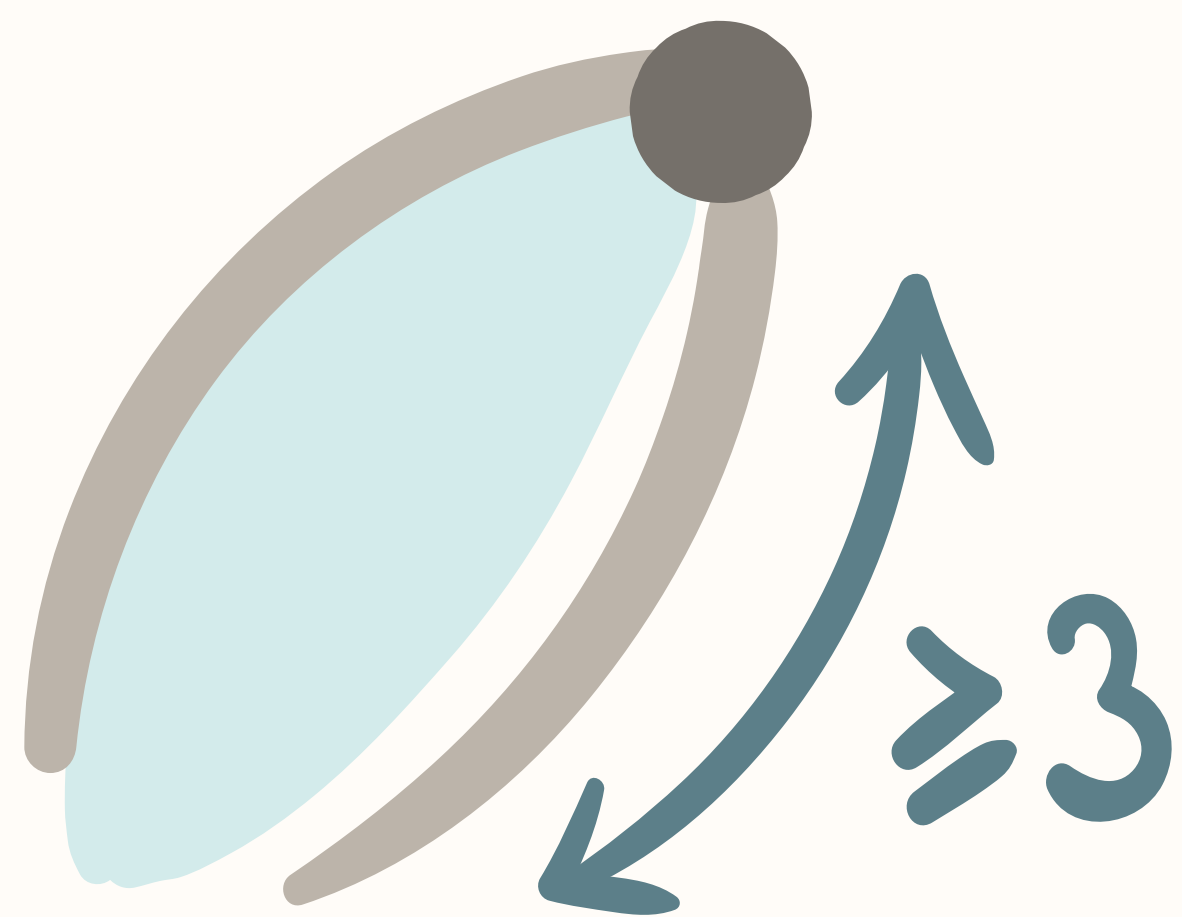
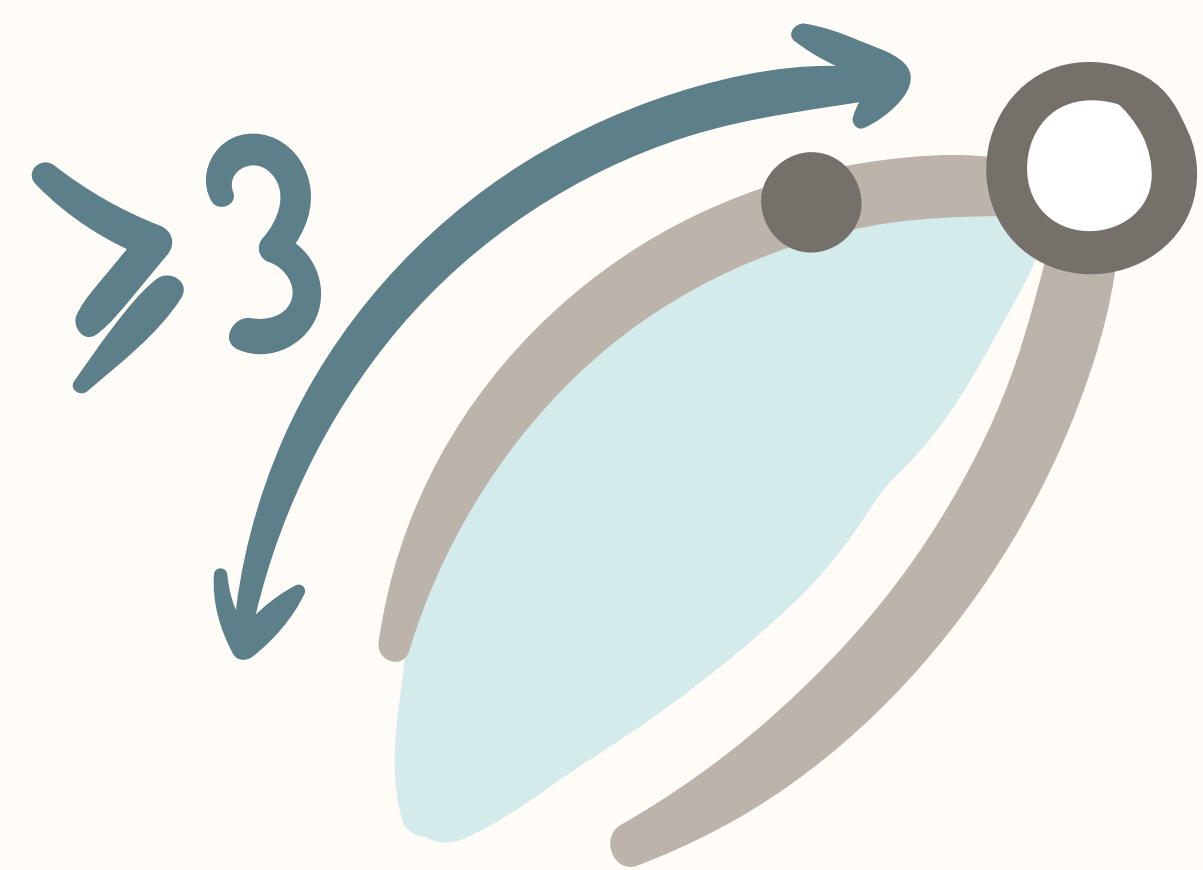
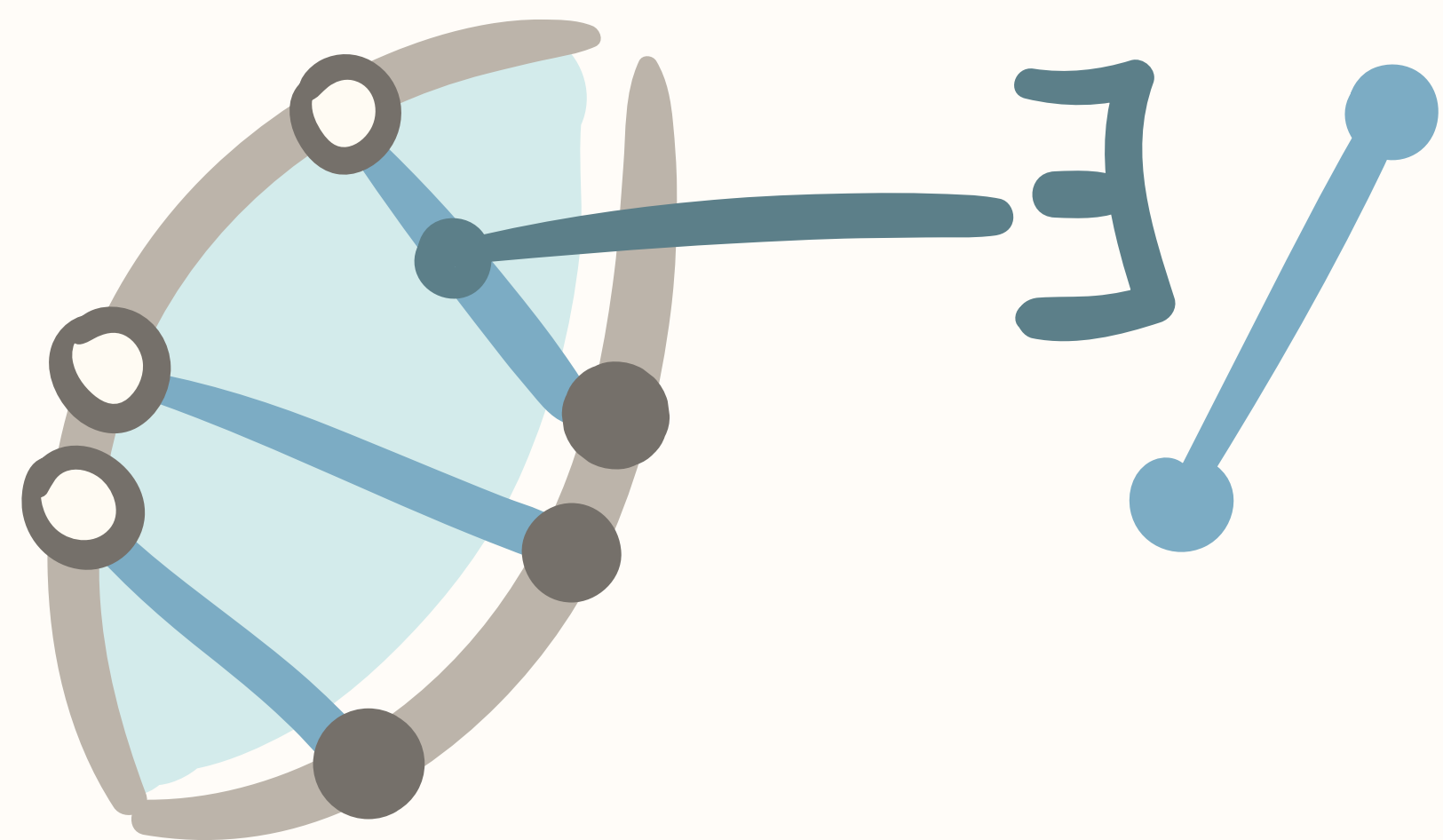
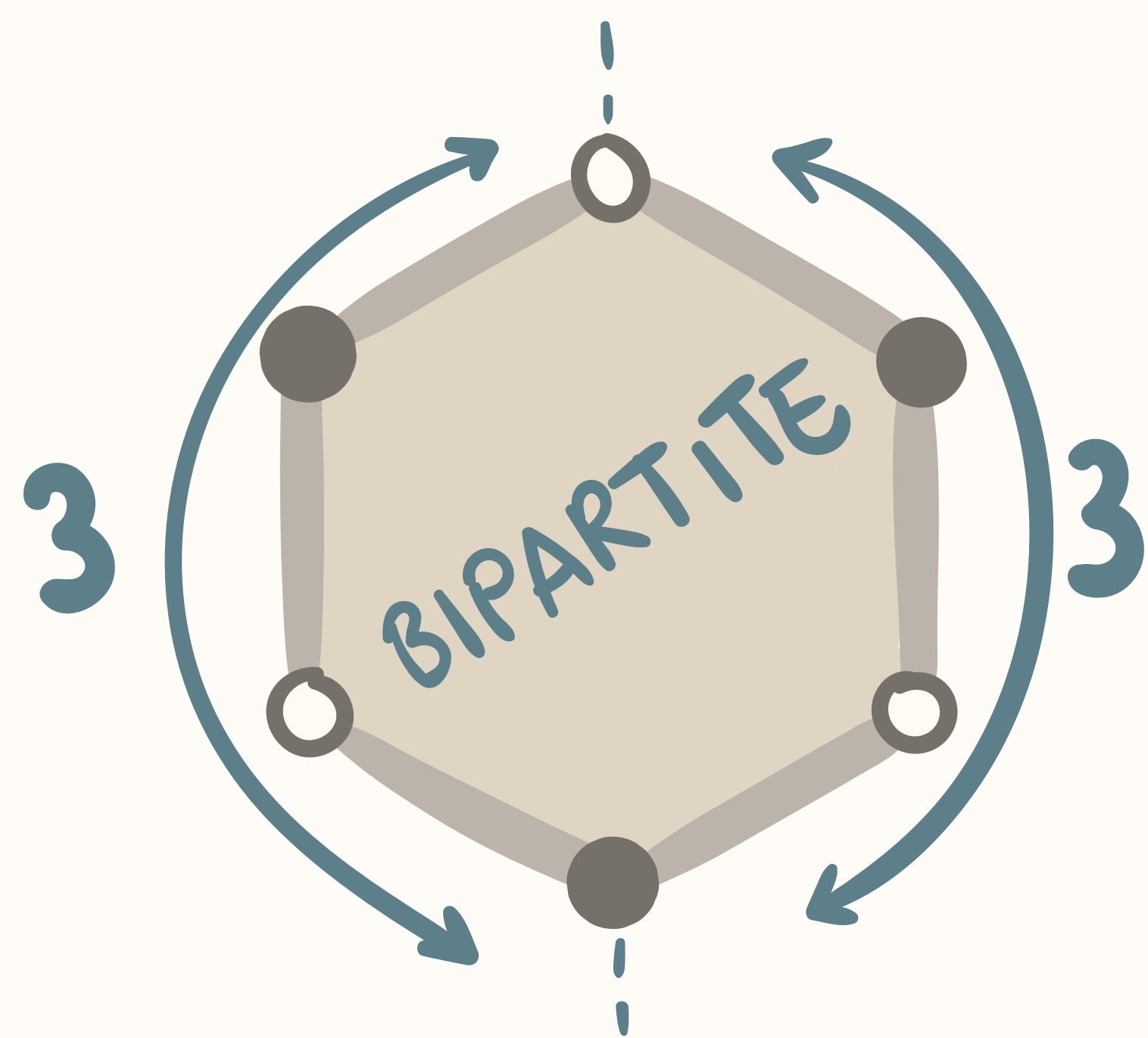
Schnyder labellings



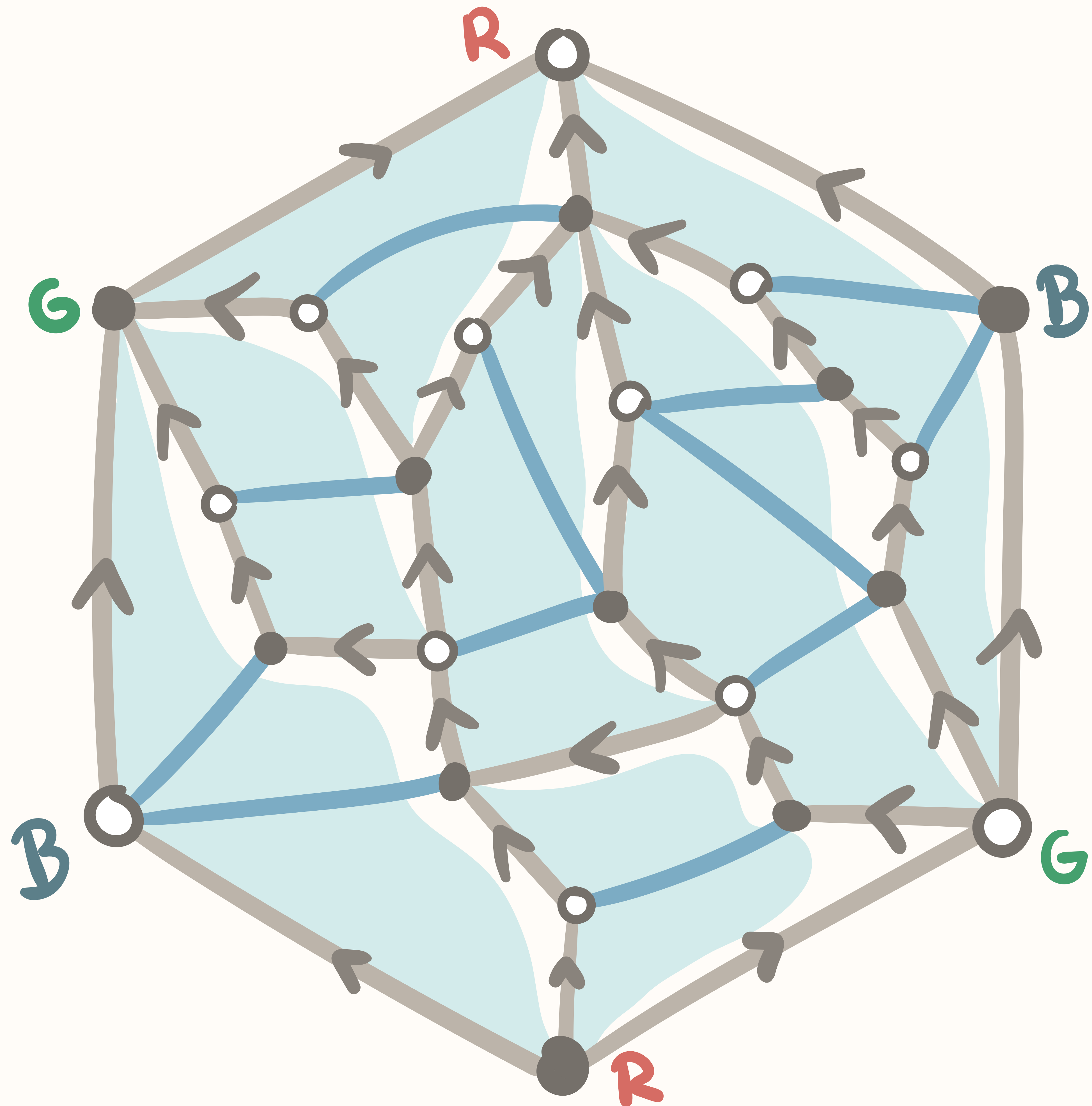
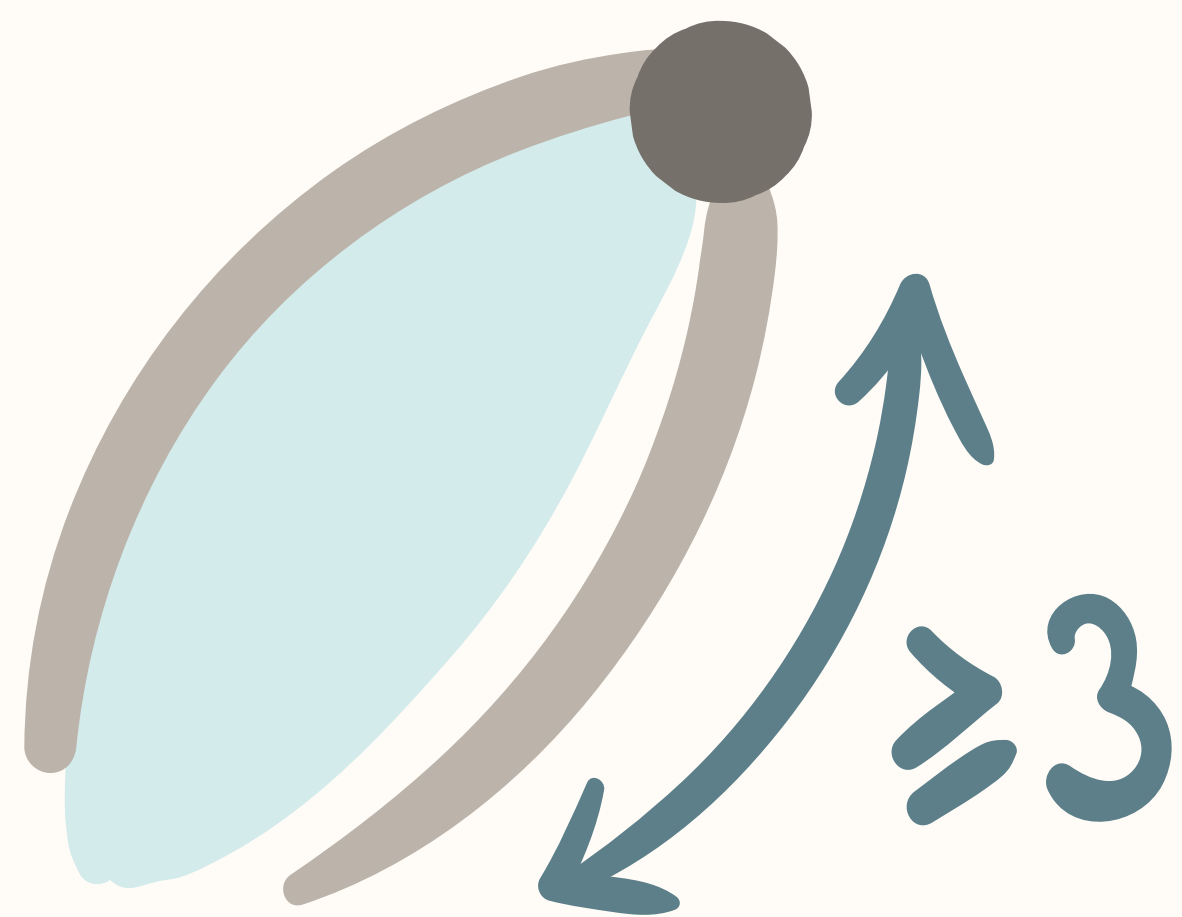
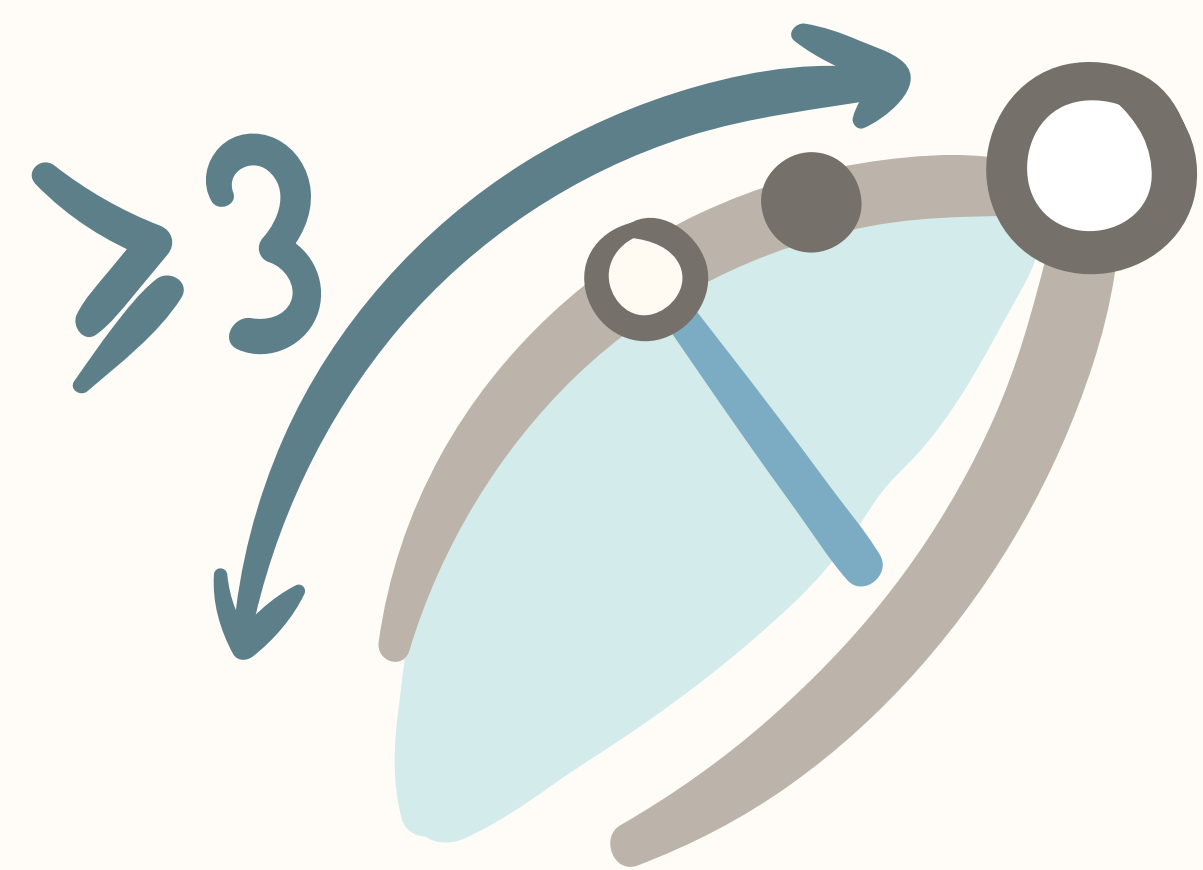
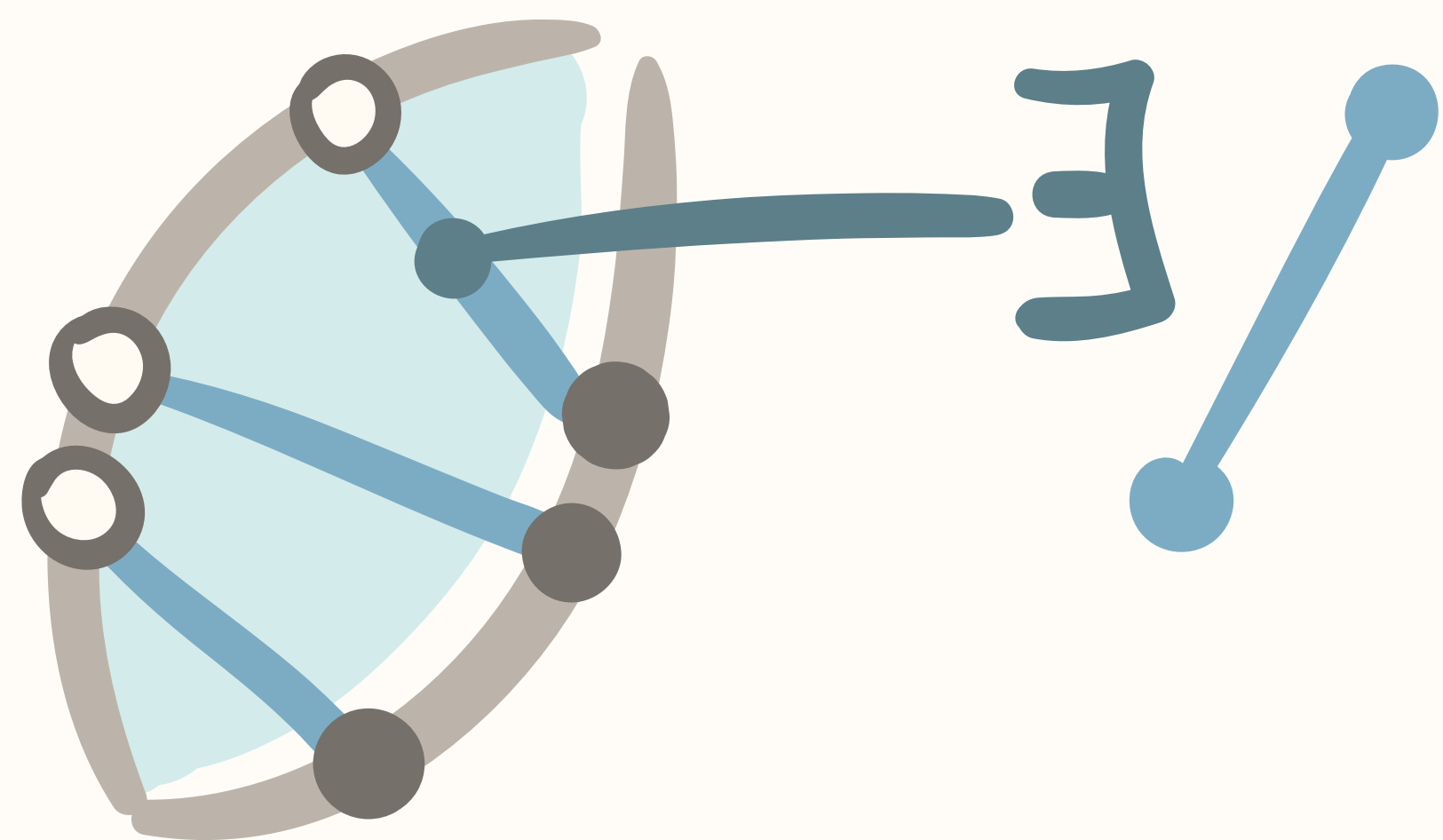
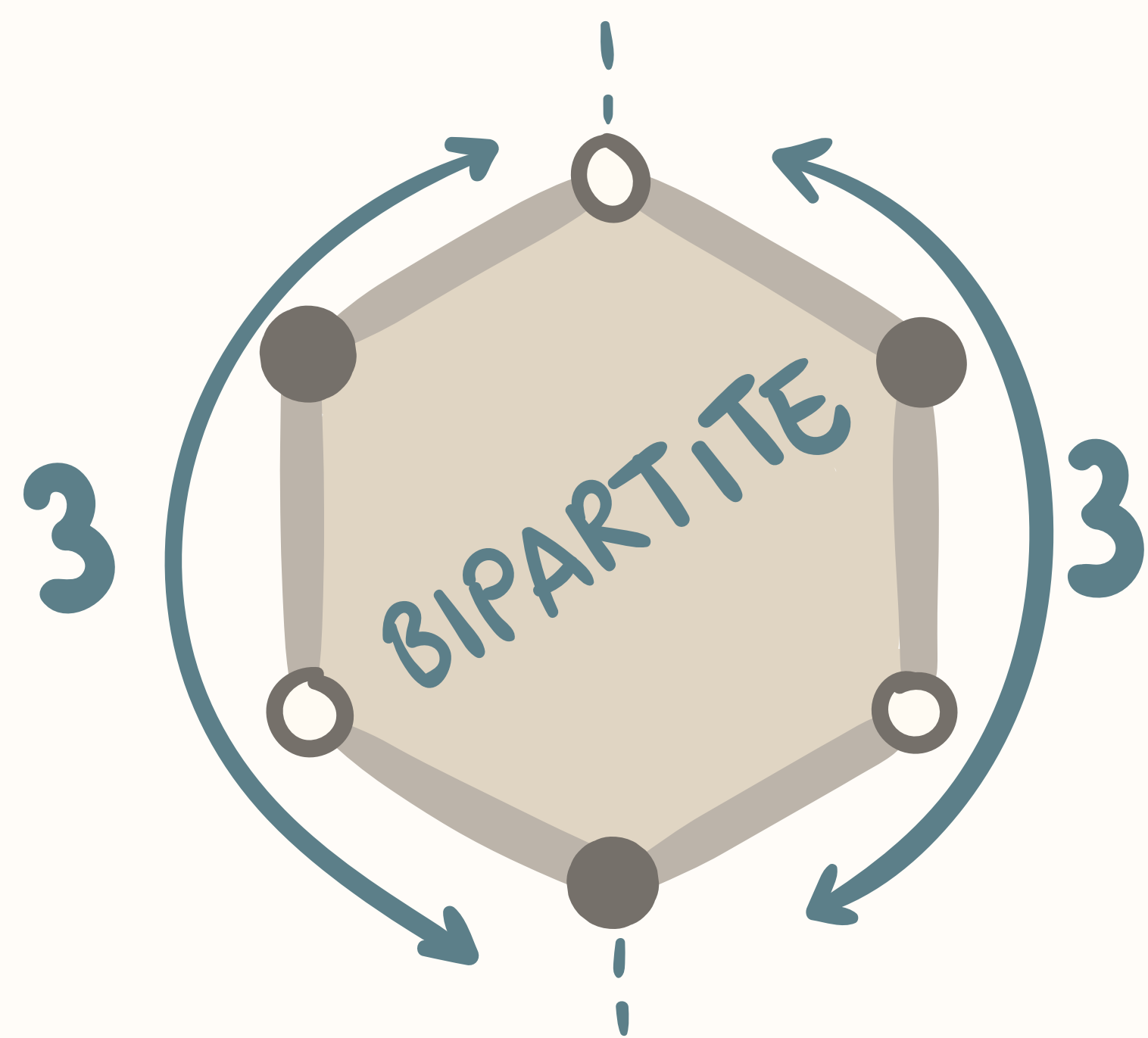
Schnyder labellings



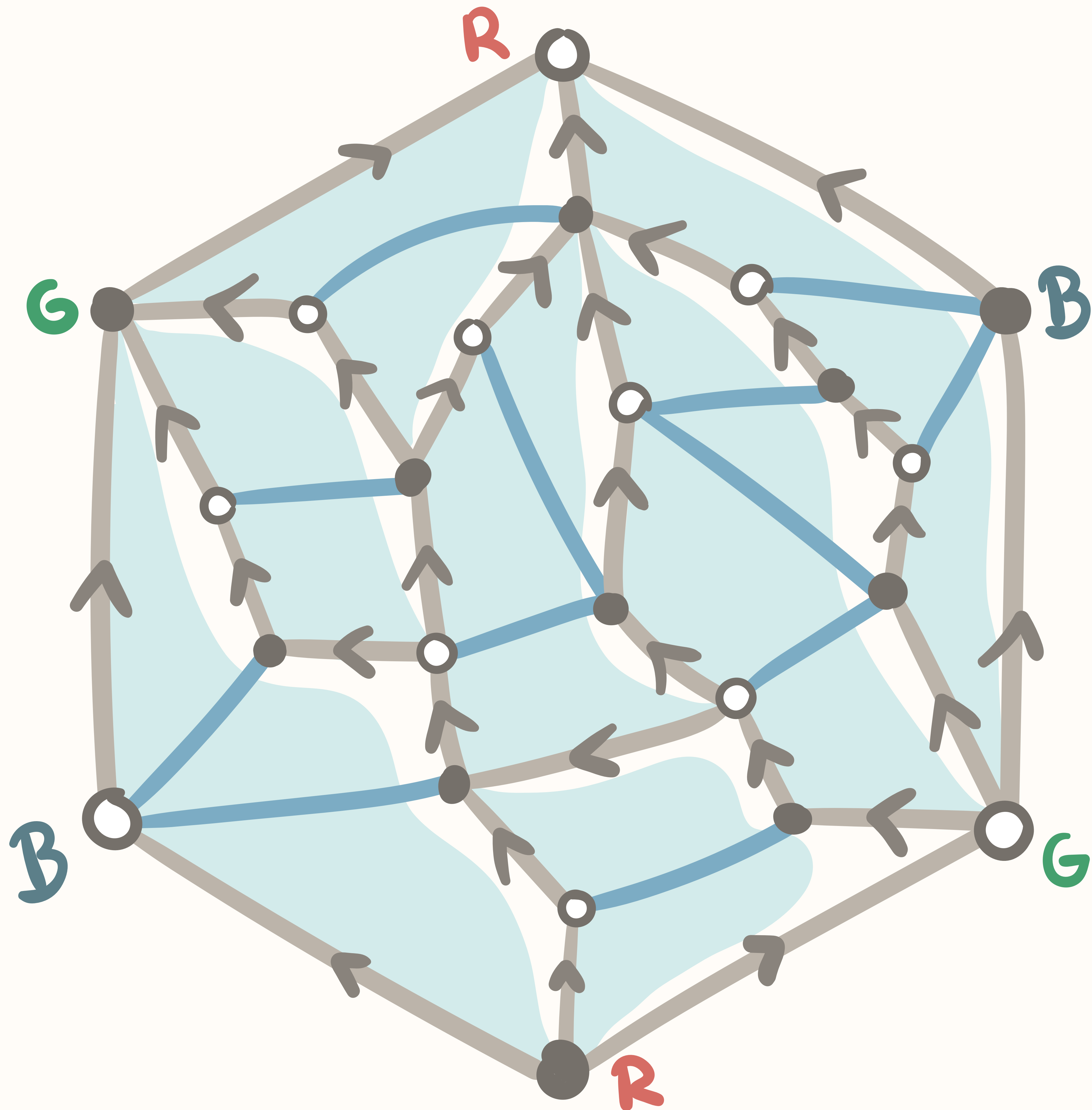
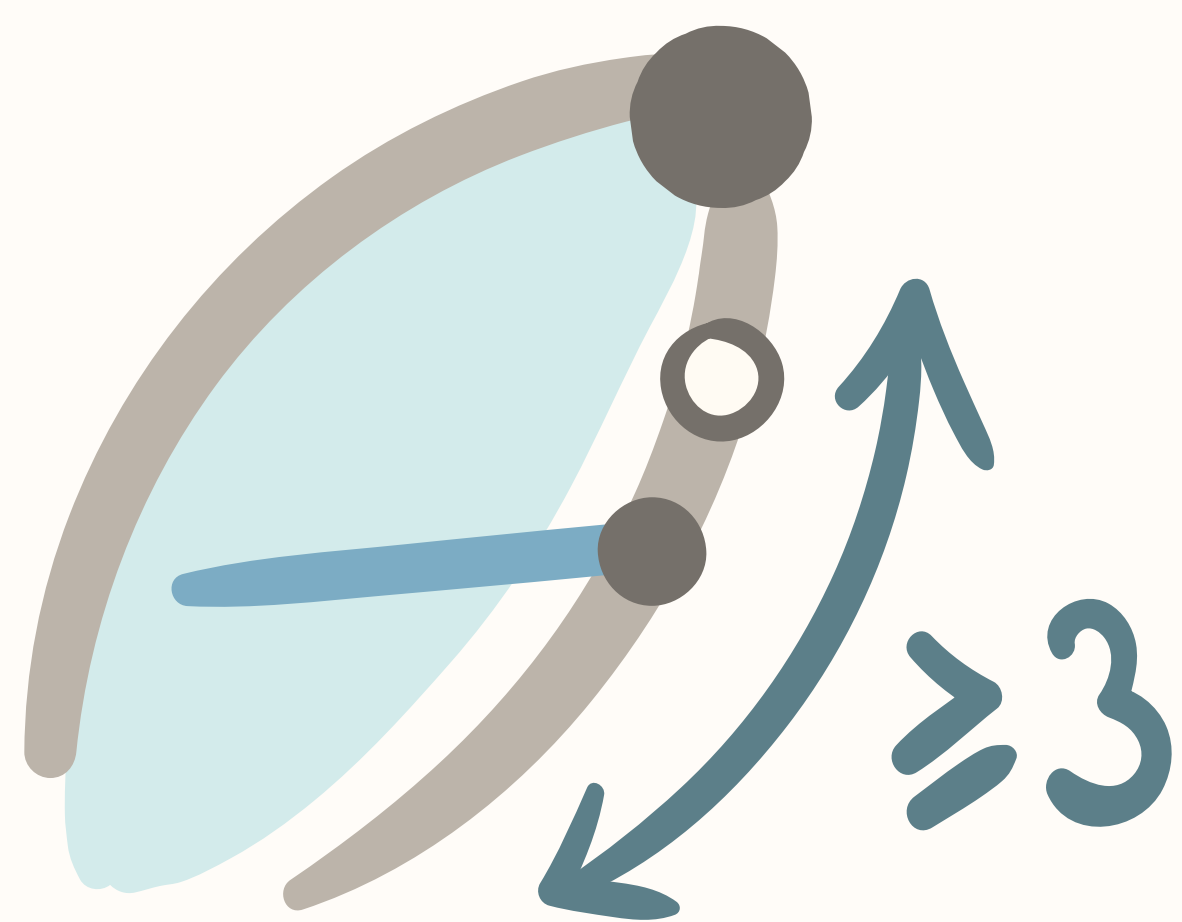
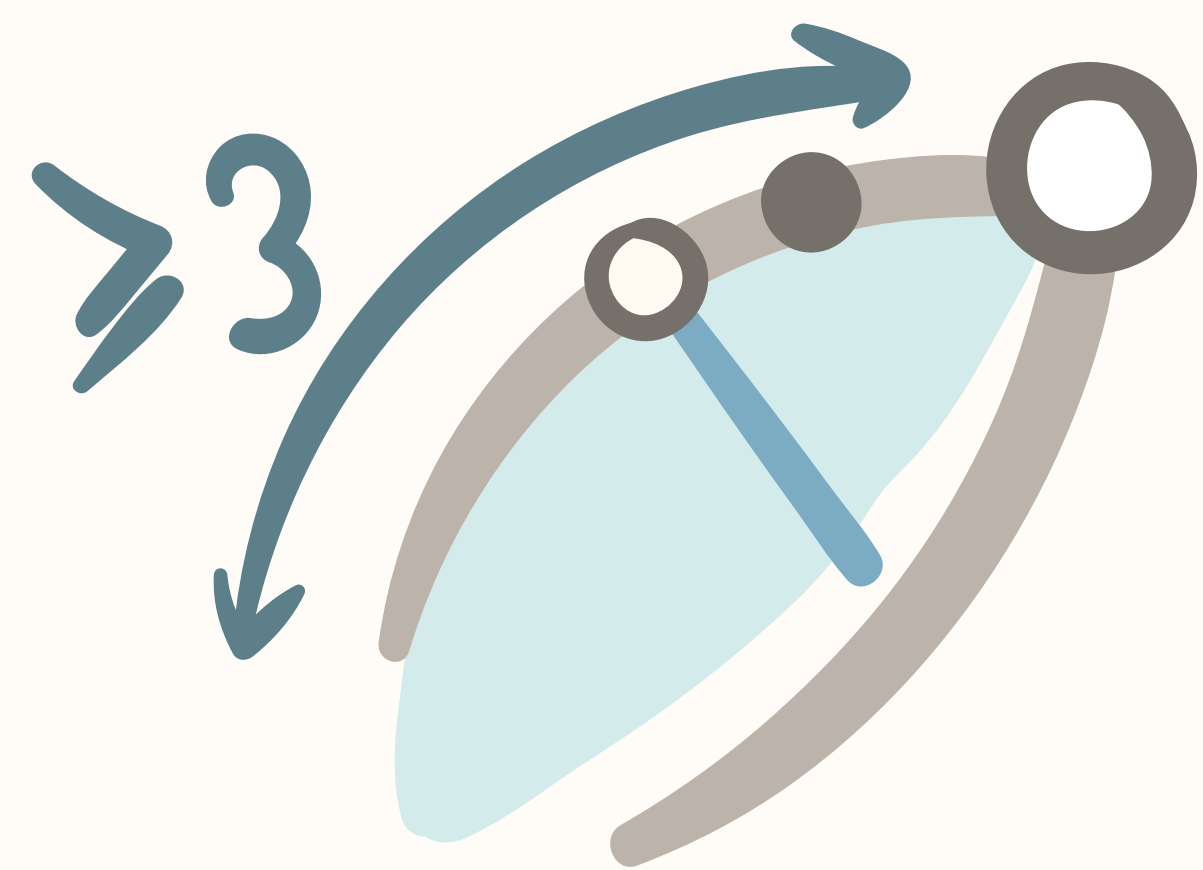
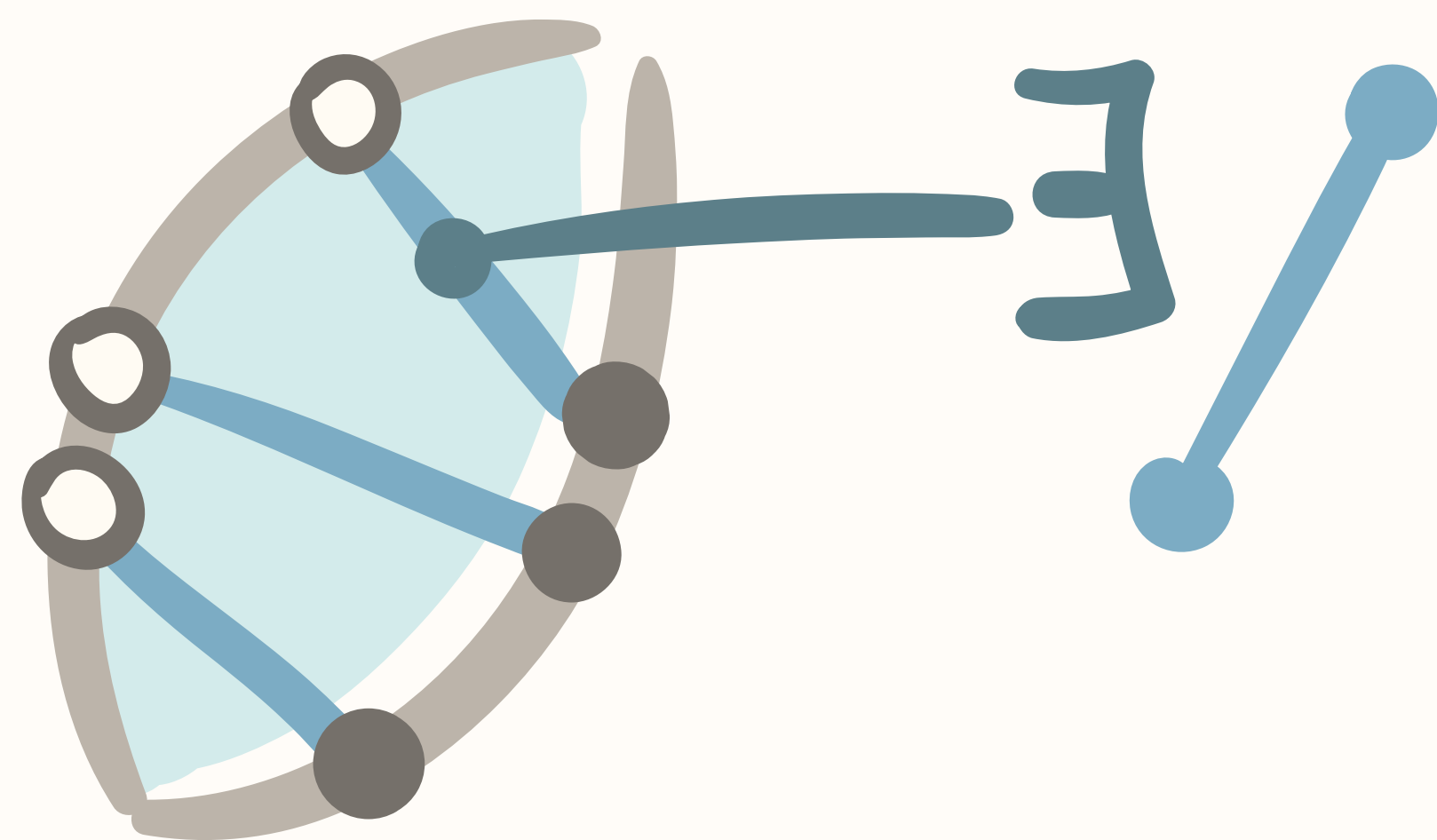
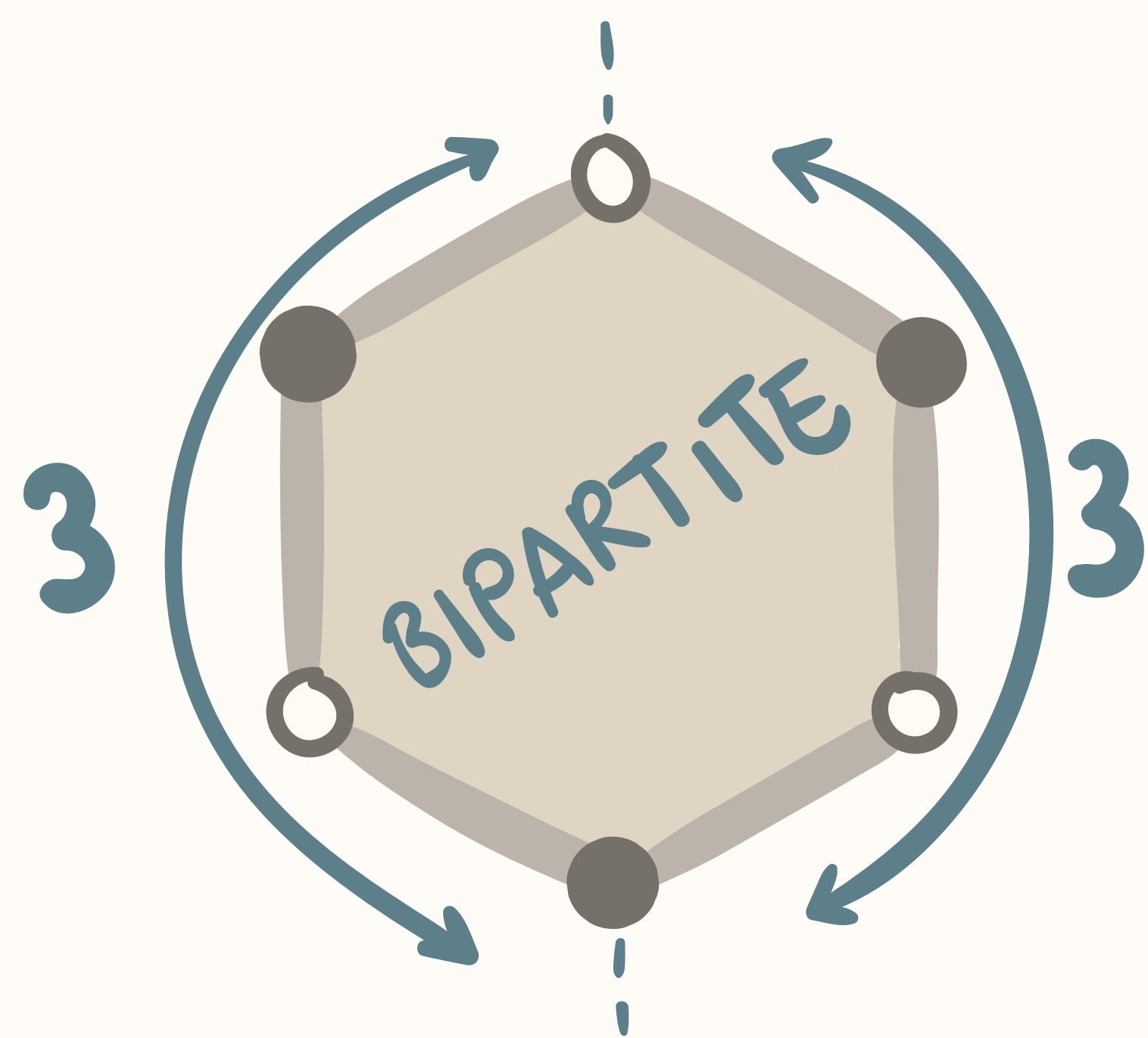
Schnyder labellings



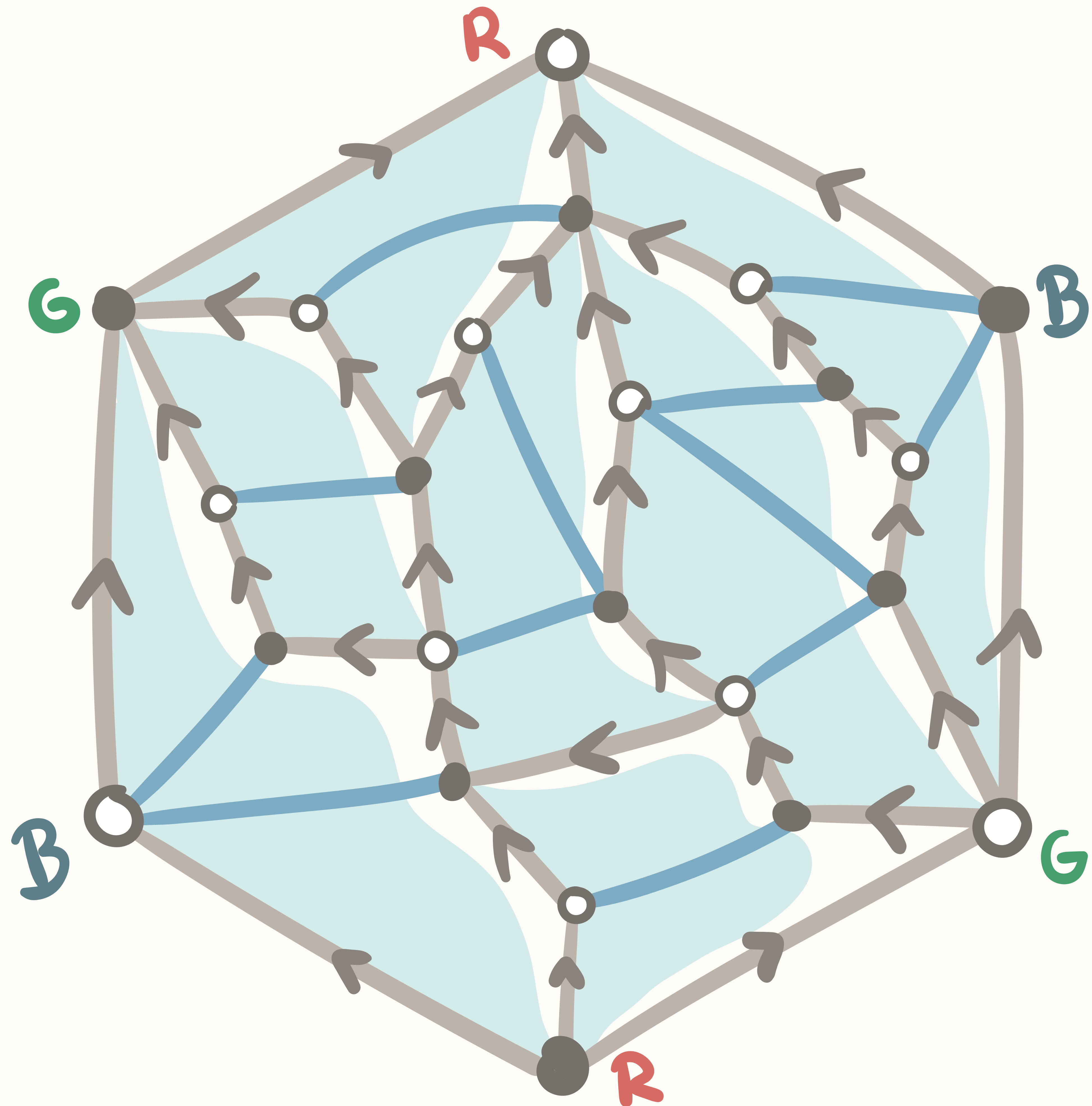
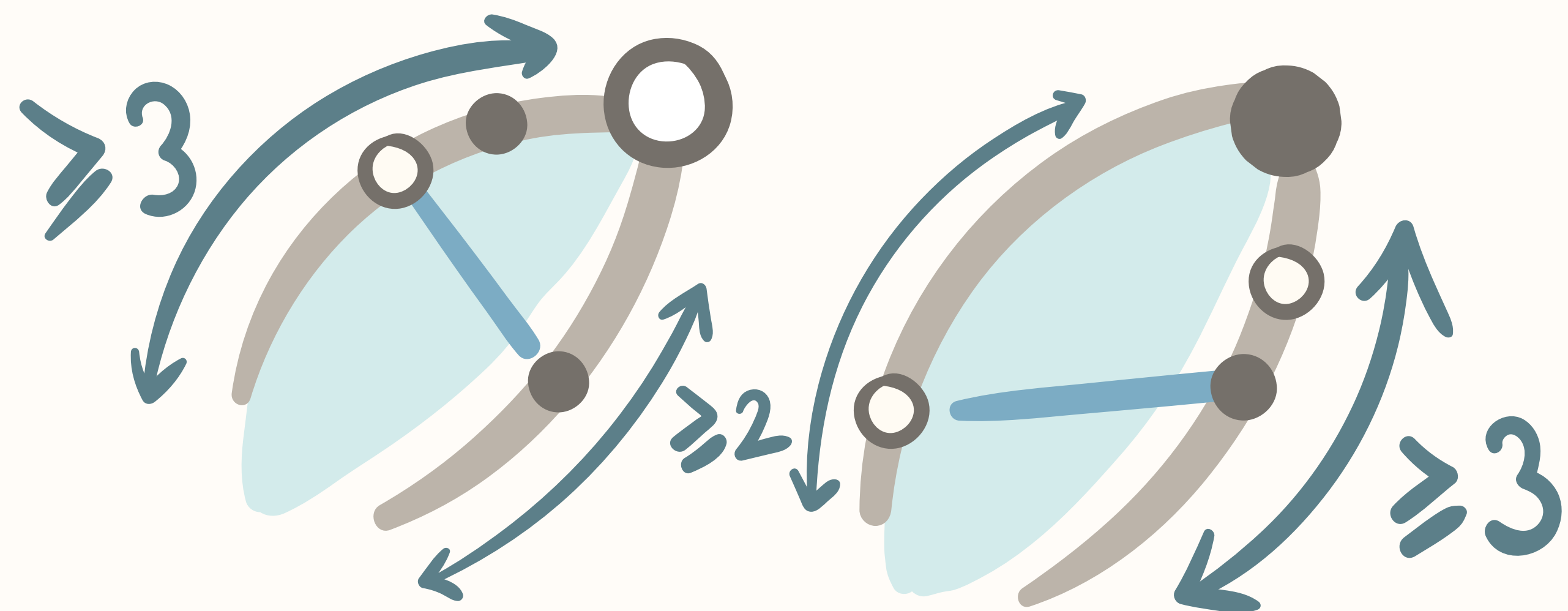
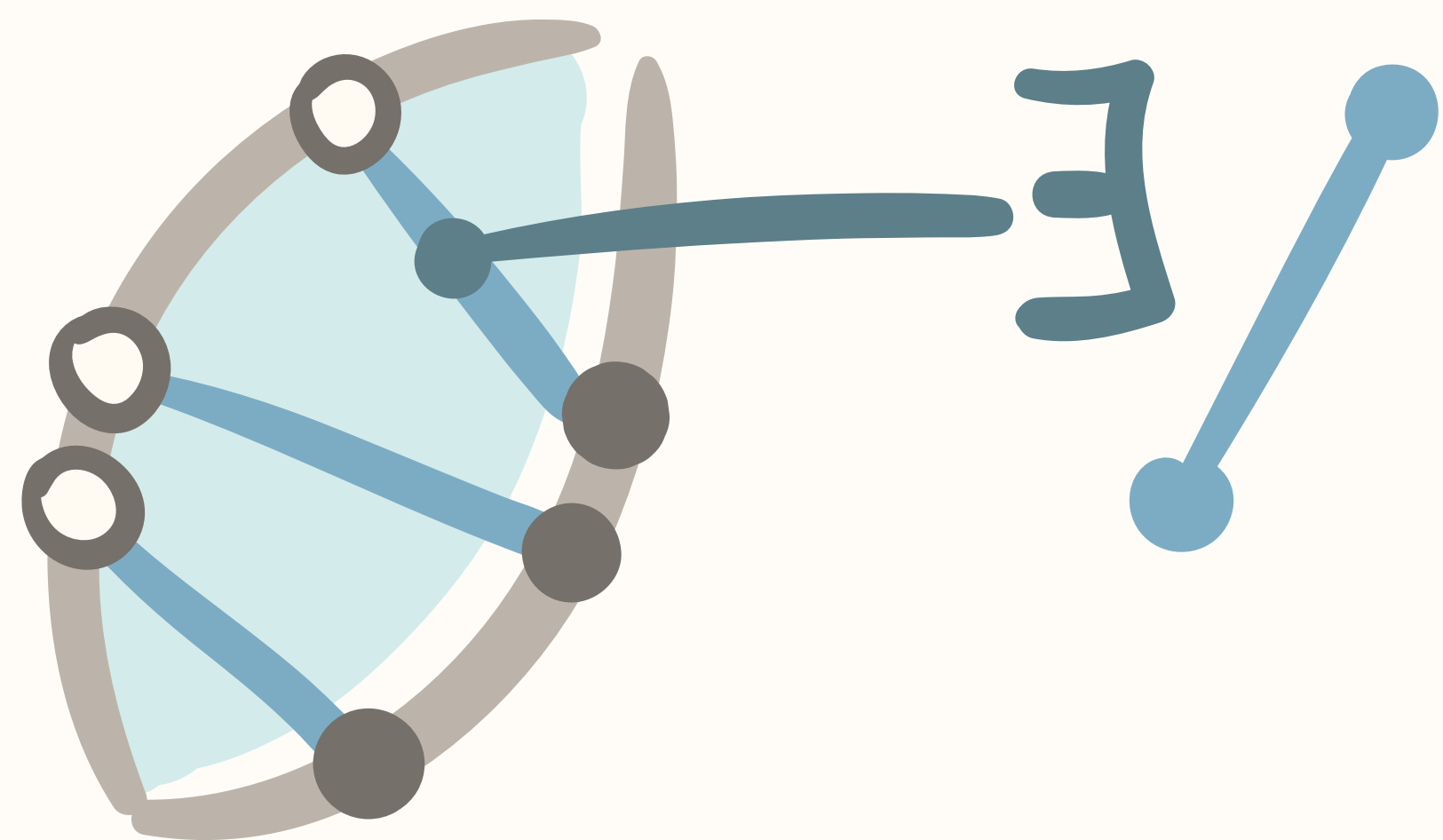
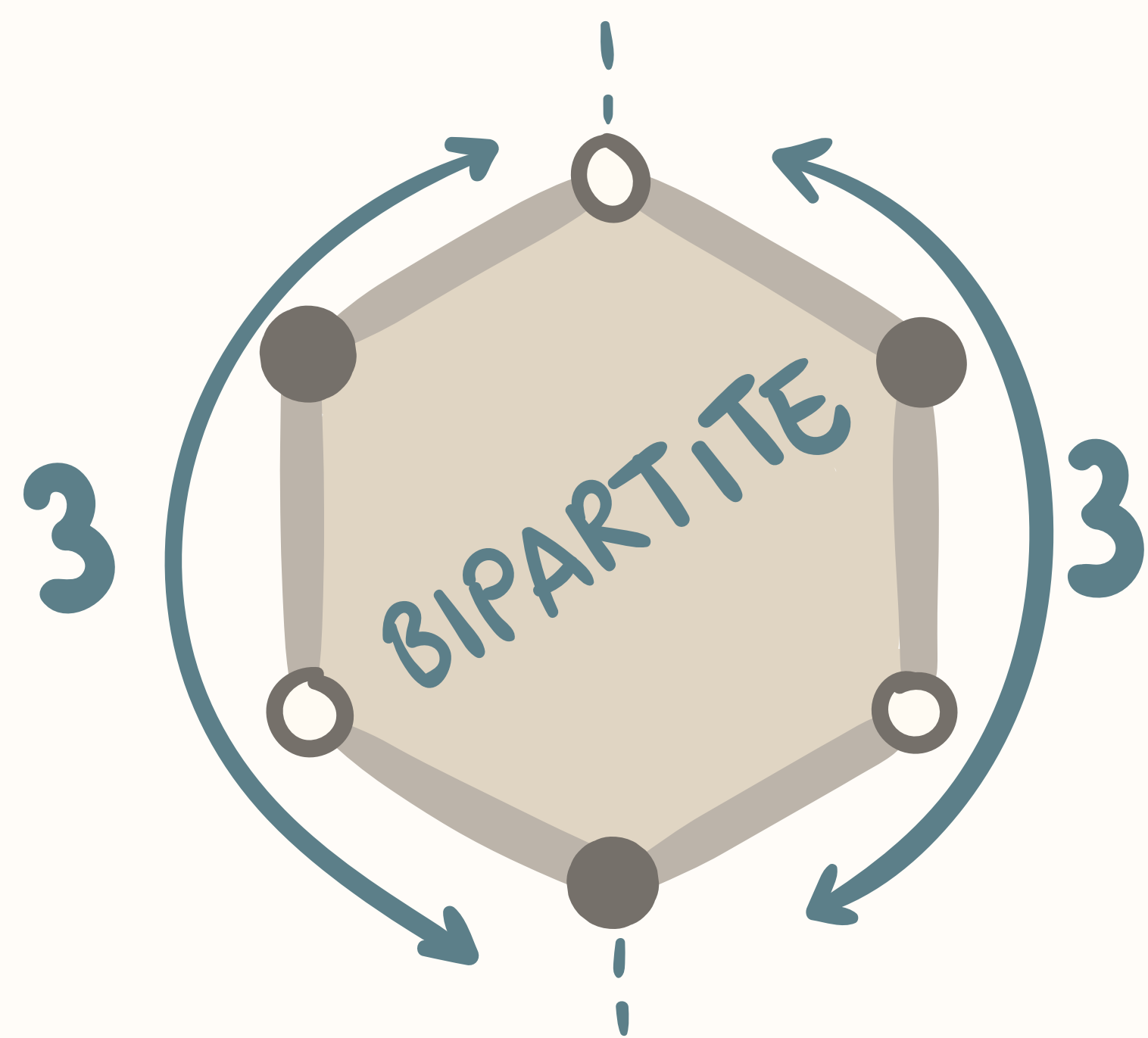
Schnyder labellings



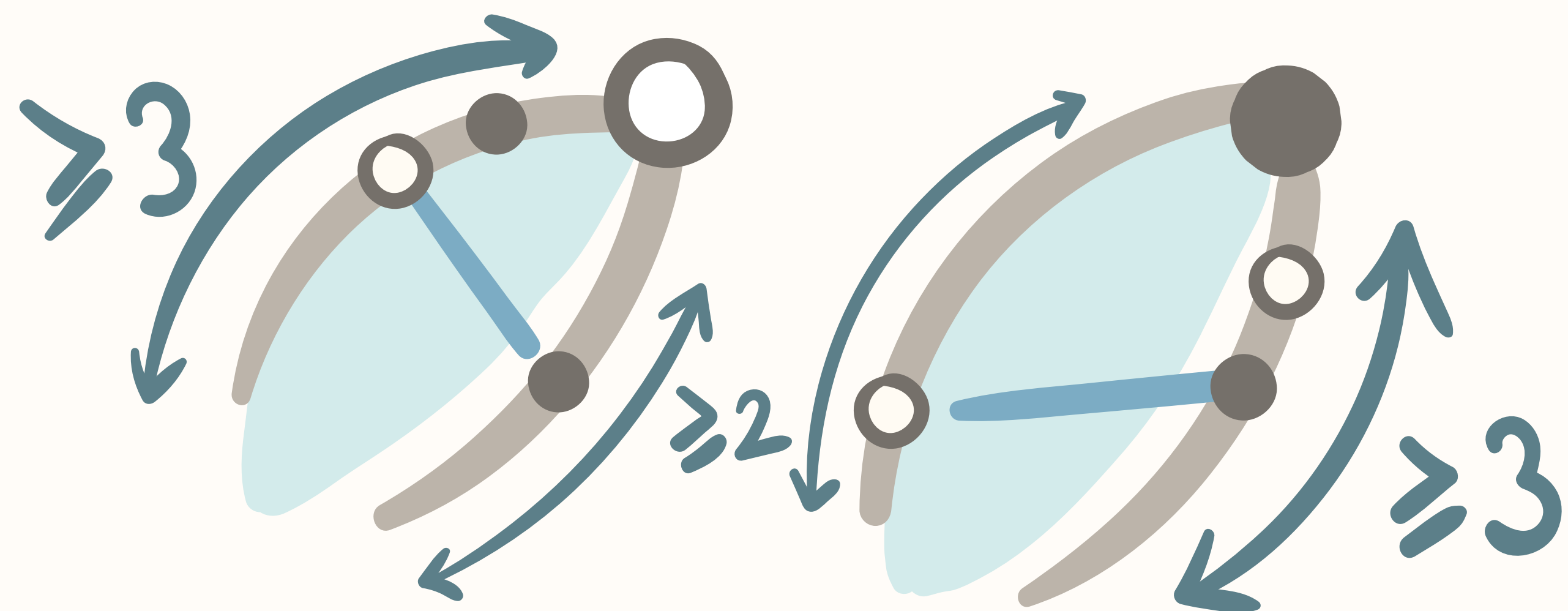
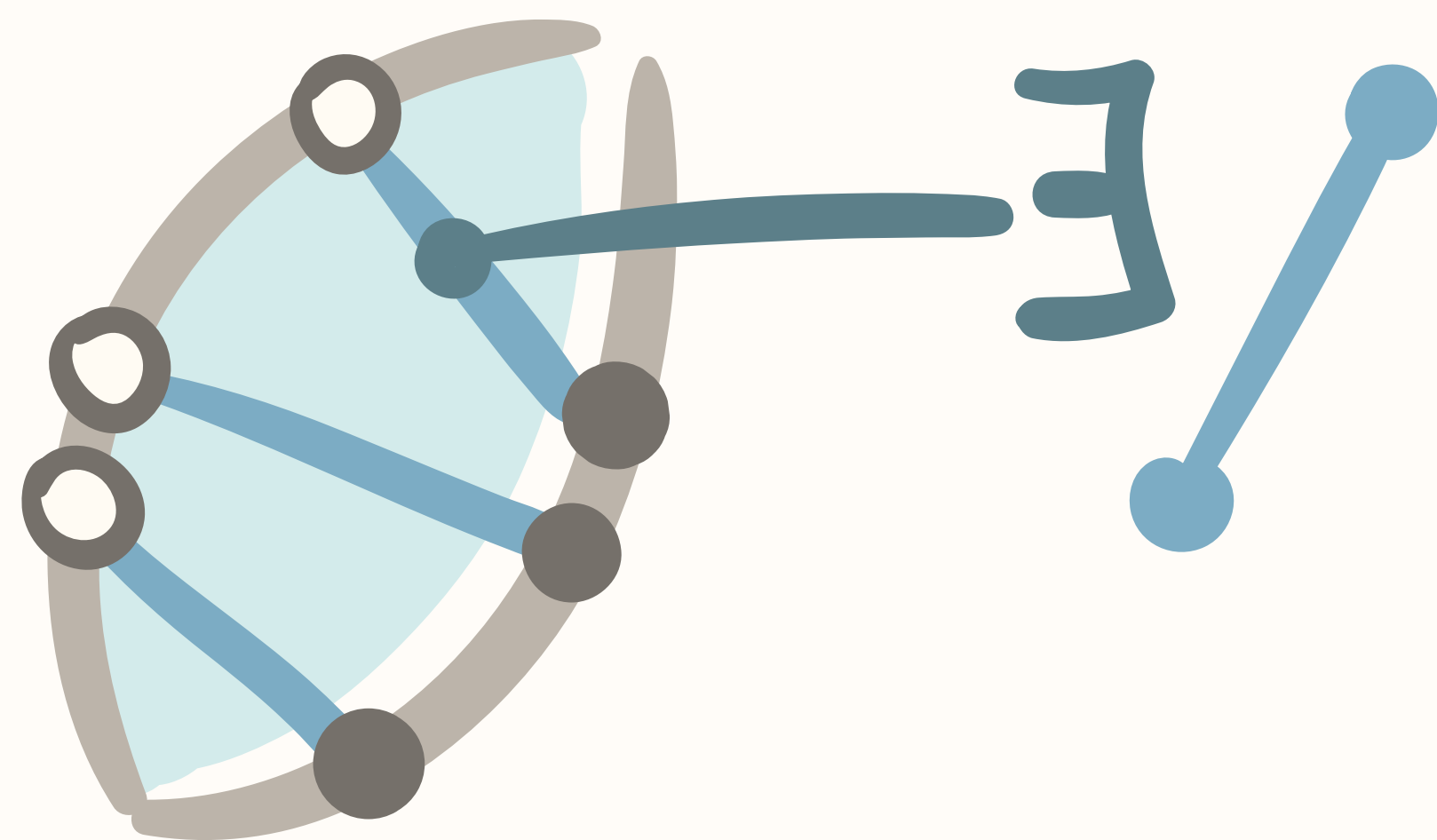
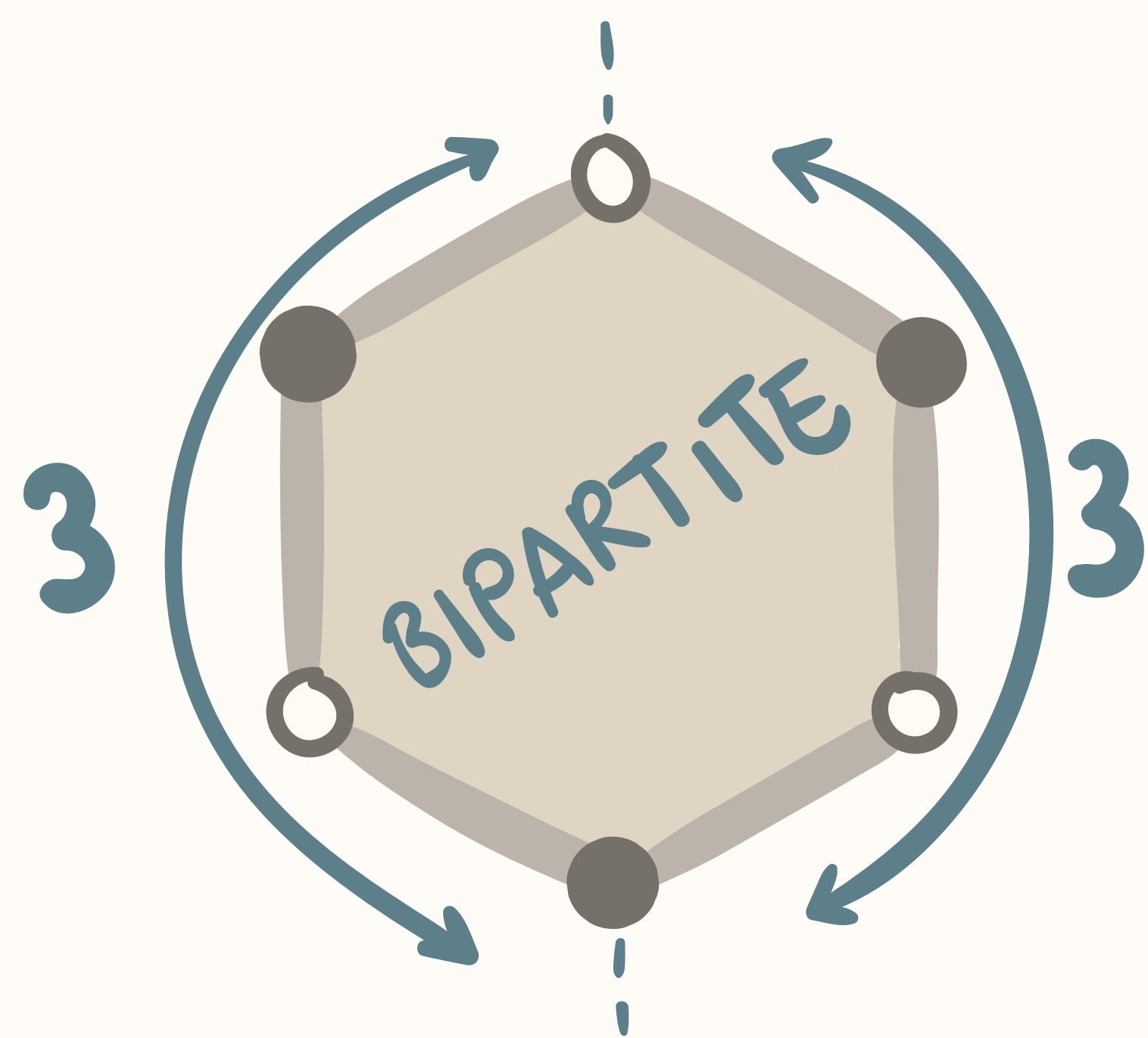
Schnyder labellings



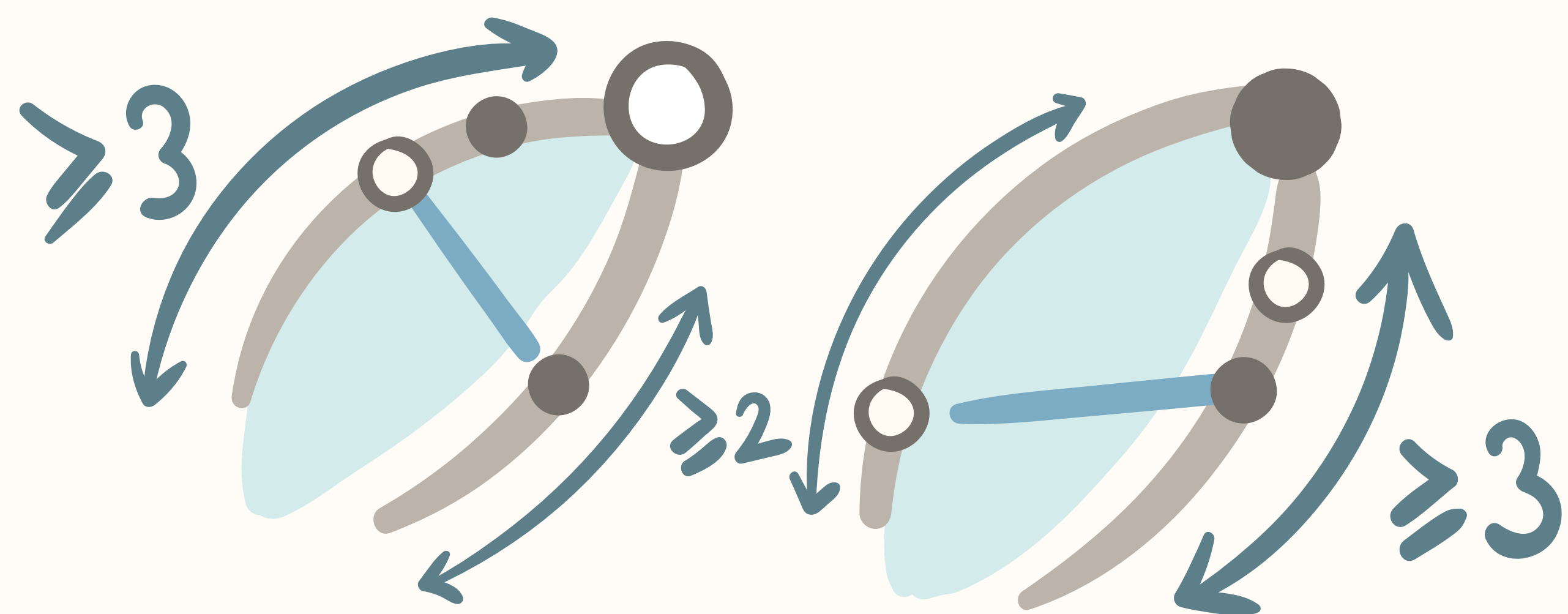
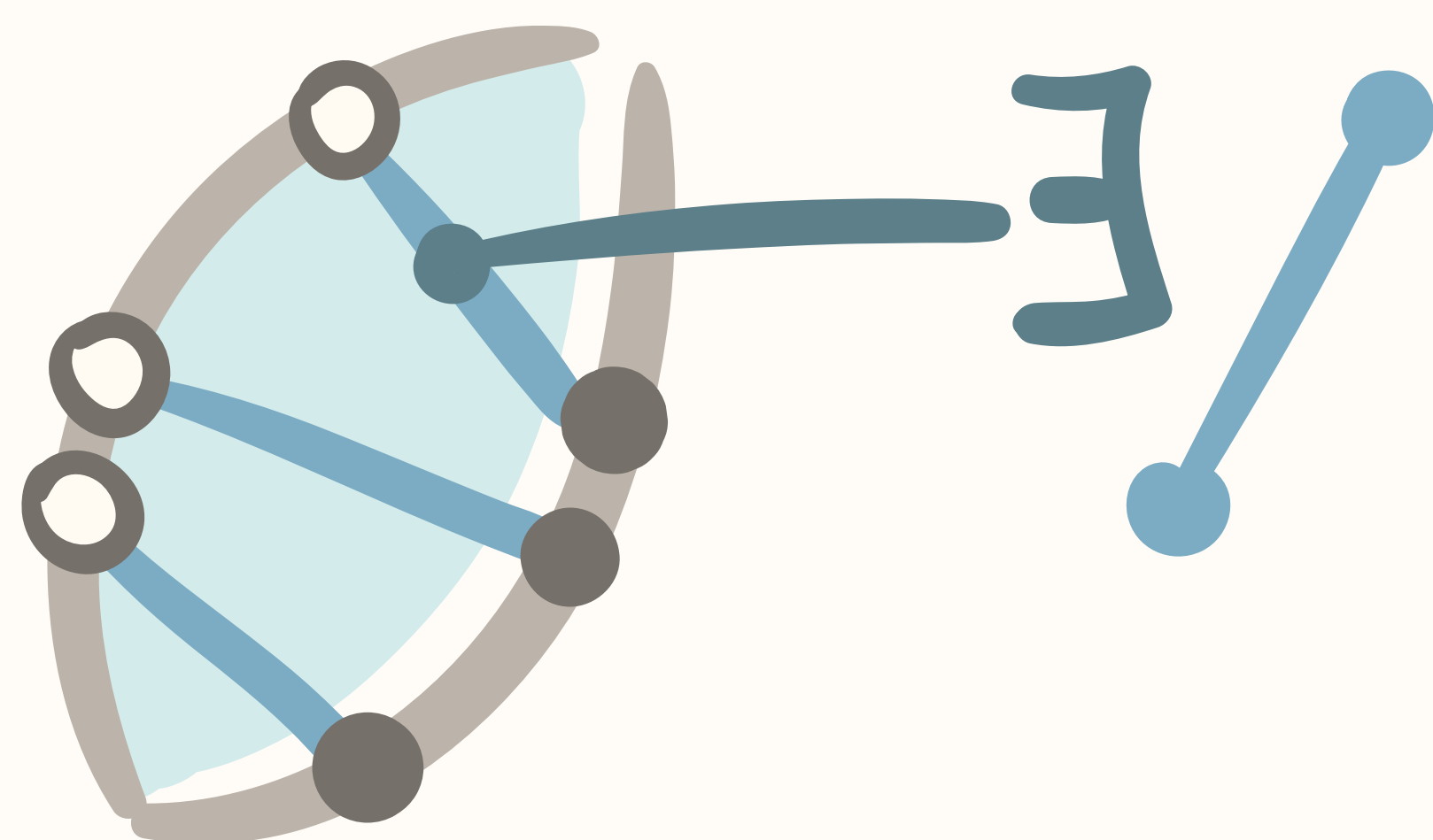
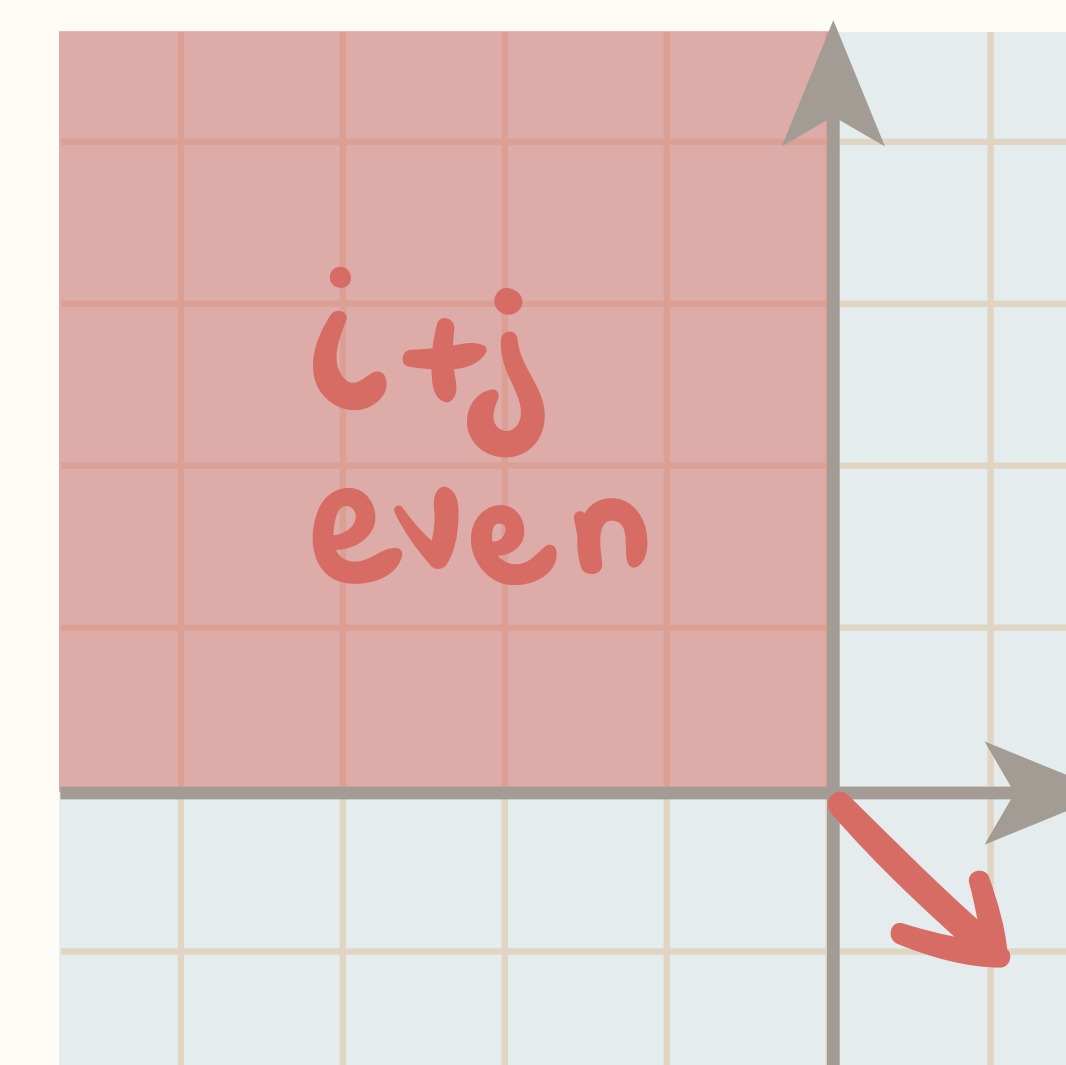
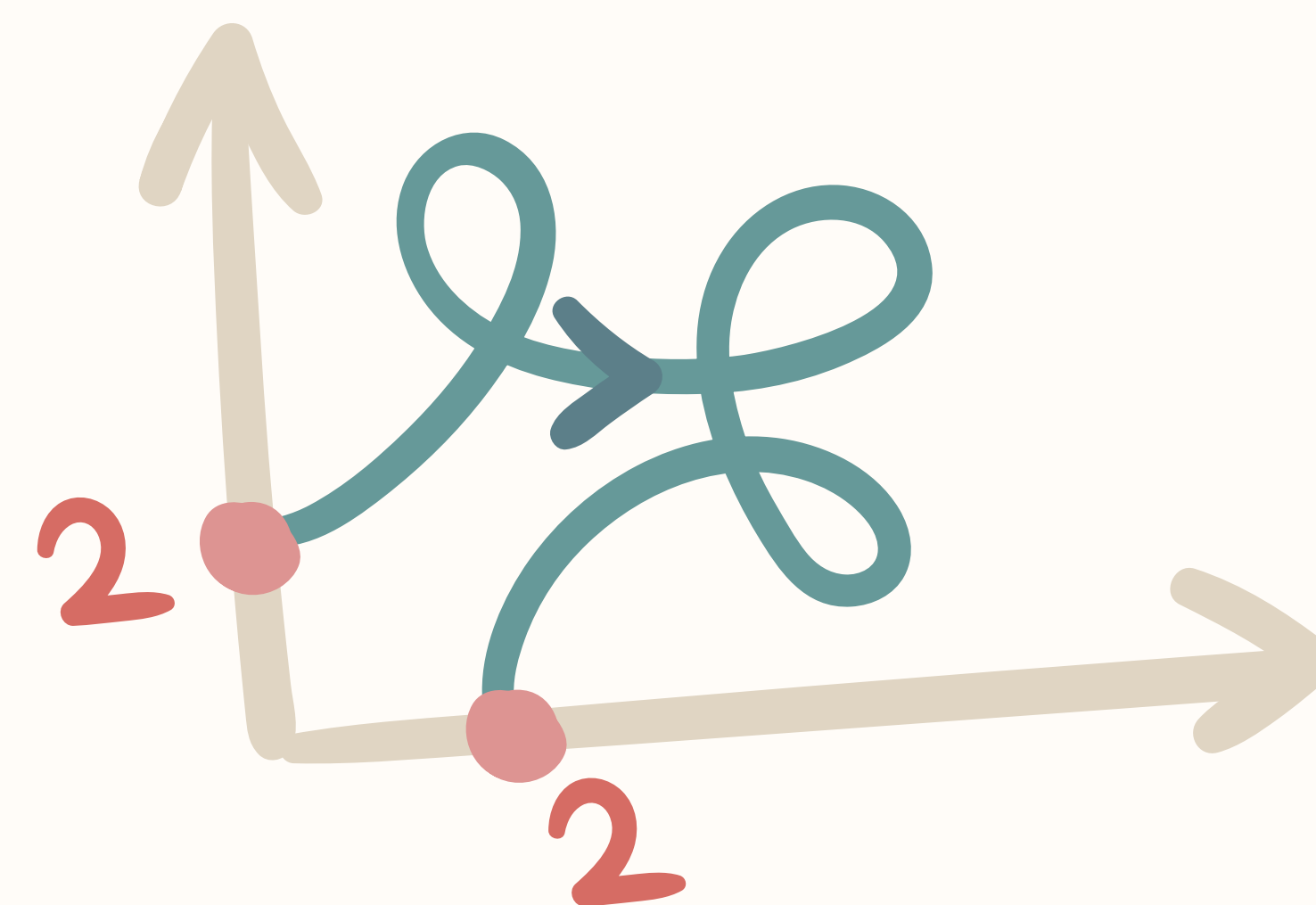
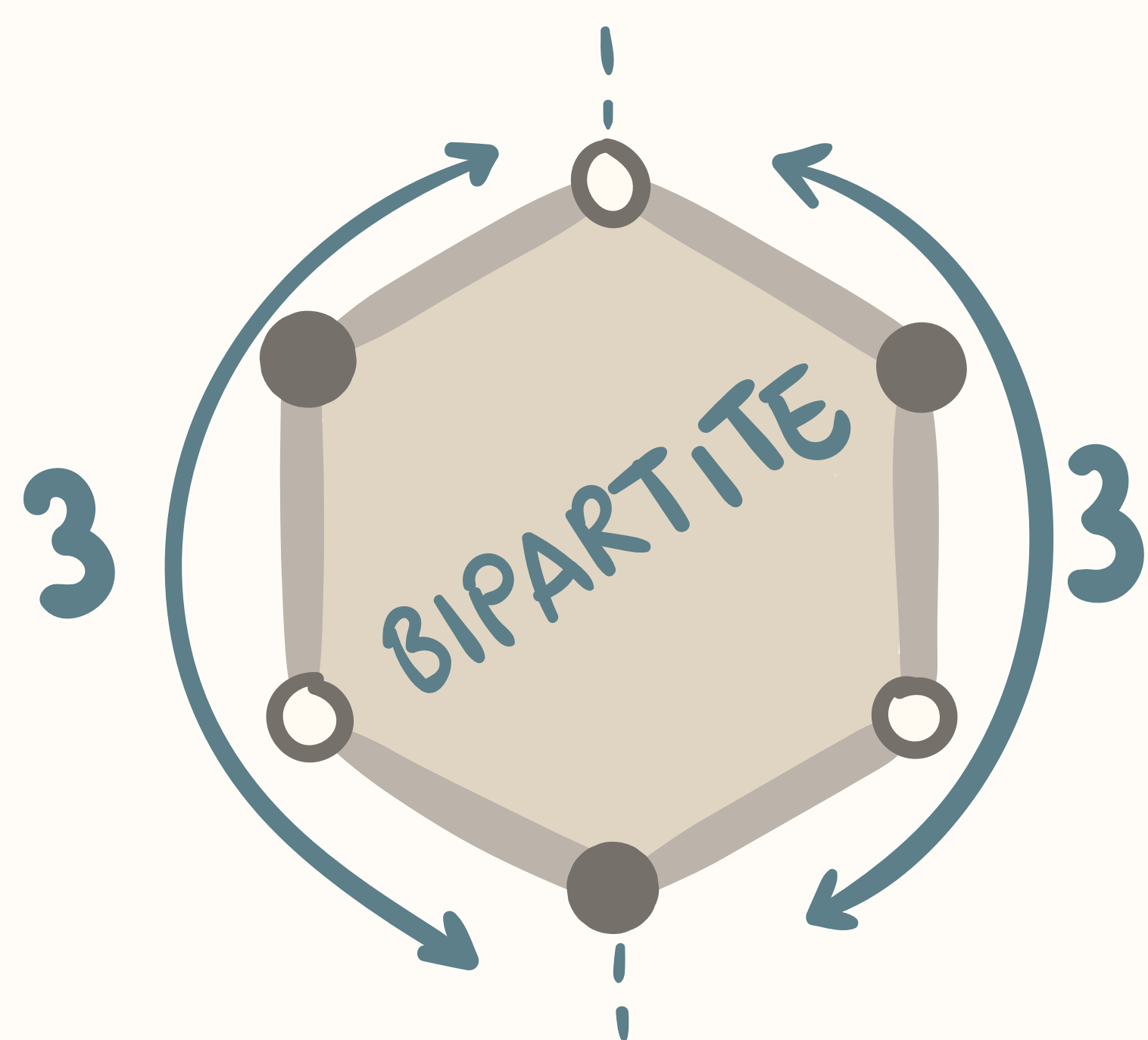
Schnyder labellings



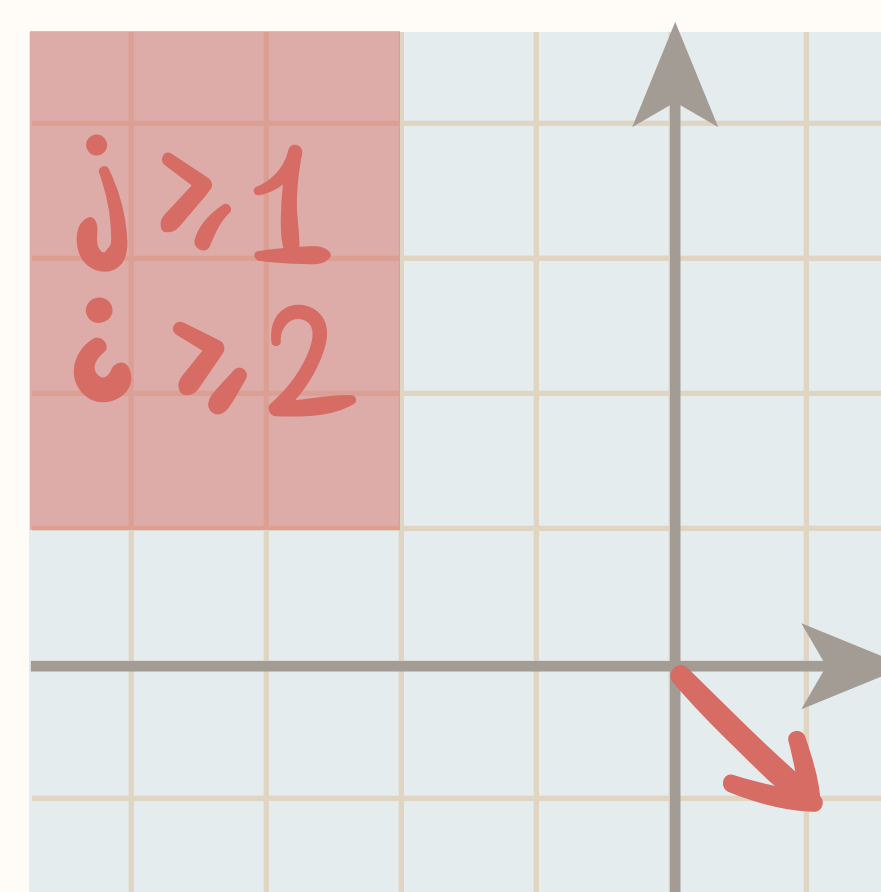
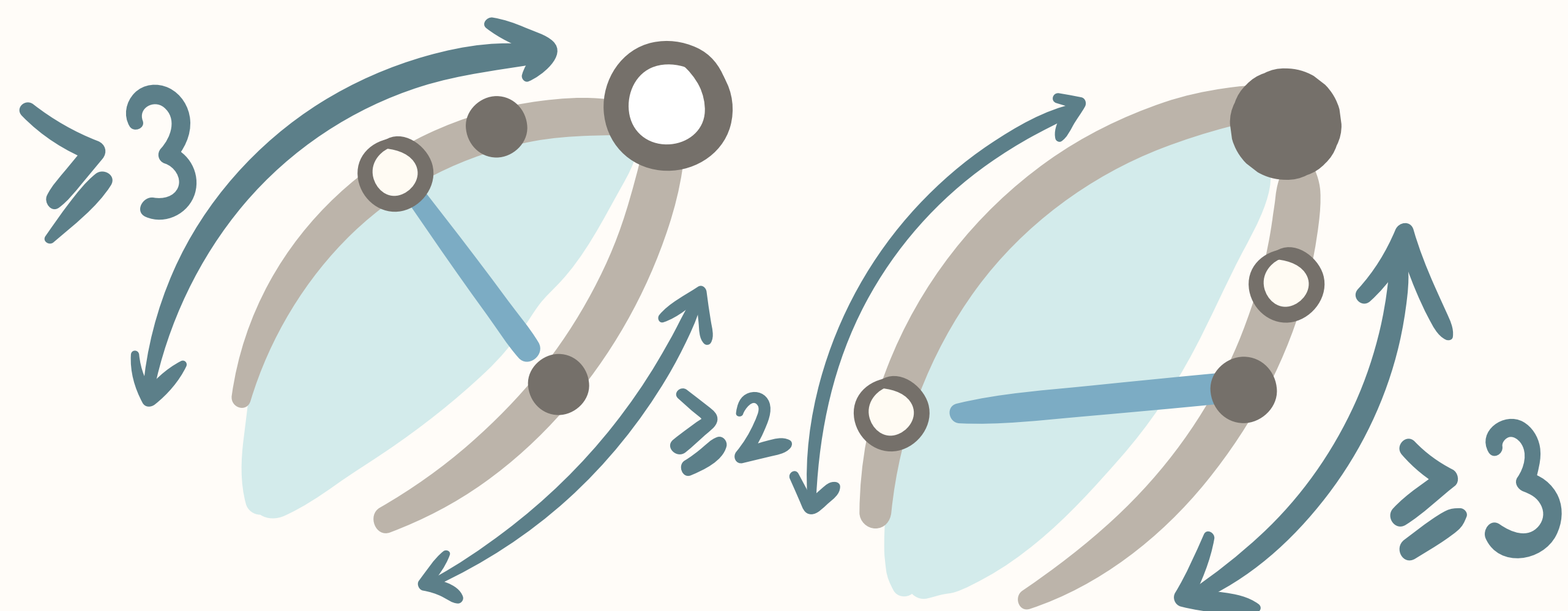
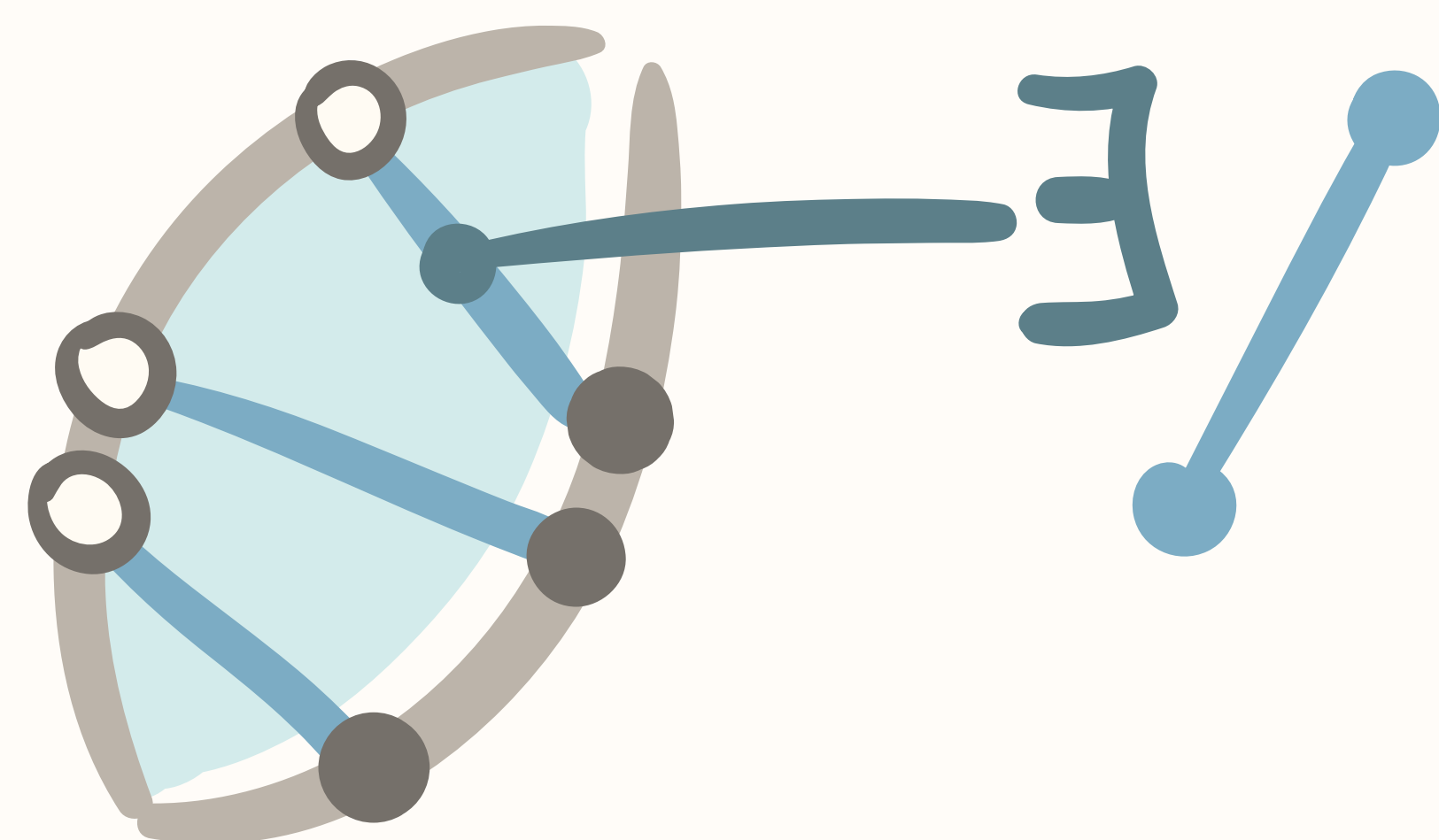
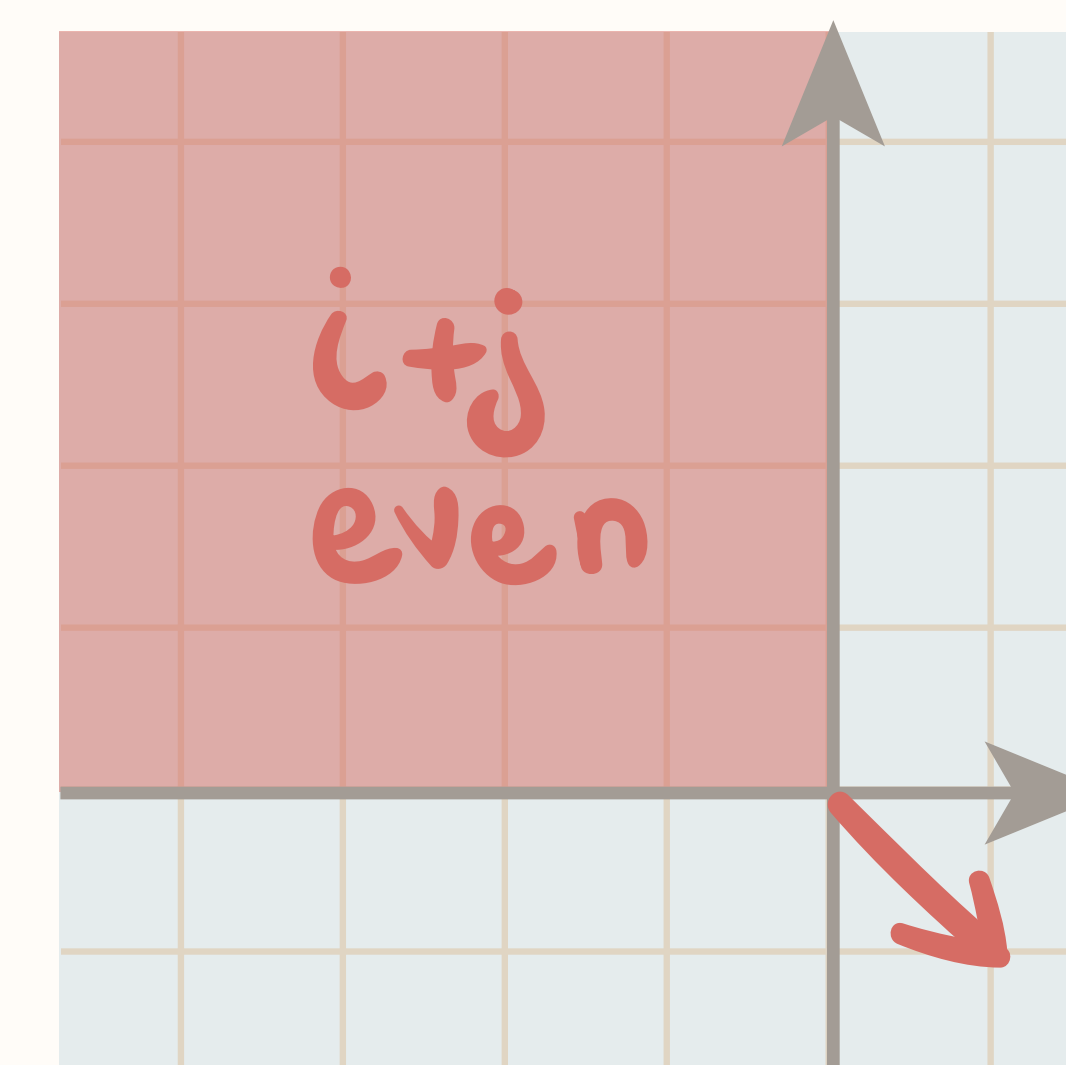
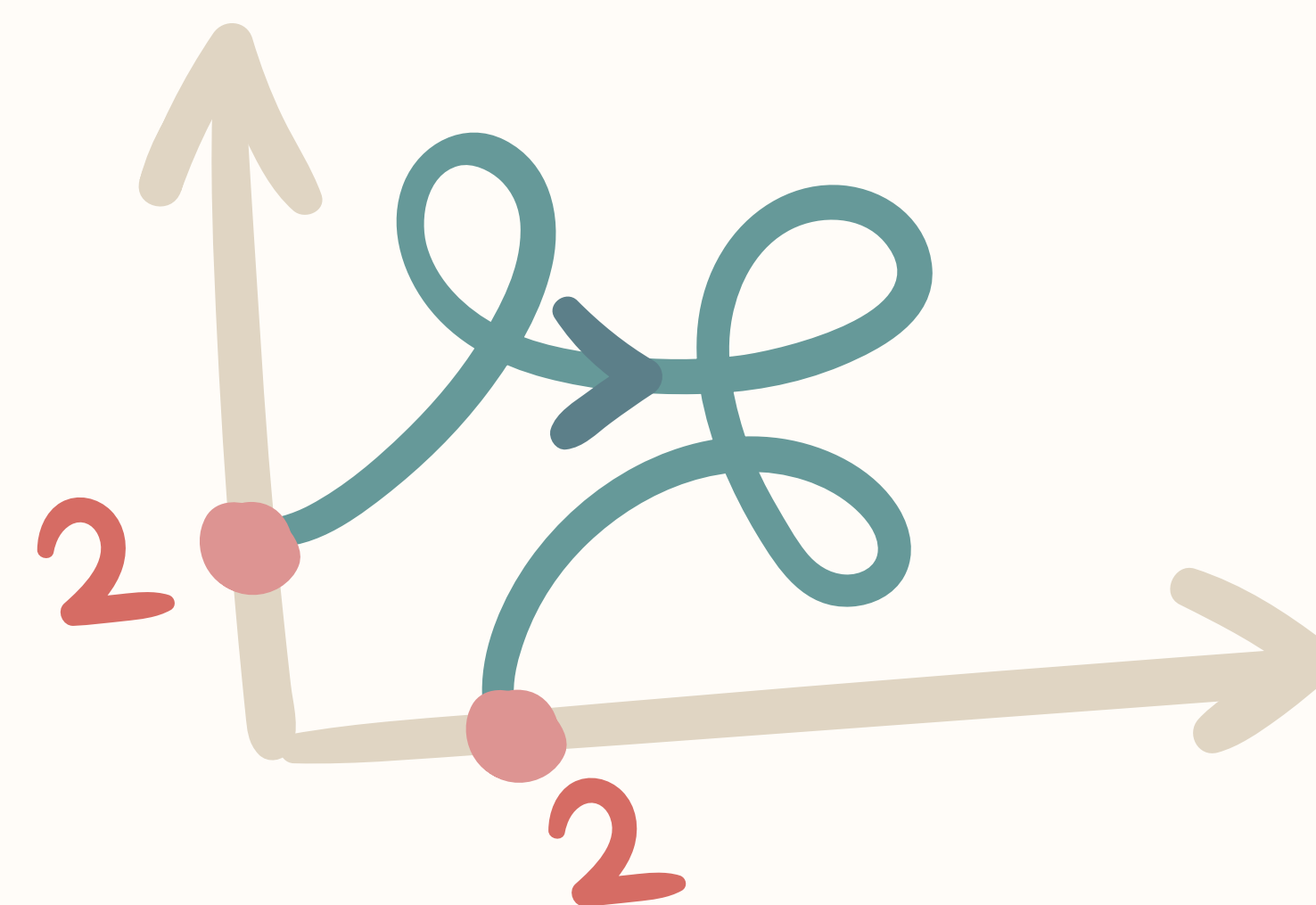
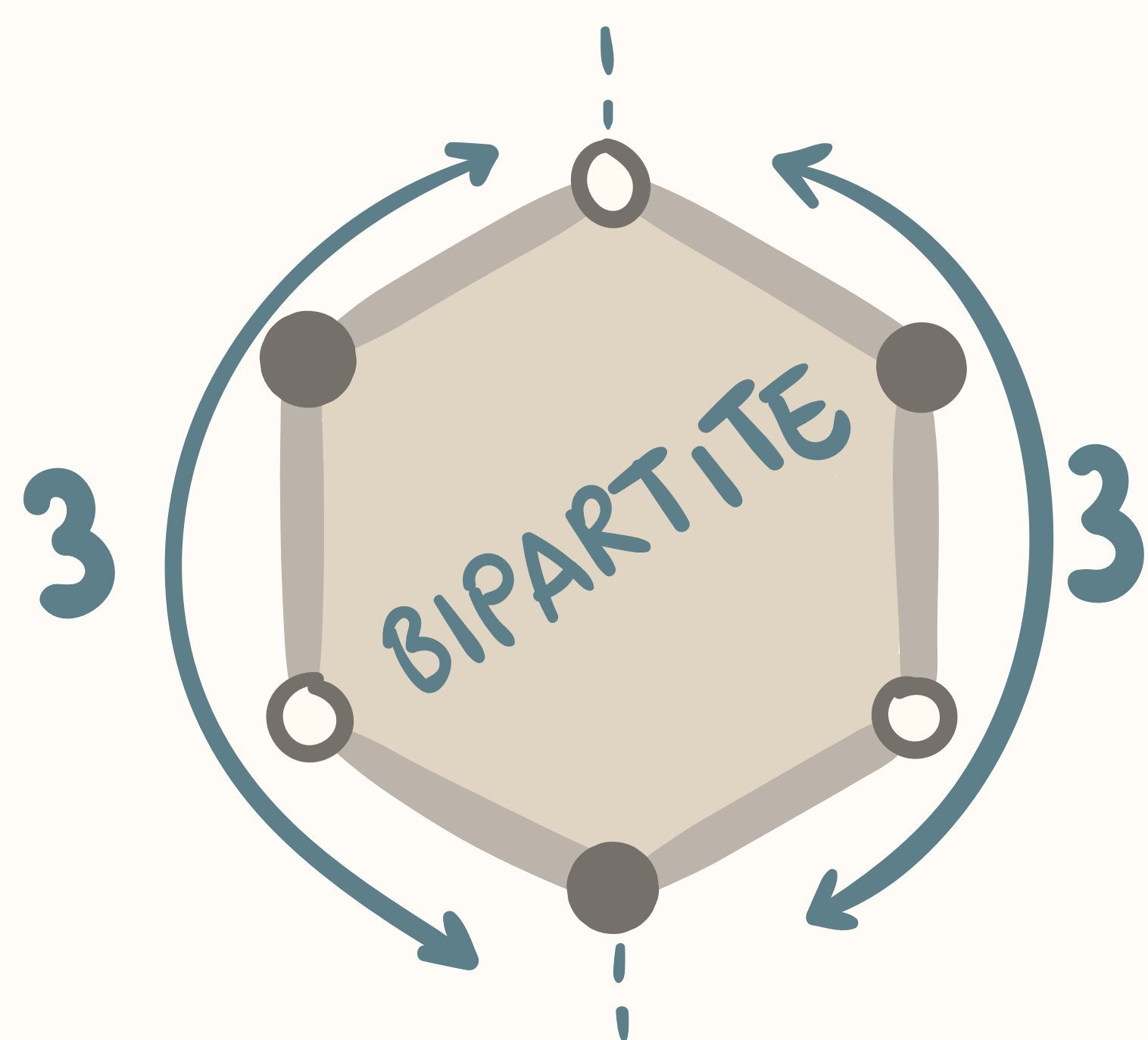
Schnyder labellings KMSW



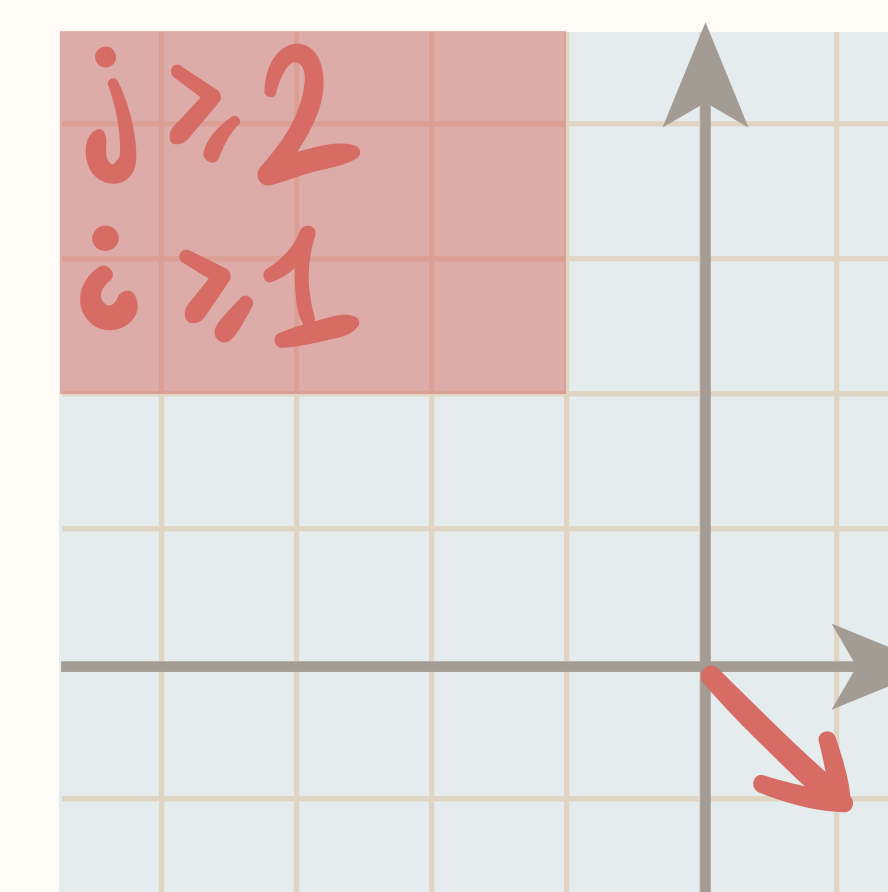
Schnyder labellings KMSW



Schnyder labellings KMSW

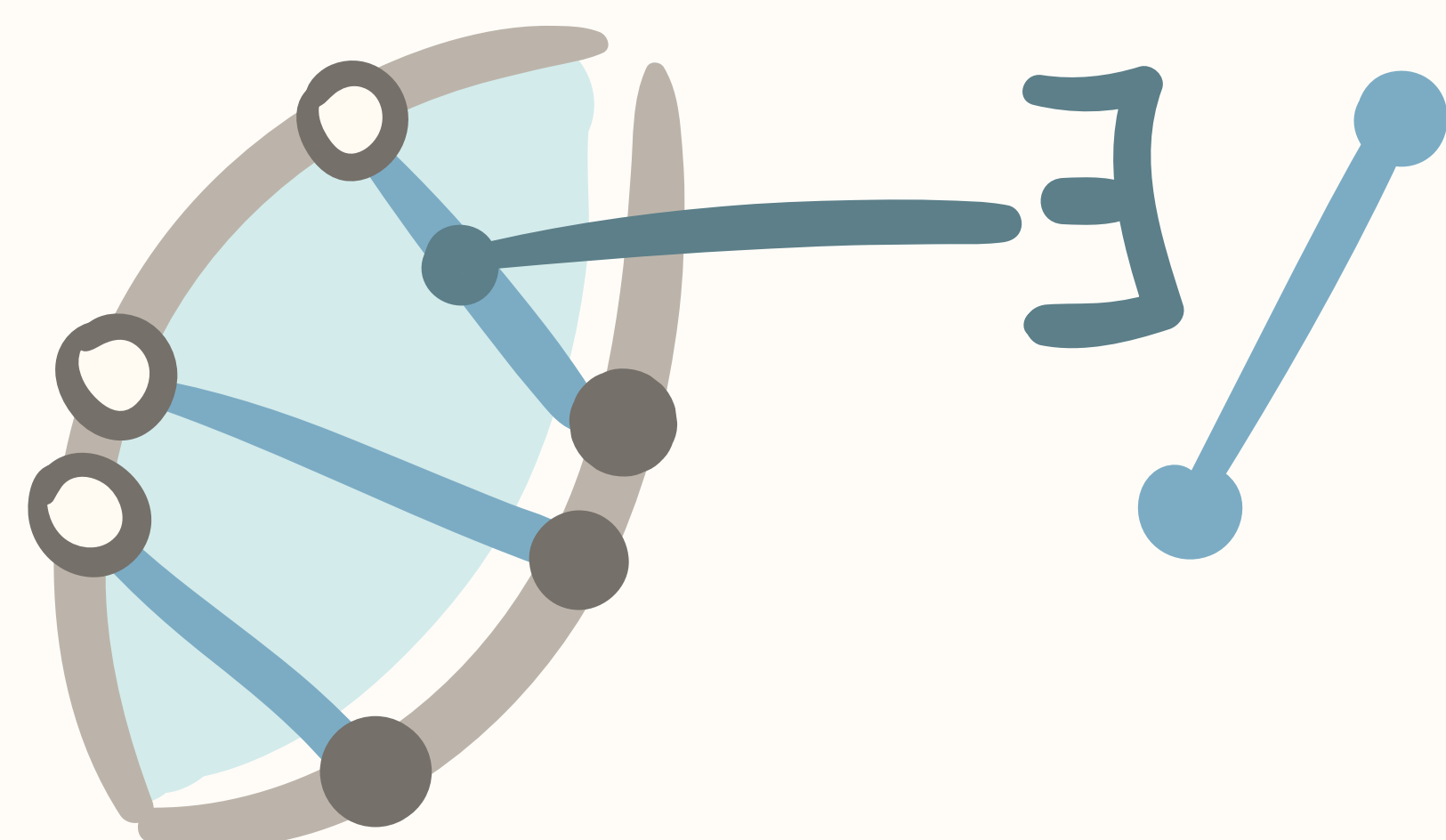
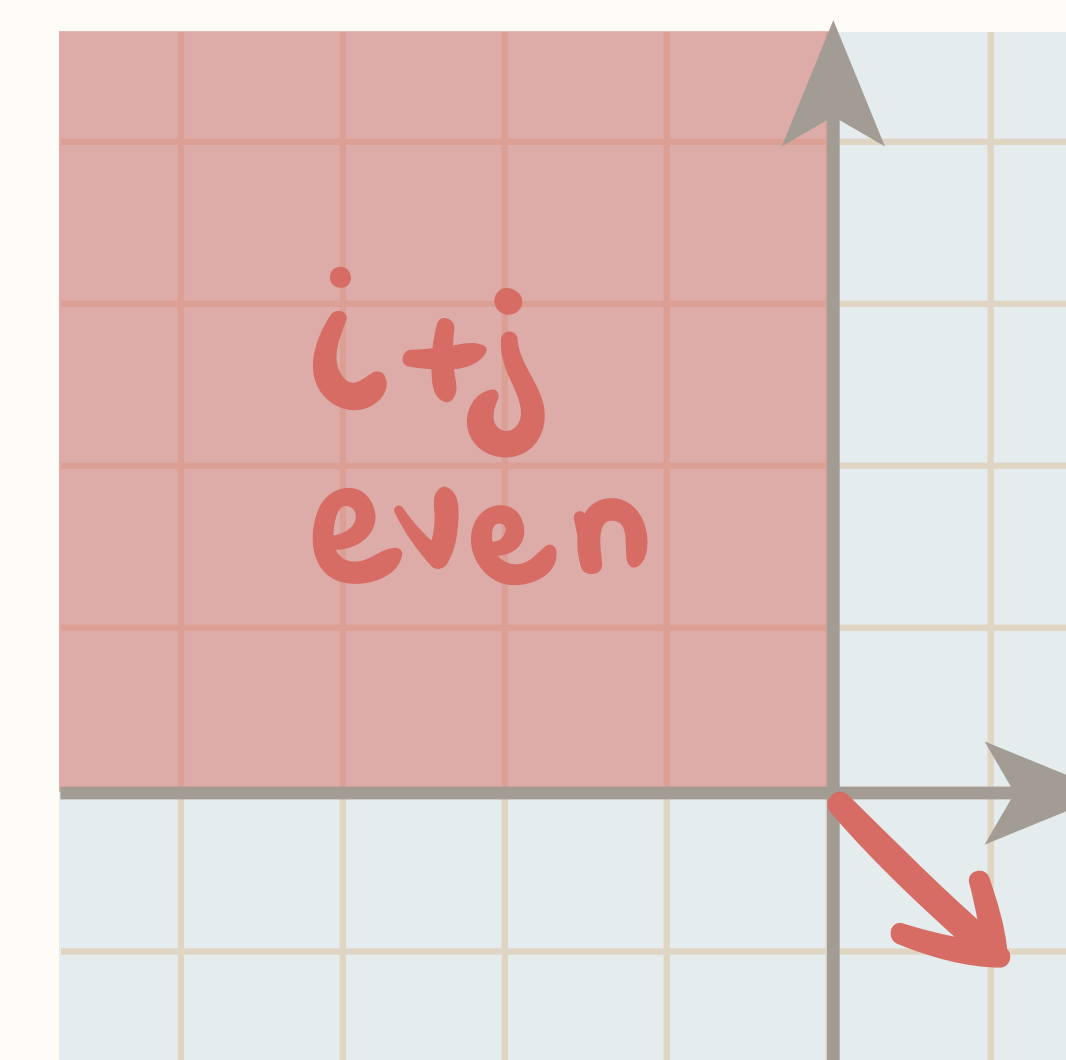
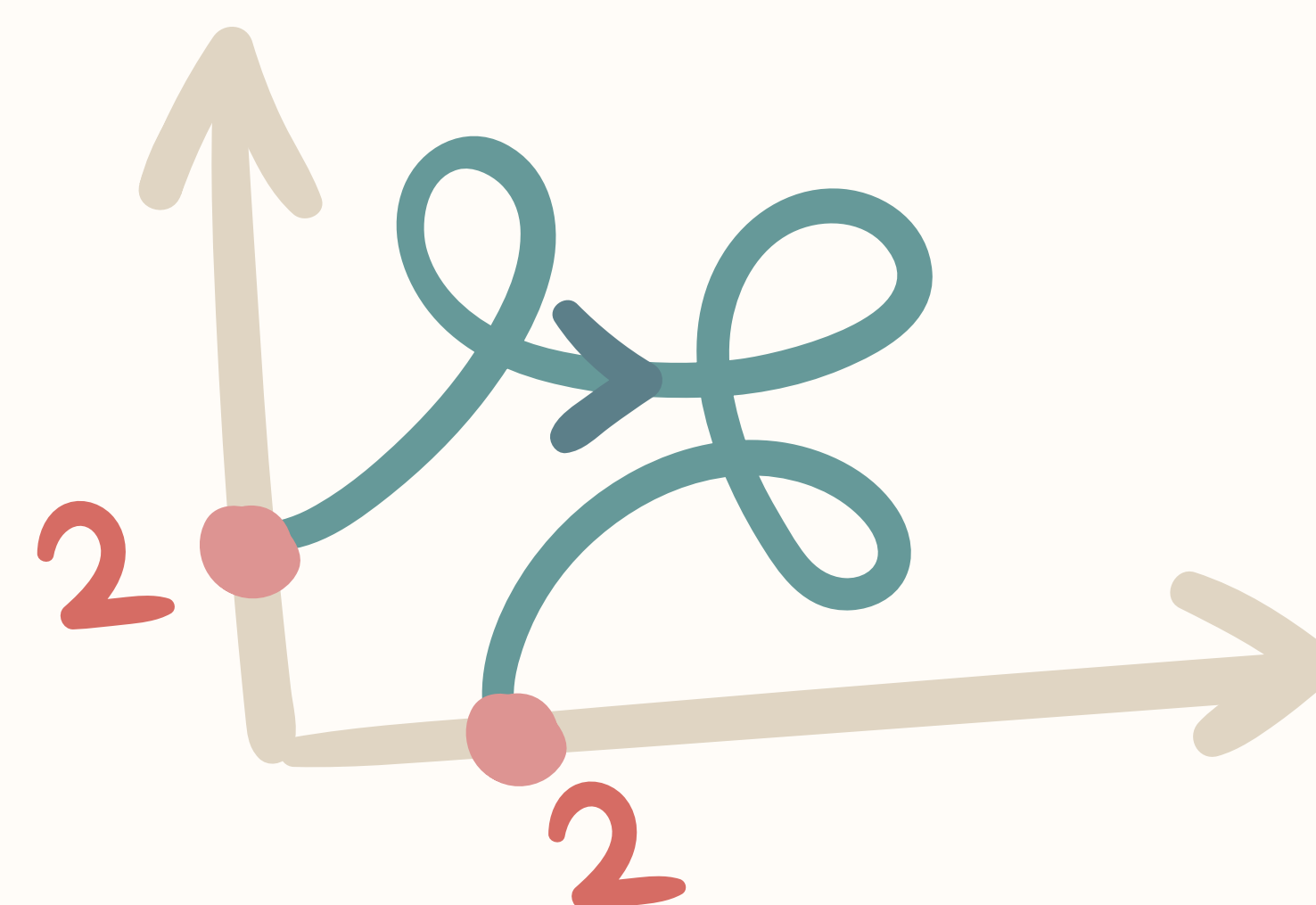
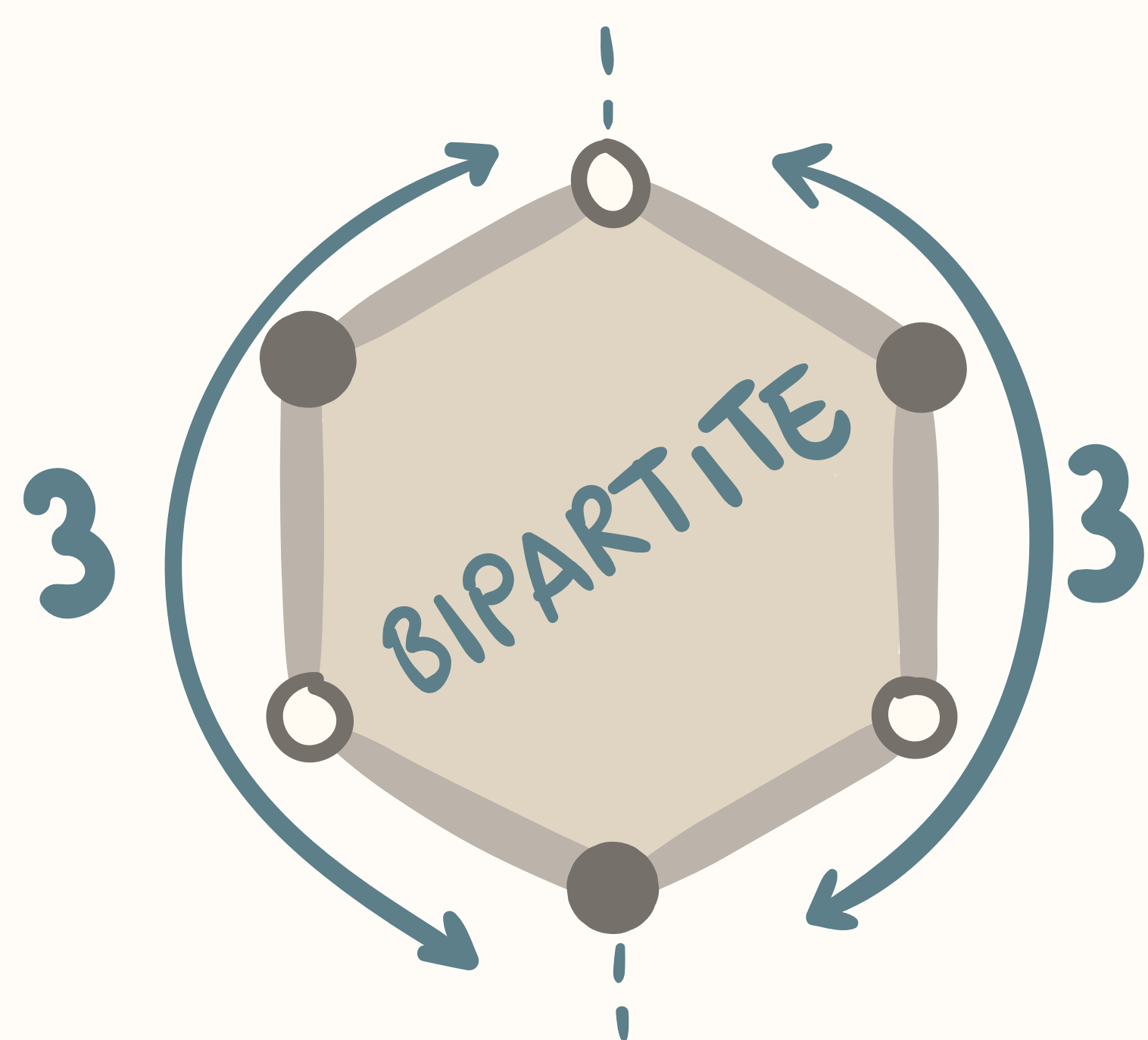


when x, y
are even

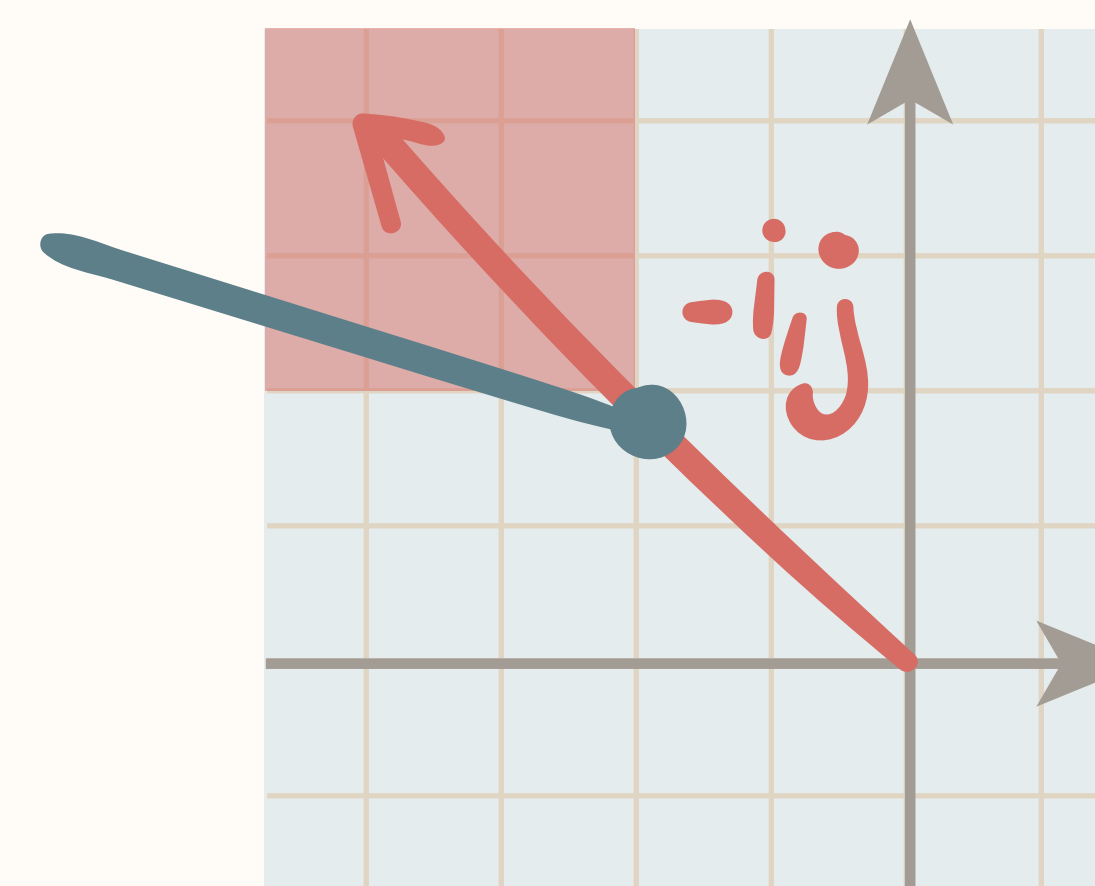


when x, y
are odd

Schnyder labellings KMSW

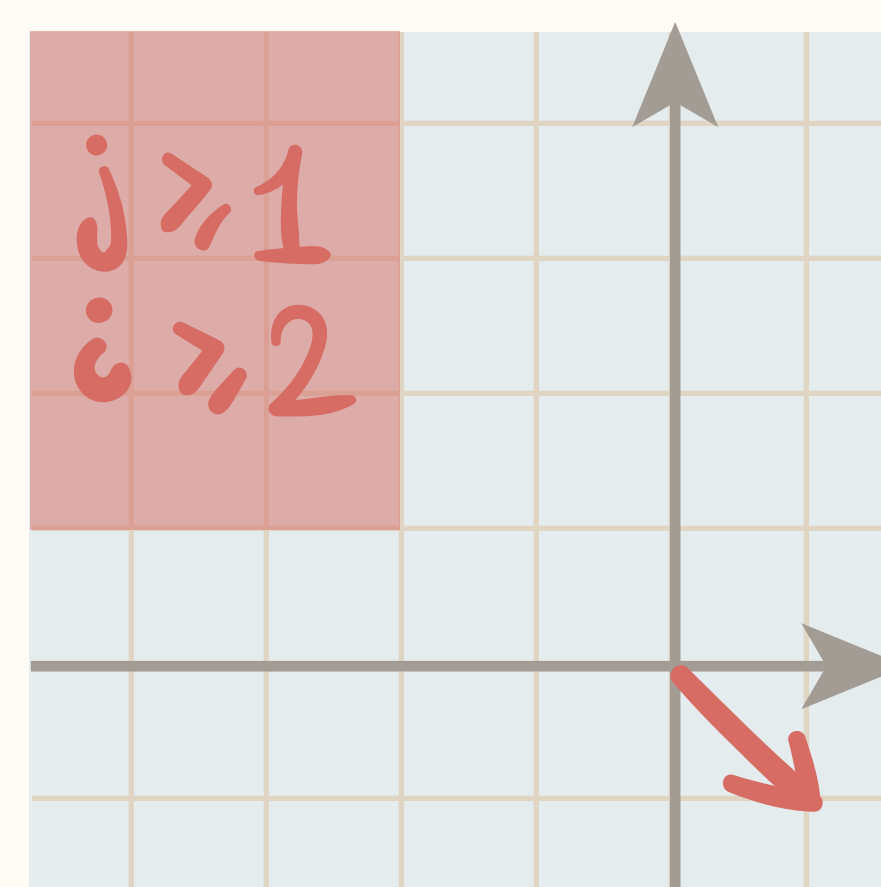
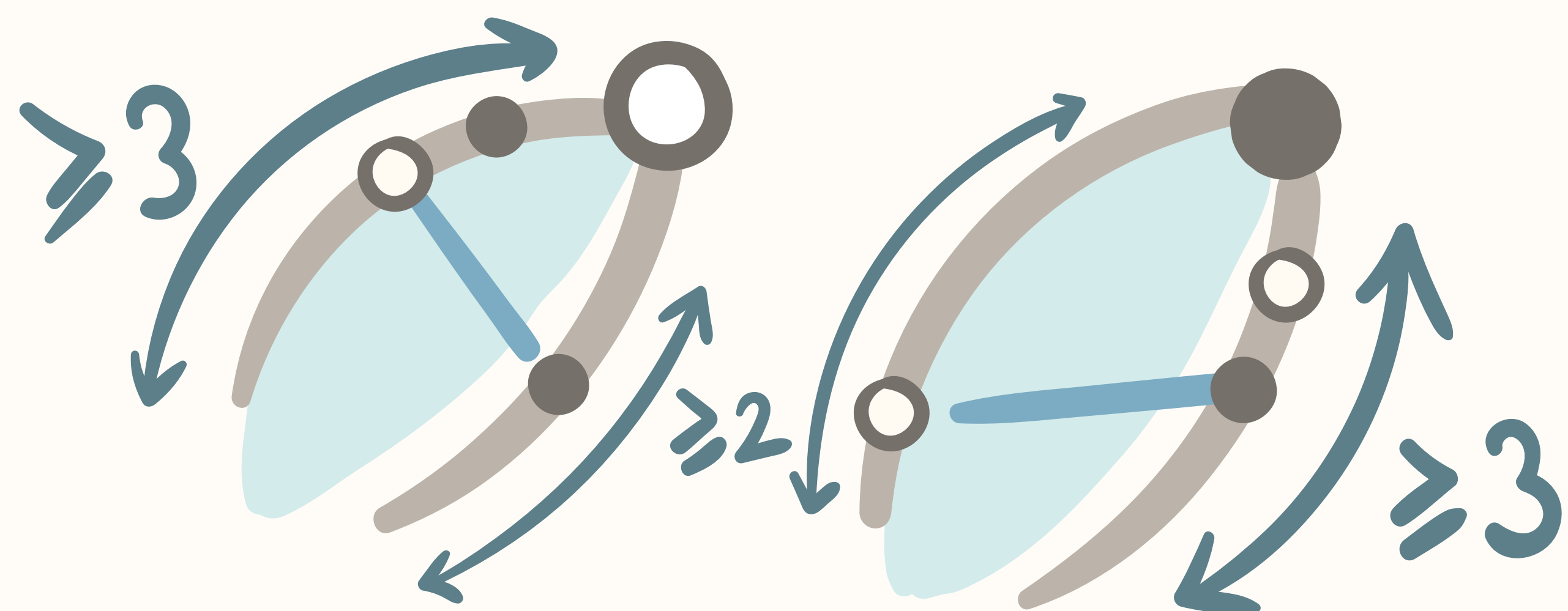


$$\binom{k+l}{k}$$

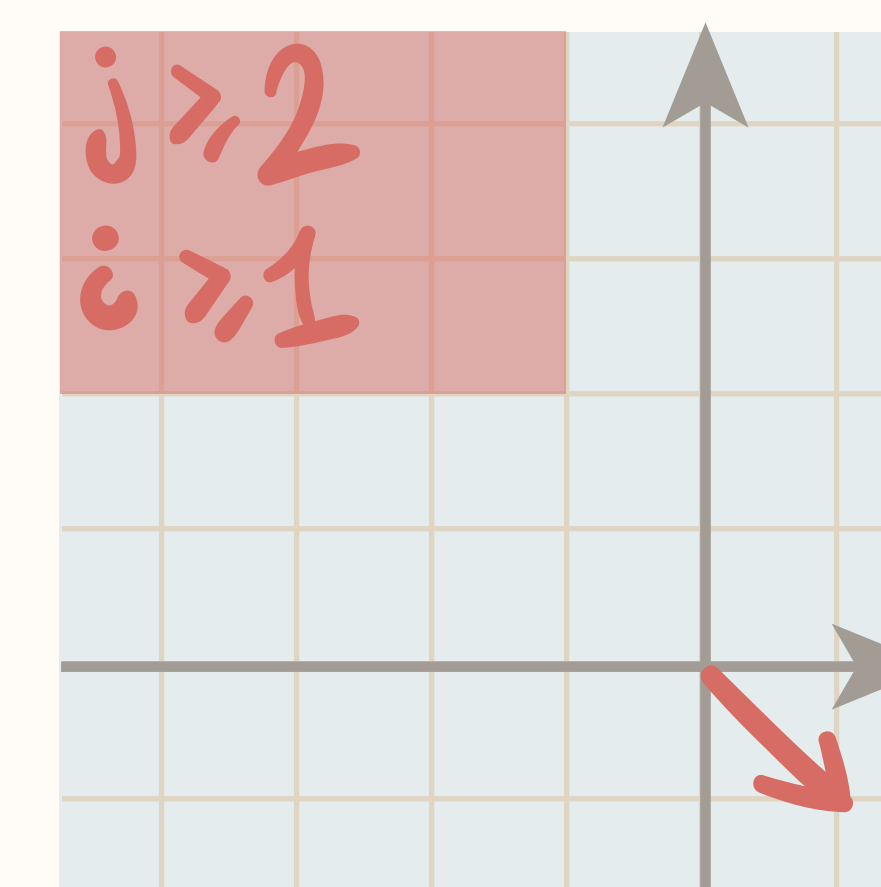


regarding
x,y parity

with
 $i = 2k+2$
 $j = 2l+2$

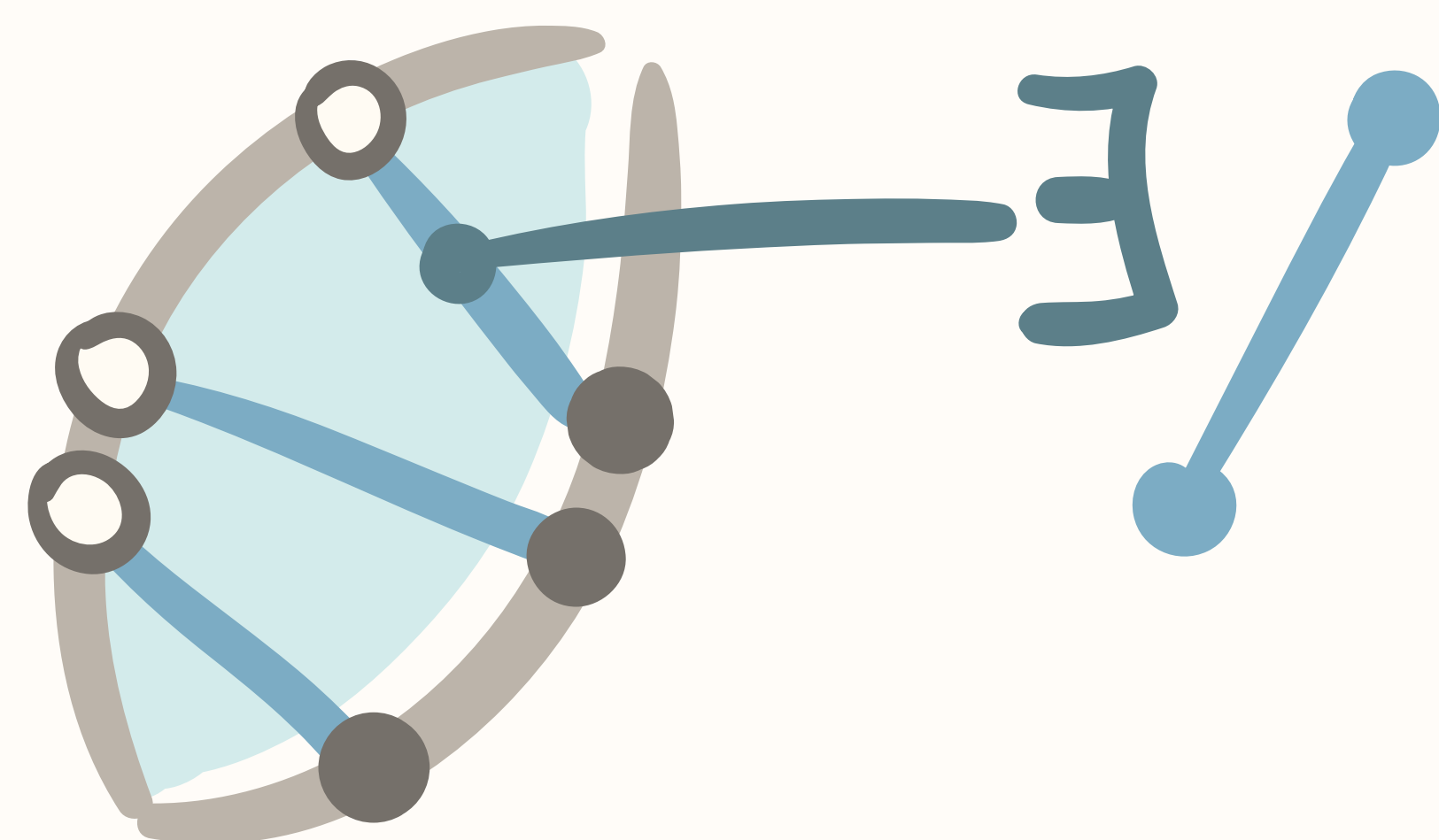
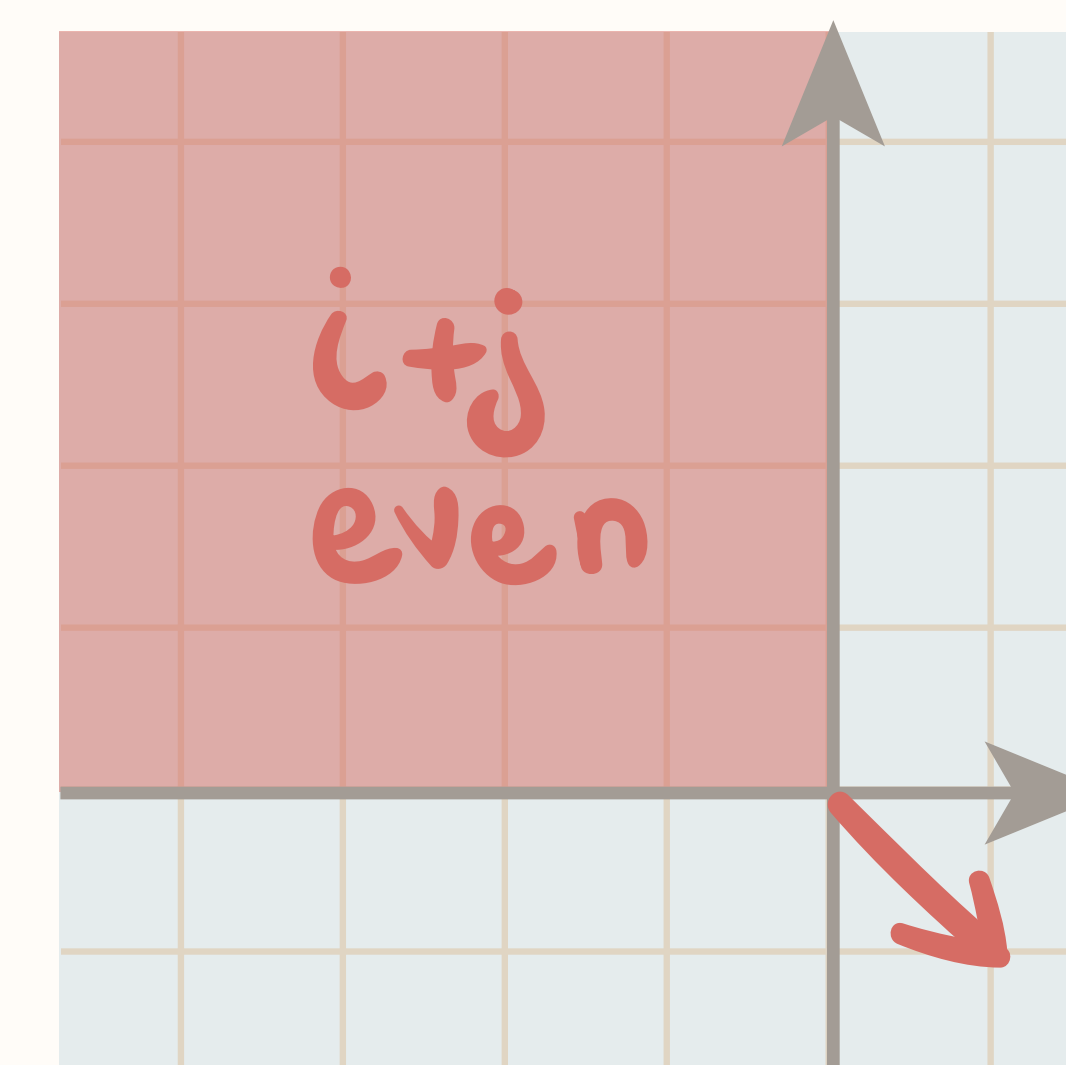
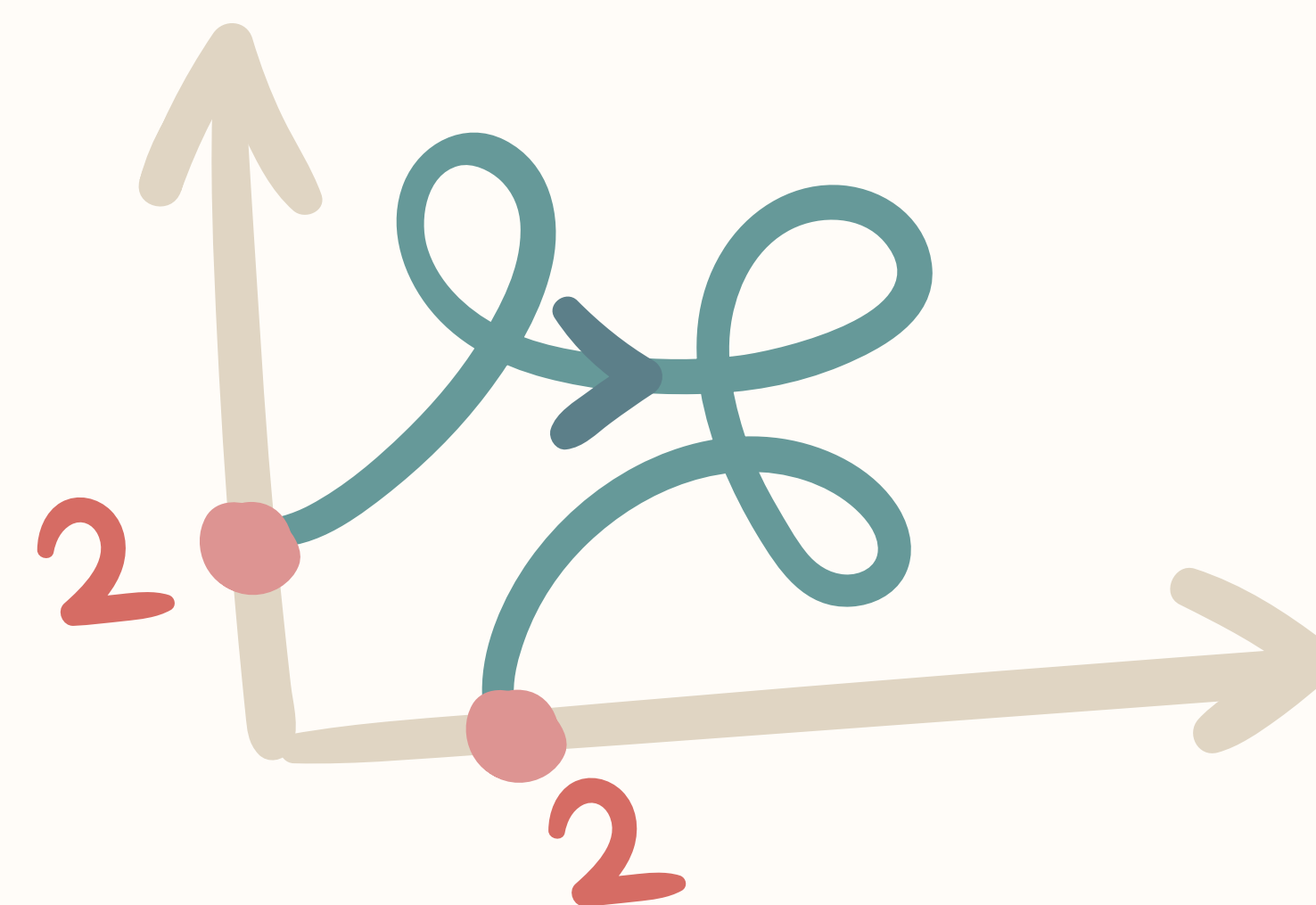
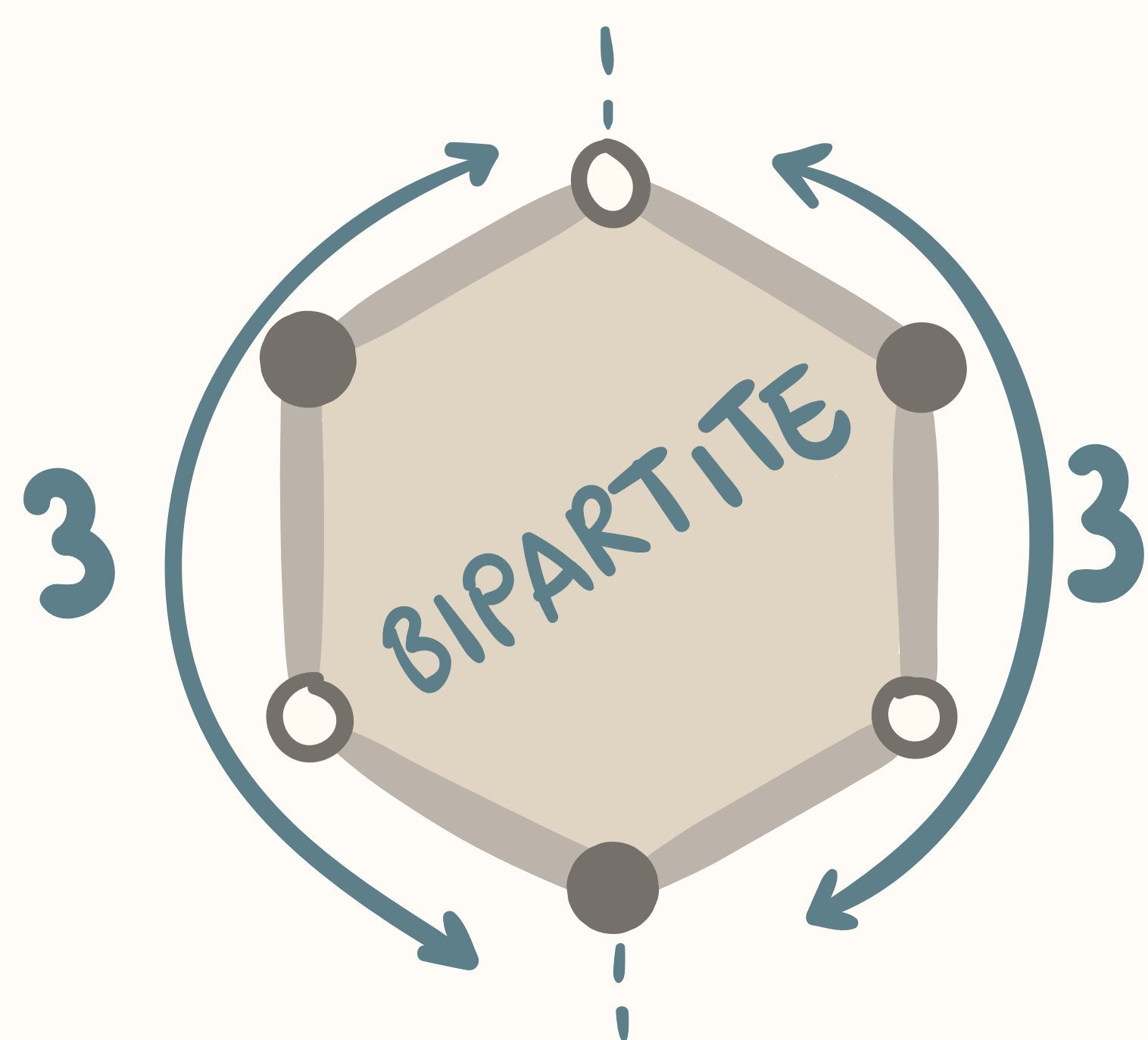


when x,y
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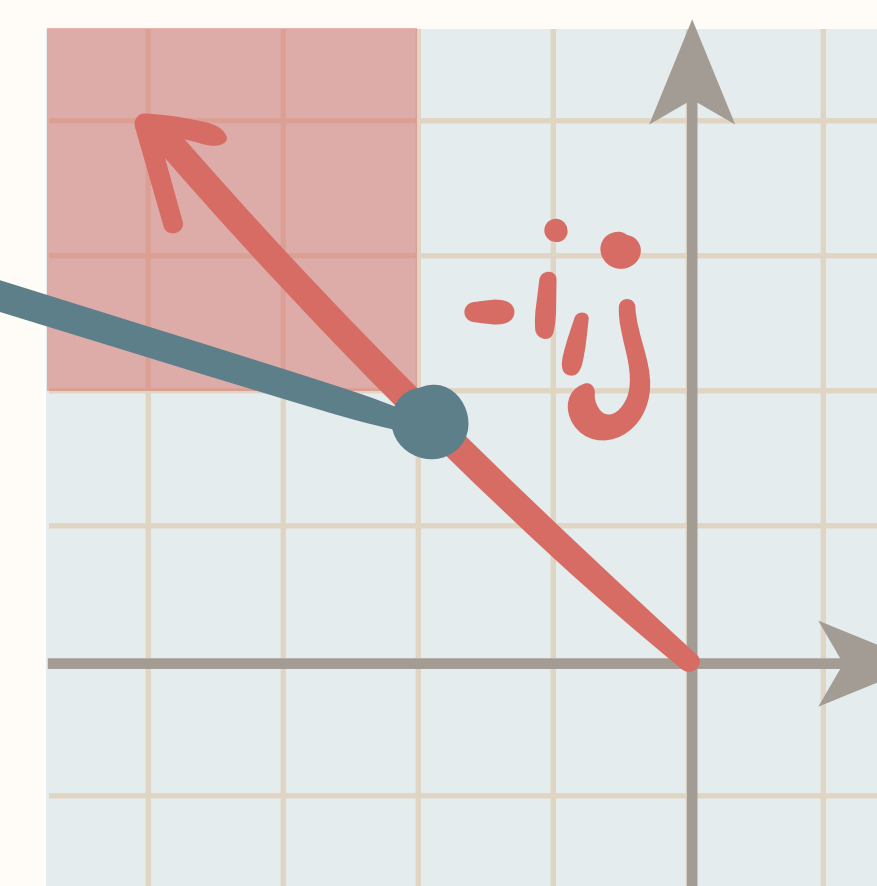


when x,y
are odd

Schnyder labellings KMSW



$$\binom{k+l}{k}$$

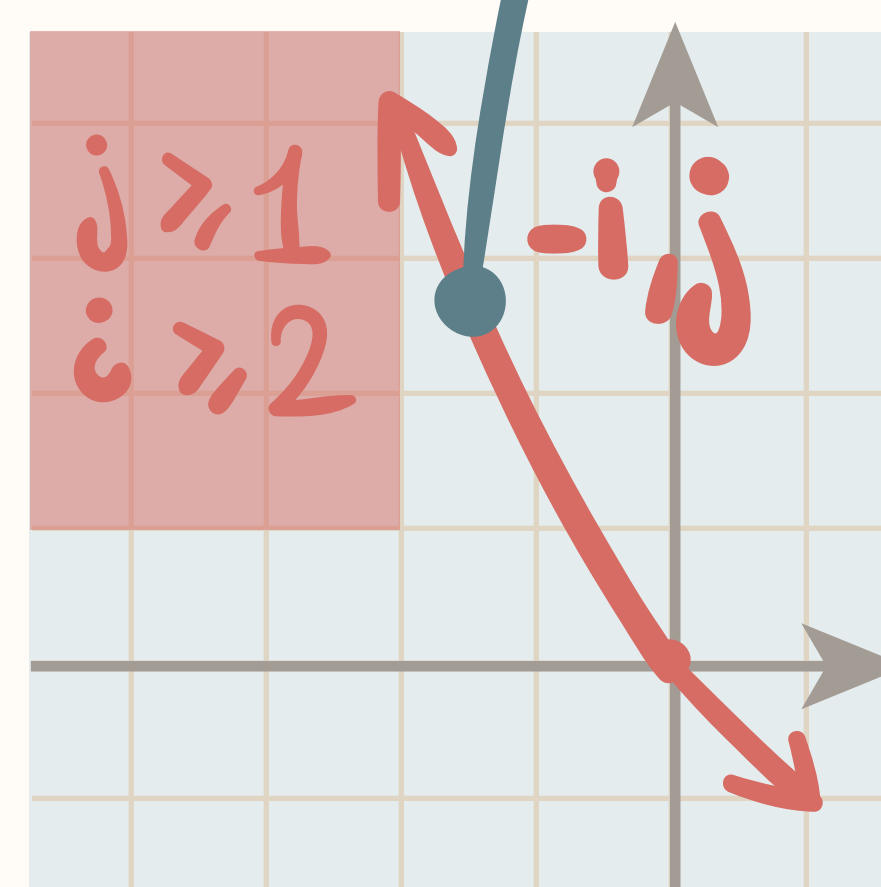
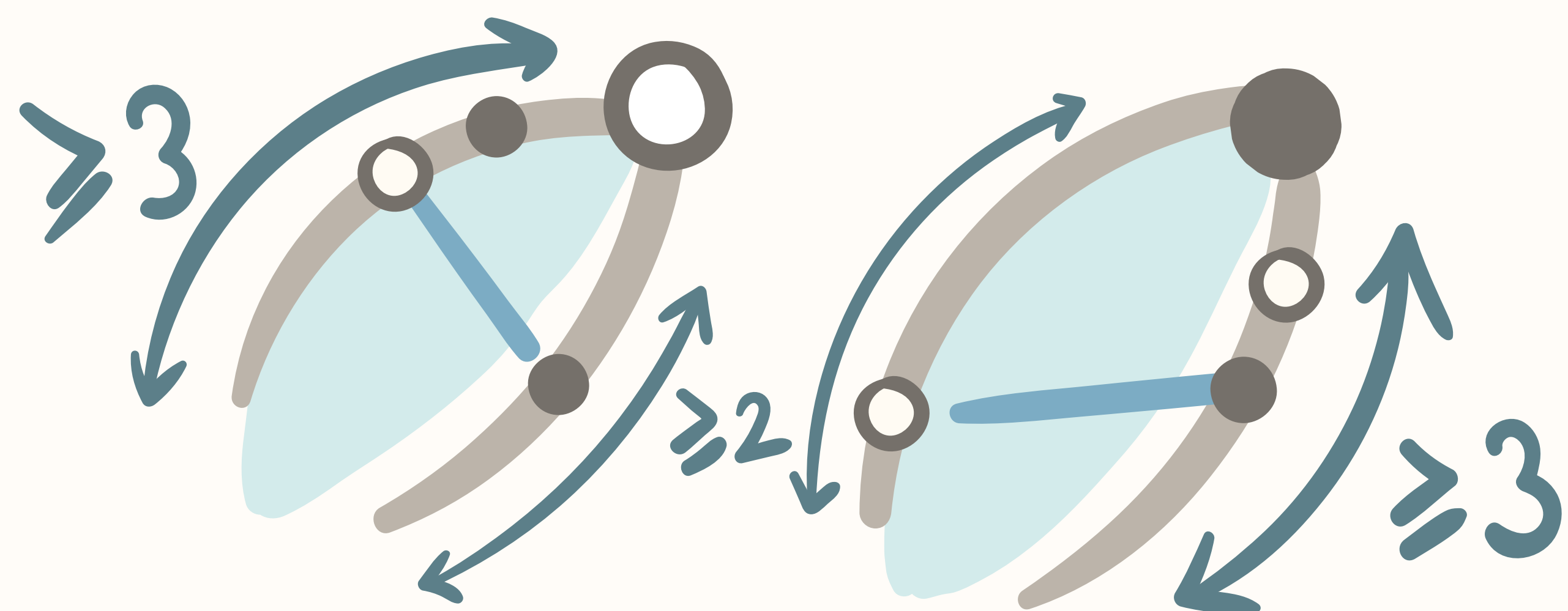


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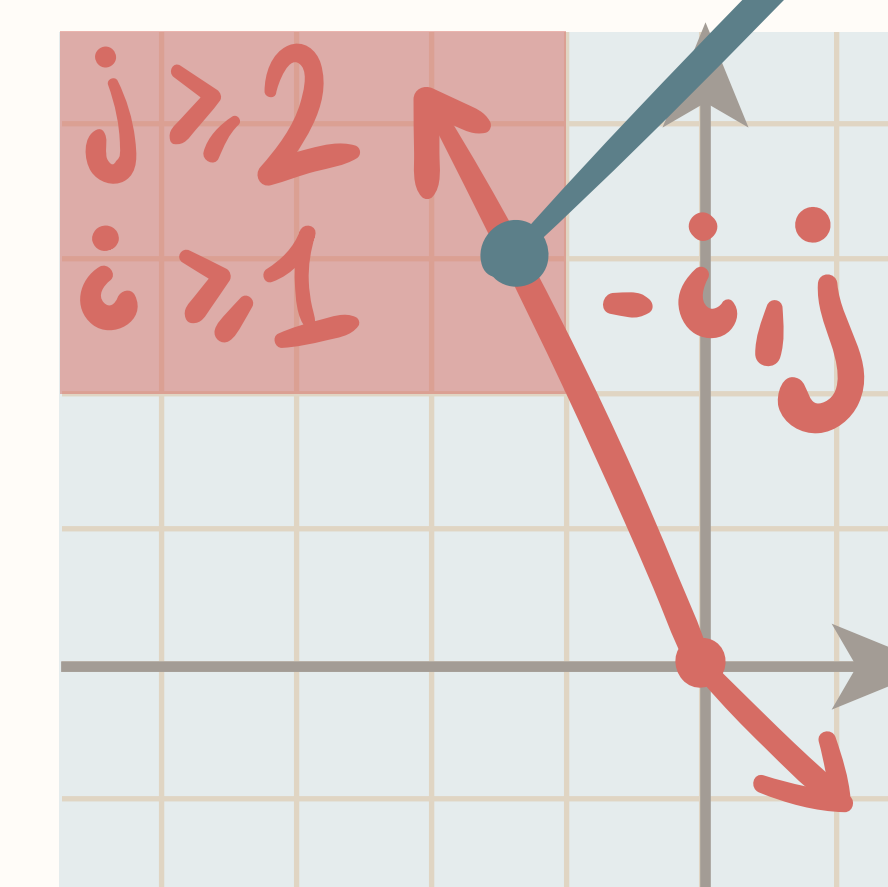
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when x,y
are even

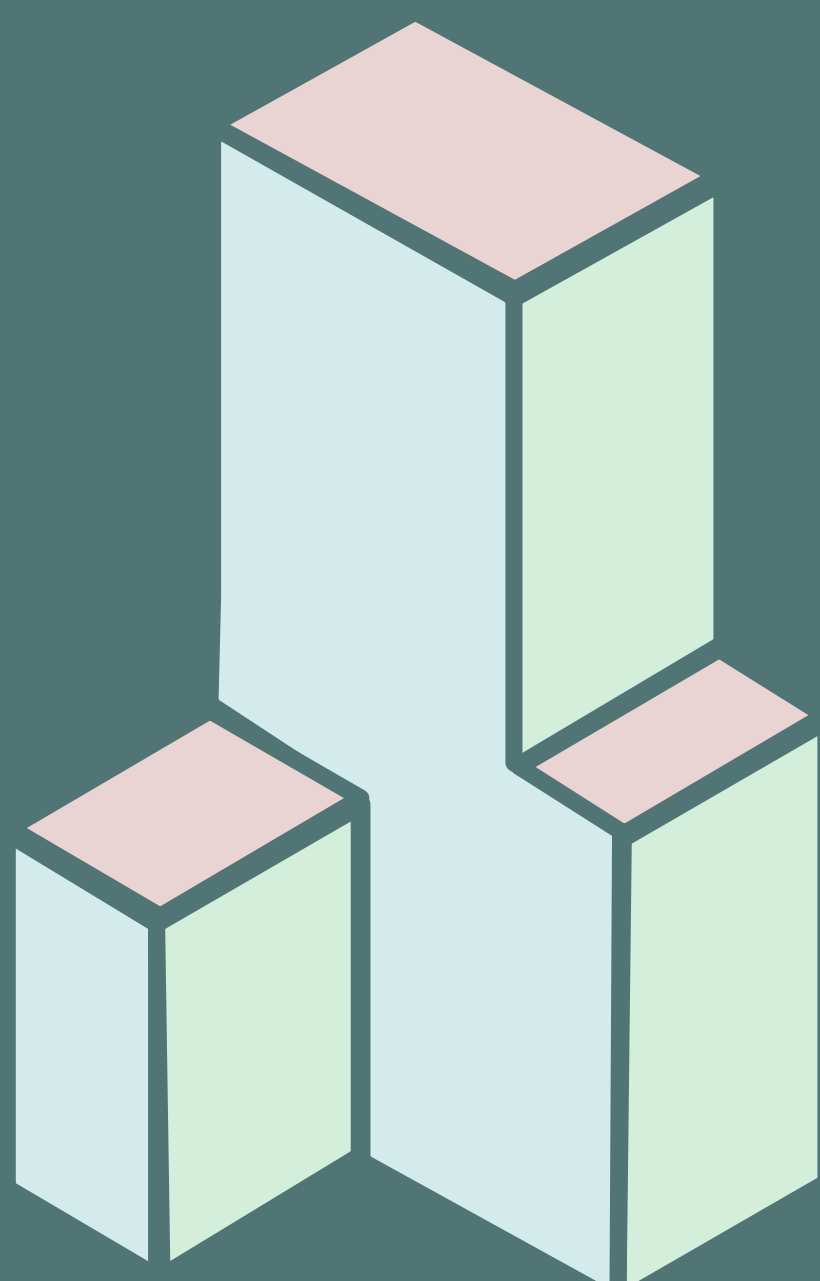


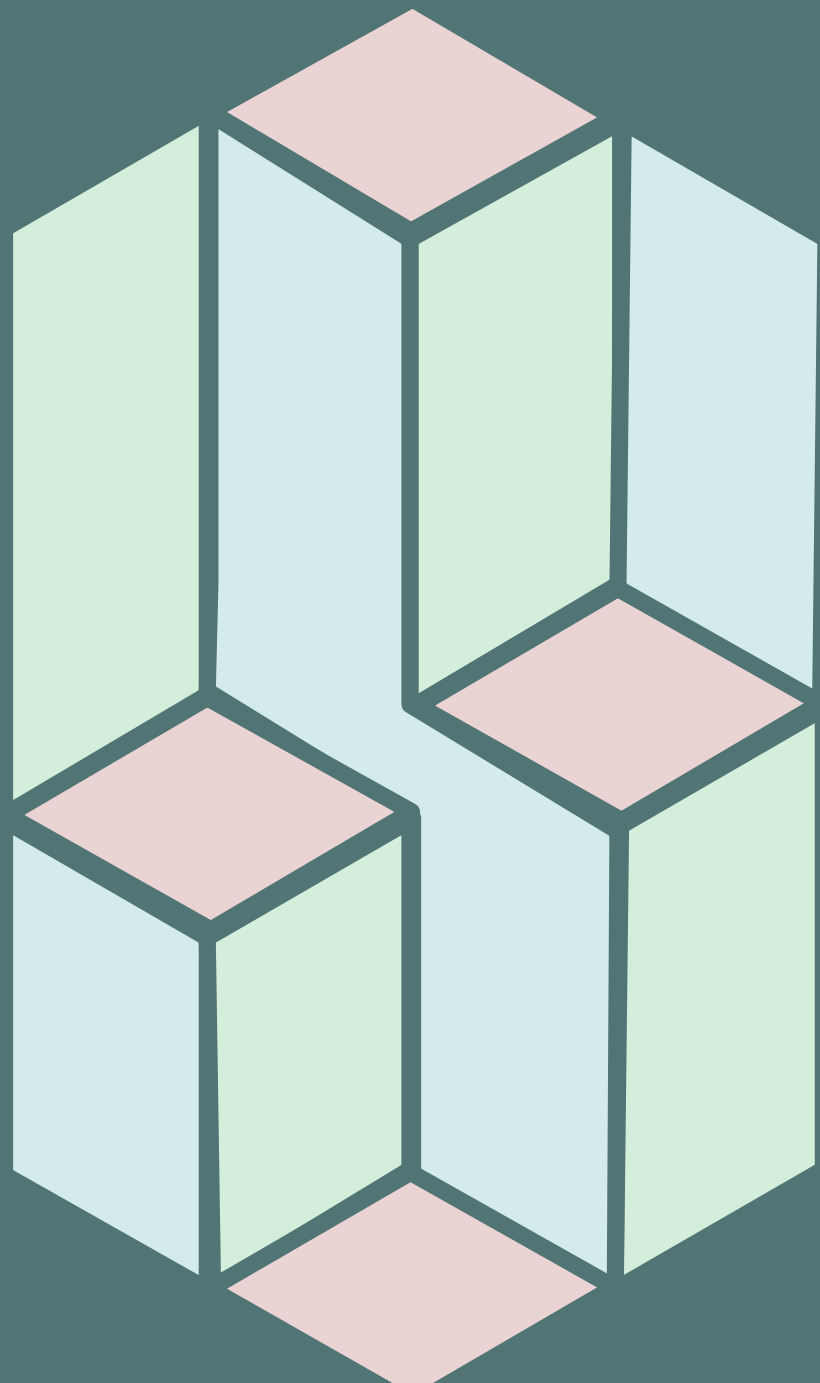
with
 $i = 2k+3$
 $j = 2l+1$



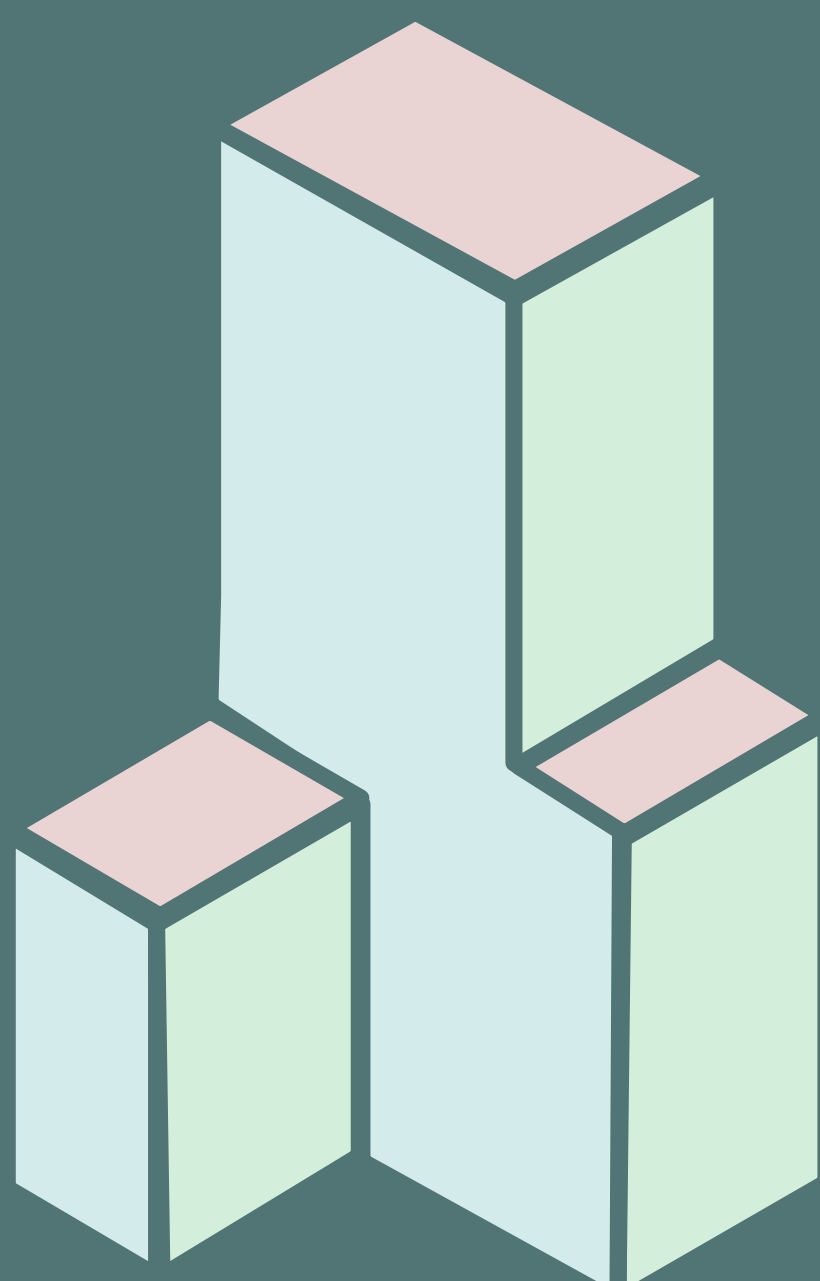


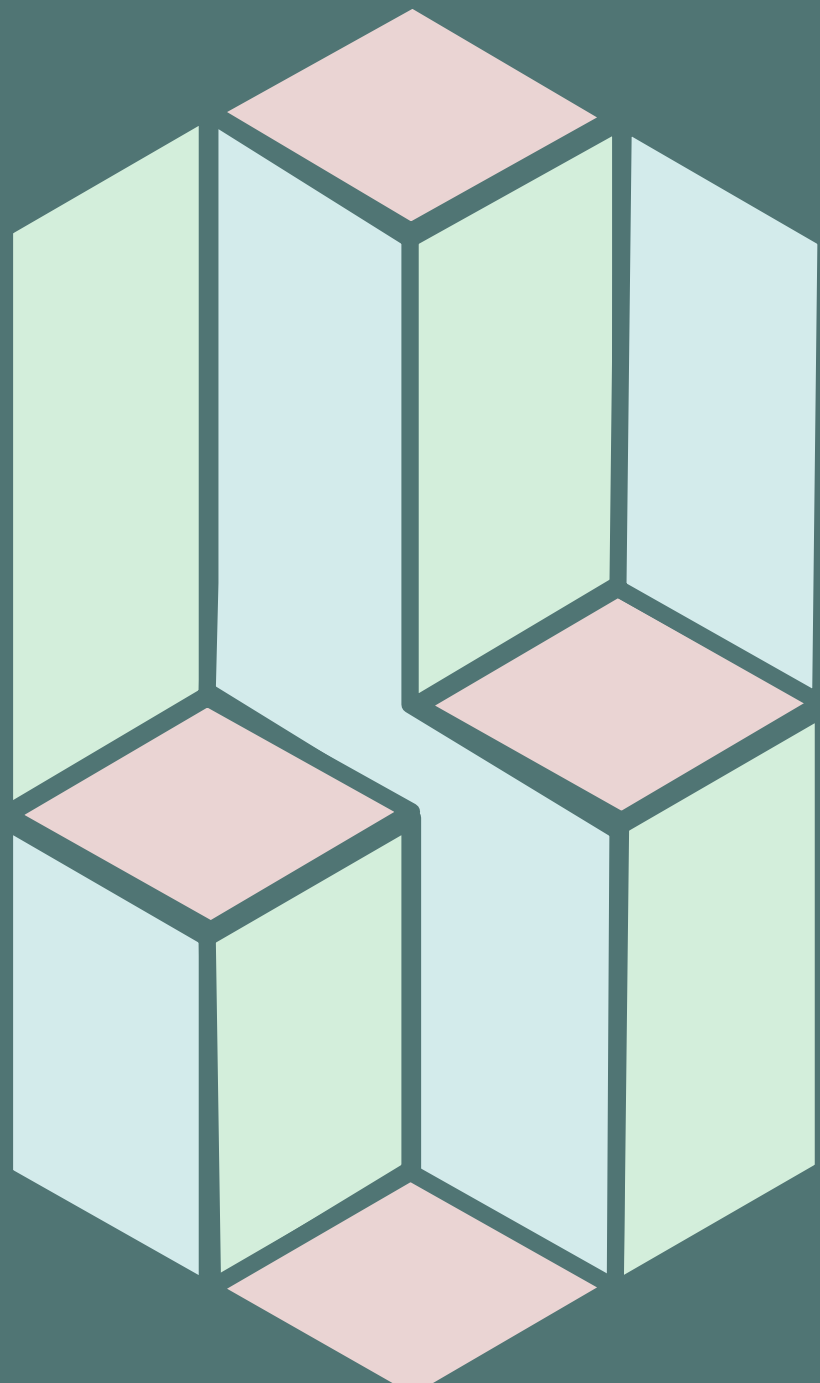

when x,y
are odd

with
 $i = 2k+1$
 $j = 2l+3$

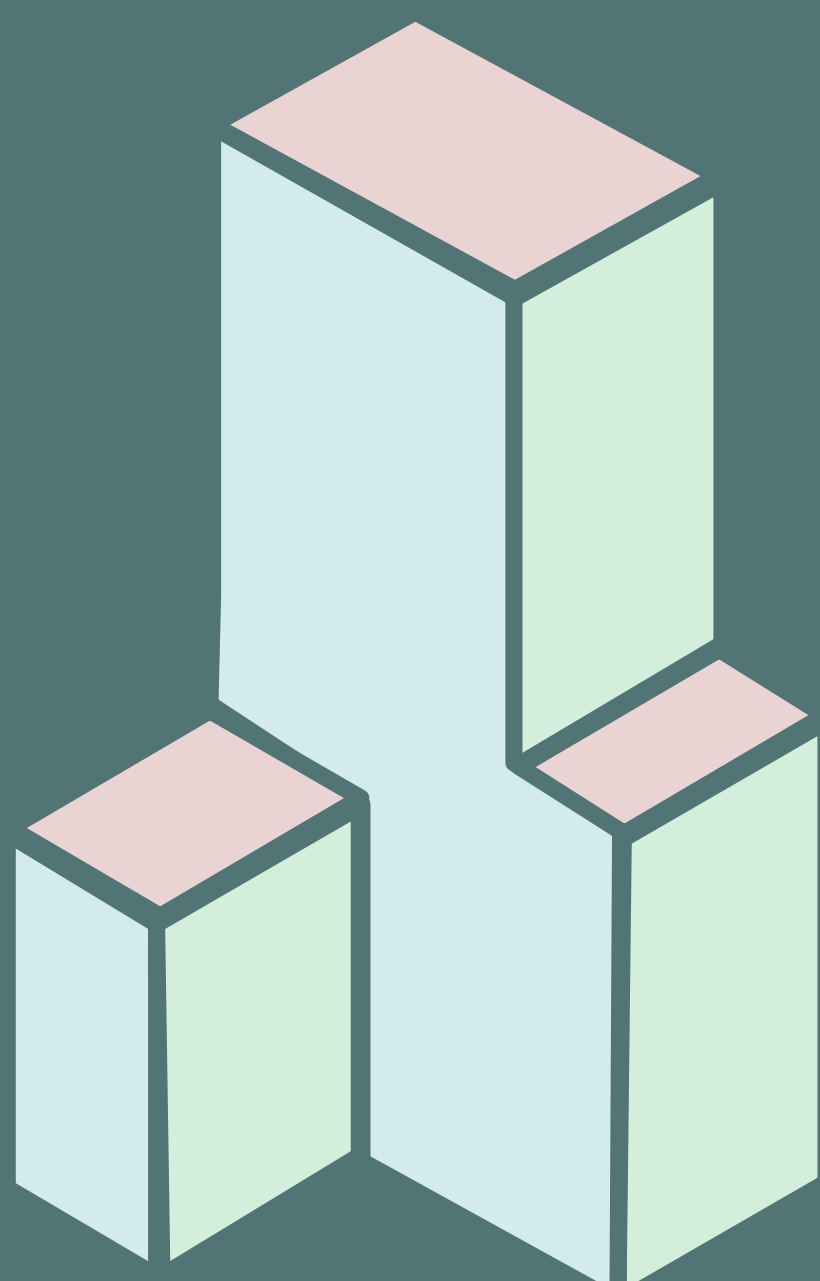


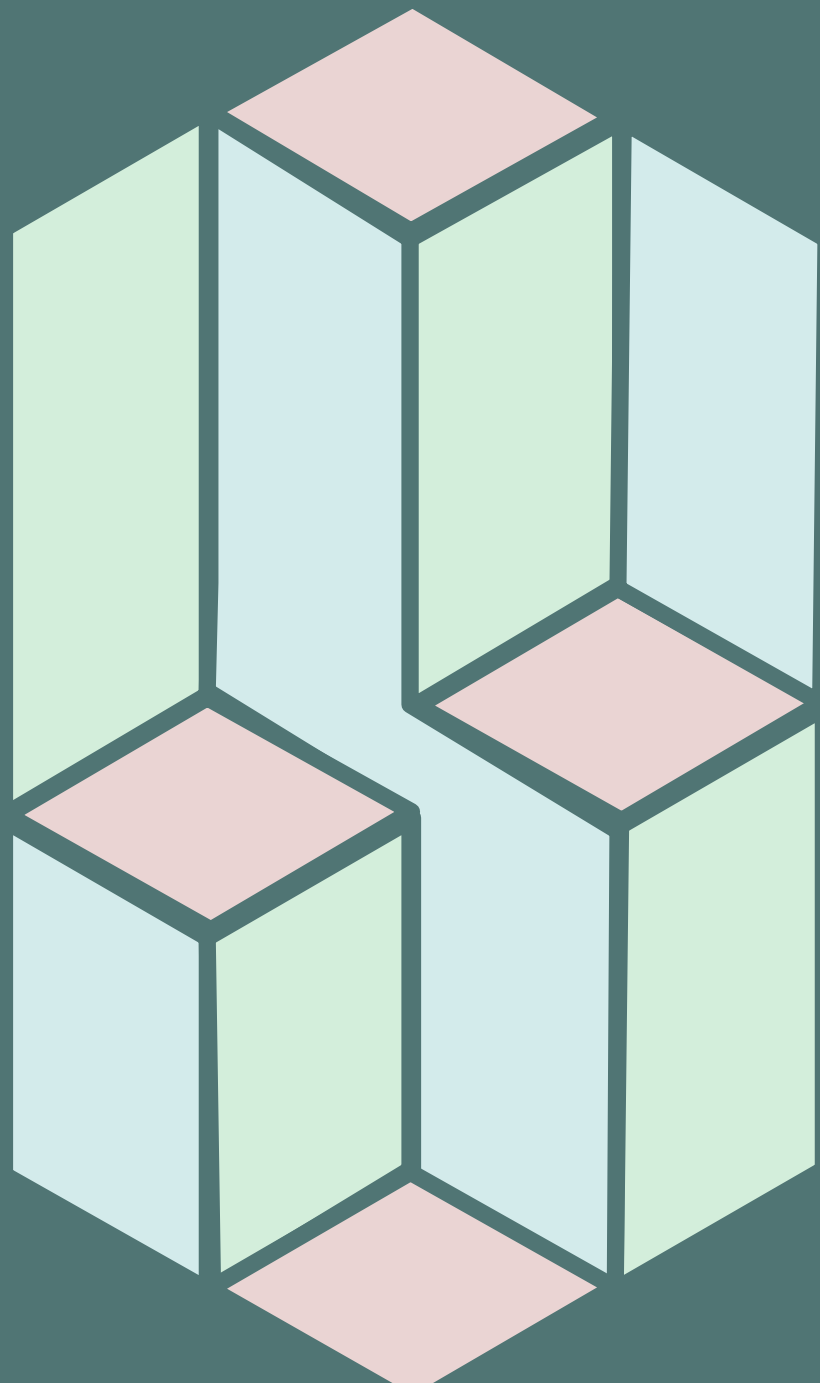

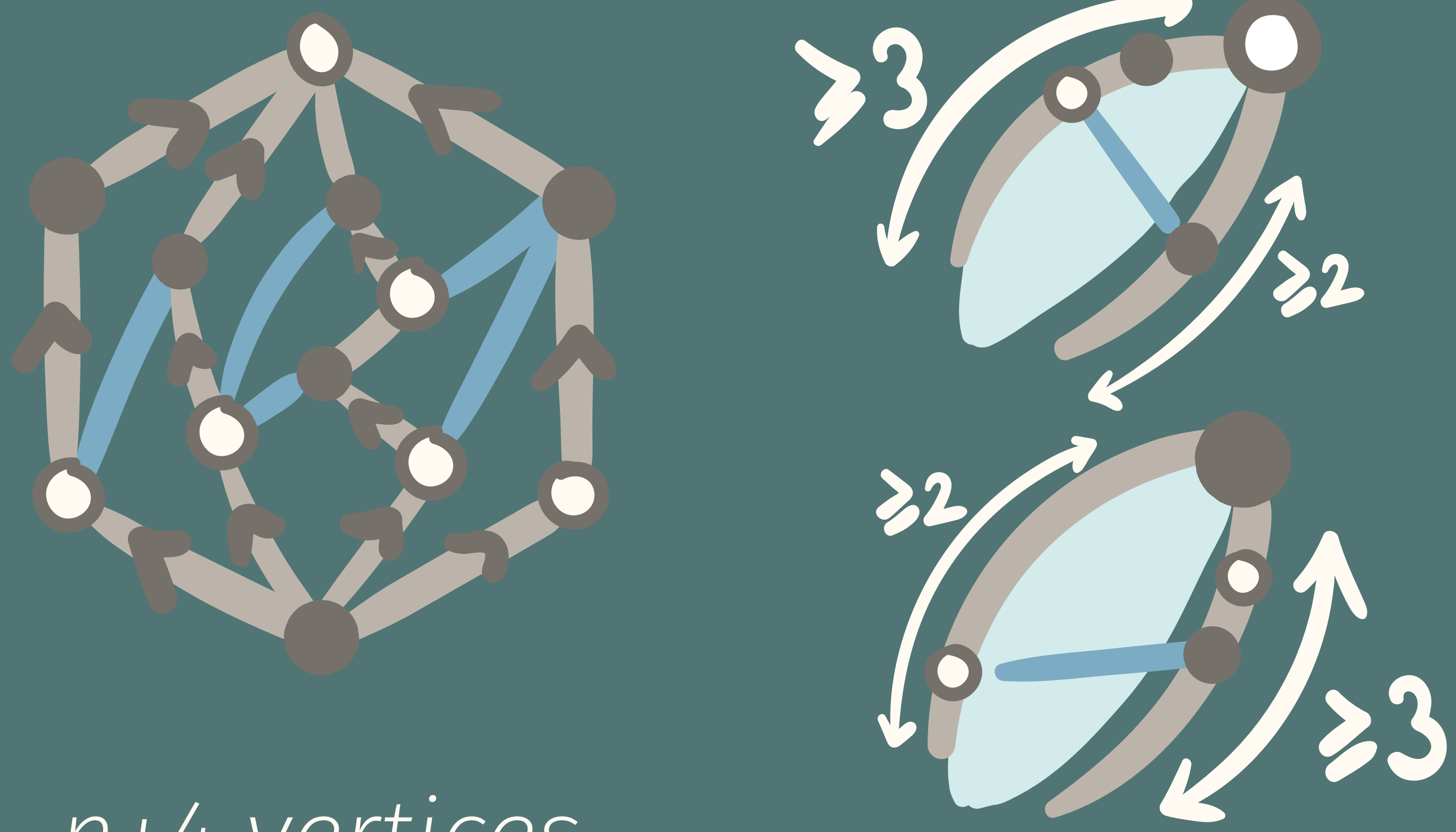
Bijections summary

<i>Model</i>	<i>Combinatorial type</i>	<i>Bipolar orient./Tandem walk</i>
 <p data-bbox="302 1078 533 1164"><i>n flats</i></p>	 <p data-bbox="821 1078 1384 1164"><i>n inner vertices</i></p>	 <p data-bbox="1643 1078 2004 1164"><i>n+1 edges</i></p>
 <p data-bbox="302 1929 533 2015"><i>n flats</i></p>		

Bijections summary

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Bijections summary

Model	Combinatorial type	Bipolar orient./Tandem walk
 <p data-bbox="302 1078 533 1164">n flats</p>	 <p data-bbox="821 1078 1384 1164">n inner vertices</p>	 <p data-bbox="1643 1078 2004 1164">$n+1$ edges</p>
 <p data-bbox="302 1929 533 2015">n flats</p>	 <p data-bbox="865 1929 1341 2015">n inner faces</p>	 <p data-bbox="1600 1929 2033 2015">$n+4$ vertices</p>

Sommaire

The KMSW Bijection

1. Application to three maps model

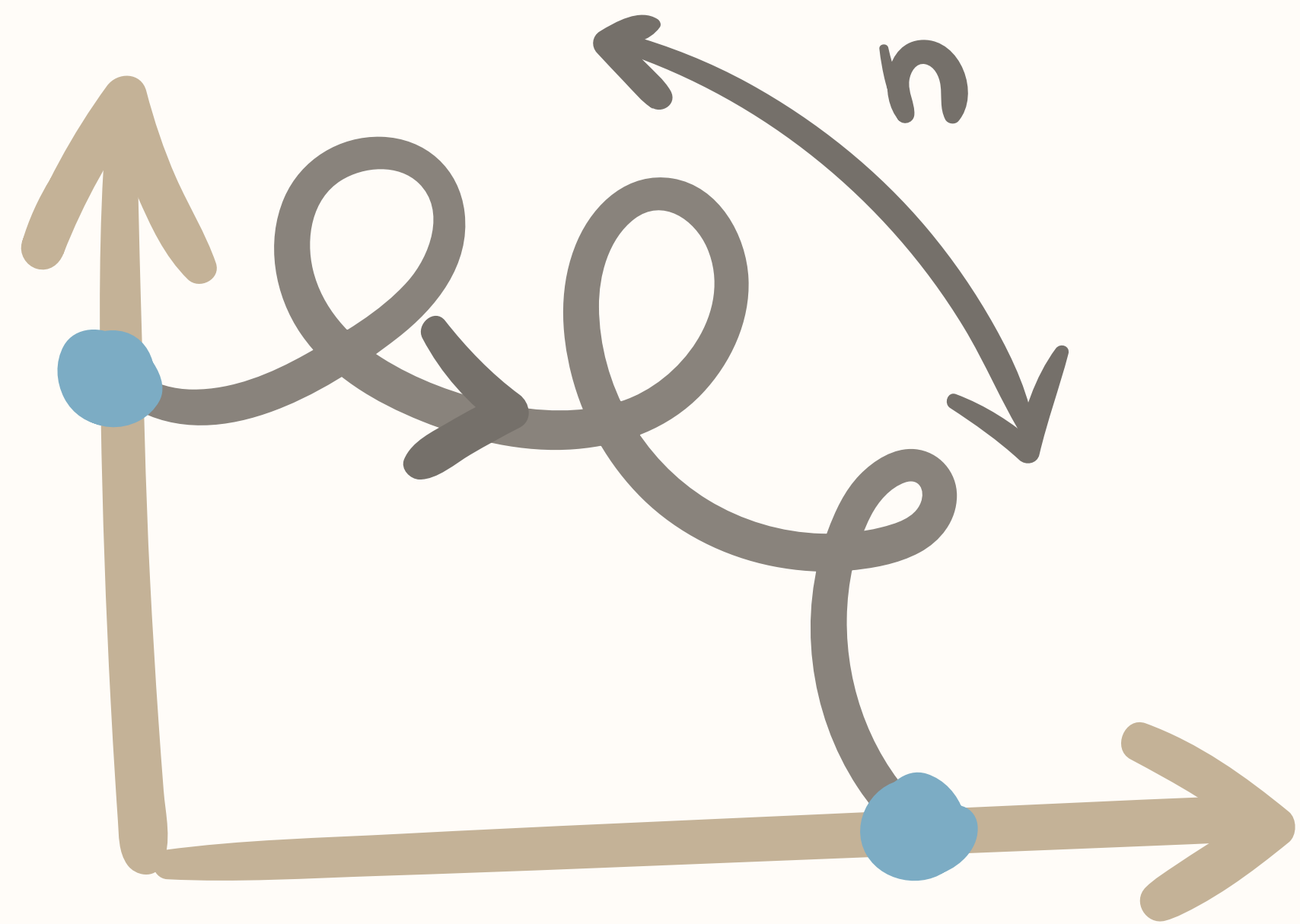
- a. Plane bipolar posets*
- b. Plane bipolar posets by vertices*
- c. Transversal structures*
- d. Asymtotoxic*

2. Interlude : plane permutations

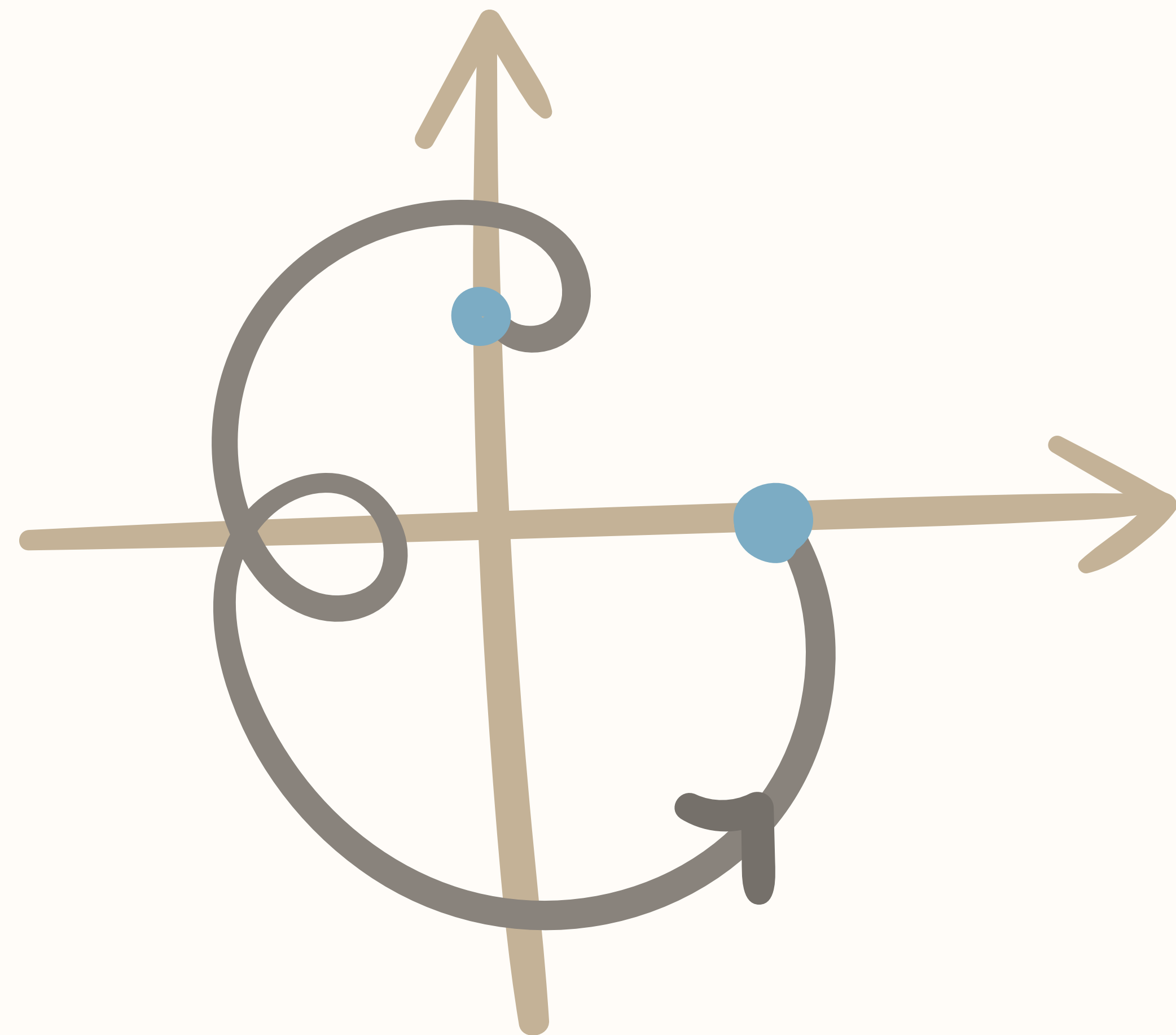
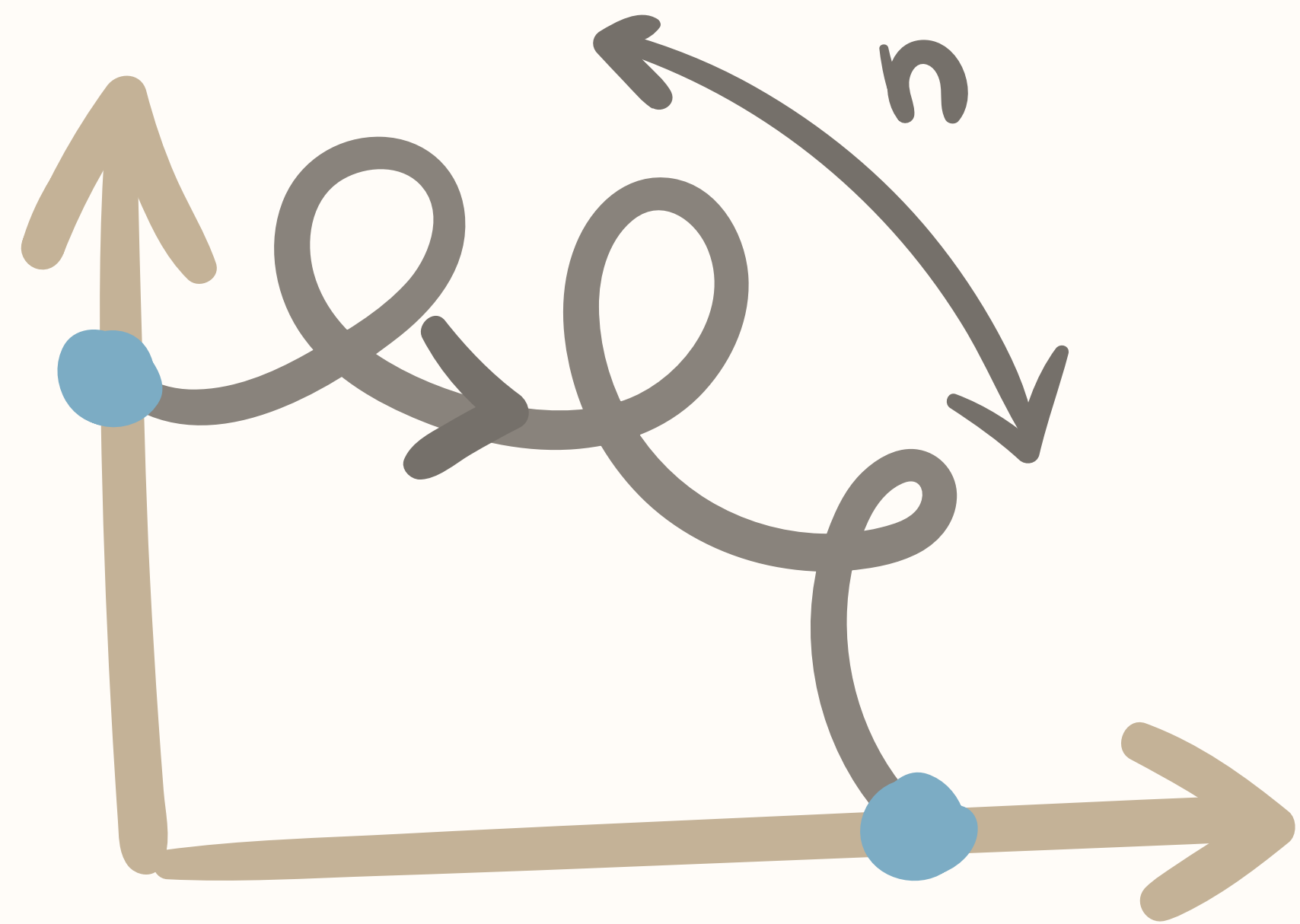
3. Application to corner polyhedra

- a. Via polyhedral orientations*
- b. Via Schnyder colorings*
- c. Asymtotoxic*

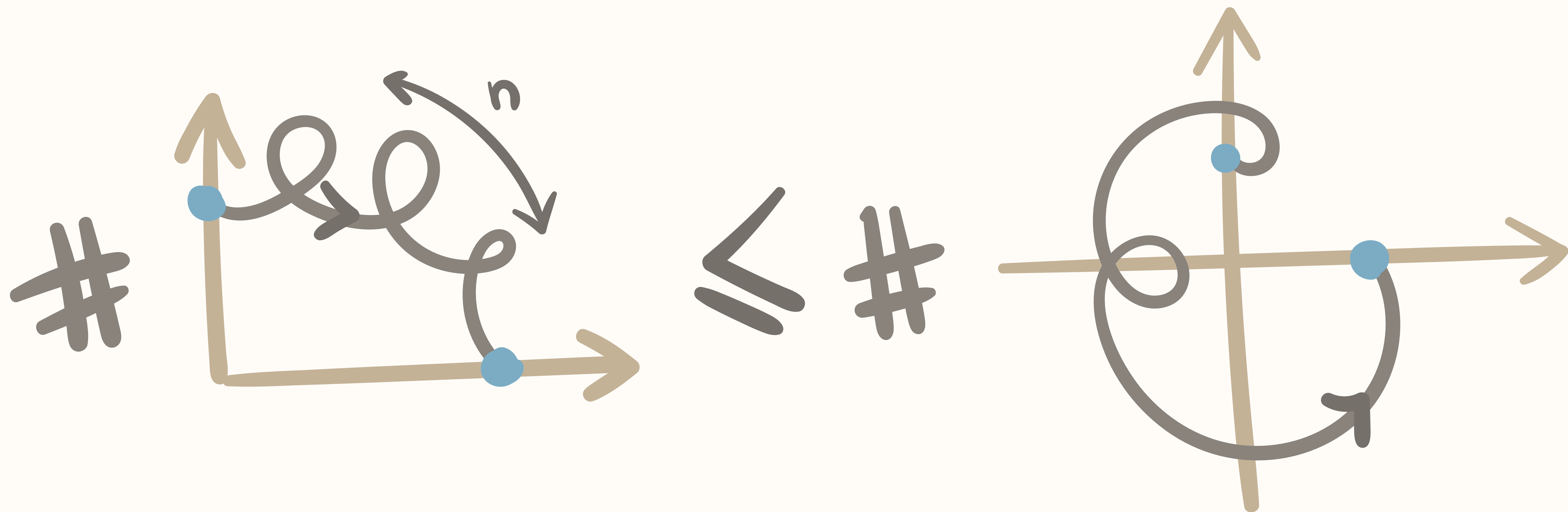
Asymptotic counting results



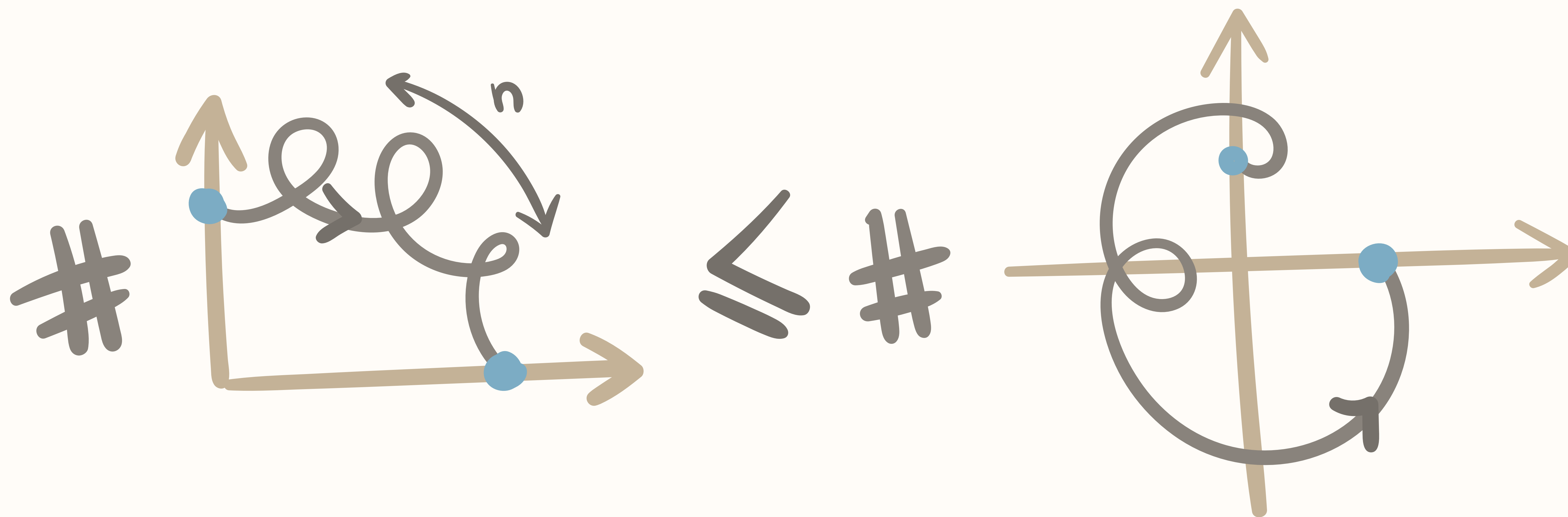
Asymptotic counting results

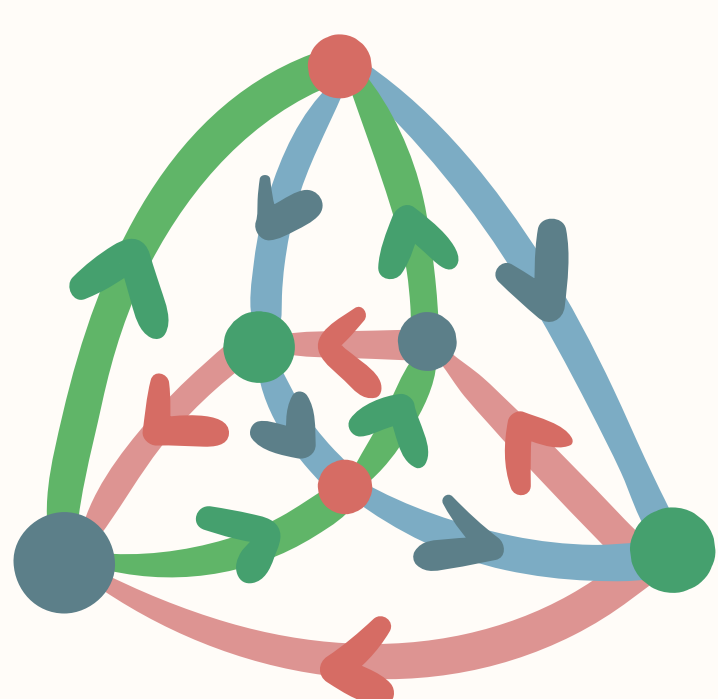


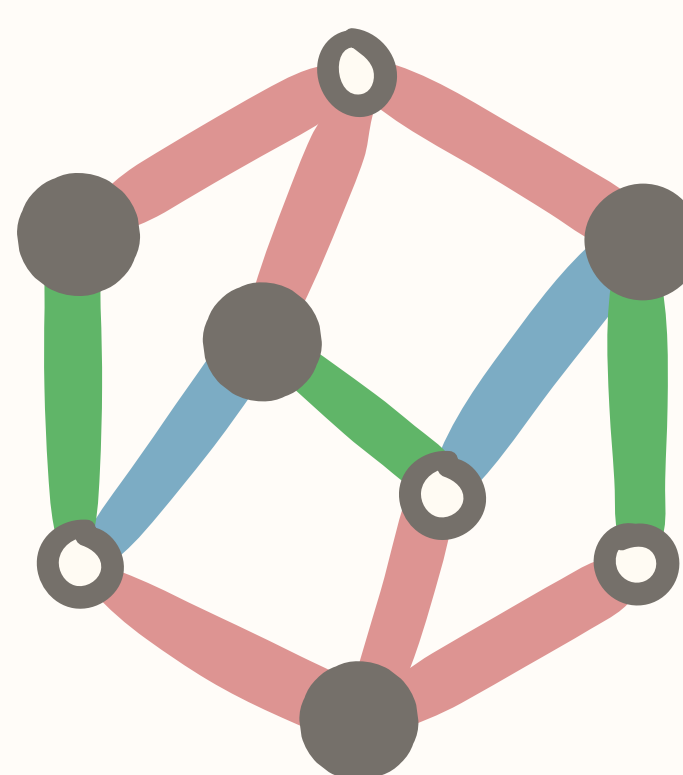
Asymptotic counting results



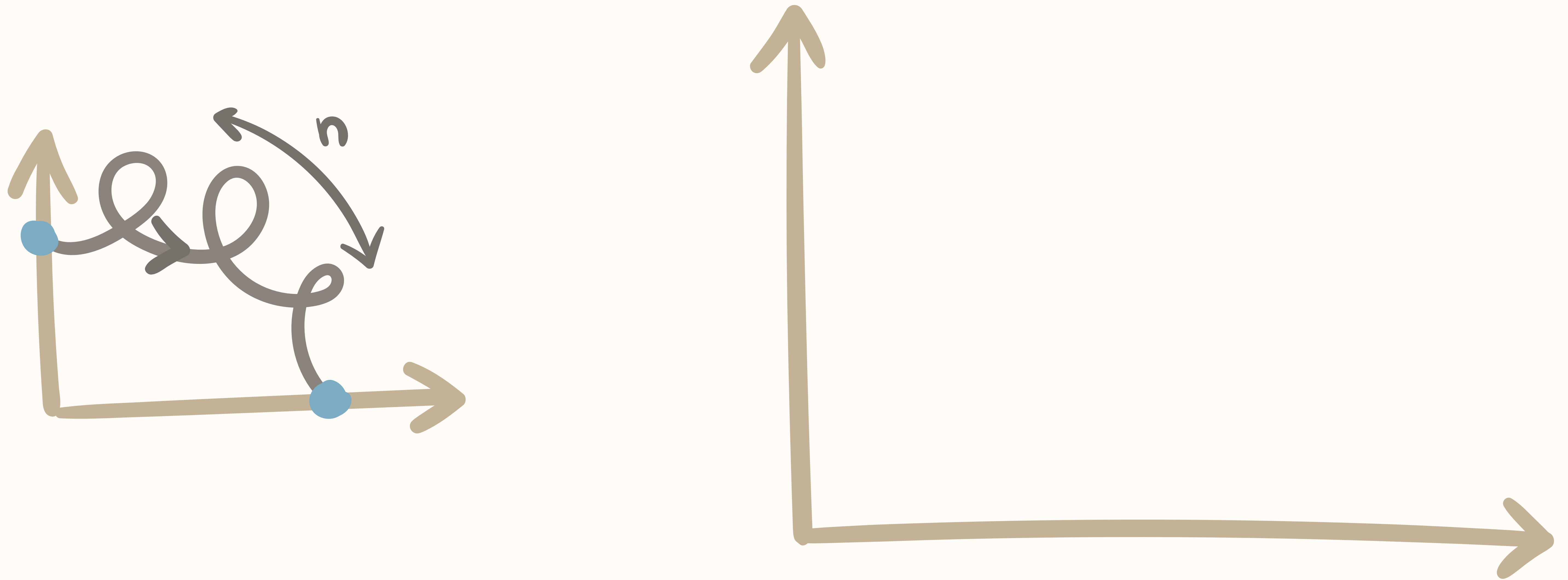
Asymptotic counting results

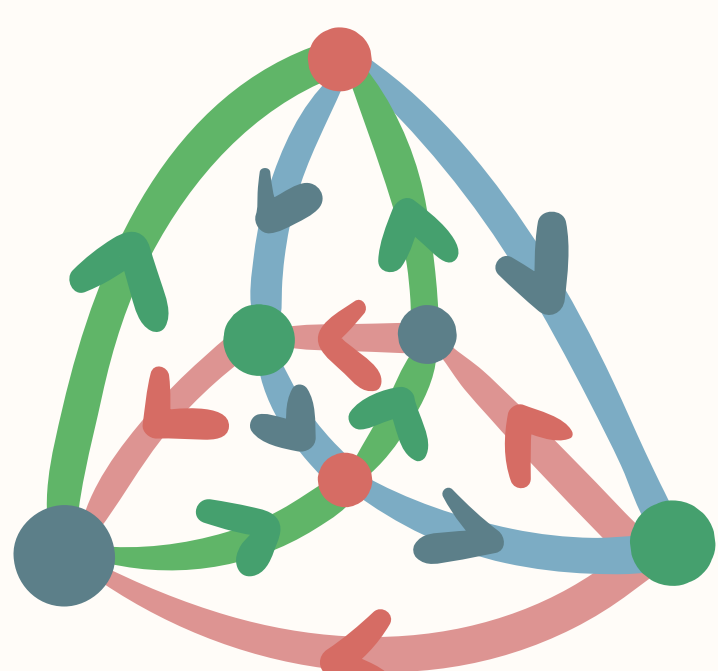


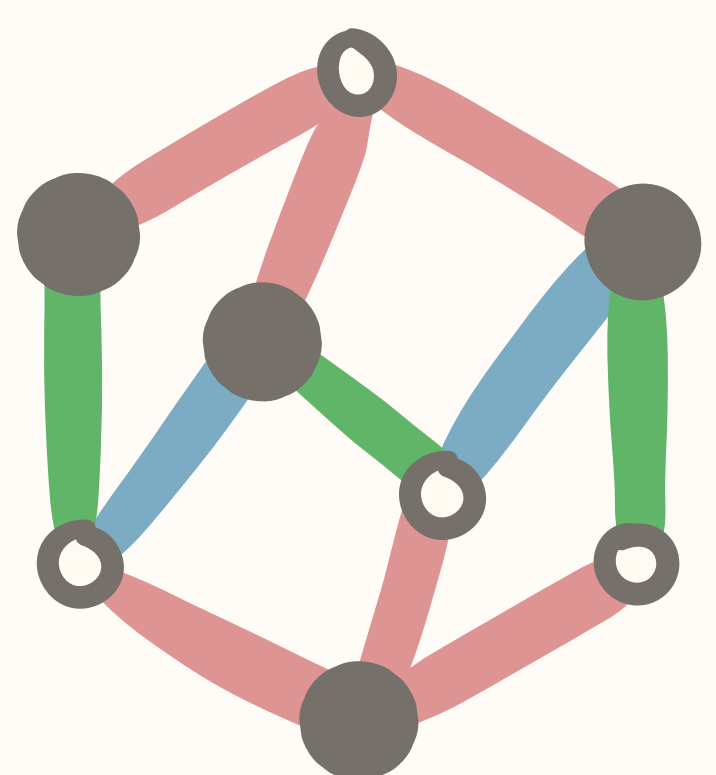
 $|_n \leq \left(\frac{9}{2}\right)^n$

 $|_n \leq \left(\frac{16}{3}\right)^n$

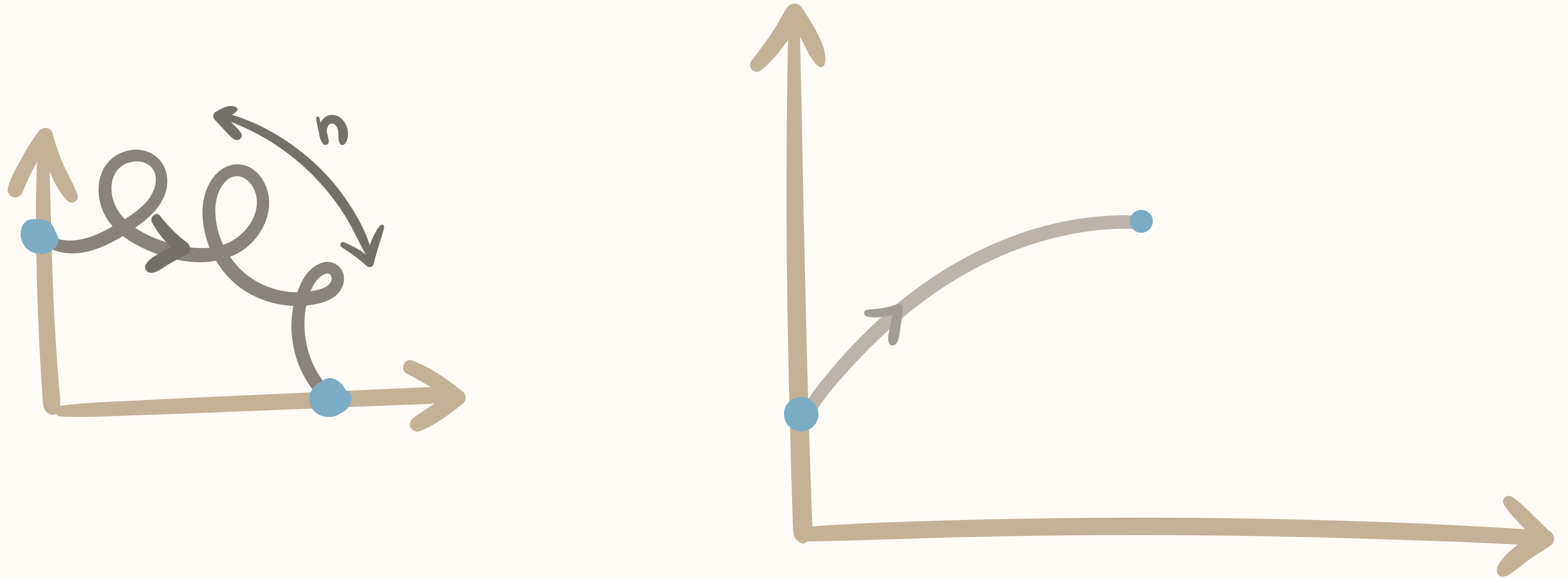
Asymptotic counting results



 $|_n \ll \left(\frac{9}{2}\right)^n$

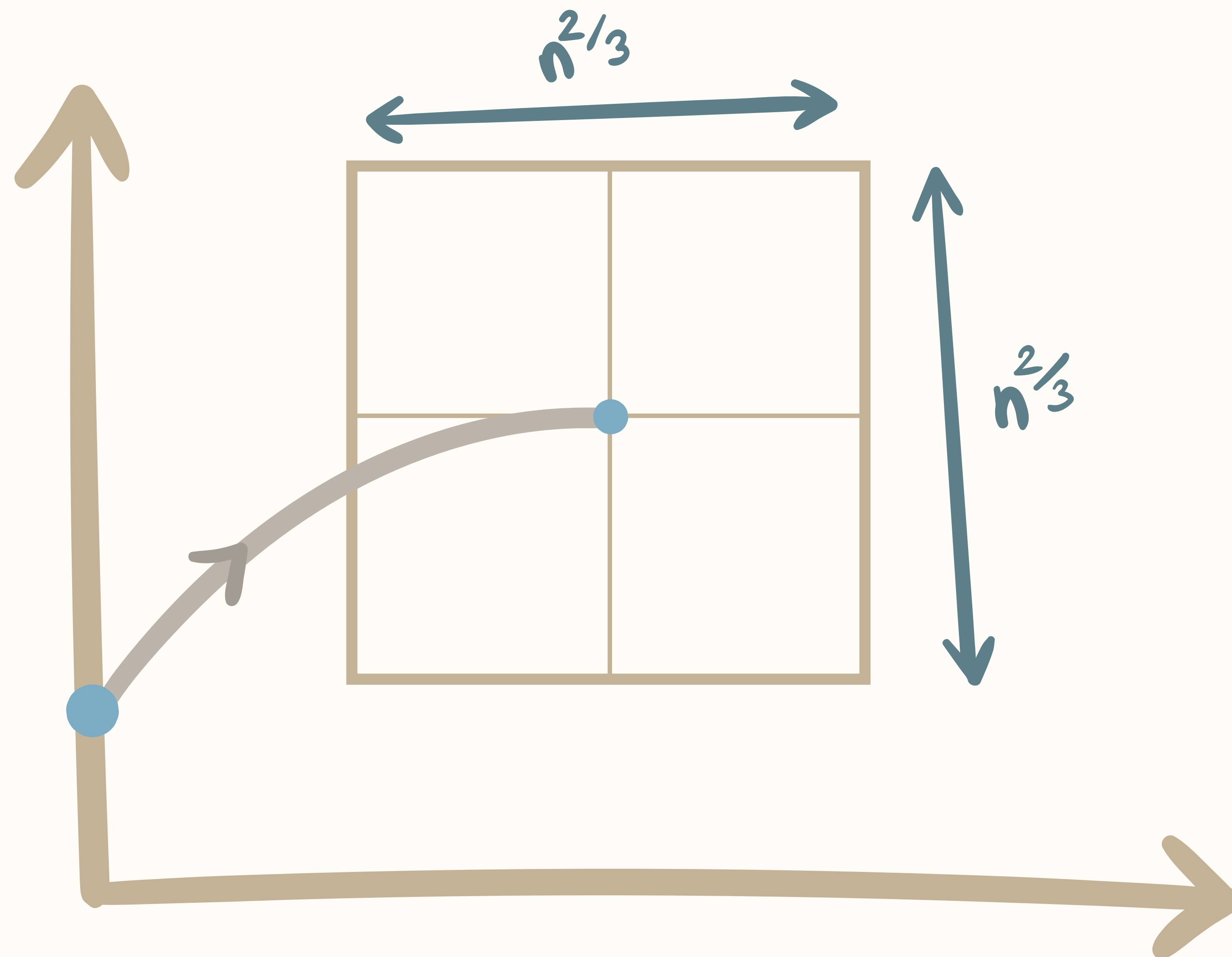
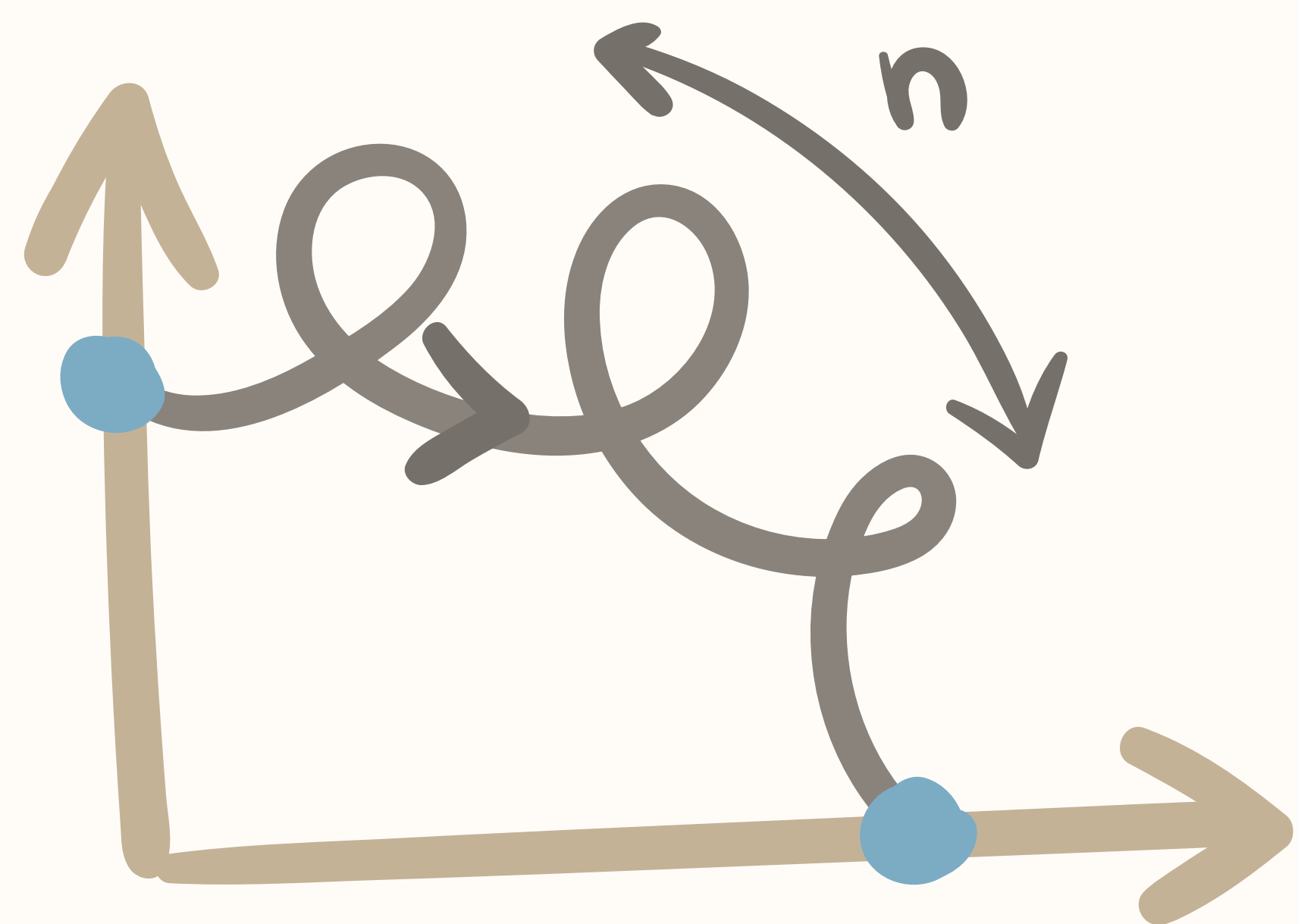
 $|_n \ll \left(\frac{16}{3}\right)^n$

Asymptotic counting results

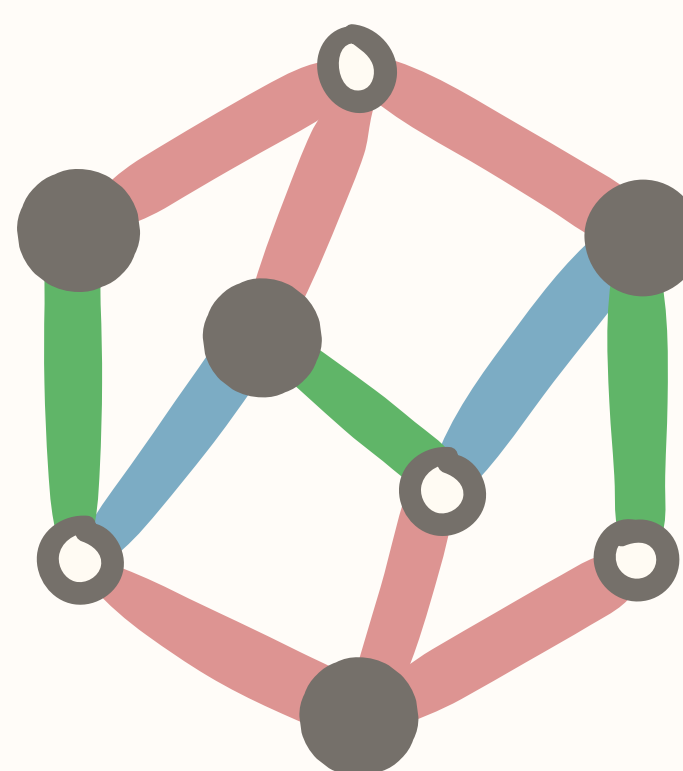


$$\# \text{ [Diagram 1] } |_{n \ll} \left(\frac{9}{2}\right)^n$$
$$\# \text{ [Diagram 2] } |_{n \ll} \left(\frac{16}{3}\right)^n$$

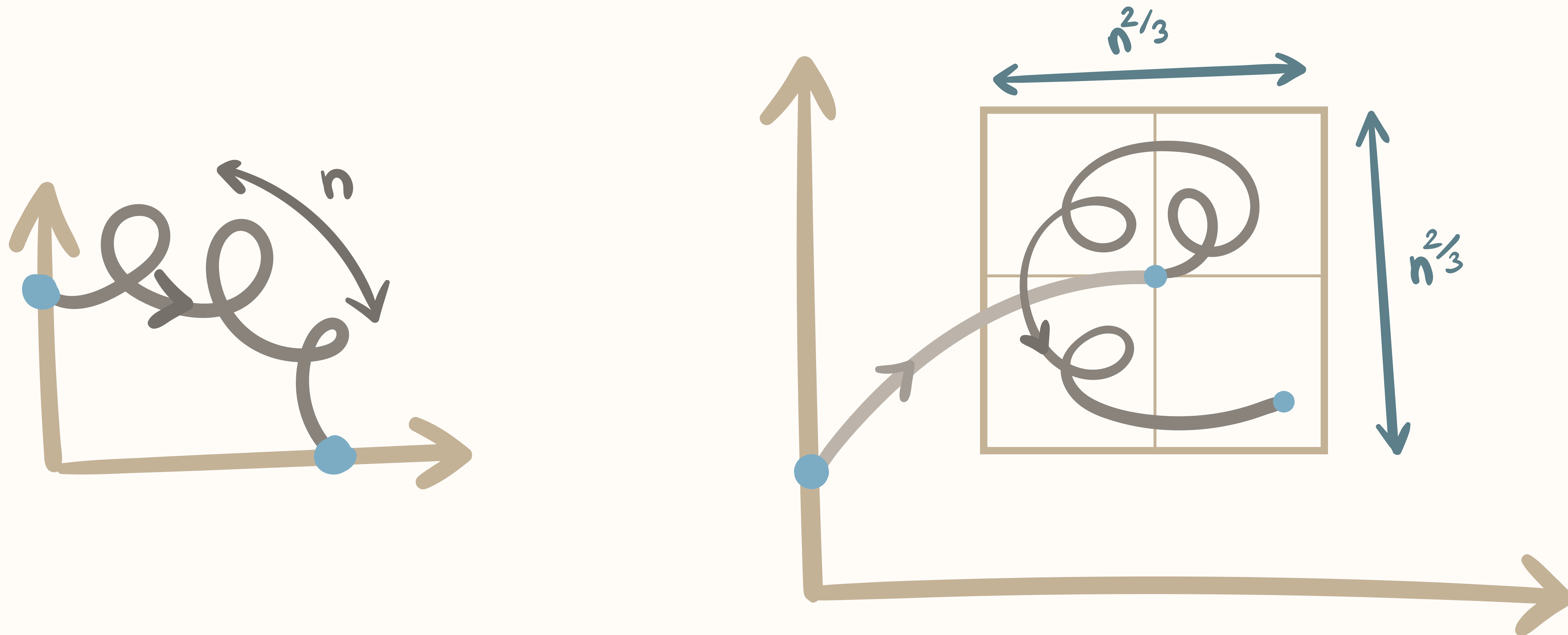
Asymptotic counting results

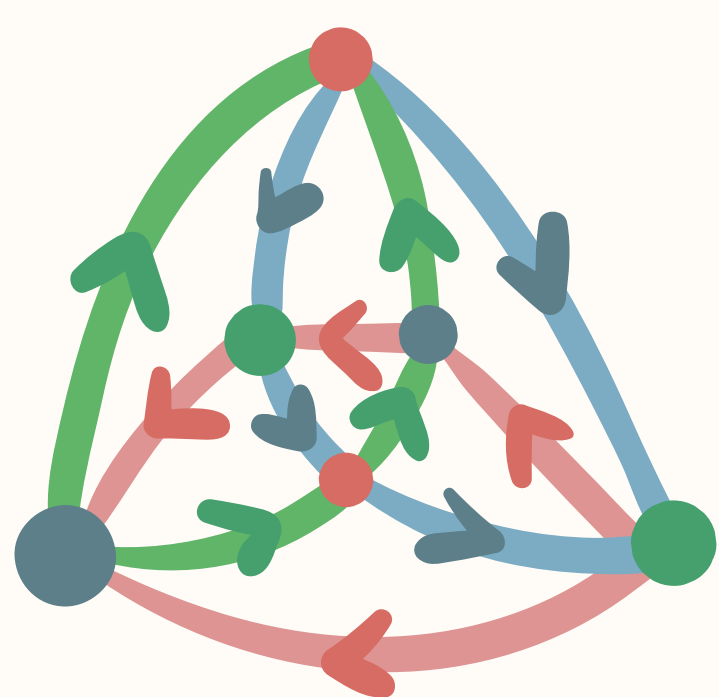


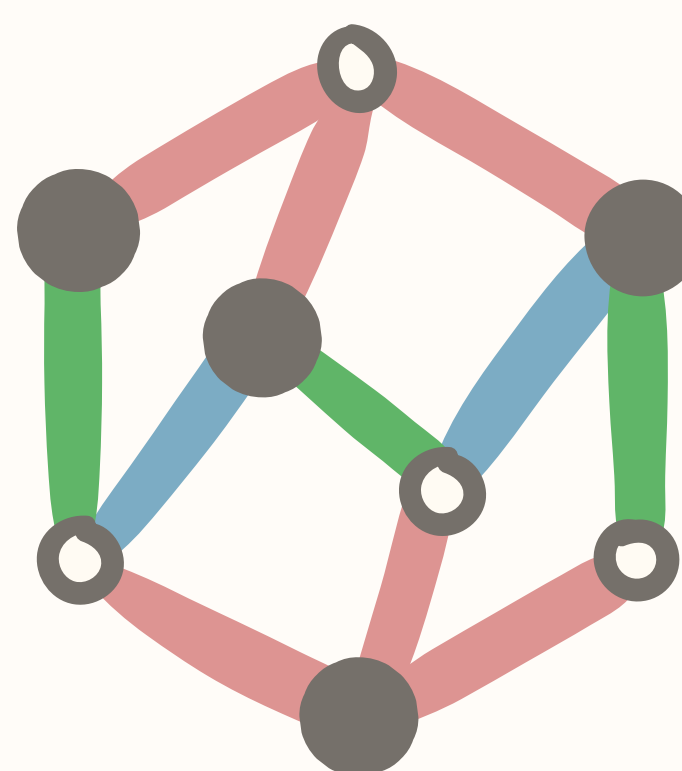
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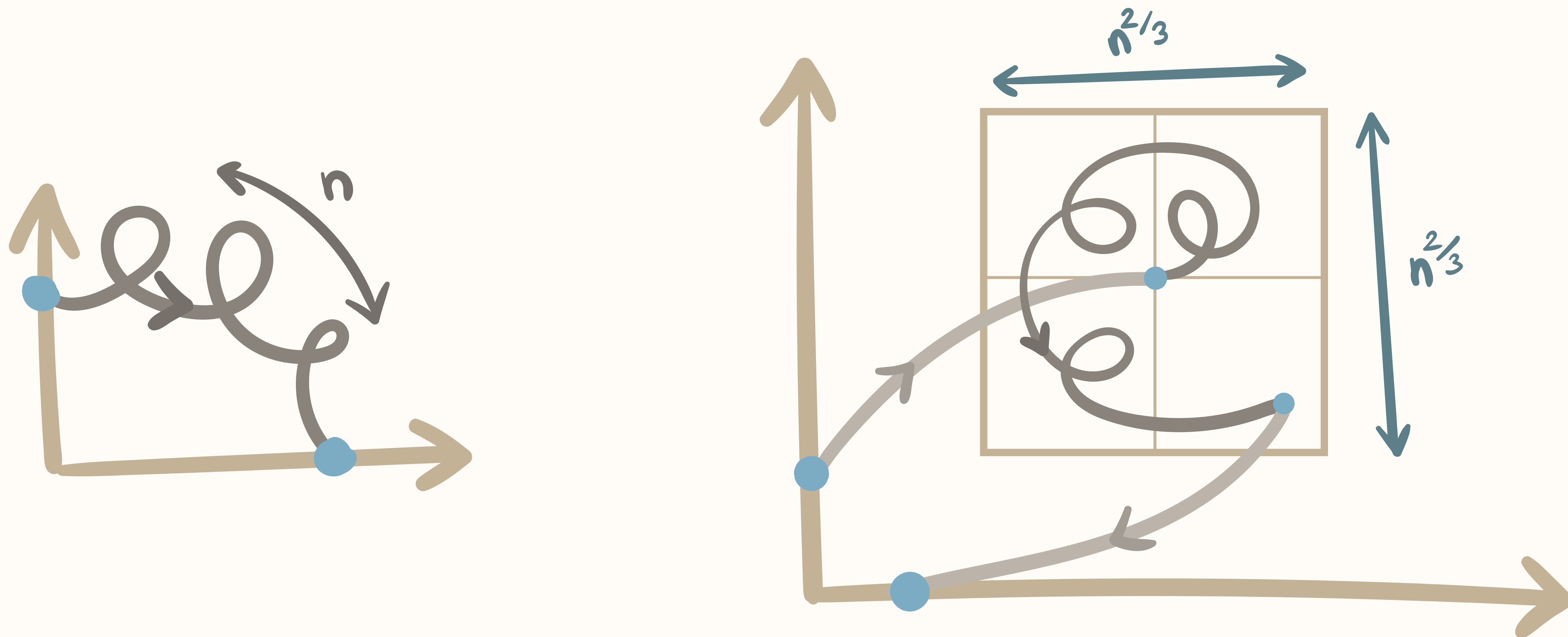
Asymptotic counting results

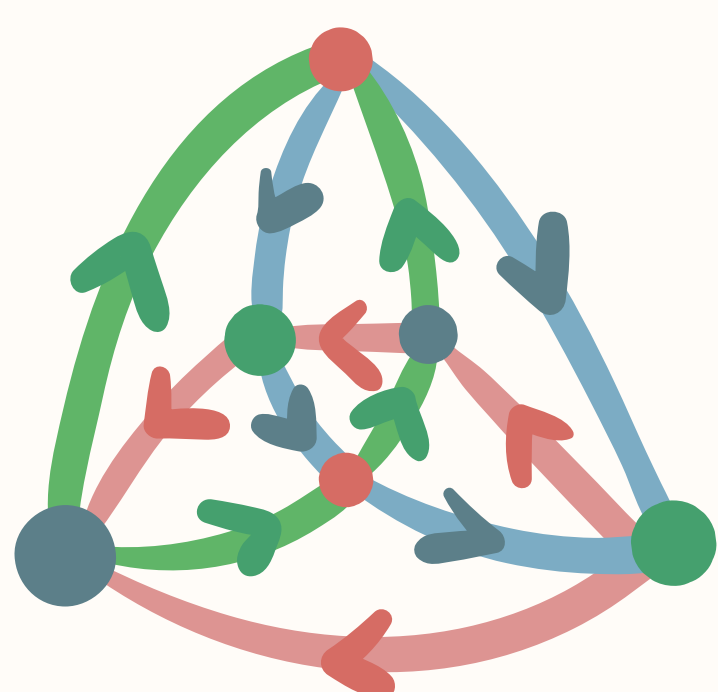


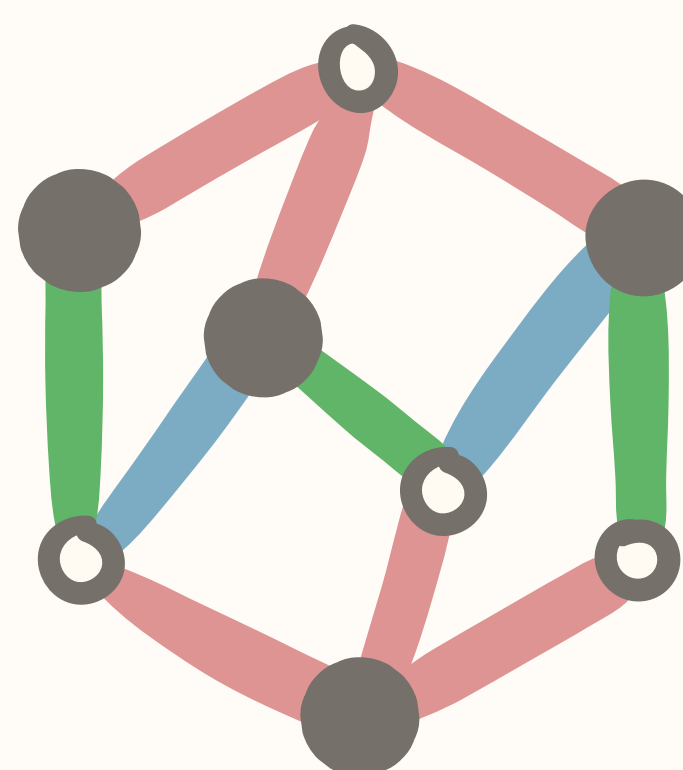
 $|_n \ll \left(\frac{9}{2}\right)^n$

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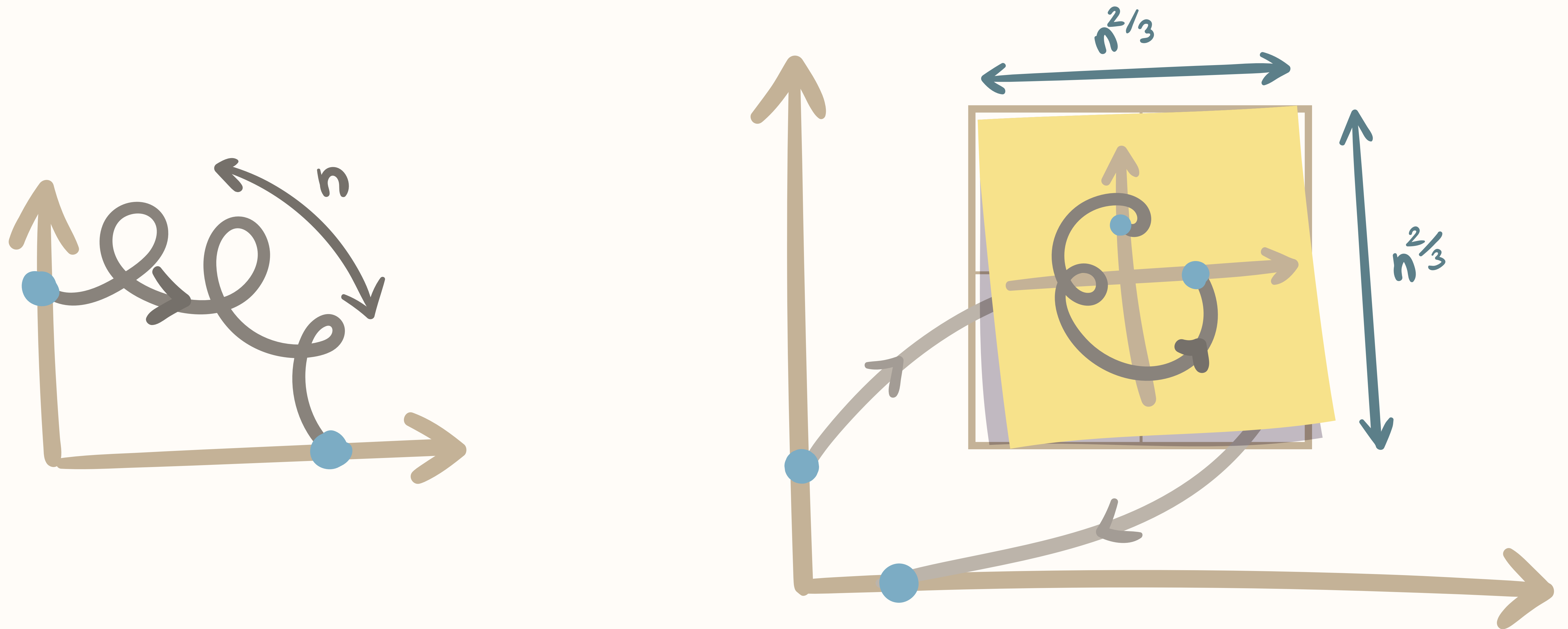
Asymptotic counting results



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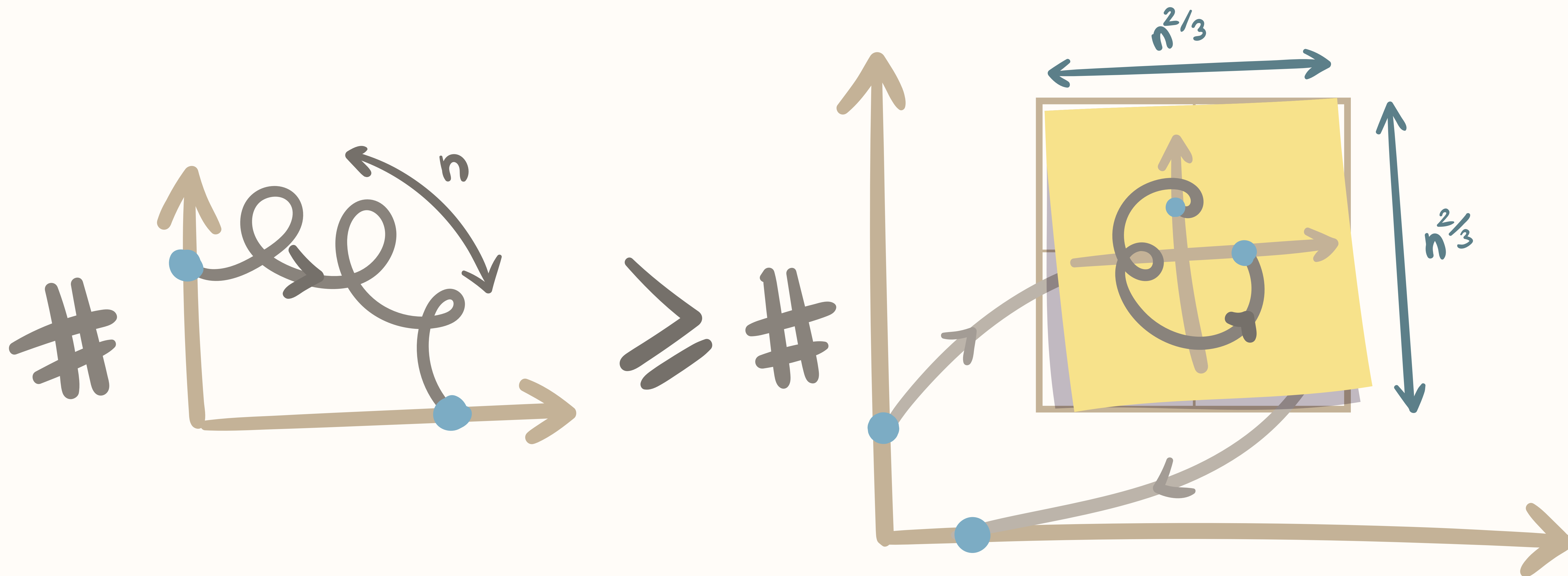
Asymptotic counting results



$$\# \text{ (looped path) } | n \leq \left(\frac{9}{2}\right)^n$$

$$\# \text{ (looped path) } | n \leq \left(\frac{16}{3}\right)^n$$

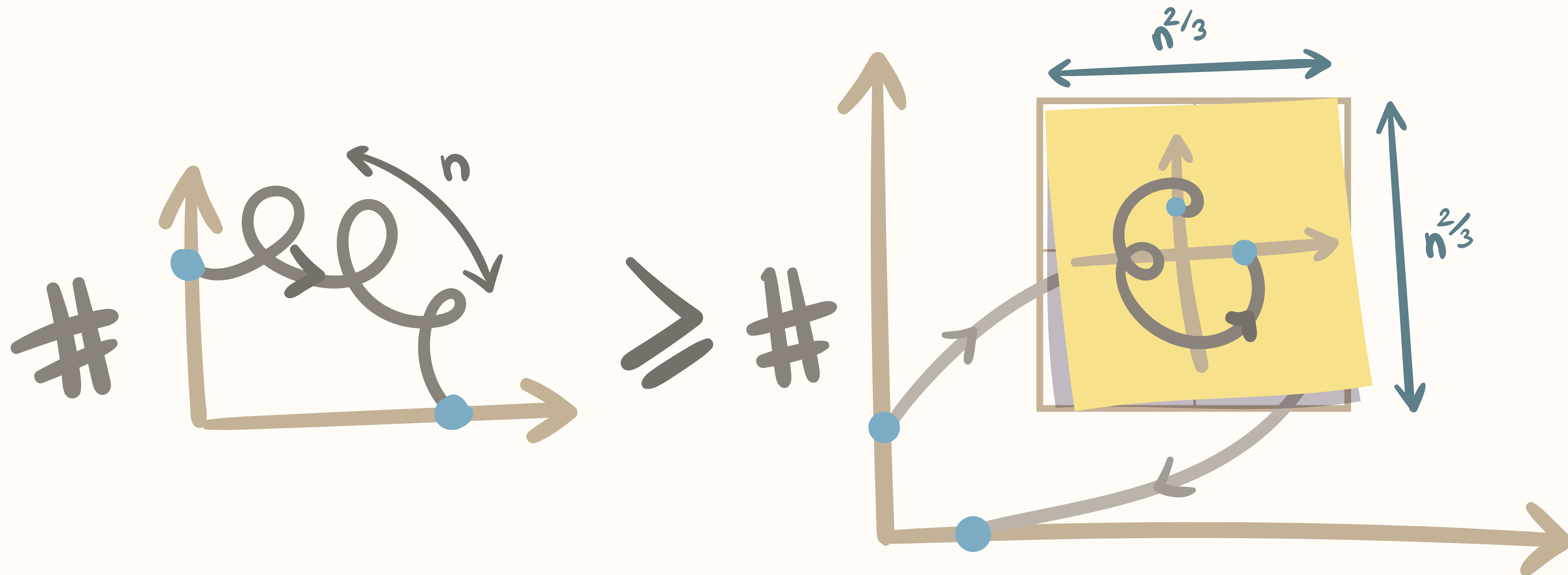
Asymptotic counting results



$$\# \text{ (path in tetrahedron) } | n \leq \left(\frac{9}{2}\right)^n$$

$$\# \text{ (path in octahedron) } | n \leq \left(\frac{16}{3}\right)^n$$

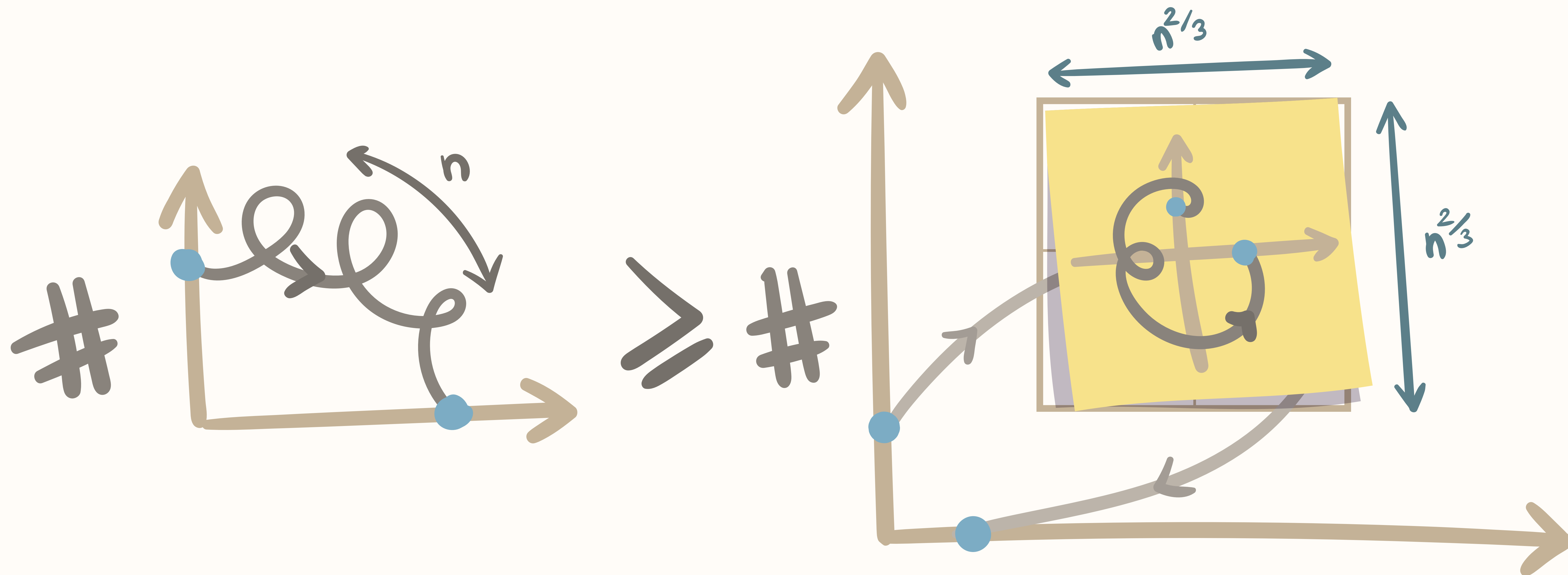
Asymptotic counting results



$$\left(\frac{9}{2}\right)^{n+o(n)} \leq \# \text{ (cubic graph) } |_n \leq \left(\frac{9}{2}\right)^n$$

$$\left(\frac{16}{3}\right)^{n+o(n)} \leq \# \text{ (triangular lattice) } |_n \leq \left(\frac{16}{3}\right)^n$$

Asymptotic counting results



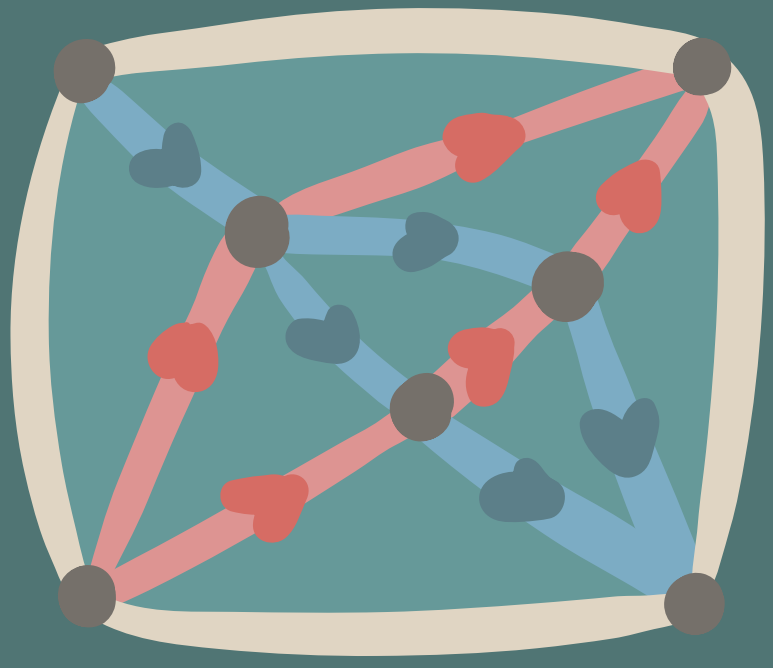
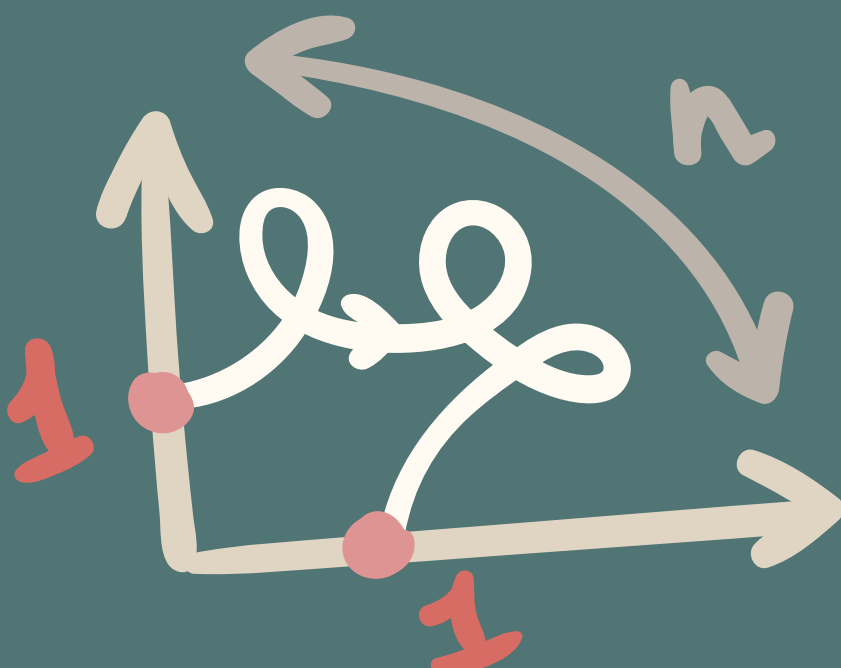
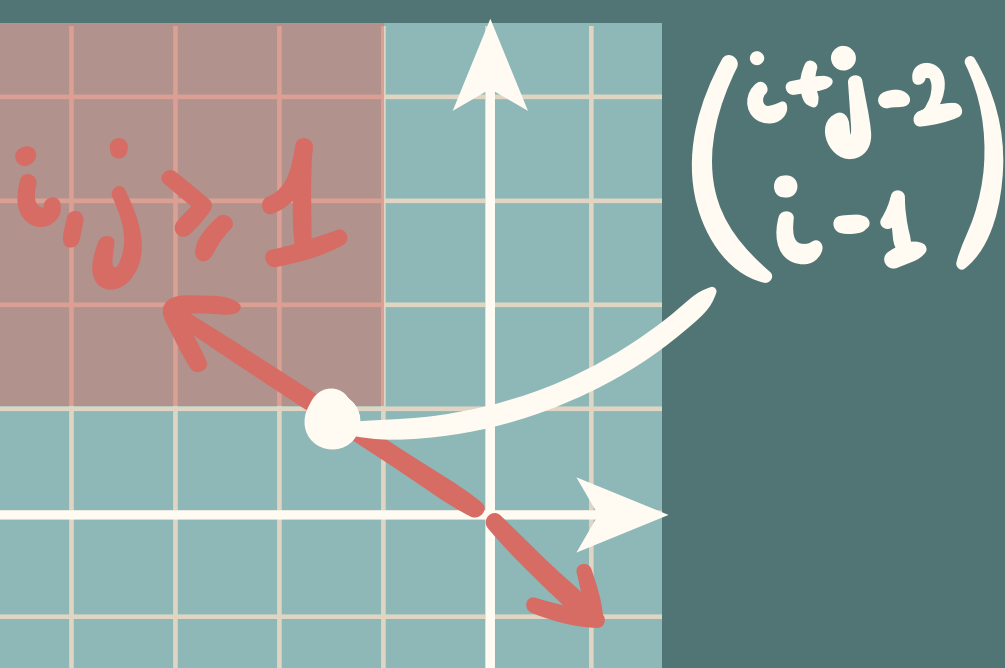

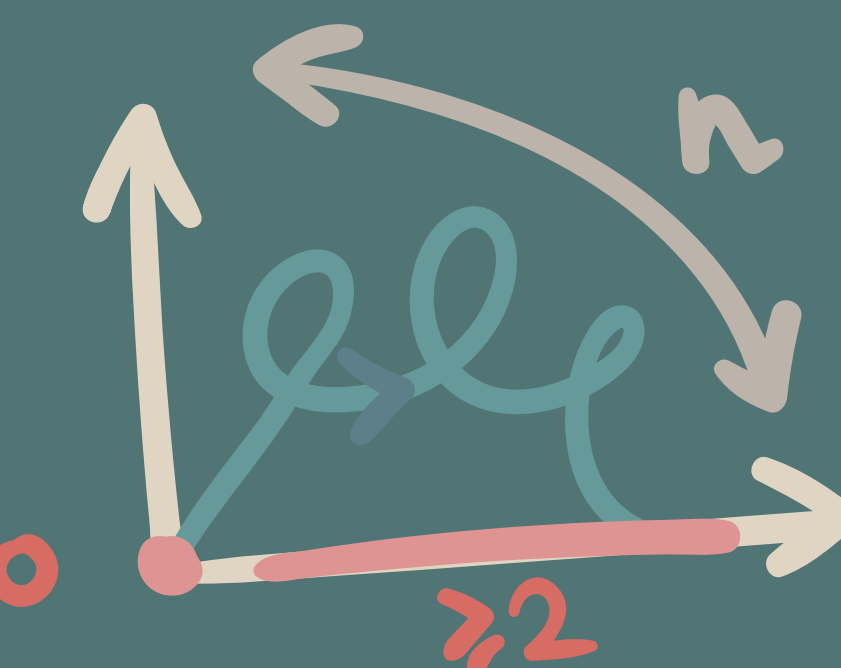
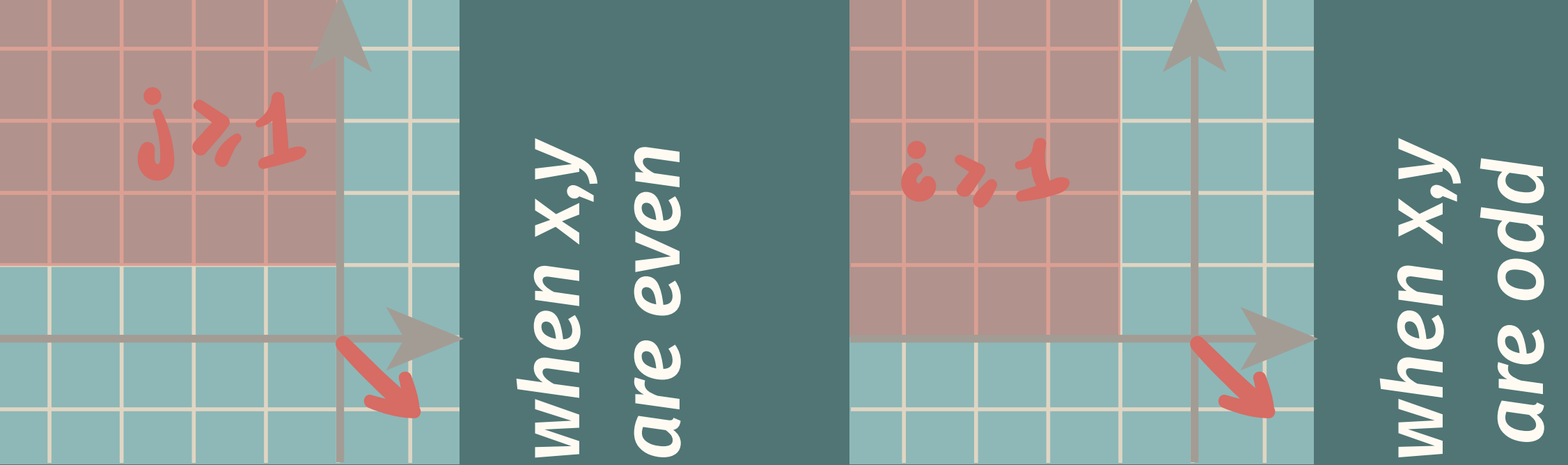

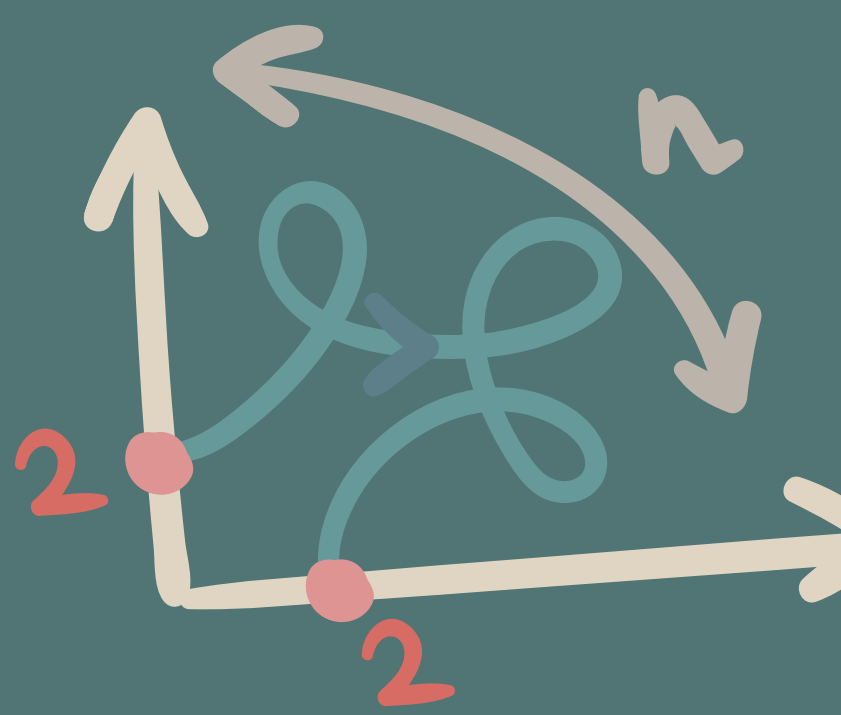
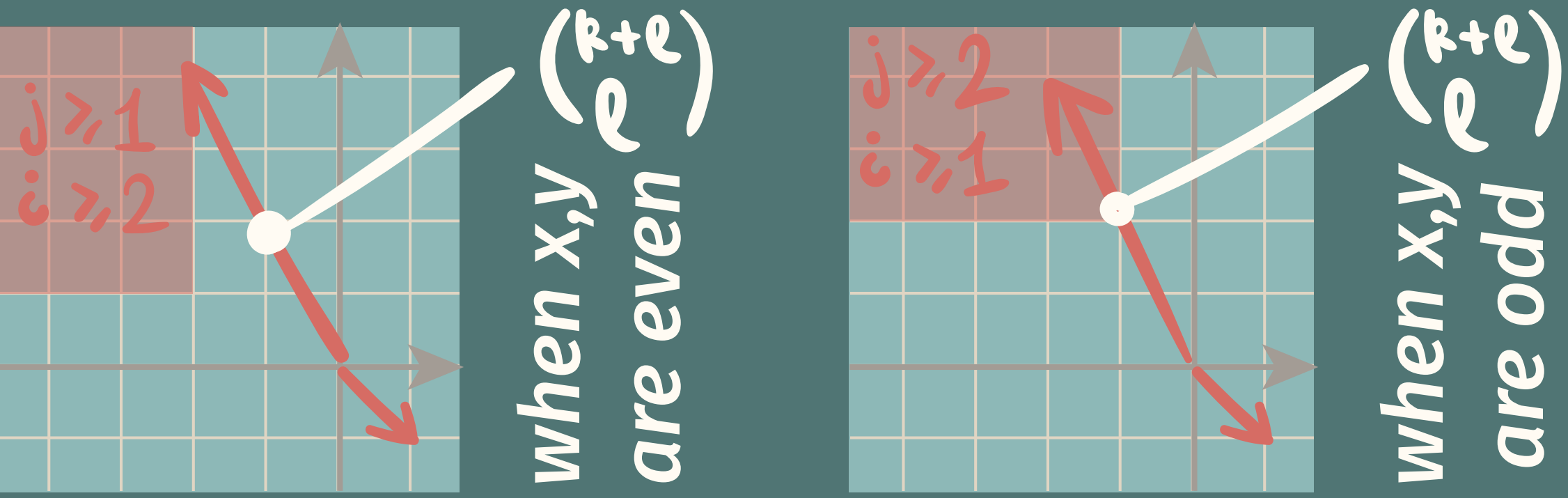

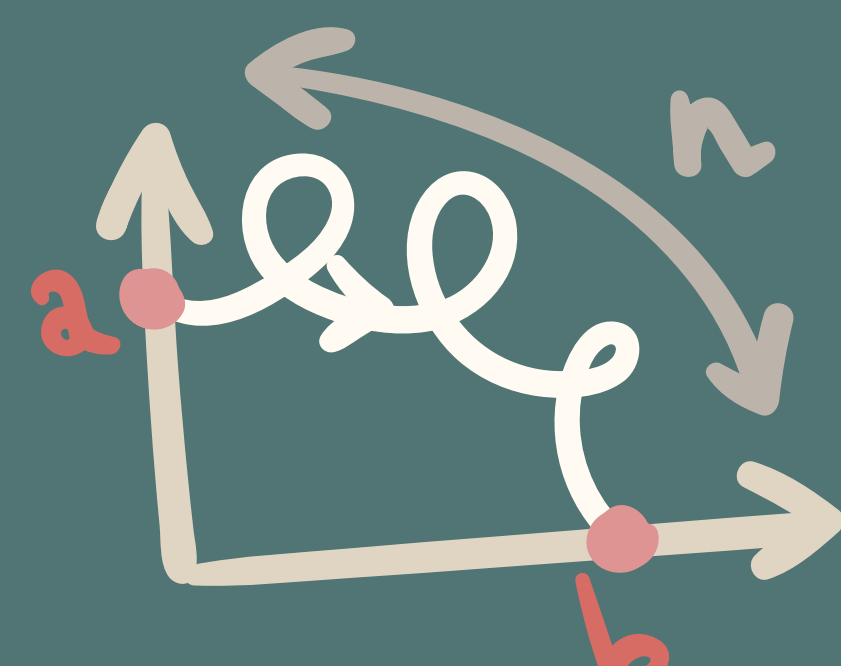
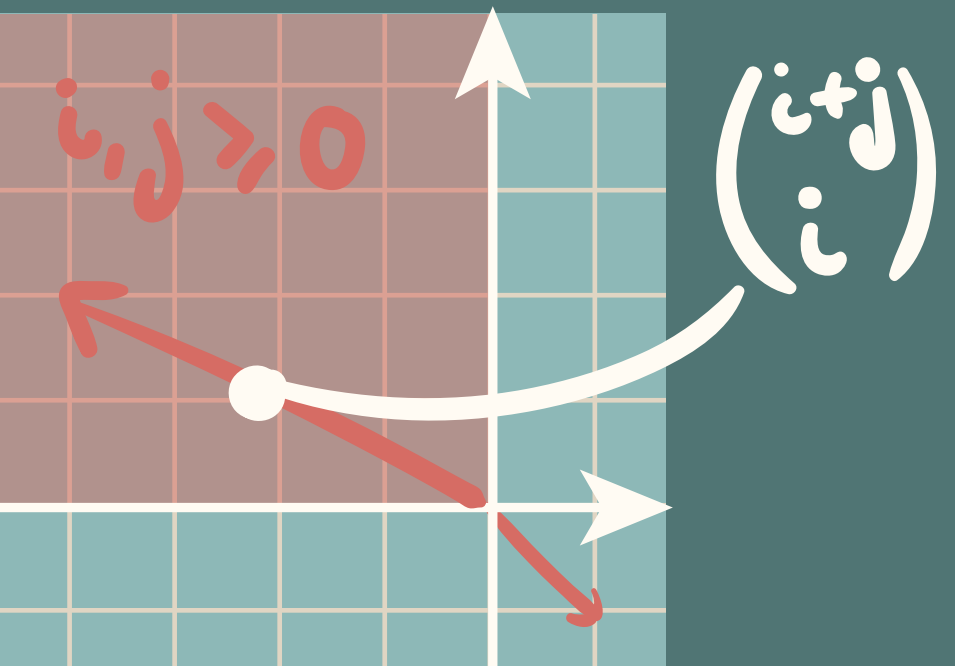
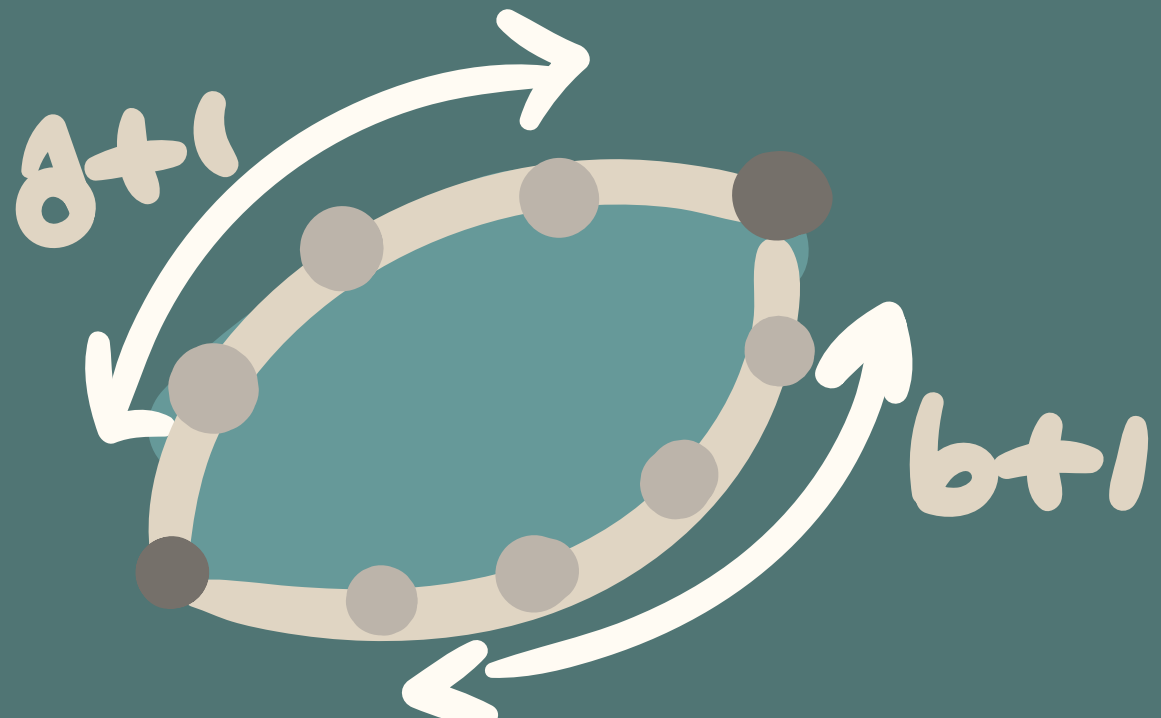
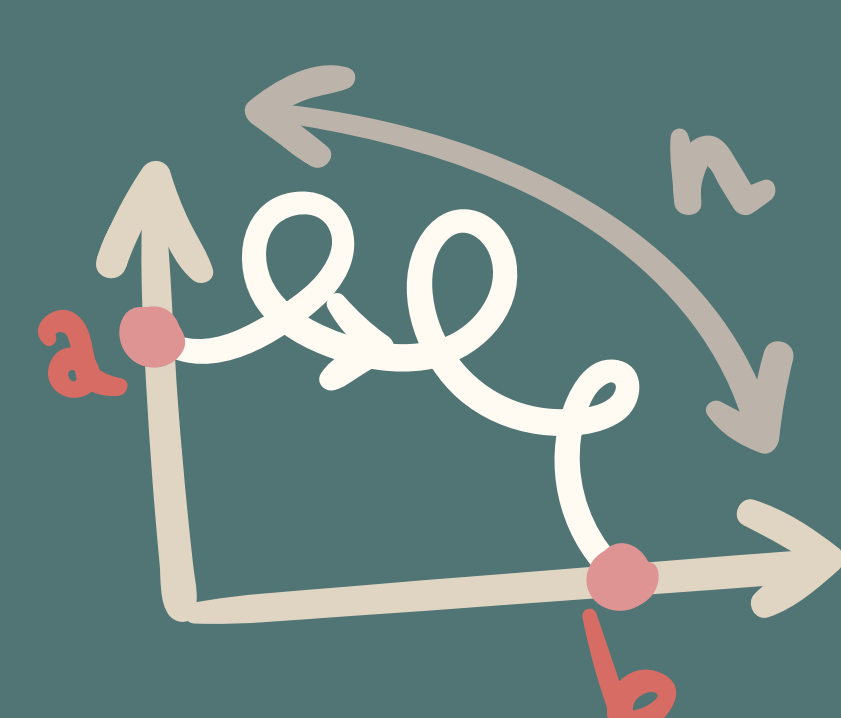
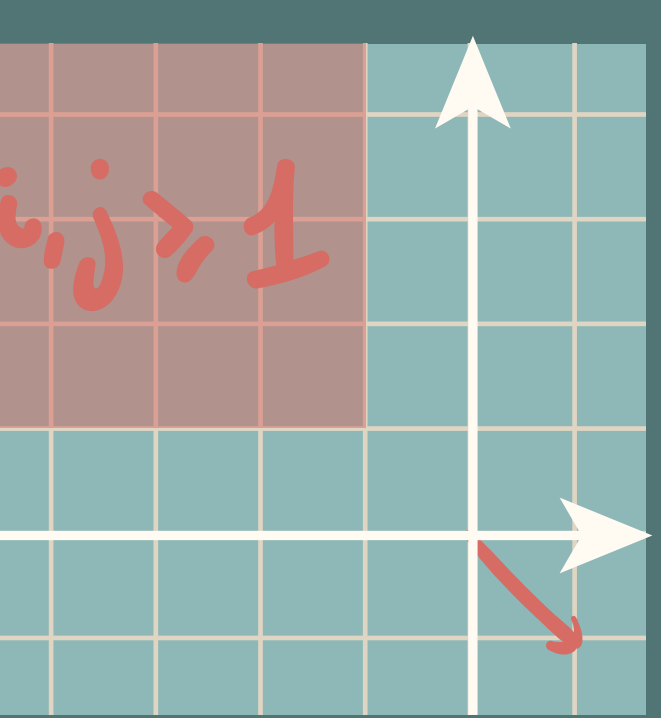
$$\left(\frac{9}{2}\right)^{n+o(n)} \leq \# \text{ (cubic lattice path) } |_n \leq \left(\frac{9}{2}\right)^n$$

$$\left(\frac{16}{3}\right)^{n+o(n)} \leq \# \text{ (triangular lattice path) } |_n \leq \left(\frac{16}{3}\right)^n$$

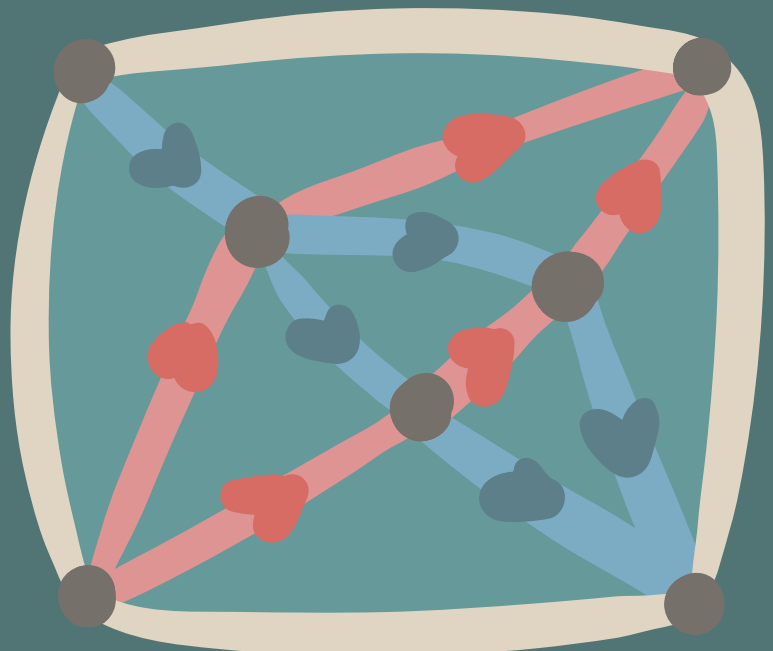




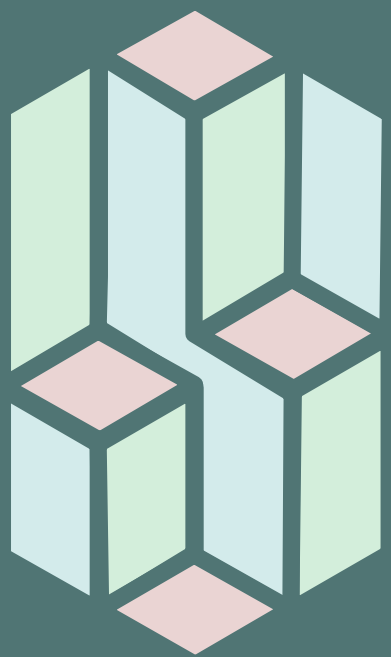
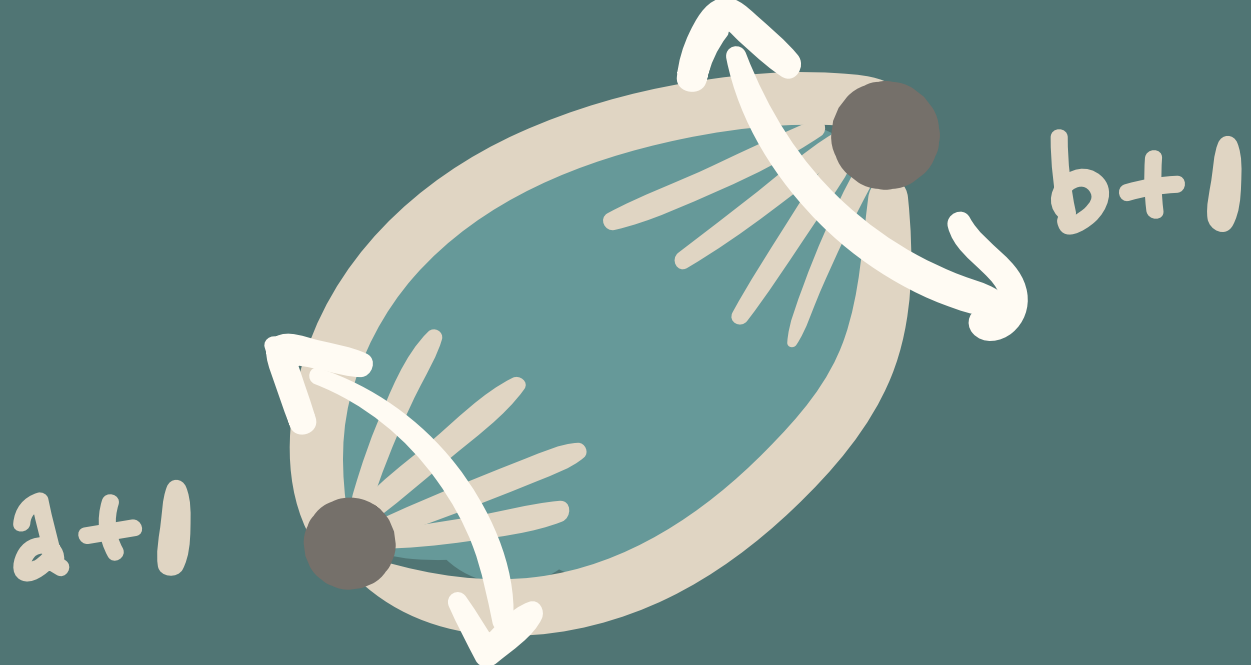
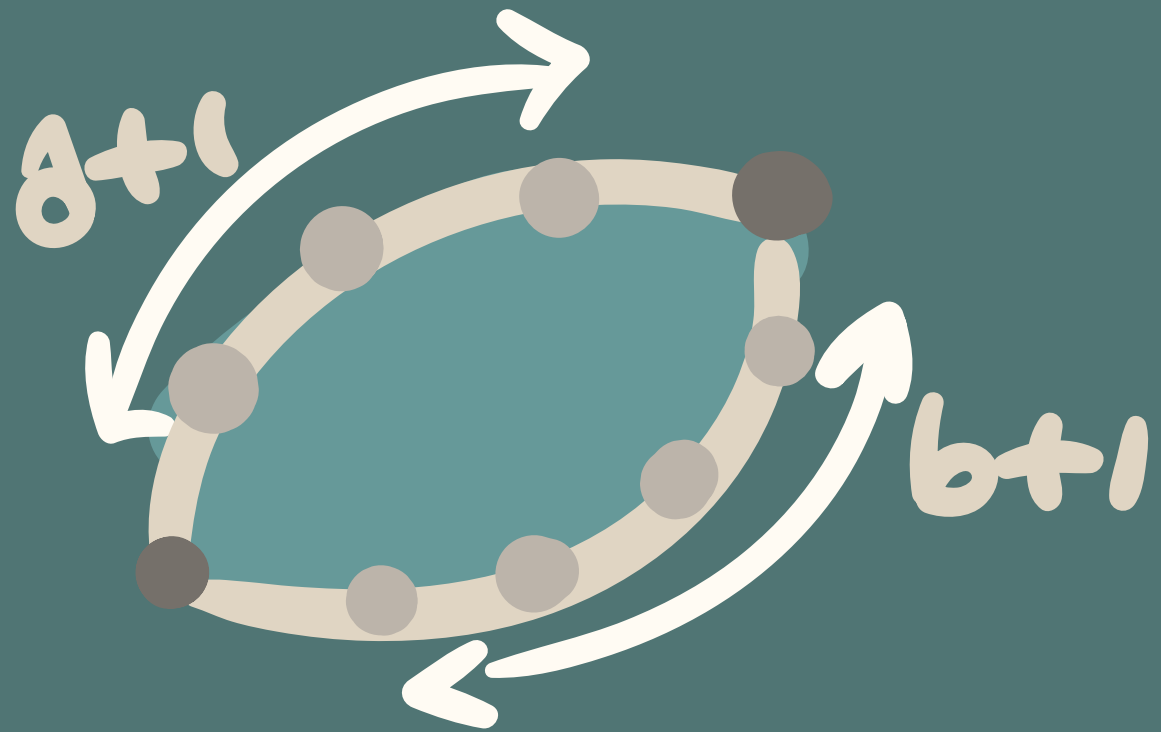
$$\lim_{n \rightarrow \infty} p_n^{1/n} = 9/2$$

$$\lim_{n \rightarrow \infty} S_n^{1/n} = 16/3$$

Summary

Model	Tandem Walks	
<p>Transversal structures <i>n blue edges</i></p> 		
<p>Polyhedral orientations <i>n inner faces</i></p> 		
<p>Schnyder colorings <i>n inner faces</i></p> 		
<p>Posets <i>n vertices</i></p> 		
<p>Posets <i>n+2 edges</i></p> 		

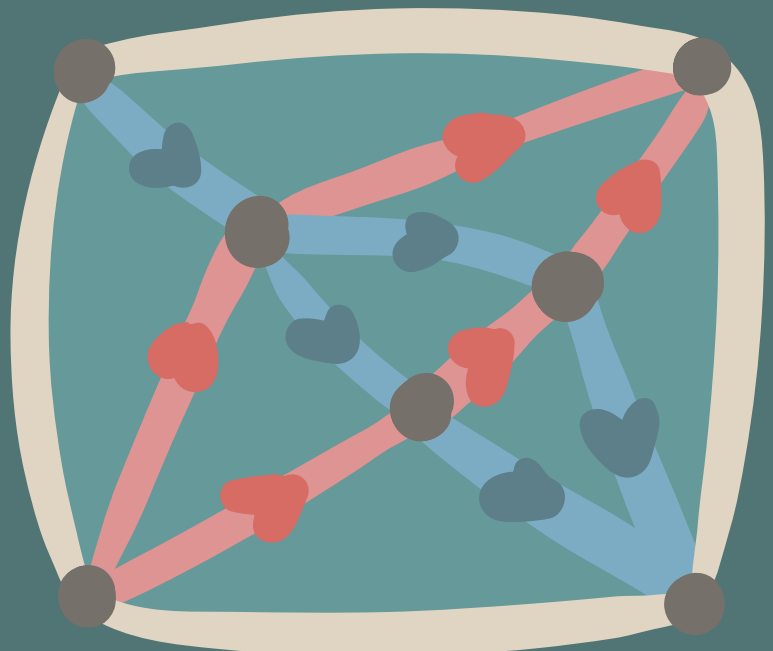
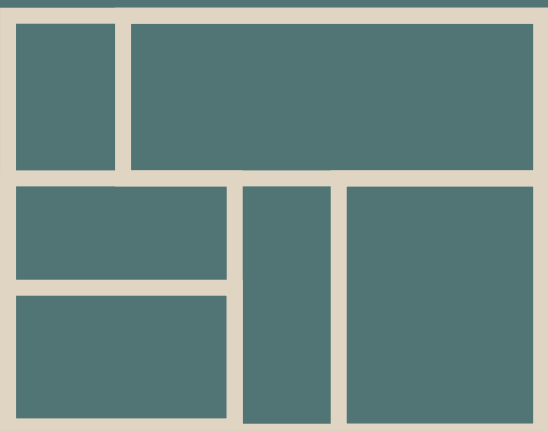



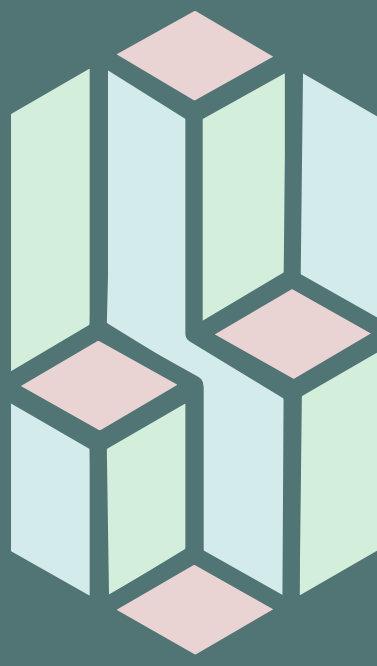
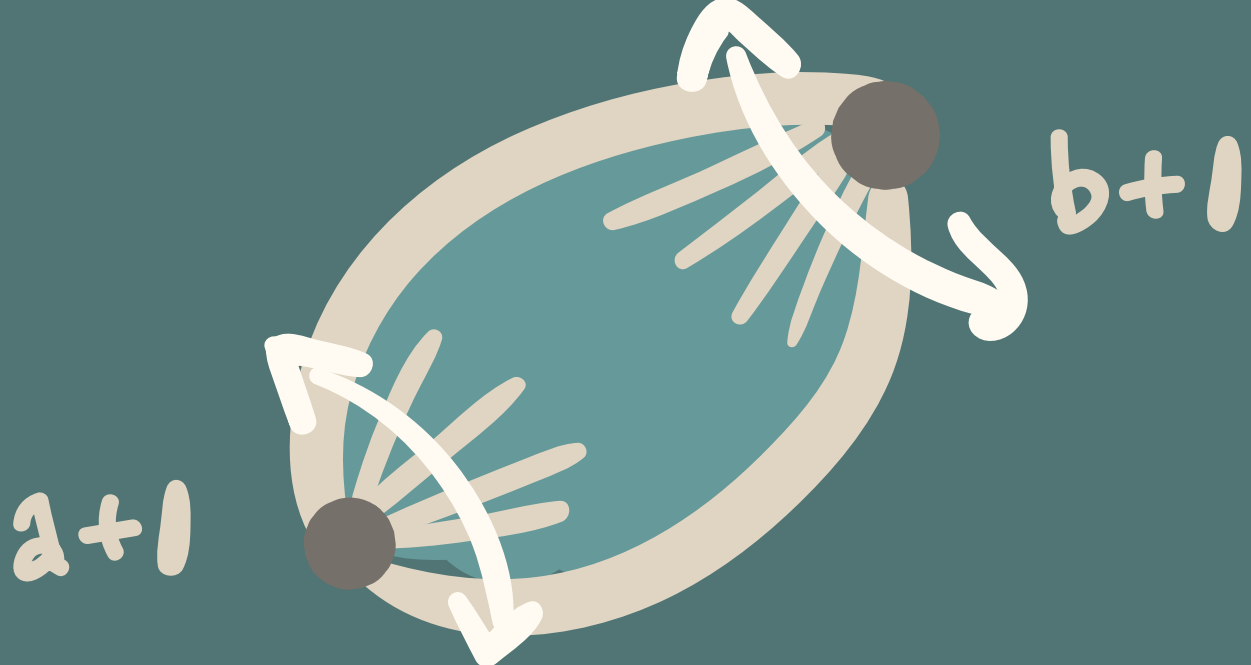
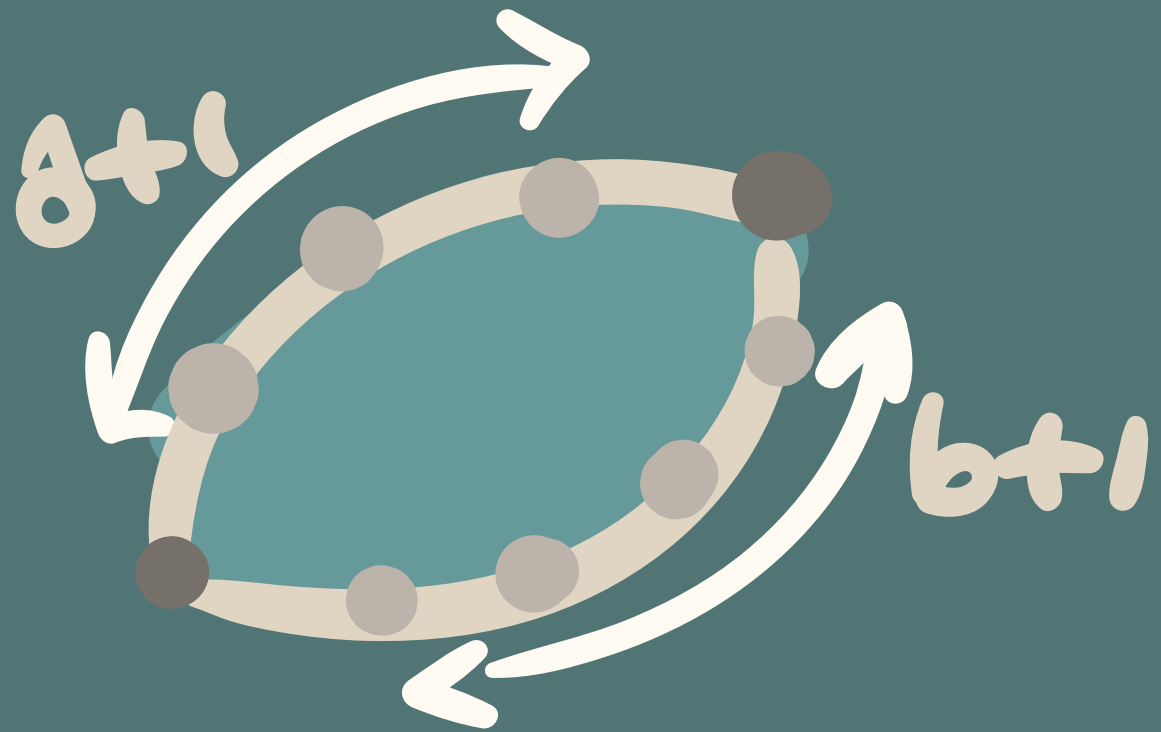
Summary

Model		Asymptotics	
Transversal structures <i>n blue edges</i>			
Polyhedral orientations <i>n inner faces</i>			
Schnyder colorings <i>n inner faces</i>			
Posets <i>n vertices</i>			
Posets <i>n+2 edges</i>			

Summary

$$a_n \sim \kappa \cdot \gamma^n n^\alpha$$

$$\alpha = 1 + \pi / \arccos(\xi)$$

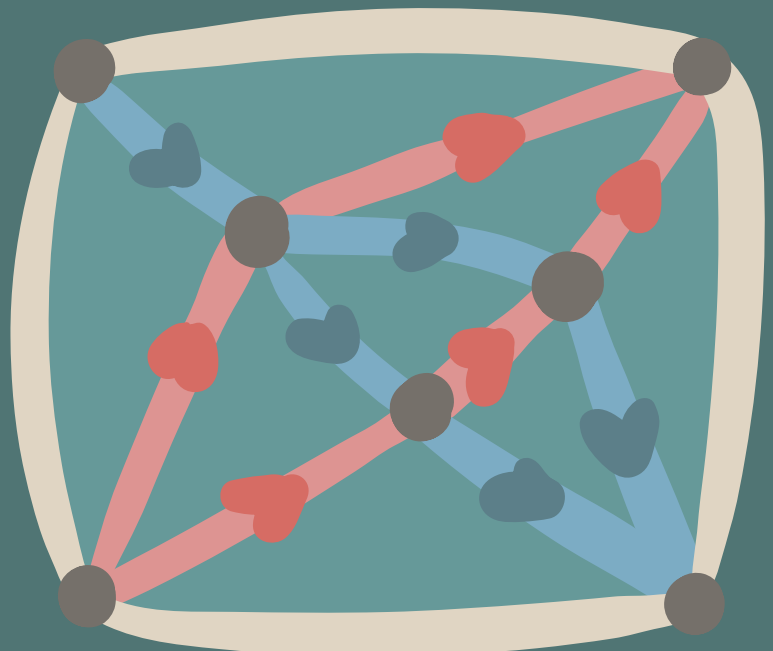
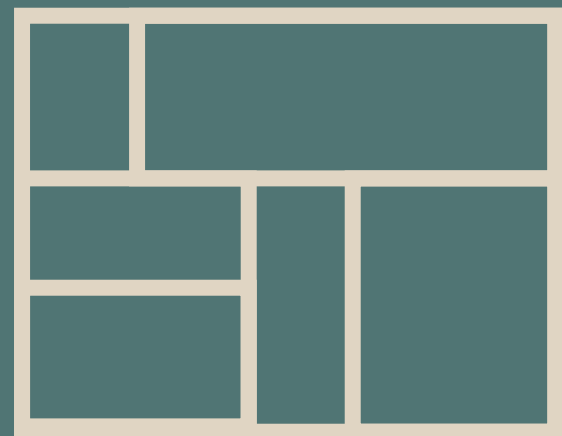



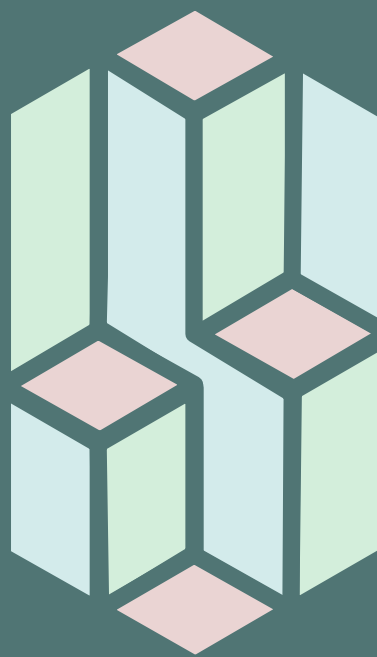
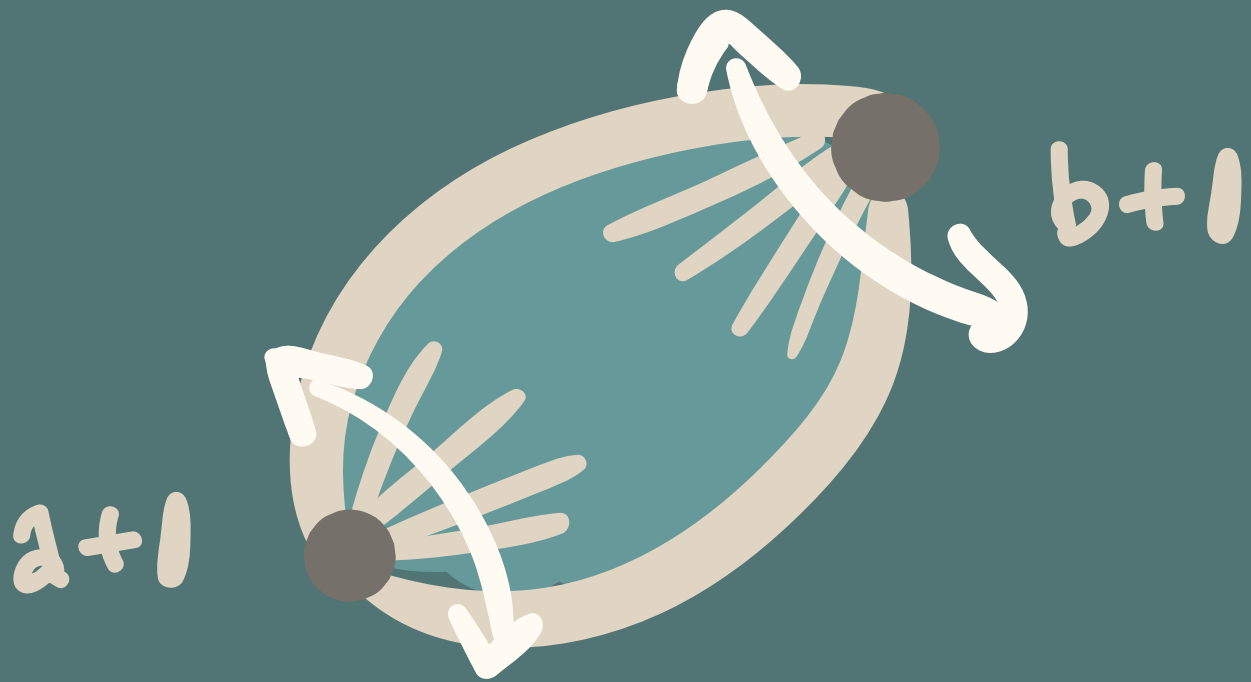
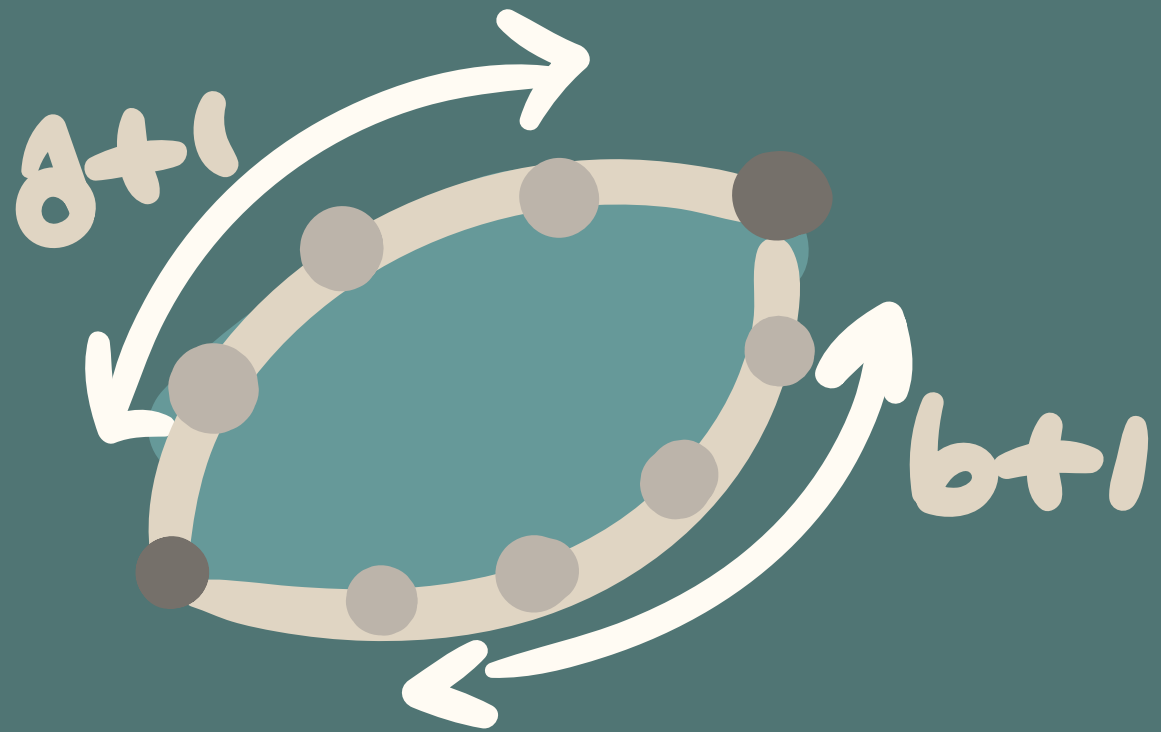
Model		Asymptotics	
Transversal structures <i>n blue edges</i> 		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations <i>n inner faces</i> 			
Schnyder colorings <i>n inner faces</i> 			
Posets <i>n vertices</i> 		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets <i>n+2 edges</i> 		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

Summary

$$\lim a_n^{1/n} = \gamma$$

$$a_n \sim \kappa \cdot \gamma^n n^\alpha$$

$$\alpha = 1 + \pi / \arccos(\xi)$$

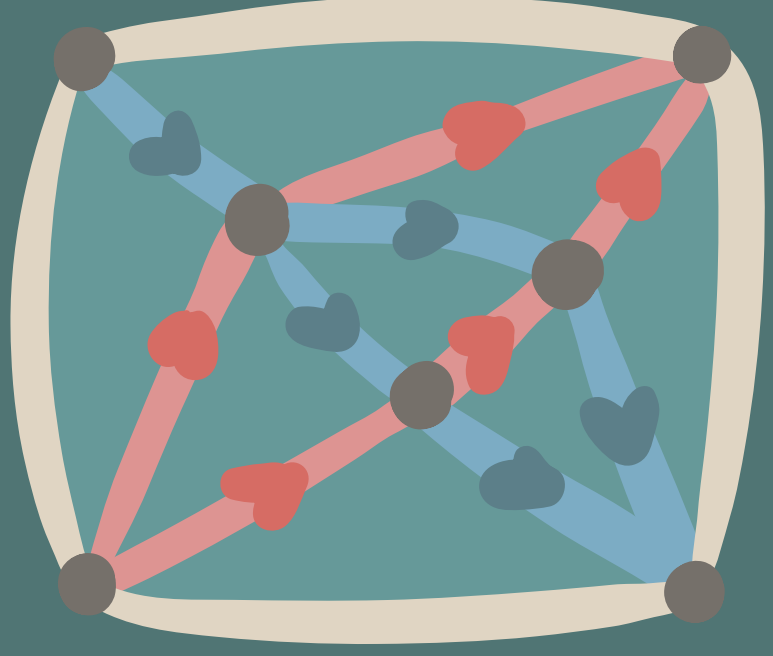
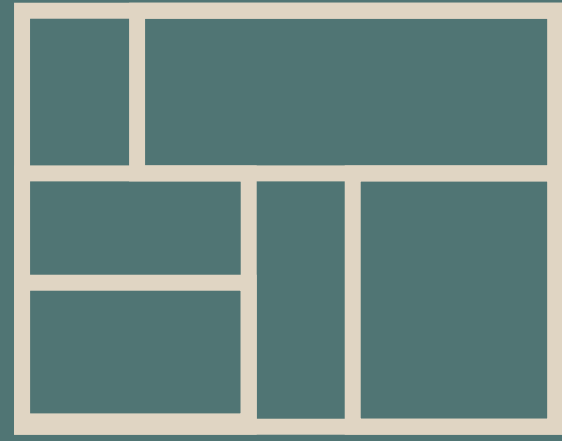



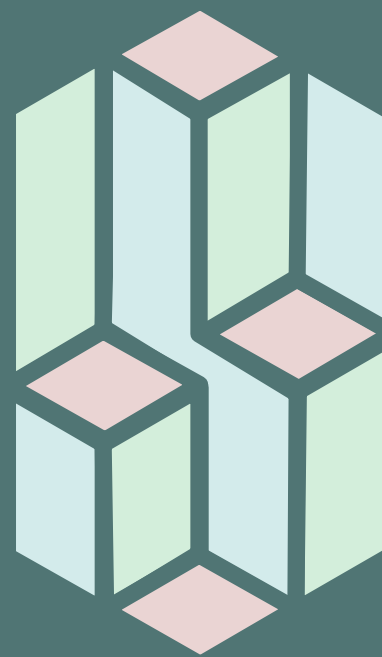
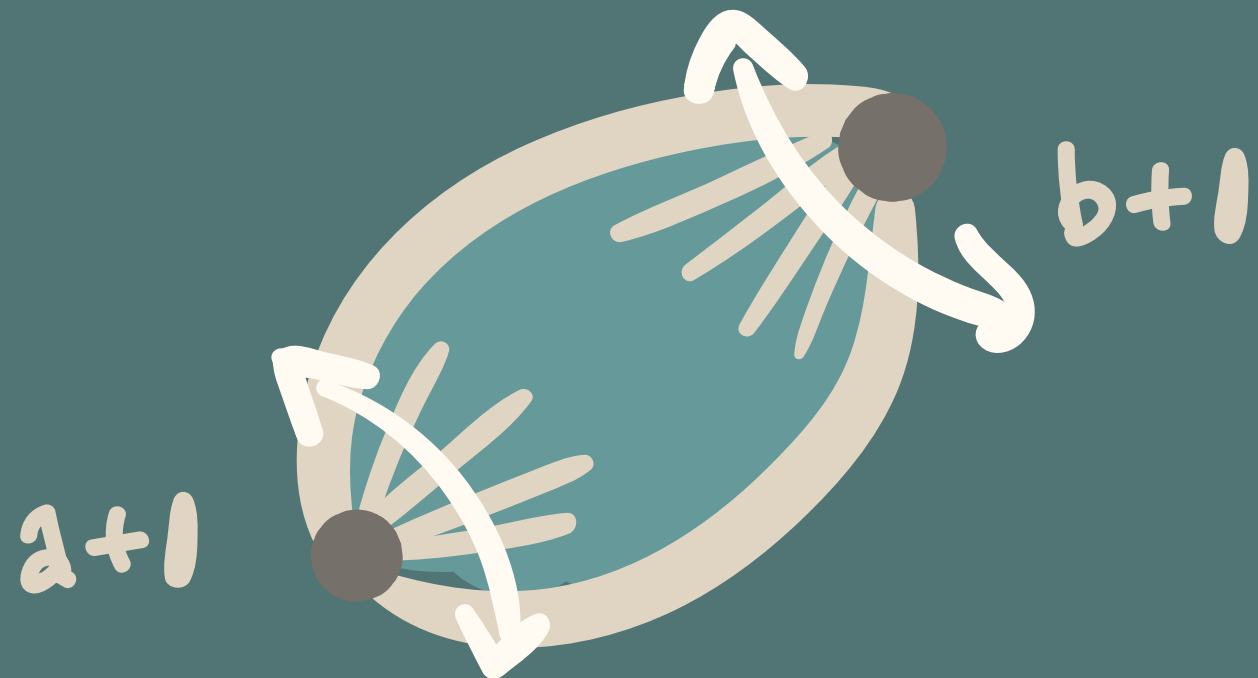
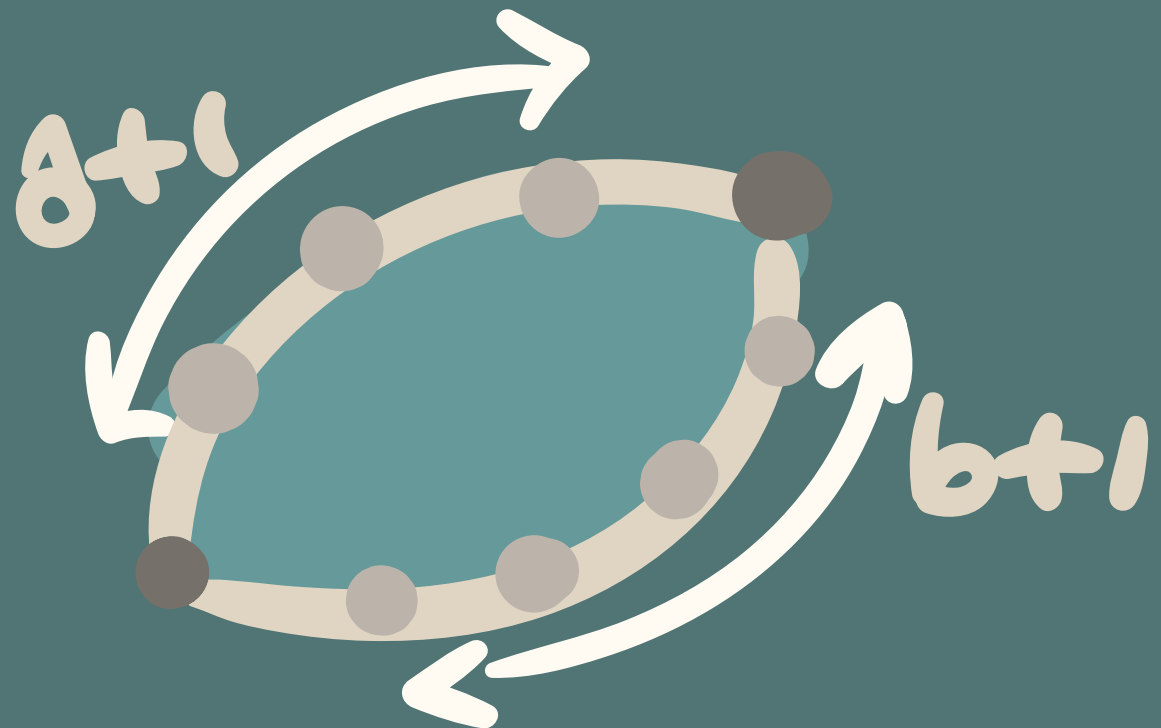
Model		Asymptotics	
Transversal structures <i>n blue edges</i> 		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations <i>n inner faces</i> 		$\gamma = 9/2$	
Schnyder colorings <i>n inner faces</i> 		$\gamma = 16/3$	
Posets <i>n vertices</i> 		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets <i>n+2 edges</i> 		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$

Summary

$$\lim a_n^{1/n} = \gamma$$

$$a_n \sim \kappa \cdot \gamma^n n^\alpha$$

$$\alpha = 1 + \pi / \arccos(\xi)$$

Model		Asymptotics	
Transversal structures <i>n blue edges</i> 		$\gamma = 27/2$	non D-Finite
		$\alpha \approx 7.21$	$\xi = 7/8$
Polyhedral orientations <i>n inner faces</i> 		$\gamma = 9/2$	(non D-Finite)
		$(\alpha \approx 4.23)$	$(\xi = 9/16)$
Schnyder colorings <i>n inner faces</i> 		$\gamma = 16/3$	(non D-Finite)
		$(\alpha \approx 6.08)$	$(\xi = 22/27)$
Posets <i>n vertices</i> 		$\gamma = (11 + 5\sqrt{5})/2$	D-Finite
		$\alpha = 6$	$\xi = (1 + \sqrt{5})/4$
Posets <i>n+2 edges</i> 		$\gamma \approx 4.80$	non D-Finite
		$\alpha \approx 5.14$	$\xi \approx 0.73$



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